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Structure of the Nonleptonic Weak Interactions of Mesons

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The limits of validity of an effective weak Hamiltonian involving an intermediate vector boson are studied by examining suitable nonleptonic weak processes. Two extreme assumptions are studied. (1) It is assumed that the dominant contribution to the weak amplitudes is given by the convergent part of the nonleptonic weak matrix elements. Current algebra and spectral-function sum rules are used to evaluate approximately the strong-interaction effects. We discuss the approximations involved in such a procedure and examine the possibility of parametrizing the diverse nonleptonic matrix elements in terms of a common weak-boson mass. Values of this mass ranging only from 3 to 5 BeV are found, notwithstanding the variety of processes considered. The dependence of the nonleptonic weak matrix element on $SU(3)$ breaking is discussed. (2) The alternative assumption is examined that the divergences in the nonleptonic matrix element are not somehow cancelled, and that they give the leading contribution to the weak amplitudes. In this case we show that given certain boundedness conditions on some scattering amplitudes, the nonleptonic matrix elements are quadratically divergent, and the cutoff momentum is independent of strong-interaction effects. From the diverse weak processes considered we find cutoff momenta ranging from 2 to 5 BeV/c. These results and their implications are discussed.

I. INTRODUCTION

A MAJOR difficulty of weak interactions is that their present description is in terms of a phenomenological theory whose exact limit of validity is unknown. One would naturally expect to obtain information about the structure of weak interactions from purely leptonic processes, in particular μ decay and lepton-lepton scattering, thus avoiding any complications due to strong interactions.[1] However, the only available data in this case concern μ decay, and so one is forced to examine the more numerous weak-interaction effects involving hadrons. The main difficulty in obtaining an idea of the validity of the phenomenological theory from processes involving hadrons is the separation of weak-interaction effects from electromagnetic or strong-interaction effects. Recently, however, techniques essentially based on the algebra of currents have indicated the possibility of evaluating the strong-interaction effects or separating the weak effects. In Sec. II, we will briefly discuss the techniques and the processes to which they may be applied, together with our form for the weak-interaction-effective Hamiltonian and the various approximations we shall make.

In Sec. III, we shall employ the concept of an asymptotic symmetry and spectral-function sum rules to evaluate the convergent part of the various relevant weak processes involving pseudoscalar mesons. In Sec. IV, on the other hand, we shall examine the most divergent part of the weak matrix elements for the processes considered in Sec. III and discuss how the use of current algebra and certain boundedness assumptions enables us to obtain results essentially independent of strong-interaction effects. Lastly, in Sec. V, we summarize and discuss our conclusions.

II. METHODS AND APPROXIMATIONS

We shall consider two methods of evaluating the nonleptonic weak matrix elements. The first allows us to determine the convergent part of an amplitude by approximately evaluating the strong-interaction effects. The second enables us to evaluate the most divergent part of an amplitude independently of strong-interaction effects. We shall only consider weak amplitudes involving pseudoscalar mesons, since it is only in this case that both techniques are applicable.

The first method consists in evaluating a vertex function or self-energy diagram at the unphysical point for which all extremal legs have zero four-momenta. The vertex, or self-energy diagram, is then expressible in terms of axial-vector- and vector-current spectral functions. These spectral functions are assumed to satisfy certain sum rules as a consequence of an asymptotic symmetry for the axial-vector- and vector-current Green's functions.[2] At the above-mentioned unphysical point, one obtains a convergent result, since all relevant divergences are proportional to the mass of the external meson. Clearly, then, the results obtained by such a procedure are physically meaningful only if in reality the divergences are cancelled by higher-order effects (weak interactions in our case) or are at most logarithmic and do not produce a significant change in the results for various reasonable values of the cutoff momentum, their contribution being negligible in either case.

However, the divergences we shall encounter in the nonleptonic weak-interaction matrix elements are quadratic and, unless cancellations occur, are expected to render all results extremely cutoff-dependent. This being the case, we may take either of the following points of view.

(a) As we have suggested above, higher-order effects eliminate the divergences. In this case, we parametrize the breakdown of the local current-current interaction by an intermediate vector boson mass. We then attempt to estimate the weak vector boson mass by the previously mentioned method, through the examination of

[1] A. Pais and G. Feinberg, Phys. Rev. **131**, 2724 (1963); A. Pais, Notes of Lectures given at the I.A.E.A. Seminar, Trieste, Italy, 1963 (unpublished).

[2] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967).

various nonleptonic weak effects. One aim is to examine the possibility that the various weak processes may be correlated in terms of approximately the same value for the weak boson mass.

(b) One may have that the matrix elements are divergent and that cancellations do not occur, the cutoff being naturally supplied by the weak, or electromagnetic, or some yet unknown, interactions. In this case, we shall assume that the most divergent part of the matrix element gives the dominant contribution to the amplitude, and we then attempt to estimate the cutoff momentum. To this end, we may employ a second technique whereby the most divergent part of the weak matrix element may be related to the equal-time commutator of two currents.[3] This method has been previously applied in the context of weak processes, in showing that, for the particular case of neutral leptonic currents in weak hadronic decays, strong-interaction effects do not eliminate the divergences due to weak interactions.[4] We shall similarly show how results essentially independent of strong-interaction effects can be obtained from the processes we shall consider and, in particular, we shall obtain estimates of the cutoff momentum.[5] Again we shall be particularly concerned with the possibility of obtaining approximately the same cutoff momentum from the diverse weak processes.

However, it could well be that physical reality lies between the above two extremes, that is, both the convergent and divergent (with cutoff) contributions are present and of comparable magnitude. If this is the case, one needs a more detailed knowledge of the structure of the hadronic weak currents beyond their equal-time commutation relations. Nevertheless, some conclusions may still be drawn, and we shall return to this point in Sec. V.

Let us now examine our form for the effective weak Hamiltonian and the conventions we shall follow. The effective hadronic weak Hamiltonian is given by

$$H_{\text{eff}}{}^{\text{w}}(0)\equiv g^2\int d^4x\, T(J_\mu(x)J_\nu{}^\dagger(0))\Delta_{\mu\nu}{}^{\text{w}}(x)\,, \quad (2.1)$$

where

$$\Delta_{\mu\nu}{}^{\text{w}}(x)=\frac{-i}{(2\pi)^4}\int d^4q\, e^{iq\cdot x}\left(\frac{\delta_{\mu\nu}+q_\mu q_\nu/m_W{}^2}{q^2+m_W{}^2-i\epsilon}\right), \quad (2.2)$$

m_W being the intermediate vector boson mass, which we shall always assume to be large relative to other masses in our weak amplitudes. The usual weak-coupling G is related to g^2 by

$$g^2/m_W{}^2=G/\sqrt{2}\,. \quad (2.3)$$

The currents J_μ are given by[6]

$$J_\mu=j_\mu{}^{1+i2}\cos\theta_V+j_{5\mu}{}^{1+i2}\cos\theta_A +j_\mu{}^{4+i5}\sin\theta_V+j_{5\mu}{}^{4+i5}\sin\theta_A\,, \quad (2.4)$$

where the $j_\mu{}^i$ ($j_{5\mu}{}^i$) are the usual vector (axial-vector) currents satisfying the current algebra[7]:

$$\left[\int j_0{}^i(x)d^3x, j_\mu{}^j(0)\right]_{x_0=0}=if_{ijk}j_\mu{}^k(0)\,, \quad (2.5)$$

$$\left[\int j_{50}{}^i(x)d^3x, j_\mu{}^j(0)\right]_{x_0=0}=if_{ijk}j_{5\mu}{}^k(0)\,, \quad (2.6)$$

$$\left[\int j_{50}{}^i(x)d^3x, j_{5\mu}{}^j(0)\right]_{x_0=0}=if_{ijk}j_\mu{}^k(0)\,, \quad (2.7)$$

and θ_V (θ_A) are the vector (axial-vector) Cabibbo angles. We shall restrict ourselves to the above effective Hamiltonian, Eq. (2.1), and not consider its local limit ($m_W{}^2\to\infty$, $g^2/m_W{}^2$ fixed). The results obtained in the latter case will be more singular and exhibit higher-order divergences. It should be stressed that we take the vector and axial-vector Cabibbo angles to be different[8] in the above effective Hamiltonian. An analysis of the experimental data for the leptonic decays of hadrons has shown that this seems to be the case.[9]

