Weak η Production and the Existence of Second-Class Currents*

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An estimate of the cross section for η production off nucleons by neutrinos is made by assuming that the process takes place via the S_{11} resonance and using the known strong and electromagnetic parameters of the resonance. If second-class weak currents exist, a pion pole contribution would compete with resonance production for this process.

In the last few years the details of the structure of the low-lying nucleonic resonances have become clearer. Information has been obtained for the strong-interaction couplings from πN phase-shift analyses,[1] and the electromagnetic couplings can be found from fits to the pion photoproduction data.[2] In this paper we show that in the case of the $N^*(1550)$,[3] $J^P=\frac{1}{2}^-$, $I=\frac{1}{2}$, which we conventionally call the S_{11}, we can use this information to make a prediction of its production cross section by neutrinos off nucleons[4] in the processes

$$\mathrm{A(i)}\quad \nu+n\rightarrow p^*+\mu^-,$$
$$\mathrm{A(ii)}\quad \bar{\nu}+p\rightarrow n^*+\mu^+,$$

where we write p^*, n^* for N^{*+}, N^{*0}. As the S_{11} is strongly coupled to the ηN channel, this calculation also estimates the cross section for

$$\mathrm{B(i)}\quad \nu+n\rightarrow \eta+p^*+\mu^-,$$
$$\mathrm{B(ii)}\quad \bar{\nu}+p\rightarrow \eta+n^*+\mu^+.$$

The Lagrangian for processes A is

$$\mathcal{L}=\tfrac{1}{2}\sqrt{2}GJ_\alpha\bar{u}_\mu\gamma_\alpha(1+\gamma_5)u_\nu+\mathrm{H.c.} \tag{1}$$

G is the Fermi coupling constant and, for simplicity, we neglect the Cabibbo angle. The weak current $J_\alpha=V_\alpha+A_\alpha$ and we now investigate its matrix element between states of n and p^*.

For the axial-vector current A_α we have

$$\langle p^*|A_\alpha|n\rangle=\bar{u}_f[F_1(k^2)\gamma_\alpha+F_2(k^2)k_\beta\sigma_{\alpha\beta}+iF_3(k^2)k_\alpha]\tau_+u_i, \tag{2}$$

where $k_\alpha=(p^*-n)_\alpha$. Similarly, for the vector current

$$\langle p^*|V_\alpha|n\rangle=\bar{u}_f[a(k^2)\gamma_\alpha\gamma_5+b(k^2)k_\beta\sigma_{\alpha\beta}\gamma_5+ic(k^2)k_\alpha\gamma_5]\tau_+u_i. \tag{3}$$

We can use partially conserved axial-vector current (PCAC) to relate the axial-vector couplings to those involving the πNN^* coupling, which we take to be

$$ihN^*\boldsymbol{\tau}N\cdot\boldsymbol{\pi}+\mathrm{H.c.} \tag{4}$$

The divergence of Eq. (2) gives

$$\langle p^*|\partial_\alpha A_\alpha|n\rangle=[(M-m)F_1(k^2)+k^2F_3(k^2)]i\bar{u}_f\tau_+u_i, \tag{5}$$

where M and m are the S_{11} and nucleon masses. This should be well approximated[5] for k^2 close to $-m_\pi{}^2$ by

$$\langle p^*|\partial_\alpha A_\alpha|n\rangle=\frac{ih\sqrt{2}\bar{u}_f\tau_+u_i}{k^2+m_\pi{}^2}f_\pi. \tag{6}$$

f_π is given approximately by a similar argument[5] as

$$f_\pi=\sqrt{2}mm_\pi{}^2g_A/g, \tag{7}$$

g and g_A are the pion-nucleon and axial-vector coupling constants ($g^2/4\pi=14.7$, $g_A=1.18$). So we have from Eqs. (5)–(7) that

$$F_1(0)=[2m/(M-m)](h/g)g_A. \tag{8}$$

We can also obtain the pion pole contribution to $F_3(k^2)$ by this method. This experiment, however, can only be carried out at energies where it is reasonable to neglect the muon mass and so neither $F_3(k^2)$ nor $c(k^2)$ contributes to the cross section.

