ward slanted segments in each interval i to $i+1$ be even. Thus one obtains, by summation,

$$n_\alpha{}^e = n_\alpha - n_\alpha{}' + 2I\,, \quad \text{(A23)}$$

which in view of (A22) is equivalent to (A21).

Once the $n_\alpha{}^e$ boost factors (A20) have been factored out of F, the remaining function F' is such that a change of sign of all factors $(v_\alpha{}^0-1)^{1/2}$ is equivalent to a change of the signs of all indices λ_α on F'. An equivalent statement is

$$(-1)^{N_i^- + N_f^-} F_{\lambda_d\lambda_c\lambda_b\lambda_a} = (-1)^{\Sigma n_\alpha{}^e} F_{-\lambda_d-\lambda_c-\lambda_b-\lambda_a}\,. \quad \text{(A24)}$$

What must be shown to prove (A24) is that the product of the three $\mathcal{C}$'s in (A16) goes into itself times $(-1)^{n^e}$ if the signs of all the λ and λ_i are reversed. The reversal of the signs of M, m, and m' takes $C_{j1/2}(J,M;m,m')$ into itself times the factor $2(J-j)=\pm1$. Thus there will be a net factor of -1 whenever $J_{\alpha i}-J_{\alpha i}{}'$ changes by a unit. Thus (A24) follows from (A21).

The distinction between $J_\alpha{}'$ and J_α is not indicated in (A4*), and the J_α's in the expression for the sign in (3.8*) should be primed. Consequently (3.6*) should read

$$F_{\Lambda_f\Lambda_i} = \eta(-1)^{N_i^- + N_f^-} F_{\Lambda_f\Lambda_i}\,, \quad \text{(A25)}$$

if we take $\eta = \eta_a\eta_b/\eta_c\eta_d$. Then $F^{\sigma_f\sigma_i}$ vanishes unless $\sigma_f = \eta\sigma_i$, if reflection invariance is maintained. The equation $p_z{}^2 = m_j{}^2$ near the end of Ref. 4 should read $p_z{}^2 = -m_j{}^2$.

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Single-Parameter Fit to Meson-Nucleon Forward Reactions*

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From the formalism combining $SU(3)$ symmetry, exchange degeneracy, and Regge poles, it is found that a single parameter can fit all the existing data in meson-nucleon reactions in the forward direction.

We present here a phenomenological study of high-energy meson-nucleon reactions in the forward direction. In this study we use the combination of $SU(3)$ symmetry and exchange degeneracy assumptions. Such a formalism was first used by Ahmadzadeh[1] in a phenomenological study of meson-nucleon and nucleon-nucleon total cross sections. In the reactions studied here, only ρ and R trajectories can be exchanged and, as we shall see, our calculation involves a single free parameter. A quite reasonable χ^2 value is obtained by fitting this one parameter to a total of 59 data points.

Assuming that only the ρ and R trajectories contribute, the various reactions can be written in the

TABLE I. Parameter and χ^2 values obtained by fitting meson-nucleon reaction data with various assumptions.

Assumptions	χ^2	α_ρ	α_R/α_ρ	$\gamma_{\rho\pi N}$	$\gamma_{R\pi N}/\gamma_{\rho\pi N}$	$\gamma_{RKN}/\gamma_{\rho KN}$	$\gamma_{\rho KN}/\gamma_{\rho\pi N}$
Perfect $SU(3)$, exchange degeneracy, α_ρ set at 0.47	88	0.47	1.0	1.56±0.02	1.0	1.0	1.0
Perfect $SU(3)$, exchange degeneracy	79	0.49	1.0	1.47	1.0	1.0	1.0
Perfect $SU(3)$, exchange degeneracy in weak form	74	0.47	0.90	1.60	1.0	1.0	1.0
Perfect exchange degeneracy	76	0.49	1.0	1.47	1.0	1.0	0.94
Perfect $SU(3)$	47	0.57	0.66	1.05	1.63	1.63	1.0
No assumptions	44	0.57	0.64	1.04	1.66	1.49	1.12