We shall use partially conserved axial-vector current (PCAC)[10] in the form

$$\partial_\mu j_{5\mu}{}^i=c\mu_i{}^2\phi^i\,, \quad \text{(not summed over } i)\,, \quad (2.8)$$

where c is the pion decay constant, μ_i the mass, and ϕ^i the field of the ith pseudoscalar. In the above form for PCAC, Eq. (2.8), we clearly have $c_\pi=c_K=c$, which is required, up to second order in $SU(3)$ breaking, by vector and axial-vector dominance of the first-spectral-function sum rules for chiral $SU(3)\otimes SU(3)$.[11] With the above form for PCAC from the $K_{\mu2}$, $\pi_{\mu2}$ rates and the $K_{e,3}{}^+$, $K_{e,3}{}^0$ rates one obtains, respectively, $\sin\theta_A=0.26$ and $\sin\theta_V=0.22$. These values are consistent with the

[3] J. Bjorken, Phys. Rev. **148**, 1467 (1966).

[4] B. L. Ioffe, in *Proceedings of the 1967 International Conference on Particles and Fields, Rochester* (Interscience Publishers, Inc., New York, 1967), p. 447; B. L. Ioffe and E. P. Shabalin, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **6**, 978 (1961) [English transl.: JETP Letters **6**, 390 (1967)].

[5] Information about the cutoff momentum from nonleptonic weak interactions, independently of strong-interaction effects, has also been obtained by F. E. Low [Comments Nucl. Particle Phys. **2**, 33 (1968)] and R. N. Mohapatra, J. Subba Rao, and R. E. Marshak [Phys. Rev. Letters **20**, 1081 (1968)]. The approach employed by the above authors, however, is different from ours.

[6] Latin indices are $SU(3)$ indices and Greek indices are space-time indices.

[7] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); also Physics **1**, 463 (1964).

[8] By this, we do not mean that the "bare" angles [no $SU(3)$ breaking] are different, but rather that $SU(3)$-breaking effects make them so. That $SU(3)$ breaking may have this effect is not too surprising, since, for the vector couplings, because of the Ademollo-Gatto theorem [M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964)], the apparent angle is expected to deviate from the bare angle only to second or higher order in the $SU(3)$ breaking; whereas, for the axial-vector coupling, one does not have the Ademollo-Gatto theorem, and one may expect effects to first order in $SU(3)$ breaking.

[9] N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. **149**, 1288 (1966).

[10] Y. Nambu, Phys. Rev. Letters **6**, 380 (1960); M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 703 (1960).

[11] R. J. Oakes, Phys. Rev. Letters **20**, 513 (1968); C. S. Lai, *ibid.* **20**, 509 (1968).

corresponding values obtained from a recent examination of the leptonic decays of baryons.[12]

The spectral functions associated with vector and axial-vector currents are

$$\Delta_{\mu\nu}{}^{ij}(q^2)=-i\int d^4x\, e^{-iq\cdot x}\langle 0|T(j_\mu{}^i(x)j_\nu{}^j(0))|0\rangle$$
$$=\int dm^2\rho_V{}^{ij}(m^2)\frac{(\delta_{\mu\nu}+q_\mu q_\nu/m^2)}{q^2+m^2-i\epsilon}-\delta_{\mu4}\delta_{\nu4}\int dm^2\frac{\rho_V{}^{ij}(m^2)}{m^2},\quad(2.9)$$

$$\Delta_{5\mu\nu}{}^{ij}(q^2)=-i\int d^4x\, e^{-iq\cdot x}\langle 0|T(j_{5\mu}{}^i(x)j_{5\nu}{}^j(0))|0\rangle$$
$$=\int dm^2\rho_A{}^{ij}(m)\frac{(\delta_{\mu\nu}+q_\mu q_\nu/m^2)}{q^2+m^2-i\epsilon}+\frac{c^2q_\mu q_\nu}{q^2+\mu_i{}^2-i\epsilon}-\delta_{\mu4}\delta_{\nu4}\int dm^2\frac{\rho_V{}^{ij}(m^2)}{m^2}.\quad(2.10)$$

We assume that these functions satisfy the first-spectral-function sum rules[2] for chiral $SU(3)\otimes SU(3)$:

$$\int_0^\infty\frac{dm^2}{m^2}[\rho_V{}^i(m^2)-\rho_A{}^i(m^2)]=c^2,\quad(2.11)$$

$$\int_0^\infty\frac{dm^2}{m^2}[\rho_V{}^i(m^2)-\rho_V{}^j(m^2)]=0.\quad(2.12)$$

We shall not make use of the second-spectral-function sum rule for $SU(3)$ breaking,[13] which, in the context of single-particle saturation, has been criticized on experimental grounds.[14] In Sec. III, we discuss the single-particle saturation of the above spectral functions $\rho_V(m^2)$, $\rho_A(m^2)$.

Another result that we shall use in Sec. IV is that the time-ordered product

$$T_{\mu\nu}(q,p_i,p_f)=i\int d^4x\, e^{iq\cdot x}\langle f|T(j_\mu{}^i(x)j_\nu{}^j(0))|i\rangle\quad(2.13)$$

satisfies[3]

$$\lim_{q_0\to\infty}q_0T_{\mu\nu}(q_0,\mathbf{q},p_i,p_f)=\int d^3x\, e^{i\mathbf{q}\cdot\mathbf{x}}\langle f|[j_\mu{}^i(0,\mathbf{x}),j_\nu{}^j(0)]|i\rangle+O(1/q_0).\quad(2.14)$$

The above result, Eq. (2.14), implies certain boundedness conditions on combinations of invariant coefficients in the spin-space decomposition of the amplitude Eq. (2.13). We shall return to this point in Sec. IV, when we shall use it to pick out the most divergent part of the nonleptonic matrix element.

We shall restrict ourselves to nonleptonic processes, since, besides the wealth of experimental data available, the diagrams contributing to the various matrix elements involve large internal momenta, thus allowing us to test the validity of the effective Hamiltonian, Eq. (2.1). In choosing which processes should be examined, we must bear in mind that the effects we wish to study are pure weak effects. Thus, among nonleptonic meson decays we may examine the $K_1{}^0\to\pi^+\pi^-$ decay, which is related to all other pure weak decays by the $\Delta I=\frac{1}{2}$ rule and PCAC.[15] It would not be meaningful to examine a process such as $K^+\to\pi^+\pi^0$, since presumably, besides being of first order in the weak interactions, it is also of second order in the electromagnetic interactions.[16] Its matrix element should then also be proportional to the fine-structure constant. However, in this case, it is not clear how electromagnetic effects may be evaluated. A process that involves electromagnetic effects that may be separated and estimated is the $K_2{}^0\to 2\gamma$ decay. We shall therefore consider this process. Lastly, concerning second-order weak effects, the $K_1{}^0$-$K_2{}^0$ mass difference is presumably a pure weak effect, and we shall therefore examine it too.[17]

To recapitulate: We shall restrict ourselves to the consideration of the $K_1{}^0\to 2\pi$ and $K_2{}^0\to 2\gamma$ decays and the $K_1{}^0$-$K_2{}^0$ mass difference in the context of the effective Hamiltonian (2.1).