We thus obtain a lower limit to the axial-vector contribution to the cross section by neglecting the axial magnetic-dipole term F_2 of which we have no knowledge. In the center-of-mass system of the reaction A, we have, following Yamaguchi,[6]

$$\sigma_A=[G^2F_1{}^2(0)q^2/\pi w^2][w^2(f_0+f_1)+pq(f_0-2f_1+f_2)+\tfrac{1}{2}(M-m)^2(f_0-f_1)], \tag{9}$$

where w is the center-of-mass energy, p and q are the initial and final momenta,

$$f_l=\frac{1}{4\pi}\int d\Omega\, f^2(k^2)\cos^l\theta,\quad F_1(k^2)=F_1(0)f(k^2),$$

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[1] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968); B. H. Bransden, P. J. O'Donnell, and R. G. Moorhouse, *ibid.* **139**, B1566 (1965); J. Cence, Phys. Letters **20**, 306 (1966); A. Donnachie, R. G. Kirsopp, A. T. Lea, and C. Lovelace, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967); L. D. Roper and R. M. Wright, Phys. Rev. **138**, B921 (1965); L. D. Roper, R. M. Wright, and B. T. Feld, *ibid.* **138**, B190 (1965).

[2] Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. **163**, 1632 (1967).

[3] We take the resonance energy from the tables in Rev. Mod. Phys. **40**, 77 (1968), but see below.

[4] This approach was used in the calculation of weak pion production via $N^*(1236)$ by S. M. Berman and M. Veltman, Nuovo Cimento **38**, 993 (1965).

[5] J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento **17**, 757 (1960).

[6] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960).

and we take

$$f(k^2)=(1+r^2k^2/12)^{-2}. \qquad (10)$$

The functions f_0, f_1, and f_2 are given by Yamaguchi[6] and we have, in the high-energy limit where $w\gg M$, that

$$\sigma_A=2G^2F_1^2(0)/\pi r^2. \qquad (11)$$

Now we calculate the vector contribution. Assuming conserved vector current (CVC), $\partial_\alpha V_\alpha=0$ and hence, from Eq. (3),

$$(M+m)a(k^2)+k^2c(k^2)=0. \qquad (12)$$

Thus we have $a(0)=0$. As $c(k^2)$ does not contribute to the cross section, we see that the only significant term is the electric-dipole transition of strength $b(0)$. As before, take $b(k^2)=b(0)f(k^2)$ and we can write $b(0)=\mu^*/2m$ where μ^* is the isovector electric-dipole moment of the transition $N\rightarrow N^*$. Now

$$\sigma_V=(G^2\mu^{*2}q^2/4\pi m^2w^2)\{2(p^2+q^2)w^2(f_0-f_1) - pq[2w^2-(M-m)^2](f_0-2f_1+f_2)\}, \qquad (13)$$

and in the high-energy limit this gives

$$\sigma_V=3G^2\mu^{*2}/\pi m^2r^4. \qquad (14)$$

The interference term between the vector and axial-vector currents gives a contribution which vanishes at high energy, so $\sigma_A+\sigma_V$ given by Eqs. (11) and (14) should give a lower limit to the high-energy total cross section for S_{11} production by neutrinos.

Now all that it is needed are evaluations of h and μ^*. This is easily accomplished in terms of the partial widths of the N^*. From the coupling (4) we obtain for $N^*\rightarrow N\pi$

$$\Gamma_\pi=3(h^2/4\pi)[(M+m)^2-m_\pi^2]q/2M^2 \qquad (15)$$

and for $N^*\rightarrow N\gamma$, assuming a pure isovector transition,[7]

$$\Gamma_\gamma=\alpha\mu^{*2}k_\gamma^3/4m^2, \qquad (16)$$

where q and k_γ are the pion and photon momenta.

The parameters of the S_{11} resonance are, unfortunately, in dispute.[8] This is mainly due to the effects caused by the η threshold lying so close to the resonance position. If we take the πN phase-shift analysis of Bareyre *et al.*[1] as our starting point we obtain[2]

$$M=1530\ \text{MeV},\quad \Gamma_\pi/\Gamma=0.41,\quad \Gamma=159\ \text{MeV}. \qquad (17)$$

To obtain a value of Γ_γ it is best to ignore background terms and the consequent instabilities[2] and compare directly the cross sections for η production by pions[9] and photons[10] in the resonance region. The threshold factors now cancel and we get

$$\Gamma_\gamma/\Gamma_\pi=3.2\times10^{-3}. \qquad (18)$$

From all this we conclude that

$$h^2/4\pi=0.036,\quad F_1(0)=0.19,\quad \mu^*=0.96, \qquad (19)$$

and so, taking $r=0.8\times10^{-13}$ cm,[11] we obtain the high-energy limits

$$\sigma_A=0.7\times10^{-40}\ \text{cm}^2,\quad \sigma_V=1.9\times10^{-40}\ \text{cm}^2, \qquad (20)$$

Thus the cross section for weak η production at high energies (subject to the S_{11} parameters being correct) should be of the order of 1.3×10^{-40} cm^2, taking $\Gamma_\eta/\Gamma\approx\frac{1}{2}$.[12] We have, of course, neglected the axial-magnetic-dipole transition F_2 and also other processes leading to η production, both of which could increase this figure.

