TABLE I. Activity table for the double group D_2.

D_2	E	$\bar{E}$	C_2 $\bar{C}_2$	C_2' $\bar{C}_2'$	C_2'' $\bar{C}_2''$	Transformation properties of scattering tensor	${}_4F_{7/2}$ levels
Γ_1	1	1	1	1	1	$\alpha_{xx}, \alpha_{yy}, \alpha_{zz}$	
Γ_2	1	1	−1	1	−1	$\alpha_{xz}\pm\alpha_{zx}$	
Γ_3	1	1	1	−1	−1	$\alpha_{yz}\pm\alpha_{zy}$	
Γ_4	1	1	−1	−1	1	$\alpha_{xy}\pm\alpha_{yx}$	
Γ_5	2	−2	0	0	0		All levels

shift of the A_{1g} mode towards lower energy. The Raman band is now peaked at 530 cm^{-1}, and a weak shoulder occurs at 523 cm^{-1}. Also present in the spectrum are two Raman lines whose intensity is dependent on the concentration of the Yb ion. We associate these bands with electronic Raman transitions of the Yb^{3+} ion.[7]

The scattering tensor for the electronic Raman transition is different from that for the vibrational transitions. First, the scattering tensor also contains antisymmetric combinations such as α_{xy} α_{yx}; and second, the interaction of the rare-earth ions is much weaker, and a "factor" group analysis does not have to be carried out. Instead, we can label the electronic levels according to the species of the D_2 point group which describes the symmetry of the environment of the lanthanide ion in the garnet crystal.[8] The electronic levels are, however, Kramers doublets, because they are the crystal-field components of a $J=\frac{7}{2}$ Russell-Saunders manifold; and in order to predict the scattering tensor the D_2 double group has to be employed. The character table and transformation properties of the components of the scattering tensor and the labeling of the electronic energy levels are shown in Table I. All electronic levels belong to the irreducible representation Γ_5, and an electronic Raman transition is allowed if in the representation product $\Gamma_5\times\Gamma_5$ a species occurs to which a component of the scattering tensor also belongs. Now $\Gamma_5\times\Gamma_5=\Gamma_1+\Gamma_2+\Gamma_3+\Gamma_4$, and consequently the Raman scattering tensor of all electronic transitions contains diagonal as well as off-diagonal elements. This is in perfect agreement with experiment. Separation of $\alpha_{yy}+\alpha_{yz}$ and $\alpha_{xy}+\alpha_{xz}$ into their individual components for the two electronic Raman transitions of Yb^{+} in YbGaG reveals that of the various scattering components, $\alpha_{yy}\neq\alpha_{yz}\neq\alpha_{xy}\neq\alpha_{xz}$, and none is equal to zero.

ACKNOWLEDGMENTS

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Coupled Motion of Vortices in Superposed Superconducting Films*

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The coupling of vortices between two superconducting tin films electrically insulated from one another (i.e., in a dc transformer) has been studied as a function of the applied perpendicular magnetic field, of the current in the primary, and of the temperature. The results are qualitatively interpreted in terms of a vortex-slipping mechanism, and they support the picture of vortex flow as the dissipation process in superconducting films.

INTRODUCTION

In 1965, Giaever showed[1] that it was possible to couple the motion of vortices present in two superposed superconducting films, electrically insulated from one another. This phenomen was detected by measuring the voltage across the secondary film, in which no external current was passing, induced by the motion of the vortices due to the passage of a dc current in the other film (the primary). Since then various authors[2–4] have investigated these dc transformers, driving currents simultaneously through the primary and secondary films, varying the thickness of the primary, and studying the influence of an external magnetic field or the impurity concentration on the coupling between the vortices in the two films. Similar experiments were

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carried out by Solomon[5] on thick foils of type-I material. The results supported the existence of normal regions moving in both layers when the perpendicular applied field was smaller than $\sim\frac{2}{3}H_{cb}$. Recently, Van Gurp's noise measurement on thick type-I foils showed the existence of at least two dissipative mechanisms, one of which was associated with viscous flow and was predominant only in the low-field region. This result was in agreement with the experiments of Brandt and Parks[6] which were able to observe directly the normal state region in type-I specimens and did not detect any motion of the normal regions.

We have studied the variation of the coupling of the vortices in two superposed tin superconducting films, electrically insulated from one another, when varying an applied perpendicular magnetic field and the dc current in one of the films.