III. CONVERGENT CONTRIBUTION TO NONLEPTONIC MATRIX ELEMENTS

In this section, we shall consider the previously mentioned processes under the assumption that the divergences in the nonleptonic weak matrix elements are cancelled by higher-order weak effects. We shall evalu-

[12] N. Brene, M. Roos, and A. Sirlin, CERN Report No. Th. 872, 1968 (unpublished).

[13] Let us note that we do not take the point of view that $SU(3)$ is a spontaneously broken symmetry, which then leads to massless scalar mesons. Indeed, in our spectral-function representation for the Green's function of the vector current, Eq. (2.9), we have neglected the effects of possible scalar mesons, since their doubtful existence is not necessary unless one uses the second spectral-function sum rule for $SU(3)$ [S. L. Glashow, H. Schnitzer, and S. Weinberg, Phys. Letters **19**, 134 (1967)]. We shall similarly neglect the occurrence of the so-called "c terms" when we continue our nonleptonic weak amplitudes to the soft-meson limit.

[14] J. J. Sakurai, Phys. Rev. Letters **19**, 803 (1967).

[15] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 865 (1966); M. Suzuki, Phys. Rev. **144**, 1154 (1966).

[16] In particular, if one assumed that it were pure first-order weak and computed the matrix element, one would find that it is proportional to the weak coupling constant and the weak-boson mass or cutoff, that is, for example, of the form kGL^2, L being the cutoff momentum and k some constant. Then, since the rate for $\Delta I=\frac{3}{2}$ processes is smaller than the usual weak rates, remarkably low values for the cutoff momentum or boson mass would be obtained.

[17] We do not consider other possibly second-order weak effects such as $K_2{}^0\to\mu^+\mu^-$, since, besides the possibility that such processes, if they occur, are in part due to electromagnetic interactions [M. A. B. Bég, Phys. Rev. **132**, 426 (1963)], only an upper limit exists for their rates. Thus, if one assumes that they are purely weak decays, on examining their matrix elements, remarkably large volumes of the boson mass or cutoff momentum would be obtained. The actual cutoff momentum or boson mass could, of course, be much lower, and the large values obtained are simply due to the fact that the experimental upper limit is large. Our aim, of course, is to find the lowest possible values of the cutoff momentum or weak-boson mass needed to maintain agreement with experiment for the various weak processes.

ate the convergent part of the nonleptonic amplitudes by examining the weak matrix elements in the limit for which all external legs have zero four-momenta. We begin by considering the $K_1^0 \to \pi^+ + \pi^-$ decay process. Let us represent the vertex function in such a manner that the nature of the approximations we shall make is explicitly exhibited.[18]

We have

$$ {}_{\text{out}}\langle \pi^+\pi^- | K_1^0 \rangle_{\text{in}} = \frac{-i}{(2\pi)^{3/2}(2q_0)^{1/2}} \int e^{iq\cdot x} d^4x \, {}_{\text{out}}\langle \pi^+\pi^- | j(x) | 0 \rangle \quad (3.1) $$

$$ = \frac{-(2\pi)^4 i\delta^4(q-p)}{(2\pi)^{3/2}(2q_0)^{1/2}} \, {}_{\text{out}}\langle \pi^+\pi^- | j(0) | 0 \rangle , \quad (3.2) $$

where $j(x) \equiv (\Box - \mu_K^2)\varphi^K$ is the K_1^0 source current and p is the total momentum of the 2π system. We now define a vertex function or scalar form factor:

$$ {}_{\text{out}}\langle \pi^+\pi^- | j(0) | 0 \rangle \equiv F(-q^2) , \quad (3.3) $$

which we shall assume satisfies a subtracted dispersion relation in $t \equiv -q^2$, which of course is the mass of the K meson,

$$ F(t) = F(0) + \frac{t}{\pi} \int_{4\mu_\pi^2}^{\infty} dt' \frac{\operatorname{Im} F(t')}{t'(t'-t-i\epsilon)} . \quad (3.4) $$

On restricting ourselves to two-pion states in the unitarity sum, we have, as a solution to the above Eq. (3.4),[18]

$$ F(t) = F(0) \exp\left(\frac{t}{\pi} \int_{4\mu_\pi^2}^{\infty} dt' \frac{\varphi(t')}{t'(t'-t)} \right) , \quad (3.5) $$

where φ is related to the two-pion s-wave phase shift. $F(0)$ is the $K_1^0 \to 2\pi$ vertex evaluated at zero K-meson mass, $q^2 = 0$. We approximate it by evaluating the vertex function, Eq. (3.3), in the soft-K-meson limit, $q_\mu = 0$. To first order in the weak interactions, we have, in the limit $q_\mu \to 0$,

$$ \langle 2\pi | H_{\text{eff}}^{\text{w}}(0) | K_1^0 \rangle \equiv M = \frac{-i}{c} \langle 2\pi | \left[\int j_{50}{}^7(\mathbf{x},0) d^3x , H_{\text{eff}}^{\text{w}}(0) \right] | 0 \rangle , \quad (3.6) $$

where we define M with the usual factors $(2\pi)^{-3/2}(2E)^{-1/2}$ omitted.

We further approximate the vertex by evaluating it in the limit for which each of the individual pions has zero four-momentum. We then have

$$ \langle \pi^+\pi^- | H_{\text{eff}}^{\text{w}}(0) | K_1^0 \rangle = \frac{1}{c^3} (\cos\theta_V \sin\theta_A - \cos\theta_A \sin\theta_V) \left(\int d^4x \langle 0 | T[j_\mu{}^K(x) j_\nu{}^K(0) - j_{5\mu}{}^K(x) j_{5\nu}{}^K(0) + 2 j_\mu{}^\pi(x) j_\nu{}^\pi(0) - 2 j_{5\mu}{}^\pi(x) j_{5\nu}{}^\pi(0)] | 0 \rangle \Delta_{\mu\nu}{}^{\text{w}}(x) \right) , \quad (3.7) $$

where we have defined $j^i(x) \equiv j^K(x)$ for $i = 4, 5, 6, 7$ and $j^i(x) \equiv j^\pi(x)$ for $i = 1, 2, 3$. We now use the spectral representations for the currents [Eqs. (2.9) and (2.10)], together with the first spectral-function sum rules [Eqs. (2.11) and (2.12)], and obtain for the above matrix element [Eq. (3.7)]

$$ \langle \pi^+\pi^- | H_{\text{eff}}^{\text{w}}(0) | K_1^0 \rangle = \frac{3g^2}{c^3(2\pi)^4} (\cos\theta_V \sin\theta_A - \cos\theta_A \sin\theta_V) \int\int d^4q dm^2 \times \frac{[\rho_A{}^K(m^2) - \rho_V{}^K(m^2) + 2\rho_A{}^\pi(m^2) - 2\rho_V{}^\pi(m^2)]}{(q^2+m_W{}^2)(q^2+m^2)} . \quad (3.8) $$

The physical decay amplitude is given by $F(-\mu_K^2)$, which is related to $F(0)$ through Eq. (3.5). We then note that, since $F(-\mu_K^2) \neq 0$, $F(0)$ cannot be zero. We shall assume that the π-π s-wave phase shifts are small, so that the amplitude $F(-\mu_K^2)$ is not too different from $F(0)$ or our approximation [Eq. (3.8)] to $F(0)$.