So far we have only considered total cross sections, but in order to identify an η in this process it will probably be necessary to make a complete resolution of the kinematics. Thus we can also consider the more sensitive effects seen in differential cross sections. In order to analyze these it is convenient to use the analysis introduced by Hand[13] for electroproduction experiments and subsequently generalized to apply to production processes initiated by neutrinos.[14] In the notation of Ref. 14 we write

$$\frac{d^3\sigma}{dE'd\omega d\Omega}=\frac{G^2k^2}{4\pi^3}\frac{E'}{E}\frac{|\mathbf{K}|}{1-\epsilon}\frac{d\sigma_w}{d\Omega} \qquad (21)$$

for the differential cross section for scattering into the muon solid angle $d\omega$ measured in the laboratory and producing an η in the angle $d\Omega$ measured in the center-of-mass frame of the final $N\eta$ system. E and E' are the neutrino and muon energies and $k_\alpha=[\mathbf{K},i(E-E')]$, all measured in the laboratory. $d\sigma_w/d\Omega$ is then proportional to the differential cross section in the center-of-mass frame for η production off a nucleon by a virtual intermediate meson (of very large mass as a meson propagator is not written into the formula) and ϵ, given by

$$\epsilon^{-1}=1+2(|\mathbf{K}|^2/k^2)\tan^2(\tfrac{1}{2}\psi), \qquad (22)$$

where ψ is the lepton angle of scattering in the laboratory, is a measure of the polarization of the virtual intermediate meson.

Now if the η is produced via the S_{11} resonance, $d\sigma_w/d\Omega$ has a very simple form. Because the resonance has $J=\frac{1}{2}$ we can write

$$(k_c/q)(d\sigma_w/d\Omega)=A+B(1-\epsilon^2)^{1/2}+(k_0^2/k^2)C\epsilon, \qquad (23)$$

where A, B, and C are functions only of k^2 and the $N\eta$ center-of-mass energy W, and do not depend either on

[7] Electric-dipole transitions would be pure isovector in the nonrelativistic quark model, for example, just as in nuclear physics. See J. S. Levinger, *Nuclear Photodisintegration* (Oxford University Press, New York, 1960).

[8] See A. T. Davies and R. G. Moorhouse [Nuovo Cimento **52**, 1112 (1967)] for a discussion of the strong parameters; see Ref. 2 for a discussion of Γ_γ.

[9] F. Bulos *et al.*, Phys. Rev. Letters **13**, 486 (1964); W. B. Richards *et al.*, *ibid.* **16**, 1221 (1966).

[10] R. Prepost, D. Lundquist, and D. Quinn, Phys. Rev. Letters **18**, 82 (1967).

[11] Assuming that $f(k^2)$ approximates to the form factors observed in nucleon electromagnetic structure.

[12] Strictly speaking, a constant Γ_η can not be defined since the resonance is so close to the $N\eta$ threshold.

[13] L. Hand, Phys. Rev. **129**, 1834 (1963).

[14] N. Dombey, Rev. Mod. Phys. (to be published).

the η center-of-mass scattering angle θ or the azimuthal angle φ relative to the lepton plane. Here the η four momentum $q_\alpha=(\mathbf{q}, iq_0)$, $k_\alpha=(\mathbf{k}_c, ik_0)$, $q=|\mathbf{q}|$, $k_c=|\mathbf{k}_c|$. A and B involve only transverse amplitudes: the electric dipole (vector) E_{0+} of strength $b(0)$ at $k^2=0$ and the magnetic dipole (axial vector) $\bar{M}_{0+}$. C involves the longitudinal-scalar monopole (axial vector) Λ_{0+} of strength $F_1(0)$ at $k^2=0$ and a longitudinal dipole (vector) L_{0+}. This approximation will be good for the same energies W for which η production by pions and photons is isotropic; that is, up to about 1570 MeV.[15]