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† National Defense Education Act Title IV Graduate Fellow.
[1] A. Ahmadzadeh, Phys. Rev. Letters **16**, 952 (1966).
[2] We have used essentially the same notation as A. Ahmadzadeh and C. H. Chan, Phys. Letters **22**, 692 (1966). Note that in the forward direction our Eqs. (1)–(5) give the same sum rules as were obtained in that paper.

form[2]

$$\frac{d\sigma}{d\Omega}(\pi^- p \to \pi^0 n) = 8(\gamma_{\rho\pi\pi}\gamma_{\rho p\bar{n}})^2 |\mathcal{R}_\rho|^2 ,$$

$$\frac{d\sigma}{d\Omega}(K^- p \to \bar{K}^0 n) = 4(\gamma_{\rho K\bar{K}}\gamma_{\rho p\bar{n}})^2 |\mathcal{R}_\rho|^2 + 4(\gamma_{RK\bar{K}}\gamma_{Rp\bar{n}})^2 |\mathcal{R}_R|^2 - 4(\gamma_{\rho K\bar{K}}\gamma_{\rho p\bar{n}})(\gamma_{RK\bar{K}}\gamma_{Rp\bar{n}})(\mathcal{R}_\rho^* \mathcal{R}_R + \mathcal{R}_\rho \mathcal{R}_R^*) ,$$

$$3\frac{d\sigma}{d\Omega}(\pi^- p \to \eta n) = 8(\gamma_{R\pi\eta}\gamma_{Rp\bar{n}})^2 |\mathcal{R}_R|^2 , \quad (1)$$

$$k[\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)] = -16\pi(\gamma_{\rho\pi\pi}\gamma_{\rho p\bar{n}}) R_\rho ,$$

$$k[\sigma_t(K^- p) - \sigma_t(K^+ p) - \sigma_t(K^- n) + \sigma_t(K^+ n)] = -16\pi(\gamma_{\rho K\bar{K}}\gamma_{\rho p\bar{n}}) R_\rho ,$$

$$k[\sigma_t(K^- p) + \sigma_t(K^+ p) - \sigma_t(K^- n) - \sigma_t(K^+ n)] = -16\pi(\gamma_{RK\bar{K}}\gamma_{Rp\bar{n}}) R_R ,$$

where $d\sigma/d\Omega$ is the differential cross section at the forward direction, σ_t is the total elastic cross section, and $\gamma_{\rho p\bar{n}}$, for example, is the ρ trajectory coupling at the $p\bar{n}$ vertex. k is the center-of-mass momentum, and finally

$$\mathcal{R}_\rho = R_\rho(1 - e^{-i\pi\alpha_\rho})/\sin\pi\alpha_\rho ,$$

$$\mathcal{R}_R = R_R(1 + e^{-i\pi\alpha_R})/\sin\pi\alpha_R , \quad (2)$$

$$R_\rho = \frac{-1}{4(\pi s)^{1/2}} \frac{\Gamma(\alpha_\rho + \frac{3}{2})}{\Gamma(\alpha_\rho + 1)} \left[\frac{s - \frac{1}{2}\sum_{i=1}^{4} \mu m_i^2}{s_0}\right]^{\alpha_\rho} ,$$

with a similar expression for R_R. Here s is the invariant energy squared and s_0 is the scale factor. As is usual we take $s_0 = (1\text{ BeV})^2$. α_ρ and α_R are the trajectory functions; all the α's and γ's are evaluated at $t=0$.