We now evaluate the matrix element, Eq. (3.8), in the context of a vector and axial-vector meson dominance assumption of the spectral functions. We approximate the spectral functions by the lowest states

$$ \rho_V{}^i(m^2) = 2c^2 m_i{}^2 \delta(m^2 - m_i{}^2) , \quad (3.9) $$

$$ \rho_A{}^i(m^2) = c^2 m_{A\,i}{}^2 \delta(m^2 - m_{A\,i}{}^2) , \quad (3.10) $$

m and m_A being the vector and axial-vector meson masses, respectively, and we have used the KSRF relation to express the spectral functions in the above form for convenience.[19] We assume the existence of axial-vector mesons $(A_1, K_A{}^*)$ [20] whose masses are related to the vector-meson masses by $m_{A\,i}{}^2 = 2m_i{}^2$.[21]

[18] See, for example, K. Nishijima, *Fundamental Particles* (W. A. Benjamin, Inc., New York, 1963), p. 276; also M. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, New York, 1964). I am grateful to Professor M. A. B. Bég for discussions and suggestions on this point.

[19] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Riazuddin and Fayazzudin, Phys. Rev. **147**, 1072 (1966). Since there is some doubt about the justification of this relation, we may accept it as an empirical observation.

[20] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[21] This may be regarded as a consequence of a chiral symmetry for each vector meson. We note that Weinberg's second sum rule, while not necessarily meaningful when applied to $SU(3)$ symmetry breaking (see Ref. 14), does not lead to any inconsistency if used only in the context of chiral symmetry. That is, $\int dm^2 [\rho_A{}^i(m^2) - \rho_V{}^i(m^2)] = 0$.

We then obtain the following expression for our matrix element, Eq. (3.8):

$$\langle\pi^+\pi^-|H_{\rm eff}{}^{\rm w}(0)|K_1{}^0\rangle=\frac{3g^2i}{2(2\pi)^2c}(\cos\theta_V\sin\theta_A-\cos\theta_A\sin\theta_V)\left(\frac{m_{K^*}{}^4}{m_W{}^2-m_{K^*}{}^2}\ln\frac{m_W{}^2}{m_{K^*}{}^2}-\frac{2m_{K^*}{}^4}{m_W{}^2-2m_{K^*}{}^2}\ln\frac{m_W{}^2}{2m_{K^*}{}^2}\right.$$
$$\left.+\frac{2m_\rho{}^4}{m_W{}^2-m_\rho{}^2}\ln\frac{m_W{}^2}{m_\rho{}^2}-\frac{4m_\rho{}^4}{m_W{}^2-2m_\rho{}^2}\ln\frac{m_W{}^2}{2m_\rho{}^2}\right),\quad(3.11)$$

that is,

$$\langle\pi^+\pi^-|H_{\rm eff}{}^{\rm w}(0)|K_1{}^0\rangle\simeq\frac{3g^2i}{2(2\pi)^2cm_W{}^2}(\cos\theta_V\sin\theta_A-\cos\theta_A\sin\theta_V)$$
$$\times\left(-m_{K^*}{}^4\ln\frac{m_W{}^2}{m_{K^*}{}^2}+2m_{K^*}{}^4\ln2+4m_\rho{}^4\ln2-2m_\rho{}^4\ln\frac{m_W{}^2}{m_\rho{}^2}\right)\quad(3.12)$$

for $m_W{}^2\gg m^2$. The above matrix element is related to the $K_1{}^0\to\pi^+\pi^-$ rate by

$$\Gamma(K_1{}^0\to\pi^+\pi^-)=\frac{1}{16\pi\mu_K}\left(1-\frac{4\mu_\pi{}^2}{\mu_K{}^2}\right)^{1/2}|M|^2,\quad(3.13)$$

and we obtain

$$|\langle\pi^+\pi^-|H_{\rm eff}{}^{\rm w}(0)|K_1{}^0\rangle|=7.85\times10^{-7}\mu_K.\quad(3.14)$$

On comparing with Eq. (3.12), we obtain

$$m_W\approx5\ {\rm BeV},\quad(3.15)$$

where we have taken $g^2/m_W{}^2=G/\sqrt{2}$ with $G=10^{-5}/m_p{}^2$, m_p being the nucleon mass, and $\sin\theta_A=0.26$, $\sin\theta_V=0.22$.

The next weak process we consider is the $K_1{}^0$-$K_2{}^0$ mass difference. To second order in the weak interactions, it is given by

$$\delta E=E(K_1{}^0)-E(K_2{}^0)={\rm Re}\tfrac{1}{2}i(2\pi)^3\int d^4x\langle\bar{K}^0|T(H_{\rm eff}{}^{\rm w}(x)H_{\rm eff}{}^{\rm w}(0))|K^0\rangle+(K^0\leftrightarrow\bar{K}^0)$$
$$={\rm Re}\tfrac{1}{2}i(2\pi)^3\int d^4x\langle\bar{K}^0|[\theta(x_0)H_{\rm eff}{}^{\rm w}(x)H_{\rm eff}{}^{\rm w}(0)+\theta(-x_0)H_{\rm eff}{}^{\rm w}(0)H_{\rm eff}{}^{\rm w}(x)]|K^0\rangle+(K^0\leftrightarrow\bar{K}^0).\quad(3.16)$$

Inserting a complete set of intermediate states,

$$\delta E={\rm Re}\tfrac{1}{2}i(2\pi)^3\sum_n\int d^4x\langle\bar{K}^0|[\theta(x_0)H_{\rm eff}{}^{\rm w}(x)|n\rangle\langle n|H_{\rm eff}{}^{\rm w}(0)+\theta(-x_0)H_{\rm eff}{}^{\rm w}(0)|n\rangle\langle n|H_{\rm eff}{}^{\rm w}(x)]|K^0\rangle+(K^0\leftrightarrow\bar{K}^0)$$
$$={\rm Re}\tfrac{1}{2}i(2\pi)^3\sum_n\int d^4x\langle\bar{K}^0|[\exp[i(p_{K_0}-p_{n_0})\cdot x_0]\theta(x_0)H_{\rm eff}{}^{\rm w}(\mathbf{x},0)|n\rangle\langle n|H_{\rm eff}{}^{\rm w}(0)$$
$$+\theta(-x_0)\exp[-i(p_{K_0}-p_{n_0})\cdot x_0]H_{\rm eff}{}^{\rm w}(0)|n\rangle\langle n|H_{\rm eff}{}^{\rm w}(\mathbf{x},0)]|K^0\rangle+(K^0\leftrightarrow\bar{K}^0),$$

where p_K and p_n are the momenta of the K mesons and the intermediate states, respectively. Taking ${\rm Im}p_{K_0}>0$, we obtain for δE,

$$\delta E=-(2\pi)^6\,{\rm Re}\sum_n\frac{\delta(\mathbf{p}_K-\mathbf{p}_n)}{p_{K_0}-p_{n_0}}\langle\bar{K}^0|H_{\rm eff}{}^{\rm w}(0)|n\rangle\langle n|H_{\rm eff}{}^{\rm w}(0)|K^0\rangle+(K^0\leftrightarrow\bar{K}^0).\quad(3.17)$$

One may be concerned about the validity of using our nonlocal effective Hamiltonian (2.1) to calculate second-order processes and obtain a result such as Eq. (3.17). We note that for $m_W{}^2\to\infty$, with $g^2/m_W{}^2$ fixed, our effective Hamiltonian (2.1) reduces to a point structure and our Eq. (3.17) is exact. Presumably, then, our treatment of the effective Hamiltonian (2.1) as a local object in the above involves a neglect of corrections of the order $m_n{}^2/m_W{}^2$ to the contribution of the nth intermediate state, m_n being the mass of the nth intermediate state. Since we shall later consider the contributions of the lowest-mass intermediate states in Eq. (3.17), for which $m_n{}^2\ll m_W{}^2$, we expect that in this case the approximation is justified. We shall later discuss the validity of our low-mass intermediate-state approximation.