EXPERIMENTAL TECHNIQUE

The geometry used is shown in Fig. 1. It is similar to the original design of Giaever[1] except for some modifications concerning the position and nature of the voltage-measuring electrodes. The dimensions of the substrate, a dust-free clean glass plate, are $\frac{1}{16}\times1\times1\frac{1}{4}$ in. (a) shows the position of the gold electrodes to which the connecting leads were indium-soldered. The two rectangular ones are used to feed the current in the primary film, the four others being used to measure the voltages across the two films. (b) depicts a primary film of tin 1700 Å thick. It is deposited by evaporation from 99.999% pure material. The thickness was determined from measured values of $H_{c\parallel}$ and $H_{c\perp}$, the parallel and perpendicular critical fields, using well-known[7] relations. The fine lines represent the trimming done on this film after the four successive evaporations have been done. It is intended to localize a region of higher current density in the primary where the vortices will be depinned from their equilibrium position. (c) is a ~200-Å SiO film evaporated on top of the primary film at room temperature using an indirect evaporation boat. (The film thickness is estimated from interferometric measurements.) The resulting insulation between the two tin films at liquid-helium temperature is greater than 10^8 Ω. (d) is the secondary, a tin film approximately 1100 Å thick. Both tin films were deposited on the substrate cooled to the temperature of liquid nitrogen. (e) displays the complete geometrical arrangement after the four evaporations and the trimming of the primary are completed.

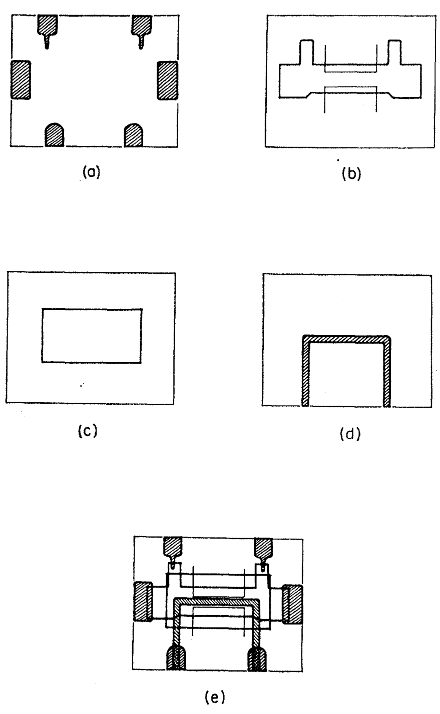

FIG. 1. Geometrical configuration of dc transformer: (a) gold electrodes, (b) primary tin film, (c) SiO layer, (d) secondary tin film, and (e) complete sample.

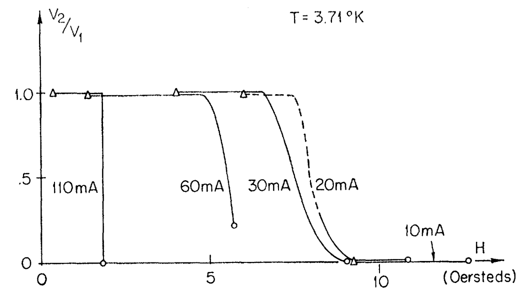

FIG. 2. Ratio of the voltages along the two superconducting films as functions of the magnetic field for different values of the current in the primary. The values $V_2/V_1=1.0$ and 0.0 have been slightly shifted to show more clearly the positions of the depinning thresholds (Δ) in the several cases. The symbols ○ indicate the values of H at which the films were driven normal as indicated by the recorder going rapidly off-scale.

The films are immersed in a liquid-He^4 bath whose vapor pressure is continuously regulated. The primary current is fed from a current-regulated power supply, and both primary and secondary voltages are simultaneously measured by conventional dc technique (limit of sensitivity 0.3 μV). After subsequent amplification, the data are finally obtained on an X-Y recorder.

DEPENDENCE OF COUPLING ON CURRENT AND MAGNETIC FIELD

In Fig. 2, we have plotted the ratio of the voltages across the two films as a function of the perpendicular magnetic field H for different values of the current I_1 in the primary film. Each curve starts on the low-field

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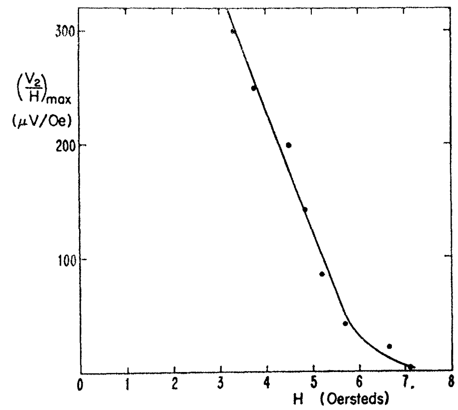