Without the exchange degeneracy and $SU(3)$ assumptions,[3] Eqs. (1) depend on the six independent parameters α_ρ, α_R,

$$\gamma_{\rho\pi N} \equiv \gamma_{\rho\pi\pi}\gamma_{\rho p\bar{n}}, \quad \gamma_{R\pi N} \equiv \gamma_{R\pi\eta}\gamma_{Rp\bar{n}} ,$$
$$\gamma_{\rho KN} \equiv \gamma_{\rho K\bar{K}}\gamma_{\rho p\bar{n}}, \quad \gamma_{RKN} \equiv \gamma_{RK\bar{K}}\gamma_{Rp\bar{n}} . \quad (3)$$

These are similar to the six parameters used by Barger and Olsson[4] in a previous analysis. Now $SU(3)$ symmetry (universality[3]) imposes the constraints

$$\gamma_{\rho\pi N} = \gamma_{\rho KN}, \quad \gamma_{R\pi N} = \gamma_{RKN} , \quad (4)$$

and exchange degeneracy gives three constraints:

$$\gamma_{\rho\pi N} = \gamma_{R\pi N}, \quad \gamma_{\rho KN} = \gamma_{RKN}, \quad \alpha_\rho = \alpha_R . \quad (5)$$

Therefore we are left with only two free parameters so far. Now we use the straight-line approximation to the trajectories. Such an approximate ρ-R trajectory based on the known masses of the ρ and A_2 resonances was first given in Ref. 1. Numerous new resonances were found by Focacci *et al.*[5] Although their spins and parities are not known, their masses fit remarkably well on the straight line connecting the ρ and A_2 resonances. This line has an intercept[5,6] of 0.47. Thus we use the additional condition

$$\alpha = \alpha_\rho = \alpha_R = 0.47 , \quad (6)$$

and we are left with only one free parameter. The experimental data[7] and our theoretical fit are given in Fig. 1. In this one-parameter fit we obtained $\chi^2 = 88$ for the total of 59 data points. We consider such a χ^2 value to be quite satisfactory, especially in view of the fact that all lower-order contributions (cuts, daughter, direct channel, etc.) have been neglected.

For the sake of completeness we consider also the following cases. We relax condition (6) and obtain a two-parameter fit with $\alpha = 0.49$ and $\chi^2 = 79$. Assuming weak exchange degeneracy with $SU(3)$, namely, relaxing the condition $\alpha_\rho = \alpha_R$, we obtain $\chi^2 = 74$ with a three-parameter fit. Exchange degeneracy without $SU(3)$ gives $\chi^2 = 76$, also with three free parameters. $SU(3)$ without exchange degeneracy gives a four-parameter fit with $\chi^2 = 47$. And, finally, a six-parameter fit without $SU(3)$ and exchange degeneracy gives $\chi^2 = 44$. The parameters obtained in each case are given in Table I.

To conclude, we have obtained a one-parameter fit to all the high-energy meson-nucleon data in the for-

[3] We wish to emphasize that the present data are not sufficient to test the full $SU(3)$ symmetry of the trajectories. In the examples we have chosen, only the ρ and R trajectories appear, and so no information can be obtained about the couplings of the other members of the trajectory multiplets (K^*, ω, etc.). Furthermore, as the same nucleon vertex is common to all of the reactions, we are really testing a very weak form of $SU(3)$, namely, the universal couplings of the ρ and R trajectories to the $I=1$ current of the pseudoscalar mesons. The same remark applies in Ref. 4.

[4] V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967).

[5] M. N. Focacci, W. Kienzle, B. Levrat, B. C. Maglić, and M. Martin, Phys. Rev. Letters **17**, 890 (1966); D. Cline, Nuovo Cimento **45A**, 750 (1966); A. Ahmadzadeh, *ibid.* **46A**, 415 (1966); S. Minami, *ibid.* **46A**, 545 (1966).

[6] Richard M. Spector, Phys. Letters **25B**, 551 (1967). In this paper also arguments have been given for a first daughter of the ρ-R trajectory based on exchange degeneracy and the particle masses.