We now restrict the intermediate-state sum to states n and $n\bar{K}K$ with n a single-particle state, in this case the

π^0 and the η. When we later consider the $K_2^0 \to 2\gamma$ decay, we shall make an analogous approximation. If the results we shall obtain for the cutoff momentum or boson mass in the two cases are in agreement, a measure of justification is given to our approximation.

In the soft-K-meson limit Eq. (3.17) becomes

$$\delta\mu^2 = \mu^2(K_1^0) - \mu^2(K_2^0) = \frac{(2\pi)^3}{2c^2} \operatorname{Re} \sum_n \langle 0 | \left[\int j_{50}^{6+i7}(\mathbf{x},0) d^3x, H_{\text{eff}}^{\text{w}}(0) \right] | n \rangle \times \langle n | \left[\int j_{50}^{6+i7}(\mathbf{x},0) d^3x, H_{\text{eff}}^{\text{w}}(0) \right] | 0 \rangle \frac{\delta(\mathbf{p}_n) 2p_{n0}}{\mu_K^2 - p_{n0}^2} + (K^0 \leftrightarrow \bar{K}^0). \quad (3.18)$$

We further approximate our matrix elements in Eq. (3.18) by evaluating them in the soft-π^0 and η limit, obtaining[22]

$$\delta\mu^2 = -\frac{g^4}{c^4} \operatorname{Re}(\sin\theta_V \cos\theta_V - \sin\theta_A \cos\theta_A)^2 \left[\frac{1}{(\mu_K^2 - \mu_\pi^2)} \left(\int d^4x \langle 0 | T[2 j_{5\mu}^\pi(x) j_{5\nu}^\pi(0) - 2 j_\mu^\pi(x) j_\nu^\pi(0) - j_\mu^K(x) j_\nu^K(0) + j_{5\mu}^K(x) j_{5\nu}^K(0)] | 0 \rangle \Delta_{\mu\nu}^{\text{w}}(x) \right)^2 + \frac{3}{(\mu_K^2 - \mu_\eta^2)} \left(\int d^4x \langle 0 | T[j_{5\mu}^K(x) j_{5\nu}^K(0) - j_\mu^K(x) j_\nu^K(0)] | 0 \rangle \Delta_{\mu\nu}^{\text{w}}(x) \right)^2 \right], \quad (3.19)$$

which, again employing the spectral representations for the currents [Eqs. (2.9) and (2.10)] together with the first spectral-function sum rules [Eqs. (2.11) and (2.12)], can be rewritten as

$$\delta\mu^2 = -\frac{9g^4}{c^4} \frac{(\sin\theta_V \cos\theta_V - \sin\theta_A \cos\theta_A)^2}{(2\pi)^8} \operatorname{Re} \left[\frac{1}{(\mu_K^2 - \mu_\pi^2)} \left(\int\int \frac{d^4q\, dm^2}{(q^2 + m_W^2)(q^2 + m^2)} [2\rho_A^\pi(m^2) - 2\rho_V^\pi(m^2) - \rho_V^K(m^2) + \rho_A^K(m^2)] \right)^2 + \frac{3}{(\mu_K^2 - \mu_\eta^2)} \left(\int\int \frac{d^4q\, dm^2}{(q^2 + m_W^2)(q^2 + m^2)} [\rho_A^K(m^2) - \rho_V^K(m^2)] \right)^2 \right]. \quad (3.20)$$

As before, approximating our spectral functions by single-particle states [Eqs. (3.9) and (3.10)], and assuming $m_W^2 \gg m^2$, we obtain

$$\delta\mu^2 = \frac{9g^4}{4(2\pi)^4 m_W^4} (\sin\theta_V \cos\theta_V - \sin\theta_A \cos\theta_A)^2 \left[\frac{1}{(\mu_K^2 - \mu_\pi^2)} \left(m_{K^*}^4 \ln\frac{m_W^2}{m_{K^*}^2} - 2m_{K^*}^4 \ln 2 - 4m_\rho^4 \ln 2 + 2m_\rho^4 \ln\frac{m_W^2}{m_\rho^2} \right)^2 + \frac{3}{(\mu_K^2 - \mu_\eta^2)} \left(m_{K^*}^4 \ln\frac{m_W^2}{m_{K^*}^2} - 2m_{K^*}^4 \ln 2 \right)^2 \right], \quad (3.21)$$

which we compare with the experimental mass difference[20] $\delta\mu = \mu(K_1^0) - \mu(K_2^0) = -0.48/\tau_s = -3.6 \times 10^{-6}$ eV. With the same parameters as used for the K_1^0 decay, we obtain

$$m_W \approx 3 \text{ BeV}. \quad (3.22)$$

The last process we must examine is the $K_2^0 \to 2\gamma$ decay. For this decay, we shall study a pole model in terms of π^0 and η poles analogously to the $K_1^0 - K_2^0$ mass difference. We then consider the following matrix element:

$$M(K_2^0 \to 2\gamma) = \int d^4x \langle 2\gamma | T(H_{\text{eff}}^{\text{w}}(x) H_{\text{em}}^{\text{eff}}(0)) | K_2^0 \rangle = (2\pi)^3 i \sum_\eta \frac{\delta(\mathbf{p}_K - \mathbf{p}_n)}{p_{K_0} - p_{n_0}} [\langle 2\gamma | H_{\text{em}}^{\text{eff}}(0) | n \rangle \langle n | H_{\text{eff}}^{\text{w}}(0) | K_2^0 \rangle + \langle 2\gamma | H_{\text{eff}}^{\text{w}}(0) | n \rangle \langle n | H_{\text{em}}^{\text{eff}}(0) | K_2^0 \rangle], \quad (3.23)$$

where $H_{\text{em}}^{\text{eff}}$ is the effective electromagnetic Hamiltonian and $M(K_2^0 \to 2\gamma)$ is defined with the usual factors $(2\pi)^{-3/2}(2E)^{-1/2}$ omitted. Again we restrict ourselves to the lowest-mass intermediate states. These are the π^0 and η

[22] The effect of $\eta\chi_0$ mixing on our final result is negligible because of its small magnitude. R. H. Dalitz and D. G. Sutherland, Nuovo Cimento **37**, 1777 (1965).

for the first term in Eq. (3.23) and the $K_2^0\eta 2\gamma$ and $K_2^0\pi^0 2\gamma$ for the second term. We shall then have

$$M(K_2^0\to 2\gamma)=\frac{i(2\pi)^3}{\mu_K-\mu_\eta}\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\eta\rangle\langle\eta|H_{\rm eff}{}^{\rm w}(0)|K_2^0\rangle+\frac{i(2\pi)^3}{\mu_K-\mu_\pi}\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\pi^0\rangle\langle\pi^0|H_{\rm eff}{}^{\rm w}(0)|K_2^0\rangle$$
$$-\frac{i(2\pi)^3}{\mu_K+\mu_\eta}\langle 0|H_{\rm eff}{}^{\rm w}(0)|\eta K_2^0\rangle\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\eta\rangle-\frac{i(2\pi)^3}{\mu_K+\mu_\pi}\langle 0|H_{\rm eff}{}^{\rm w}(0)|\pi K_2^0\rangle\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\pi^0\rangle. \quad (3.24)$$

