Comments and Addenda

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Spin-Parity Analysis of the Low-Mass (1.1–1.4 BeV) $K^*\pi$ System in $K^-p \to \bar{K}^{*0}\pi^-p$ at 5.5 BeV/c†

J. C. PARK AND S. KIM
University of Illinois, Urbana, Illinois

(Received 21 June 1968)

We suggest a method of spin-parity analysis and apply it to low-mass $K^*\pi$ systems in $K^-p \to \bar{K}^{*0}\pi^-p \to K^-\pi^+\pi^-p$ at 5.5 BeV/c. We find that the $K^*\pi$ system in each of the mass regions 1.1–1.2, 1.2–1.3, and 1.3–1.4 BeV is consistent only with $J^P=1^+$. We then determine the corresponding density-matrix elements, the strength of the K^*-helicity-0 componenent, and that of the S-wave component.

In our earlier paper,[1] we described gross features of the $K^*\pi$ system, in connection with the diffraction dissociation process, of the reaction[2]

$$5.5\ \text{BeV}/c \quad K^-p \to \bar{K}^{*0}(890)\pi^-p$$
$$\searrow K^-\pi^+$$
$$(1511\ \text{events};\ \sigma=0.41\pm0.03\ \mu\text{b}). \quad (1)$$

We then suggested, in agreement with many other experimental groups,[3] that the $K^*\pi$ system in the mass region 1.1–1.4 BeV behaves like a genuine resonance with J^P consistent with 1^+ or 2^-. The aim of the present paper is twofold: (a) First, our analysis of decay angular distributions shows that the $K^*\pi$ system in this mass region can only be consistent with 1^+. The method used here is to analyze the $K^*\pi$ system, using both the normal to the three-body decay plane and the K^* helicity axis. The ratios of moments of the polar angular distributions associated with the two analyzers are independent of production mechanism and do, in our data, strongly discriminate against J^P other than 1^+. (b) The second result has to do with the question of substructures in the mass region 1.1–1.4 BeV. The $M(K^*\pi)$ spectrum of our data (Fig. 1) does not exhibit significant substructures in this mass region. In contrast to the K^+p data at 5 BeV/c,[4] the use of various cuts in $\Delta^2(p)$ and/or KK scattering angle does not clearly resolve our data into distinct peaks.[5] We show, for instance, the spectrum with $\Delta^2(p)>0.1$ (BeV/c)^2 (the shaded distribution in Fig. 1), in which there are at best some suggestions for two peaks: one centered at $\sim$1360 and the other at $\sim$1430 MeV. The following study in three successive 100-MeV intervals of the mass region 1.1–1.4 BeV shows that decay angular distributions of these subregions are essentially identical. Thus, if there are dominant $K^*\pi$ resonances, then they are indistinguishable in terms of their production and decay properties (apart from the slowly decreasing $m=0$ polarization with increasing mass).

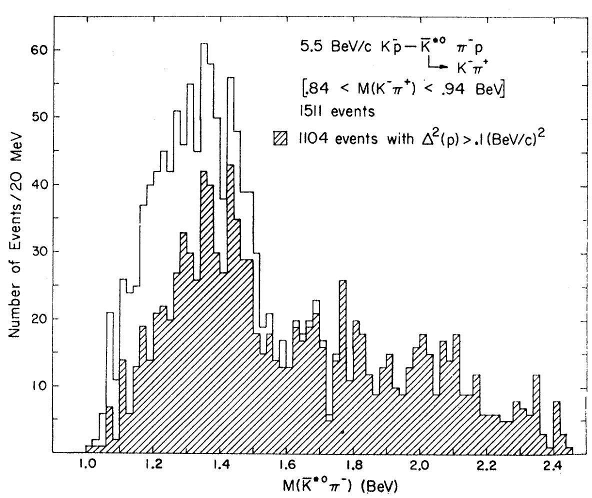

FIG. 1. Mass spectra of $\bar{K}^{*0}\pi^-$ system from reaction 1. The cross-hatched area corresponds to events with $\Delta^2(p)>0.1$ (BeV/c)^2.

† Work supported in part by the U. S. Atomic Energy Commission.

[1] J. C. Park, S. Kim, G. Chandler, G. Ascoli, and E. L. Goldwasser, Phys. Rev. Letters **20**, 171 (1968).