FIG. 3. Plot of $(V_2/H)_{max}$, a quantity proportional to the maximum vortex velocity or viscous drag sustained in the secondary without any appreciable slipping of vortices, as a function of the applied perpendicular magnetic field at $T=3.71$°K. ($T_c=3.85$ and 3.81°K for these films.)

end at the depinning threshold for the given current, indicated by a triangular point. Below that field value, both V_2 and V_1 are zero. The high-field limit of each curve, indicated by a circle, is set by the point where heating effects in the primary have driven V_1 off scale, rising rapidly.

For large currents I_1 such that the depinning of the vortices in the primary film could occur at relatively small H fields, we observed the simultaneous appearance of equal voltages in both films ($V_2/V_1=1$). With increasing magnetic field, the two voltages increased while remaining equal to each other, up to a value of H where the heating in the films, immersed in liquid helium, led to a discontinuous transition to the normal state, V_1 jumping to its normal resistive value and V_2 going discontinuously to zero. This behavior is illustrated in Fig. 2 by the curve taken with $I_1=110$ mA.

In the high-field region associated with small currents I_1, the ratio of the voltages induced by the moving vortices in the two superconducting films is decreased. We assume that this is due to a partial slipping of the vortices of the primary with respect to those of the secondary. The origin of this slipping can be understood as follows: As H is increased, the magnetic field becomes continually more homogeneous, both because the fluxons are packed more closely together so they overlap more and because the superconducting order parameter and hence the currents causing field variations are decreased. Since it is the inhomogeneity of the field which provides the gradient of the coupling energy, eventually this gradient will fall below that required to overcome the pinning in the secondary. At that point, V_2 falls to zero. Presumably the observed gradual fall of V_2/V_1 may be due to macroscopic inhomogeneities of pinning strength over the film.

This decrease in V_2/V_1 is shown, for instance, by the curve taken with I_1 equal to 30 mA, where the voltage ratio stays equal to unity up to 6.5 G, after which there is a gradual slipping of the vortices and V_2/V_1 decreases continuously to zero. The same behavior is shown for $I_1=20$ mA, although because of the smaller values of V_1 and V_2 no precise measurements could be done in the field region just beyond the depinning threshold (dotted part of that curve). An extreme case is the curve $I_1=10$ mA where even though the flow regime in the primary is observed from 9 to 12.5 G (where the film becomes normal), no coupling with the secondary is observed.

If, as we assume, the coupling between the vortices decreases with increasing H, the maximum viscous drag which can be exerted by the secondary without causing slipping will also decrease when the magnetic field is increased. This was tested in the following way:

The viscous drag force F on a vortex in the secondary is expressed in terms of a viscosity coefficient characteristic of the viscous flow regime by

$$F=-\eta v,$$

where the mean vortex velocity v is related to the measured longitudinal voltage V_2 by

$$V_2=El=(v/c)Bl,$$

where l is the length of the trimmed region, E is the longitudinal electric field, and B is the magnetic induction field in the film. Thus $(V_2/B)\approx(V_2/H)$ should be proportional to the viscous drag per vortex exerted by the secondary. As a measure of the coupling strength, we take the maximum viscous drag which can be sustained without slipping, i.e., with $V_2/V_1=1$. In Fig. 3, we have plotted this quantity $(V_2/H)_{max}$ versus H. In the small H (high-current) region, heating effects intervene, leading to a discontinuous jump to the normal values, before slipping occurs. Hence no sensible results could be plotted in that region. However, in the higher-field region, there is a reasonably well-defined onset of slipping, and the location of this is plotted in Fig. 3. The marked decrease with increasing magnetic field of the maximum flow velocity without slipping is evident.

The decoupling of the vortices between the two films can also be detected by the increase of slope of

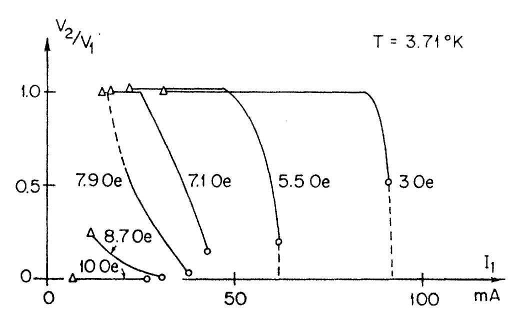

FIG. 4. Ratio of the voltage along the two superconducting films as a function of the current in the primary for different values of the applied perpendicular magnetic field. Explanations given with Fig. 2 apply here also.

the $V(I)$ characteristics of the primary film, since the dynamic resistance of the primary is dependent upon the viscous drag exerted by the moving vortices in the secondary, but such data are not presented here.