As before, we evaluate the nonleptonic matrix element in the limit for which the K_2^0, and then the π^0 and η, are soft. We then obtain

$$M(K_2^0\to 2\gamma)=\frac{3g^2 i}{(2\pi)^4c^2}(\cos\theta_A\,\sin\theta_A-\cos\theta_V\,\sin\theta_V)\bigg(\frac{1}{(\mu_K{}^2-\mu_\pi{}^2)}$$
$$\times\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\pi^0\rangle\int\int d^4q dm^2\frac{[\rho_A{}^K(m^2)+2\rho_A{}^\pi(m^2)-2\rho_V{}^\pi(m^2)-\rho_V{}^K(m)^2]}{(q^2+m_W{}^2)(q^2+m^2)}$$
$$+\frac{3}{(\mu_K{}^2-\mu_\eta{}^2)}\langle 2\gamma|H_{\rm em}{}^{\rm eff}(0)|\eta\rangle\int\int d^4q dm^2\frac{[\rho_A{}^K(m^2)-\rho_V{}^K(m^2)]}{(q^2+m_W{}^2)(q^2+m^2)}\bigg). \quad (3.25)$$

The above matrix element (3.25) is related to the $K_2^0\to 2\gamma$ width Γ_{K_2} by

$$\Gamma_{K_2}=(1/16\pi\mu_K)|M(K_2^0\to 2\gamma)|^2, \quad (3.26)$$

and similarly the π^0 and η to 2γ matrix elements occurring in Eq. (3.25) are related to the $\pi^0\to 2\gamma$ and $\eta\to 2\gamma$ widths. As before, limiting our spectral functions to one-particle states, Eqs. (3.9) and (3.10), and using $\Gamma_{K_2}=(9.9\pm 2.1)\times 10^{-18}$ MeV,[23] and 0.74×10^{-5} MeV, 0.87×10^{-3} MeV for the π^0, $\eta\to 2\gamma$ widths, respectively, we evaluate m_W, obtaining

$$m_W\approx 3\text{ BeV}. \quad (3.27)$$

This result is consistent with our previous values, Eqs. (3.15) and (3.22).

Let us note a few points about our nonleptonic matrix elements. As we have seen, they depend on the difference of the vector and axial-vector Cabibbo angles, and are therefore presumably of first order in $SU(3)$ breaking.[8] Hence, as is expected, they will vanish in the $SU(3)$ limit.[24] Our procedure in this section has been to continue first the strange pseudoscalar mesons and then the nonstrange pseudoscalars to the soft limit. The results then obtained depend on $SU(3)$ symmetry breaking only through the difference of the vector-current and axial-vector-current Cabibbo angles. If, on the other hand, we had continued to the soft limit for the strange pseudoscalars last, we would have arrived at matrix elements which depend on the $SU(3)$ breaking also through the combination of the spectral functions obtained.[25]

We may then conclude by remarking that, within the context of our rather drastic approximations, we have consistently found a weak boson of mass in the range 3–5 BeV independently of the radically different nature of the processes considered.

IV. DIVERGENT CONTRIBUTION TO NONLEPTONIC MATRIX ELEMENTS

In Sec. III, we have approximately evaluated the convergent part of the nonleptonic matrix element, and we assumed that the divergent part is cancelled by higher-order weak effects or, for some other reason, gives a negligible contribution. The purpose of this section is to examine the possibility that the most divergent part of the nonleptonic matrix element is not cancelled and gives the dominant contribution to the weak amplitudes.

Let us consider a typical nonleptonic matrix element involving two different spin-zero states i, f:

$$\langle f|H_{\rm eff}{}^{\rm w}(0)|i\rangle=\frac{-ig^2}{(2\pi)^4}\int\int d^4x d^4q\frac{(\delta_{\mu\nu}+q_\mu q_\nu/m_W{}^2)}{q^2+m_W{}^2-i\epsilon}e^{iq\cdot x}\langle f|T(J_\mu(x)J_\nu{}^\dagger(0))|i\rangle. \quad (4.1)$$

[23] J. W. Cronin *et al.*, Phys. Rev. Letters **18**, 25 (1967).
[24] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964); N. Cabibbo, *ibid.* **12**, 62 (1964).
[25] See, for example, S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967). Presumably, if one demands that the amplitudes evaluated at the same unphysical point by the two previously mentioned procedures should coincide, certain constraints between $SU(3)$ breaking in the spectral functions, weak-boson mass, and the difference between vector-current and axial-vector-current Cabibbo angles will be obtained.

If the integral over the internal momentum q is divergent, then the most divergent contribution will come from the $q_\mu q_\nu/m_W^2$ term in the weak vector-boson propagator. We shall comment later on the $\delta_{\mu\nu}$ terms.

Thus, restricting ourselves to the longitudinal terms in the weak-boson Green's function, we have

$$\langle f|H_{\text{eff}}^{w}(0)|i\rangle \simeq \frac{g^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{q_\nu e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\langle f|\partial_\mu T(J_\mu(x)J_\nu^\dagger(0))|i\rangle$$

$$=\frac{g^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{q_\nu e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\langle f|\{\delta(x_0)[J_0(x),J_\nu^\dagger(0)]+T(\partial_\mu J_\mu(x)J_\nu^\dagger(0))\}|i\rangle. \quad (4.2)$$

The first term vanishes because of the d^4q integration, and one then has

$$\langle f|H_{\text{eff}}^{w}(0)|i\rangle = \frac{ig^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\partial_\nu\langle f|T(\Phi(x)J_\nu^\dagger(0))|i\rangle$$

$$=\frac{ig^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\langle f|\{\delta(x_0)[\Phi(x),J_0^\dagger(0)]$$

$$+i(p_i-p_f)_\nu T(\Phi(x)J_\nu^\dagger(0))-T(\Phi(x)\Phi^\dagger(0))\}|i\rangle. \quad (4.3)$$

We have defined $\Phi(x)=\partial_\mu J_\mu(x)$, and p_i and p_f are the momenta of the initial and final states, respectively.

Further, we have

$$\langle f|H_{\text{eff}}^{w}(0)|i\rangle = \frac{ig^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\langle f|\{\delta(x_0)[\partial_0 J_0(x),J_0^\dagger(0)]$$

$$+i(p_i-p_f)_\nu T(\Phi(x)J_\nu^\dagger(0))-T(\Phi(x)\Phi^\dagger(0))\}|i\rangle$$

$$=\frac{ig^2}{m_W^2(2\pi)^4}\int\int d^4x d^4q \frac{e^{iq\cdot x}}{q^2+m_W^2-i\epsilon}\langle f|\{-\delta'(x_0)[J_0(x),J_0^\dagger(0)]$$

$$+i(p_i-p_f)_\nu T(\Phi(x)J_\nu^\dagger(0))-T(\Phi(x)\Phi^\dagger(0))\}|i\rangle. \quad (4.4)$$

In the above, if one is willing to accept Eq. (2.14), the most divergent contribution will come from the equal-time commutator term, or the first term on the right-hand side of Eq. (4.4). Let us see this by examining the following scattering amplitude[4]:

$$T_\nu(p_i,p_f,q)\equiv i\int d^4x\, e^{iq\cdot x}\langle f|T(\Phi(x),J_\nu^\dagger(0))|i\rangle \equiv Ap_{i\nu}+Bp_{f\nu}+Cq_\nu. \quad (4.5)$$