[2] K^* events are defined by the mass cut $0.84<M(K^-\pi^+)<0.94$ BeV. As mentioned in Ref. 1, the sample is clean with background no more than 15%.

[3] G. Goldhaber, A. Firestone, and B. C. Shen, Phys. Rev. Letters **19**, 972 (1967), in addition to references given in Ref. 1.

[4] G. Bassompierre *et al.*, Phys. Letters **26B**, 30 (1967).

[5] S. Kim, J. C. Park, G. Chandler, G. Ascoli, and E. L. Goldwasser, in Proceedings of the Third Topical Conference on Resonant Particles, Athens, Ohio, 1967 (to be published).

TABLE I. $\langle P_2\rangle$ moments, q_2 test for different J^p, and χ^2 fit for 1^+. The entries in parentheses are for the data with $\Delta^2(p)>0.1$ $(\mathrm{BeV}/c)^2$.

$M(K^*\pi)$ interval (BeV)	No. of events	a_2^{II} [a] $5\langle P_2(\cos\theta_2)\rangle$	a_2^{I} $5\langle P_2(\cos\theta_1)\rangle$	a_2 $5\langle P_2(\cos\beta)\rangle$	Observed q_2 [b] a_2^{I}/a_2	Calculated q_2 [c] 1^+	2^-	3^+
1.1–1.2	152	0.02 ±0.18	−0.05 ±0.18	0.97±0.14	−0.05 ±0.18	−0.02 ±0.18	−0.81±0.07	−0.92±0.05
	(66)	(−0.06 ±0.28)	(0.002±0.25)	(−0.96±0.19)	0.003±0.27)	(0.06 ±0.28)	(−0.78±0.10)	(−0.89±0.08)
1.2–1.3	226	0.22 ±0.15	−0.15 ±0.15	−0.75±0.12	0.20 ±0.28	−0.22 ±0.15	−0.88±0.06	−0.98±0.05
	(123)	(0.49 ±0.22)	(−0.01 ±0.21)	−0.54±0.17	(0.02 ±0.38)	(−0.49 ±0.22)	(−0.99±0.10)	(−1.07±0.08)
1.3–1.4	269	0.001±0.13	0.06 ±0.14	−0.53±0.13	−0.12 ±0.25	−0.001±0.13	−0.80±0.05	−0.91±0.04
	(168)	(−0.001±0.16)	(−0.04 ±0.17)	(−0.40±0.18)	(0.10 ±0.45)	(0.001±0.16)	(−0.80±0.06)	(−0.91±0.05)

$M(K^*\pi)$ interval (BeV)	χ^2 for individual distribution $\cos\theta_2$	$\cos\theta_1$	$\cos\beta$	φ_2	φ_1	φ	Parameters f_0	ρ_{00}	ρ_{1-1}	χ^2 $\cos\theta'$	Parameter $\lvert S\rvert^2$
1.1–1.2	10.0	7.5	2.8	13.2	13.5	4.1	0.32±0.04	0.96±0.06	−0.15±0.11	11.9	0.83±0.14
1.2–1.3	5.6	10.0	7.6	5.2	12.5	7.5	0.35±0.04	0.87±0.06	−0.11±0.09	5.4	0.85±0.10
1.3–1.4	12.3	2.7	11.7	8.5	17.9	7.9	0.34±0.04	0.67±0.08	−0.07±0.09	6.6	1.20±0.28

[a] $\langle P_L\rangle\equiv(1/N)\sum_i^N P_L(x_i)$, where i refers to events. The error is estimated as $\delta a_L=(2L+1)(\langle P_L^2\rangle-\langle P_L\rangle^2)^{1/2}/\sqrt{N}$.
[b] The error δq_2 includes the correlation between the $\cos\theta_1$ and $\cos\beta$ distributions.
[c] The error for calculated q_2 is due only to that of f_0 determined from $a_2^{\mathrm{II}}=3f_0-1$. Explicitly, $q_2=1-3f_0$ for 1^+, $(1+f_0)/(f_0-2)$ for 2^-, and $(6+2f_0)/(5f_0-9)$ for 3^+.