In Fig. 4, we have represented the variation of the voltage coupling ratio between the two films as a function of the current for different values of H. As discussed previously, if H is small, the coupling is independent of the current as long as heating effects are negligible. At higher fields, however, the coupling between vortices can be affected by the current (see curves at 7–9 Oe) since, once the pinning is overcome, an increase of the current produces a higher velocity of the vortices and as a consequence a greater viscous drag in the secondary. Thus, increase of I will favor the slipping of the vortices between the two films.[8]

If the driving current was passed through film (d) of Fig. 1, a voltage could be observed in the new secondary (b) but the coupling ratio was much smaller than in the reverse situation. As discussed by Solomon,[5] this can be understood as follows: around the edge of the film (b) no Lorentz force acts and the vortices are driven only by the gradient of the magnetic field energy in that region. Typically the ratio of the voltages in the two films was approximately 0.05 and increased with decreasing H. Since no appropriate potential contacts were available on film (d) to allow isolation of the voltage drop along the central section overlapping the narrowed part of the film (b), it is likely that this measurement underestimates the coupling effectiveness in that region.

EFFECT OF TEMPERATURE

The effect of the temperature can be understood by noticing that as in ordinary single thin films, the pinning of the vortices is highly temperature-dependent. Upon lowering the temperature from T_c, it was necessary to increase the driving force in order to initiate the flow regime in the primary. We now consider separately the two cases: constant current and constant field.

At constant current, a lowering of the temperature must then be accompanied by an increase of the magnetic field value so as to increase the driving force enough to maintain the same flow situation in the primary. This necessary increase of H produces a more homogeneous field in the insulating SiO layer and thus favors a decoupling of the vortices between the two films. This is illustrated in Table I where we have presented the maximum vortex flow velocity sustainable in the secondary for a given current in the primary as a function of temperature. We see the corresponding increase of $H_{\text{depinning}}$, the magnetic field at the threshold, of $H_{\max}$, the field corresponding to the maximum flow velocity without slipping, and the induced decrease of $(V_2/H)_{\max}$.

TABLE I. Temperature dependence of film coupling parameters. All values refer to primary current $I_1=90$ mA. The critical temperatures of the films were 3.85°K (primary) and 3.81°K (secondary). $H_{\max}$ is the field at which (V_2/H) has its maximum value $(V_2/H)_{\max}$. In this sample the length of the region in which the flux flow occurs is 1 cm, so the actual vortex velocity in cm/sec is numerically 100 times the ratio $V/B\approx V/H$ in μV/Oe.

T (°K)	$H_{c\perp,1}$ (Oe)	$H_{\text{depinning}}$ (Oe)	$H_{\max}$ (Oe)	$(V_2/H)_{\max}$ (μV/Oe)
3.71	16	0.7	3.3	300
3.60	28	4	11	40
3.49	41	20	22	5
3.36	56	34	No coupling	0

At constant field, maintenance of a threshold condition with a lowering of the temperature required an increase of the current and hence of the dissipation. Heating instabilities were a limiting factor below approximately 3.4°K, but it was interesting to note that in low fields the large voltage fluctuations of V_1 associated with these instabilities were accompanied by synchronous identical voltage fluctuations of V_2, as judged visually by observing side-by-side meters indicating V_1 and V_2.

CONCLUSION

To summarize, the observed variation of the ratio of the voltages V_2/V_1 in thin-film superconducting dc transformers is attributed to the slipping of the vortices between the two films. The results of our experiments on the effects on this ratio of the current, the magnetic field, and the temperature can be qualitatively understood on that basis. In our discussion, we have assumed that in these films a flux flow process gives rise to all observed voltages. This is consistent with the noise measurements of Van Ooijen and Van Gurp[9] on type-II superconducting vanadium foils and contrasts with the more complex properties of thick foils of type-I superconductors.[5,10]

[8] The reduction of the order parameter Δ by the current would increase the effective penetration depth, duplicating one effect of the magnetic field. By making the field more homogeneous, this would favor the slipping of the vortices. In view of the relatively small current densities used, however, we expect this effect to be unimportant.

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