Proceeding as before, we have

$$q_\nu T_\nu = -\int d^4x\, e^{iq\cdot x}\langle f|\{\delta(x_0)[\partial_\mu J_\mu(x),J_0^\dagger(0)]+i(p_i-p_f)_\nu T(\Phi(x)J_\nu^\dagger(0))-T(\Phi(x)\Phi^\dagger(0))\}|i\rangle. \quad (4.6)$$

We now note that the functions A, B, and C in Eq. (4.5) depend on q through the invariants q^2 and $p\cdot q$. Large values of these invariants can be obtained by considering the limit $q_0\to\infty$ and $\mathbf{q}$ fixed. In such a limit, as for Eq. (2.14), we have

$$\lim_{q_0\to\infty} q_0 T_0 = \int d^4x\, e^{iq\cdot x}\delta(x_0)\langle f|[\Phi(x),J_0^\dagger(0)]|i\rangle + O(1/q_0)$$

$$= q_0 p_{i0}A + Bq_0 p_{f0} + Cq_0^2 = -q\cdot p_i A - Bp_f\cdot q - Cq^2$$

$$= -q_\nu T_\nu. \quad (4.7)$$

Hence, if we accept the validity of the expansion [Eq. (2.14)] and compare Eqs. (4.6) and (4.7), we see that the most divergent contribution in Eq. (4.4) will come from the equal-time commutator term. We further note that in Eq. (4.1) the contribution from the $\delta_{\mu\nu}$ term in the weak vector-boson Green's function, by Eq. (2.14), is also not expected to contribute to the leading divergence (quadratic) in Eq. (4.4).

We then obtain the following expression for the *leading* divergence of the nonleptonic matrix element:

$$\langle f|H_{\text{eff}}{}^{w}(0)|i\rangle=-\frac{ig^2}{m_W{}^2(2\pi)^4}\int\int d^4x d^4q\frac{e^{iq\cdot x}}{q^2+m_W{}^2-i\epsilon}\delta'(x_0)\langle f|[J_0(x),J_0{}^{\dagger}(0)]|i\rangle$$
$$=\frac{ig^2}{m_W{}^2(2\pi)^4}\int\frac{d^4q}{q^2+m_W{}^2}\langle f|\partial_0\tilde{J}_0(0)|i\rangle$$
$$=-\frac{\pi^2g^2}{m_W{}^2(2\pi)^4}L^2\langle f|\partial_0\tilde{J}_0(0)|i\rangle, \tag{4.8}$$

where we defined a cutoff momentum L and

$$\delta(x_0)[J_0(x),J_0{}^{\dagger}(0)]=\delta^4(x)\tilde{J}_0(0). \tag{4.9}$$

In obtaining Eq. (4.8), we have made use of the fact that the matrix element $\langle f|\partial_0 J_0|i\rangle$ is independent of the weak-boson momentum. As we can then see, the strong interactions do not lead to a damping of the nonleptonic matrix element.

Let us now apply the above considerations to the weak nonleptonic processes we have previously considered. We first examine the $K_1{}^0\to\pi^+\pi^-$ matrix element

$$M=\langle\pi^+\pi^-|H_{\text{eff}}{}^{w}(0)|K_1{}^0\rangle=-\frac{i}{(2)^{1/2}c}\langle\pi^-|\left[\int d^3x\, j_{50}{}^{1-i2}(\mathbf{x},0),\, H_{\text{eff}}{}^{w}(0)\right]|K_1{}^0\rangle, \tag{4.10}$$

where we have taken the soft-pion limit for one of pions in order to obtain a final expression in terms of known matrix elements; also, our states are normalized as before. The above limit does not decrease the degree of the divergence; indeed, proceeding as before, we obtain

$$\langle\pi^+\pi^-|H_{\text{eff}}{}^{w}(0)|K_1{}^0\rangle=\frac{\pi^2 ig^2}{\sqrt{2}m_W{}^2c(2\pi)^4}L^2(\cos\theta_A\sin\theta_V+\cos\theta_V\sin\theta_A)\langle\pi^-|\partial_0 j_0{}^{4-i5}|K_1{}^0\rangle$$
$$=+\frac{\pi^2g^2L^2}{2\sqrt{2}m_W{}^2c(2\pi)^4}\mu_K(\cos\theta_A\sin\theta_V+\cos\theta_V\sin\theta_A)\langle\pi^-|j_0{}^{4-i5}|K_1{}^0\rangle, \tag{4.11}$$

that is,

$$|M|=|\langle\pi^+\pi^-|H_{\text{eff}}{}^{w}(0)|K_1{}^0\rangle|=\frac{3GL^2\mu_K{}^2(\cos\theta_A\sin\theta_V+\cos\theta_V\sin\theta_A)}{32(2\pi)^2\sqrt{2}c}\{f_++\tfrac{1}{3}f_-\}, \tag{4.12}$$

when f_+ and f_- are defined by

$$\langle\pi^0|j_\mu{}^{4-i5}|K^+\rangle=(1/\sqrt{2})[f_+(P_K+P_\pi)_\mu+f_-(P_K-P_\pi)_\mu]. \tag{4.13}$$

Information on f_+/f_- can be obtained from the $K_{\mu3}$ decays. Unfortunately, the experimental situation is unclear.[26] For the sake of argument, we shall take $f_+/f_-\sim-0.8$, as obtained from muon polarization measurements,[27] and shall neglect form-factor variations. Then on comparing Eq. (4.11) with the experimental value of the matrix element (3.14), we obtain a cutoff momentum

$$L\approx 5\text{ BeV}/c. \tag{4.14}$$

Let us now examine the $K_1{}^0$-$K_2{}^0$ mass difference (3.17). Here, we directly apply our result (4.8). As before, we restrict our intermediate-state sum in Eq. (3.17) to the π^0, η and $\pi^0K\bar{K}$, $\eta K\bar{K}$ states. We then obtain

$$\delta\mu^2=-\frac{G^2}{64(2\pi)^4}(\cos\theta_V\sin\theta_V+\cos\theta_A\sin\theta_A)^2L^4\left\{+\frac{(\mu_K{}^2-\mu_\pi{}^2)^2}{\mu_\pi(\mu_\pi-\mu_K)}\right.$$
$$\times\left[1+\left(\frac{(\mu_K-\mu_\pi)}{(\mu_K+\mu_\pi)}\frac{f_-}{f_+}\right)^2\right]+\frac{(\mu_K{}^2-\mu_\pi{}^2)^2}{\mu_\pi(\mu_\pi+\mu_K)}\left[1+\left(\frac{\mu_K+\mu_\pi}{\mu_K-\mu_\pi}\frac{f_-}{f_+}\right)^2\right]$$
$$\left.+\frac{3(\mu_K{}^2-\mu_\eta{}^2)^2}{\mu_\eta(\mu_\eta-\mu_K)}\left[1+\left(\frac{1}{3}\frac{\mu_\eta-\mu_K}{\mu_\eta+\mu_K}\frac{f_-}{f_+}\right)^2\right]+\frac{3(\mu_K{}^2-\mu_\eta{}^2)^2}{\mu_\eta(\mu_\eta+\mu_K)}\left[1+\left(\frac{1}{3}\frac{\mu_K+\mu_\eta}{\mu_\eta-\mu_K}\frac{f_-}{f_+}\right)^2\right]\right\}, \tag{4.15}$$

[26] L. B. Auerbach, A. K. Mann, W. K. McFarlane, and F. J. Sciulli, Phys. Rev. Letters **19**, 464 (1967).