Definition of angles. In the rest frame of the $K^*\pi$ system we introduce the Gottfried-Jackson axes $\hat{z}=\hat{K}_{\mathrm{inc}}$ and $\hat{y}=\mathbf{p}\times\mathbf{K}_{\mathrm{inc}}/|\mathbf{p}\times\mathbf{K}_{\mathrm{inc}}|$, the K^* helicity axes $\hat{z}_H=\hat{K}^*$ and $\hat{y}_H=\hat{z}\times\hat{z}_H/|\hat{z}\times\hat{z}_H|$, the normal to the three-body decay plane $\hat{n}=\mathbf{K}\times\pi^-/|\mathbf{K}\times\pi^-|$, and $\hat{K}'=(\hat{K}$ in the K^* rest frame). We then consider the azimuth and polar angles $\varphi_1\theta_1$ of z_H with respect to xyz, $\varphi_2\theta_2$ of K' with respect to $x_Hy_Hz_H$, and redundantly, the azimuth and polar angles $\alpha\beta$ of n and those $\varphi'\theta'$ of K', with respect to xyz. In the mass range of our interest, $1.1<M(K^*\pi)<1.4$ BeV, the angle θ' is approximately equal to the usual KK scattering angle; the difference is due to the Lorentz transformation to the K^* rest frame of the incident K, which is used to define the z axis.

Formalism. It is well known[6] that the K^* decay has the form

$$W(\cos\theta_2)=\tfrac{1}{2}[1+a_2^{\mathrm{II}}P_2(\cos\theta_2)], \tag{2a}$$

with $a_2^{\mathrm{II}}=3f_0-1$, where f_0 is the decay parameter for K^* helicity 0.[7] Furthermore, for the $K^*\pi$ system with a unique J^P, one can derive the following expression for the ratio $q_L\equiv a_L^{\mathrm{I}}/a_L$, with $a_L^{\mathrm{I}}\equiv(2L+1)\langle P_L(\cos\theta_1)\rangle$ and $a_L\equiv(2L+1)\langle P_L(\cos\beta)\rangle$:

$$q_L=\left(-1+f_0\frac{L(L+1)}{L(L+1)-2J(J+1)}\right)\Bigg/\sum_{k=-J}^{J}r_k(-)^k\frac{\langle L0|JJk-k\rangle}{\langle L0|JJ1-1\rangle}, \tag{3}$$

where[8] $k\equiv\mathbf{J}\cdot\hat{z}$ and

$$r_k=[2/J(J+1)]\{[(d/d\beta)d_{k0}{}^J(\beta)]^2\}_{\beta=\pi/2} \quad \text{for} \quad \epsilon\equiv P(-)^{J-1}=-1$$
$$=[d_{k0}{}^J(\tfrac{1}{2}\pi)]^2\{f_0+[2k^2/J(J+1)](1-f_0)\} \text{ for } \epsilon=+1.$$

Our method of discriminating against a given J^P consists of determining f_0 from a_2^{II} [Eq. (2a)] and then comparing the q_L predicted by Eq. (3) with the corresponding experimental ratio.

Finally, we shall use the following explicit forms for $J^P=1^+$ to test the consistency:

$$W(\varphi_2)=\tfrac{1}{2}[1-(1-f_0)\tfrac{1}{2}(1-3\rho_{00})\cos2\varphi_2], \tag{2b}$$

$$W(\cos\theta_1)=\tfrac{1}{2}[1+(1-3f_0)\tfrac{1}{2}(1-3\rho_{00})P_2(\cos\theta_1)], \tag{2c}$$

$$W(\varphi_1)=\tfrac{1}{2}[1+(1-3f_0)\rho_{1-1}\cos2\varphi_1], \tag{2d}$$

$$W(\cos\beta)=\tfrac{1}{2}[1+\tfrac{1}{2}(1-3\rho_{00})P_2(\cos\beta)], \tag{2e}$$

$$W(\alpha)=\tfrac{1}{2}[1+\rho_{1-1}\cos2\alpha], \tag{2f}$$

$$W(\cos\theta')=\tfrac{1}{2}[1-\tfrac{1}{2}(1-3\rho_{00})\times(2|S|^2+\tfrac{1}{5}|D|^2)P_2(\cos\theta')], \tag{2g}$$

where ρ_{ij} are the density-matrix elements and S (D) stands for the S (D)-wave decay parameters[9] for the $K^*\pi$ system.