It is well known that, in the photoproduction and especially in the electroproduction[16] of positive pions, the exchange of a π^+ plays a very important role, even at energies where direct-channel resonances are involved. This is because the pion has such low mass that the corresponding pole in the amplitude is only just outside the physical region. One way that the preceding analysis would break down would be if the weak-vector current V_α has a nonzero matrix element between π^- and η; i.e.,

$$\langle\eta|V_\alpha|\pi^-\rangle=\alpha(k^2)q_\alpha+\beta(k^2)k_\alpha. \tag{24}$$

The main theoretical interest of this term is that only if second-class vector currents[17] exist would nonzero values of $\alpha(k^2)$ and $\beta(k^2)$ be possible.[18] Up to now we have not considered whether the currents involved had second-class components; in fact, as we considered the matrix elements of V_α and A_α between different nucleonic states in Eqs. (2) and (3), each term of the right-hand side can have mixed G parity in principle.

The computations of σ_A and σ_V of Eqs. (11) and (14), however, clearly apply to both reactions A(i) and A(ii). So here we have tacitly assumed that the currents involved were first class. If second-class currents are involved, $F_1(0)$, $F_2(0)$, and $b(0)$ would be, in general, different for reactions A(i) and A(ii) and so the asymptotic total cross sections for N^* production off nucleons by neutrinos and antineutrinos would be different. But a clearer test would be to look for the pion pole in weak η production.

If this pole term is present, it may well be dominant just as it is in the electroproduction of π^+ at near-forward angles. By this mechanism

$$\frac{k_c}{q}\frac{d\sigma_w}{d\Omega}=\frac{g^2q^2}{4\pi^2W^2}\frac{|\alpha(k^2)|^2t}{(t-m_\pi^2)^2}\Big[\tfrac{1}{2}\sin^2\theta\,(1+\epsilon\cos2\varphi)$$
$$+(k_0^2/k^2)\epsilon(\lambda-\cos\theta)^2$$
$$-(k_0/k)[2\epsilon(1+\epsilon)]^{1/2}(\lambda-\cos\theta)\sin\theta\cos\varphi\Big], \tag{25}$$

where $k=(k^2)^{1/2}$, $\lambda=k_cq_0/k_0q$, and

$$t=m_\eta^2-k^2+2qk_c\cos\theta-2q_0k_0. \tag{26}$$

We have again neglected the muon mass. Note that for $k^2\approx m_\eta^2-m_\pi^2$ the pion pole is almost reached in the forward direction ($\theta=0$) even at quite low energies. So a large forward peak is expected with this model, as well as a specific φ dependence. We have not assumed here that V_α is conserved, as there is no reason why a second-class vector current should be conserved.

Even with just a few events, an analysis of the θ and φ distributions given in Eqs. (23) and (25) should distinguish between η production via the S_{11} and through a peripheral pion. Also, the dependence on the energy W is distinctive; for isobar production there should be a distinct bump at around $W=1550$ MeV[19]; in the other case, the distribution is flat and should reach much higher energies.

Finally it is possible that the dominant production is via the S_{11} and that the pion-pole contribution is small. For example, in exact $SU3$, no second-class currents coupling η and π^- are possible, so that these currents would have to arise from symmetry breaking and thus would presumably lead to small effects.[18] It could still be possible to see the pole term as an interference form at small angles if enough events were available. The most striking way that this could occur would be in the coefficient C of Eq. (23). It would now involve a small additional angle-dependent term proportional to $L_{0+}/(t-m_\pi^2)$.

The transverse amplitudes also interfere with the pion pole but these interference terms are proportional to $\sin^2\theta$ and so will be damped at forward angles. New terms proportional to $E_{0+}\epsilon\sin^2\theta\cos2\varphi/(t-m_\pi^2)$ and $\bar{M}_{0+}\epsilon\sin^2\theta\sin2\varphi/(t-m_\pi^2)$ as well as small additions to A and B are generated this way.

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[15] Actually a small admixture of higher waves is seen even at resonance in η production by pions (see Refs. 8 and 9). But by the time the experiments suggested here can be carried out, details such as these will be well understood and the appropriate modifications can be made in the analysis.

[16] C. Mistretta *et al.*, Phys. Rev. Letters **20**, 1523 (1968).

[17] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

[18] P. Singer, Phys. Rev. **139**, B483 (1965); L. B. Okun' and I. S. Tsukerman, Zh. Eksperim. i Teor. Fiz. **47**, 349 (1964) [English transl.: Soviet Phys.—JETP **20**, 232 (1965)]; F. A. Berends and P. Singer, Nuovo Cimento **46**, 90 (1966).

[19] The peak occurs for η production at a higher energy than the resonance energy indicated by the πN phase-shift analyses. This is just a threshold effect.