[27] The measurement of $\xi=f_-(0)/f_+(0)$ using polarization is less sensitive to form-factor variations, and probably gives a better experimental value. We do not consider the possibility of large variations from the above value at $t=(m_K-m_\pi)^2$. See also D. Cutts, R. Stiening, C. Wiegand, and M. Deutsch, Phys. Rev. Letters **20**, 955 (1968).

and on comparing this with the experimental mass splitting, we arrive at the remarkably low value of the cutoff momentum,

$$L \approx 2\ \text{BeV}/c. \qquad (4.16)$$

One could of course expect that higher-mass intermediate states in Eq. (3.17) would alter our result, Eq. (4.16). However, Eq. (4.14) does not depend on any intermediate-state truncation.

Let us now, as before, examine the $K_2^0 \to 2\gamma$ decay in order to check the consistency of the intermediate-state truncation. Here, again, we directly use Eq. (4.9). We note that, since $\tilde{J}_0 \sim \lambda_6$ and $K_2^0 \sim \lambda_6$, our nonleptonic matrix element depends only on $SU(3)$ breaking, that is, on f_- in Eq. (4.13).

Our Eq. (3.24) then becomes

$$|M(K_2^0 \to 2\gamma)| = +\frac{L^2 G}{16\sqrt{2}\pi^2}(\cos\theta_V \sin\theta_V + \cos\theta_A \sin\theta_A) \times \left(\frac{f_-}{\sqrt{3}}\langle 2\gamma | H_{\text{em}}^{\text{eff}}(0) | \eta\rangle + f_- \langle 2\gamma | H_{\text{em}}^{\text{eff}}(0) | \pi^0\rangle\right), \qquad (4.17)$$

from which, proceeding as before, using the experimental values for the π, K, η to 2γ matrix elements, we obtain a cutoff momentum

$$L \approx 2\text{–}3\ \text{BeV}/c, \qquad (4.18)$$

a result in agreement with our previous results. Let us then note that, notwithstanding the various, rather drastic approximations made, we have obtained values for the cutoff momentum within a remarkably small range, that is, 2–5 BeV/c.

Once one accepts the algebra of currents and the boundedness assumptions for amplitudes implied by Eq. (2.14), one finds that the leading divergence for nonleptonic decays is not damped by strong interactions. The value then obtained for the cutoff momentum from nonleptonic weak decays is much lower than the cutoff momentum for weak interactions usually obtained from considering the unitarity bound to the first-order Born approximation of $\nu + l \to \nu + l$ ($\sim$300 BeV/c in the center-of-mass system).

V. SUMMARY AND CONCLUSIONS

We have considered all nonleptonic matrix elements that contribute to pure weak effects or processes for which the electromagnetic effects can be estimated. From the first alternative we considered for the nonleptonic matrix elements (Sec. III), we found values of the weak vector-boson mass ranging from 3–5 BeV. Considering the severe approximations we have made, the values obtained are not inconsistent. It is perhaps superfluous to remark that even if assumptions employed in Sec. III are correct, we do not mean that the physical weak boson mass should lie in the above range, but rather that the nonleptonic matrix elements give consistent values of m_W at the unphysical point at which they are evaluated.

We note that if we consider the limit $m_W^2 \to \infty$ (keeping g^2/m_W^2 fixed, and assuming we can interchange the limit and the order of integration) for a typical nonleptonic matrix element, e.g., Eq. (3.8), the weak nonleptonic amplitudes are divergent unless the spectral functions satisfy certain additional moment relations such as

$$\int [\rho_V{}^i(m^2) - \rho_A{}^i(m^2)] m^2 dm^2 = 0. \qquad (5.1)$$

Then, if the local current-current coupling, or Fermi theory, can be considered as the above limit of our weak Hamiltonian (2.1), we may conclude that unless Eq. (5.1) is satisfied, it does not lead to finite nonleptonic matrix elements.

The alternative possibility for the nonleptonic matrix elements, Sec. IV, leads to consistent values for the cutoff momenta obtained from the various processes considered. These values range from 2–5 BeV, and are much lower than the result obtained from the unitarity bound for purely leptonic processes. Moreover, we have seen that the cutoff is independent of strong-interaction effects. These results were obtained by assuming Eq. (2.14). This is equivalent to assuming unsubtracted dispersion relations for all our amplitudes. Formally, our nonleptonic matrix element, Eq. (4.1), may be related to an off-shell scattering amplitude[28,29] of weak vector bosons onto hadrons. Then we can deduce information about the high-energy behavior of the various amplitudes by examining the predictions of the Regge model.[30] It is easy to see that our scattering amplitude will have contributions from invariant amplitudes behaving like $\nu^{\alpha(0)}$ with $\alpha(t)$ a trajectory with the quantum numbers $P=(-1)^J$, $I=\frac{1}{2}$, $S=1$, which is presumably the trajectory associated with the K_V meson, a member of the same $SU(3)$ multiplet as the A_2 $[\alpha_{A_2}(0) > 0]$. Thus, we expect that some of our invariant amplitudes probably need subtractions. Presumably this will only make the divergences worse.[31] The above arguments are, in any case, only valid if there are no fixed poles ($J=0$ in this case) in any of our amplitudes, which cannot be excluded *a priori*.

[28] G. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).
[29] We assume the existence of only c-number Schwinger terms.
[30] The arguments are similar to the ones employed for electromagnetic self-energies. H. Harari, Phys. Rev. Letters **17**, 1303 (1966).
[31] M. B. Halpern, Phys. Rev. **163**, 1611 (1967).

For the second alternative we considered, in which the divergences are not cancelled by higher-order effects and the high-energy contribution is not somehow damped, it is not clear how meaningful is the contribution of the low-mass intermediate states in Eq. (4.1). However, one may ask, since we have found small values for the cutoff momentum, whether one should also include the low-energy contribution, as, for example, determined in Sec. III, in the total matrix element before comparing it with the experimental value, in order to determine the cutoff momentum. These two contributions will, in general, interfere. Remembering that we are essentially interested in determining as low an upper limit for the cutoff momentum as possible, we see that, in the case in which they interfere destructively, we shall only increase the upper limit by a factor of $\sqrt{2}$ and, of course, in the other we shall obtain even lower values for the upper limit. In either case, the order of magnitude of the upper limit for the cutoff momentum that we have found is essentially unchanged. There are also terms involving logarithmic divergences; however, they are of the form $\ln(L/m_W)$ and, since we have found $L\sim m_W$, presumably they are small.

Let us now examine the significance of the low cutoff momentum we have found. It implies an effective dimensionless weak-interaction coupling $G_{\rm eff}\sim(g^2/m_W^2)L^2\sim10^{-3}$. This of course means that higher-order weak effects will be damped by a factor 10^{-3} with respect to the preceding order. Thus a possibly second-order weak process, such as $K_2^0\rightarrow\mu^+\mu^-$, would have the same order of magnitude as the same process obtained through first-order weak and second-order electromagnetic[17] interactions. In general, in such a case one expects difficulty in separating weak and electromagnetic higher-order effects. From our previous considerations, since the cutoff cannot be given by strong interactions, we expect that it may arise, for example, from some property of the weak interactions (such as a self-coupling of the weak boson) or from the presence of some unknown interaction. A particularly suggestive possibility is that it arises from the electromagnetic properties of the weak boson,[4] especially since their higher-order contributions are not too different in order of magnitude. Indeed, if we expect the effective weak interaction to be smaller than the electromagnetic interactions, then $(\pi^2g^2/m_W^2)L^2=(\pi^2G/\sqrt{2})L^2<\alpha$, that is, $L\lesssim9$ BeV.