[7] (a) $(d\sigma/d\Omega)$ $(\pi^- p \to \pi^0 n)$: P. Falk-Vairant *et al.* The data are quoted in G. Hohler *et al.*, Phys. Letters **21**, 223 (1966); I. Mannelli *et al.*, Phys. Rev. Letters **14**, 408 (1965); A. V. Sterling *et al.*, *ibid.* **14**, 763 (1965). (b) $(d\sigma/d\Omega)$ $(K^- p \to \bar{K}^0 n)$: P. Astbury *et al.*, Phys. Letters **23**, 396 (1966). (c) $(d\sigma/d\Omega)$ $(\pi^- p \to \eta n)$: O. Guisan *et al.*, Phys. Letters **18**, 200 (1965). We used a branching ratio of 0.33. (d) $\sigma_t(\pi^- p) - \sigma_t(\pi^+ p)$: W. Galbraith *et al.*, Phys. Rev. **138**, B913 (1965); K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967). Note that the data used here are slightly different from those used by Barger and Olsson in Ref. 4. There the data of Foley *et al.* were not available. Furthermore, the authors in Ref. 4 claimed a systematic error in the Galbraith data which they used [W. Galbraith *et al.*, in *Proceedings of the Second Topical Conference on Resonant Particles, Athens, Ohio*, 1965, edited by B. A. Munir (Ohio University, Athens, Ohio, 1965), p. 522]. To avoid such possible systematic errors, we have used the Galbraith data from the *Physical Review* instead. The two sets of data used here are consistent with each other. (e) $\sigma_t(K^- p) - \sigma_t(K^+ p) - \sigma_t(K^- n) + \sigma_t(K^+ n)$: W. Galbraith *et al.*, Phys. Rev. **138**, 913 B(1965). (f) $\sigma_t(K^- p) + \sigma_t(K^+ p) - \sigma_t(K^- n) - \sigma_t(K^+ n)$: W. Galbraith *et al.*, Phys. Rev. **138**, B913 (9165).