Comparison with data. In Fig. 2, we show angular distributions (folded) for the mass region 1.1–1.4 BeV in 100-MeV intervals. All the distributions considered have no significant moment, $\langle P_L\rangle$ with odd L. (That is, the distributions do not exhibit significant asymmetry before folding. For the calculation of moments, note Ref. a of Table I.) Also, we do not see any strong moments for $L>2$. Similar remarks apply to azimuth distributions. Note that the three regions exhibit angular distributions whose gross characteristics are quite alike. We therefore choose to analyze each region without background considerations.[10]

For all three regions the $\cos\theta_2$ distribution is flat ($a_2^{\mathrm{II}}\approx0$ as shown in Table I), giving $f_0\approx\frac{1}{3}$. This value is expected for an S-wave $K^*\pi$ system.[11] The nonvanish-

[6] For general formalism, the reader is referred to the lecture by J. D. Jackson, in *Higy Energy Physics*, edited by C. DeWitt and M. Jacob (Gordon and Breach Science Publishers, Inc., New York, 1966).

[7] The decay parameters $f_\lambda\equiv|F_\lambda|^2$, where λ is the helicity of K^*, are normalized as $f_0+2f_1=1$.

[8] $\langle J_3m_3|J_1J_2m_1m_2\rangle$ stands for the Clebsch-Gordan coefficient for the decomposition, $J_3=J_1+J_2$. The given formula relating r_k to f_λ follows essentially from the identity

$$R(\varphi_1\theta_1\varphi_2)=R(\alpha\beta\gamma)R(\pi,\tfrac{1}{2}\pi,\tfrac{1}{2}\pi),$$

where $R(\alpha\beta\gamma)$ is the rotation through Euler angles $\alpha\beta\gamma$.

[9] S and D are related to the parameters F_λ, already introduced, as $S=(\sqrt{\tfrac{1}{3}})F_0+(\sqrt{\tfrac{1}{3}})2F_1$, and $D=-(\sqrt{\tfrac{2}{3}})F_0+(\sqrt{\tfrac{2}{3}})F_1$. We use the normalization $|S|^2+|D|^2=1$.

[10] We note, as in Ref. 1, that the $K^*\pi$ system due to the diffraction dissociation process consists mainly of $J^P=1^+$ component.

[11] It is interesting to point out that in the region 1.4–1.5 BeV of our data the $\cos\theta_2$ distribution shows the weak presence of a $\sin^2\theta_2$ term; a_2^{II} has the value -0.35 ± 0.15 for this region and again vanishes (0.04 ± 0.23) in the region 1.5–1.6 BeV. This effect can be understood to be due to the $K_V(1420)$ production in our data.

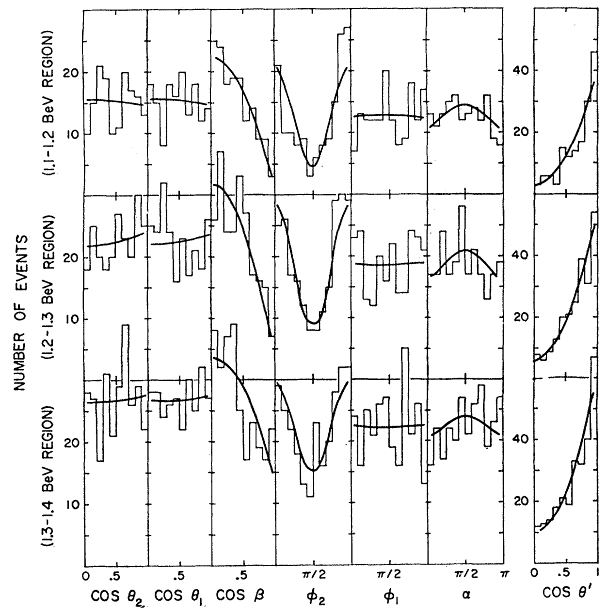

FIG. 2. Angular distributions for events in three $M(\bar{K}^{*0}\pi^-)$ regions, 1.1–1.2, 1.2–1.3, and 1.3–1.4 BeV. See text for the definition of angles. The solid lines are fitted curves expected for the decay of $J^P=1^+$ $K^*\pi$ system.

ing f_0, of course, implies $\epsilon=+1$ (or $J^P=0^-, 1^+, 2^-, \cdots$). Also, for these regions $a_2^{\mathrm{I}}\approx 0$, which is consistent with the dominance of the S-wave component. On the other hand, a_2 has large negative values indicating that the polarization is mostly $m=0$ if $J^P=1^+$, as is apparent in Eq. (2e). (The large value of a_2, of course, rules out $J^P=0^-$.)

The comparison, in Table I, of the observed ratio $q_2=a_2^{\mathrm{I}}/a_2$ with calculated values shows strongly that the angular distributions for all three regions cannot be consistent with any single J^P other than 1^+.

To test the consistency of 1^+, we make a χ^2 fit of the expressions (2a)–(2f) to the corresponding distributions.[12] The results (given in Table I and shown by smooth curves in Fig. 2) are quite acceptable in all three regions: With 62 degrees of freedom the total χ^2 is 51.1 (for the interval 1.1–1.2 BeV), 48.4 (1.2–1.3 BeV), and 60.9 (1.3–1.4 BeV), corresponding, respectively, to the confidence level of 0.84, 0.90, and 0.52. Finally, the amount of S-wave component for each region ($|S|^2$ in Table I) is determined by fitting Eq. (2g), with the value of ρ_{00} in Table I, to the $\cos\theta'$ distribution. The resulting χ^2 is given in Table I and the fitted curve in Fig. 2.

We remark parenthetically that the same analysis for events with $\Delta^2(p)>0.1$ $(\mathrm{BeV}/c)^2$ yields similar results, an essential part of which is given in Table I: The observed value of q_2 favors only 1^+ for all three regions. The χ^2 fit of the 1^+ hypothesis has the confidence level of 0.75 (1.1–1.2 BeV), 0.72 (1.2–1.3 BeV), and 0.82 (1.3–1.4 BeV). All the fitted parameters are, within error, identical to the case without the Δ^2 cut, except ρ_{00}, which has smaller value.[13]

The fitted values of various parameters confirm our earlier observations: The decay properties of the three regions, each of which is consistent only with $J^P=1^+$, are essentially identical. The only distinction appears to be the behavior of ρ_{00}, which has the maximum value in the lowest-mass region and then gradually decreases for higher $K^*\pi$ mass. Note that for S-wave $K^*\pi$ systems the separation of substructures into polar and equatorial events in terms of the KK scattering angle (as was done, for example, in Ref. 4) implies corresponding fluctuations in the value of ρ_{00}.[14]

ACKNOWLEDGMENTS

We would like to thank Professor G. Ascoli, Professor E. L. Goldwasser, and Professor U. E. Kruse for continued guidance and support. We gratefully acknowledge the combined effort of the zero-gradient synchrotron and bubble chamber staffs at the Argonne National Laboratory and that of the scanning and measuring crew at the University of Illinois, which helped make this experiment possible.[15]

[12] The procedure used was to minimize the sum of χ^2 for each distribution. This procedure is, strictly speaking, not correct, since the correlation effects among different projections are ignored. However, we obtain essentially identical results if we consider individual distributions as, for example, $f_0=\frac{1}{3}(1+a_2^{\mathrm{II}})$, $\rho_{00}=\frac{1}{3}(1-2a_2)$, and $\rho_{1-1}=2\langle\cos 2\alpha\rangle$.

[13] We obtain $\rho_{00}=1.06\pm0.07$ (1.1–1.2 BeV), 0.73 ± 0.09 (1.2–1.3 BeV), and 0.59 ± 0.08 (1.3–1.4 BeV).

[14] This is best seen by noting that Eq. (2g) reduces, for an S-wave $K^*\pi$ system, to $\frac{3}{2}[\rho_{00}\cos^2\theta'+\rho_{11}\sin^2\theta']$. Furthermore, as described in the text, the distribution in terms of the KK scattering angle is practically identical to that in terms of our $\cos\theta'$.

[15] After this work was completed, F. Bomse *et al.* [Phys. Rev. Letters **20**, 1519 (1968)] published their evidence for $J^P=1^+$ for the $K^*\pi$ system in the 1.3-BeV region of the reaction $K^+p \to K^{*+}p$ at 5.44 BeV/c.