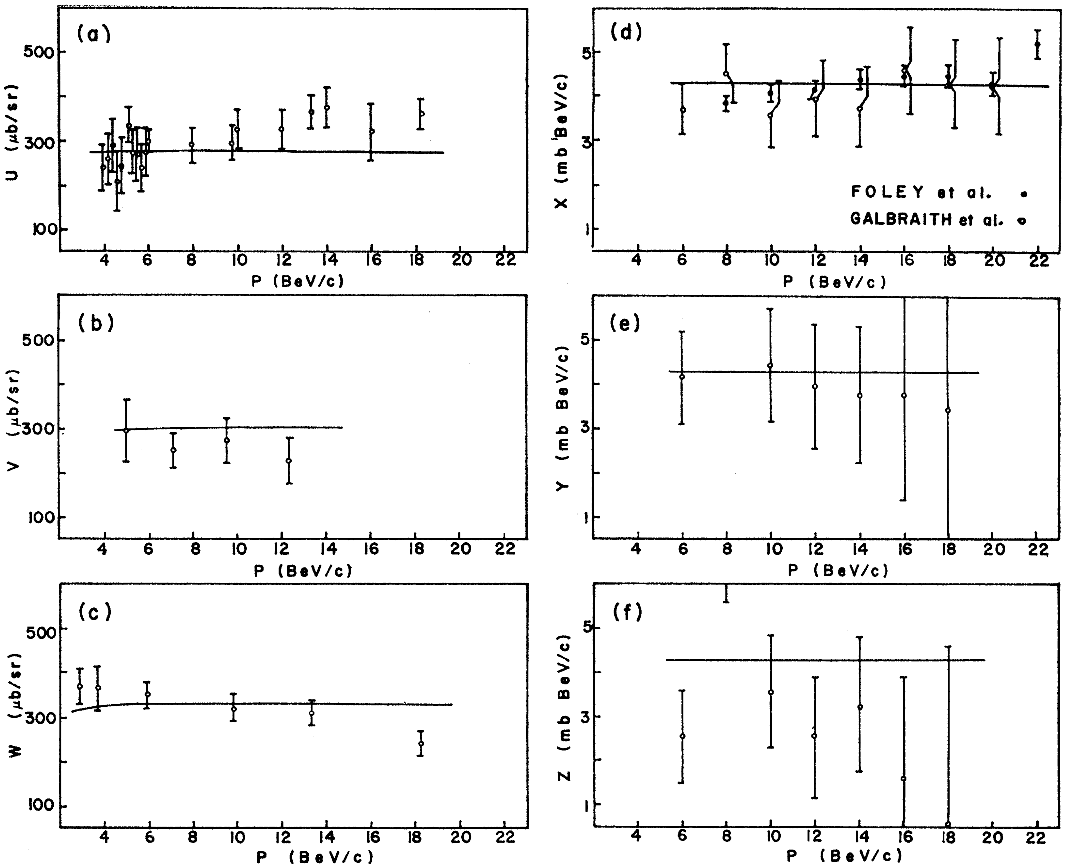

FIG. 1. Curves for our one-parameter fit to high-energy meson-nucleon reaction data ($\alpha_\rho=0.47$, $\chi^2=88$) versus the laboratory momentum P. (a) $U\equiv(d\sigma/d\Omega)\ (\pi^-p\to\pi^0n)$, (b) $V\equiv(d\sigma/d\Omega)\ (K^-p\to\bar{K}^0n)$, (c) $W\equiv3(d\sigma/d\Omega)\ (\pi^-p\to\eta n)$, all at 0°; and (d) $X\equiv k[\sigma_t(\pi^-p)-\sigma_t(\pi^+p)]$, (e) $Y\equiv k[\sigma_t(K^-p)-\sigma_t(K^+p)-\sigma_t(K^-n)+\sigma_t(K^+n)]$, and (f) $Z\equiv k[\sigma_t(K^-p)+\sigma_t(K^+p)-\sigma_t(K^-n)-\sigma_t(K^+n)]$.

ward direction. This result shows that the experimental data are in good agreement with the hypothesis of exchange degeneracy of the ρ and R trajectories and their couplings, combined with the assumption of universal coupling of these trajectories to the $I=1$ current of pseudoscalar mesons. Our result also agrees very well with the ρ-R trajectory intercept obtained from the known masses of the resonances.

The combination of $SU(3)$ and exchange degeneracy, sometimes combined with various other assumptions, has been used by numerous authors to obtain sum rules.[1,2,8] The extent of agreement of these sum rules with experiment depends on the additional assumptions made. The idea of using exchange degeneracy to predict daughter trajectories has also been used by some authors.[6,9] Recently, exchange degeneracy and $SU(3)$ have been utilized by Mandelstam[10] in a new bootstrap dynamical model involving superconvergence relations.

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[8] See, for example, A. Ahmadzadeh, Phys. Letters **22**, 96 (1966); **22**, 669 (1966); C. A. Levinson, N. S. Wall, and H. J. Lipkin, Phys. Rev. Letters **17**, 1122 (1966); Jong-Ping Hsu and S. Okubo, Phys. Letters **24B**, 179 (1967); R. C. Arnold, Phys. Rev. **153**, 1506 (1967); A. Ahmadzadeh and R. J. Jacob, *ibid.* **160**, 1359 (1967); Yuval Ne'eman and John D. Reichert, Phys. Rev. Letters **18**, 1226 (1967); A. Borgese, F. Buccella, M. Colocci, and E. Celeghini, Nuovo Cimento **49**, 199 (1967).

[9] L. Sertorio and M. Toller, Phys. Rev. Letters **19**, 1146 (1967).

[10] Stanley Mandelstam, Phys. Rev. **166**, 1539 (1968); also invited paper, American Physical Society Winter Meeting, Pasadena, 1967 (unpublished).