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Derivation of Intermultiplet Mass Formulas from the Chiral $SU(3)\otimes SU(3)$ Charge Algebra

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We have derived intermultiplet mass formulas by using an $SU(3)\otimes SU(3)$ charge algebra (involving time derivatives) and an approximation [called the $SU(3)$ approximation] previously proposed. The approximation assumes that the $SU(3)$ charge operator V_K [defined, for example, in a quark model, by $V_{K^-}=\int d^3x\, \bar{q}(x)\gamma_4(\lambda_4-i\lambda_5)q(x)/2$] acts as an $SU(3)$ generator even in broken symmetry in an appropriately chosen infinite-momentum limit. We have obtained, besides the Gell-Mann–Okubo mass formulas including first-order mixing, intermultiplet mass formulas such as (i) $m_{K^{*0}}{}^2-m_{\rho^0}{}^2=m_{K^+}{}^2-m_{\pi^+}{}^2=m_{K^{**+}}{}^2-m_{A_2^+}{}^2$, (ii) $m_{\rho^-}\simeq m_{\omega^0}$, (iii) f-f' mixing angle $\theta\simeq 30°$ from 2^+ and 0^- mass formulas, and (iv) $SU(6)$-type decuplet-octet baryon quadratic mass formulas. A brief discussion is presented about the $J^{PC}=1^{++}$ and 1^{+-} meson mass formulas.

I. INTRODUCTION

RECENTLY, we have proposed an approximation[1,2] [called the $SU(3)$ approximation] that seems useful in discussing broken $SU(3)$ symmetry. In this paper, we wish to show that, combined with the use of a chiral $SU(3)\otimes SU(3)$ charge algebra, this approximation is also able to yield intermultiplet mass formulas (in addition to the Gell-Mann–Okubo formulas) that agree with experiment rather well.

II. $SU(3)$ APPROXIMATION

Our approximation concerns the matrix elements of the operator V_K, which is an $SU(3)$ generator in the symmetry limit. Let us illustrate our approximation with a simple example. Consider a diagonal matrix element of the operator V_K [for example, we denote

$$V_{K^-}=-i\int d^3x\, V_4{}^{K^-}(x)$$
$$=\int d^3x\, \bar{q}(x)\gamma_4(\lambda_4-i\lambda_5)q(x)/2$$

in a quark model] responsible for the K_{l3} decay,

$$\langle\pi^0(\mathbf{p}')|V_\mu{}^{K^-}(0)|K^+(\mathbf{p})\rangle=(4p_0p_0')^{1/2}(-1/\sqrt{2})\times[F_+(q^2)(p+p')_\mu+F_-(q^2)(p-p')_\mu], \quad (1)$$

where $q=p-p'$. The form factors of the diagonal matrix elements of V_K, which are multiplied by *nonvanishing* [in the $SU(3)$ limit] kinematical factors, are called the $SU(3)$ form factors of $V_\mu{}^K(x)$. In the $SU(3)$ limit, where q^2 is necessarily zero, the $SU(3)$ form factor $F_+(0)$ takes the $SU(3)$ value of 1, whereas $F_-(0)=0$. In the real world both $F_+(q^2)$ and $F_-(q^2)$ are renormalized. However, according to the Ademollo-Gatto theorem,[3] at zero momentum transfer ($q^2=0$) $F_+(0)$ is renormalized only in second order of the symmetry-breaking interaction (whose strength will be symbolically denoted by ϵ in the following), whereas $F_-(0)$ receives a renormalization in the lowest order, i.e., $F_+(0)\simeq 1+O(\epsilon^2)$ and $F_-(0)\simeq O'(\epsilon)$. In order to explain our $SU(3)$ approximation and to introduce a necessary modification of Ademollo-Gatto theorem when particle mixing takes place, we recapitulate a proof of this theorem in a simple example along the lines first discussed by Fubini and co-workers.[4] Consider a commutator $[V_{K^+},V_{K^-}]=V_{\pi^0}+\sqrt{3}V_{\eta^0}$ sandwiched between the states $\langle\pi^-(\mathbf{q})|$ and $|\pi^-(\mathbf{q})\rangle$ with $|\mathbf{q}|=\infty$. We insert a complete set of intermediate states between the factors V_{K^+} and V_{K^-} and extract from them the K^0 state. For the form factors of $\langle\pi^-|V_{K^-}|K^0\rangle$ and $\langle K^0|V_{K^+}|\pi^-\rangle$ we use the expression given by (1) modified by $SU(2)$ symmetry. By noting that the contribution of the F_- form factor to the term $\langle\pi^-|V_{K^-}|K^0\rangle\times\langle K^0|V_{K^+}|\pi^-\rangle$ vanishes in the limit $|\mathbf{q}|=\infty$, we then obtain $F_+(0)=1+O(\epsilon^2)$, where $O(\epsilon^2)$ is proportional to the expression

$$\textstyle\sum_b \langle\pi^-|V_{K^+}|b\rangle\langle b|V_{K^-}|\pi^-\rangle - \sum_c \langle\pi^-|V_{K^-}|c\rangle\langle c|V_{K^+}|\pi^-\rangle$$

(where b and c are the appropriate intermediate states other than the K^0 state). The $O(\epsilon^2)$ term is clearly of

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[1] S. Matsuda and S. Oneda, Phys. Rev. **158**, 1594 (1967); **171**, 1743 (1968).
[2] S. Matsuda and S. Oneda, University of Maryland Technical Report No. 785, 1968 (unpublished); Bull. Am. Phys. Soc. **13**, 122 (1968); S. Matsuda, thesis and University of Maryland Technical Report No. 768, 1967 (unpublished).
[3] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).
[4] S. Fubini and G. Furlan, Physics (N. Y.) **1**, 229 (1965); S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965); G. Furlan, F. Lannoy, C. Rossetti, and G. Segrè, *ibid.* **40**, 597 (1965).

the order ϵ^2, since V_K is an $SU(3)$ generator in the symmetry limit. This is an example of the Ademollo-Gatto theorem. We emphasize that this theorem holds only for the $SU(3)$ form factors of V_K in the zero-momentum-transfer limit. Therefore the renormalization of the $SU(3)$ form factor of V_K seems to be minimum at zero momentum transfer. Our approximation assumes that the renormalization of the $SU(3)$ form factors of V_K at zero momentum transfer [which is at least of the order $O(\epsilon^2)$] is small and negligible compared with other symmetry-breaking effects (which appear in the masses and the physical coupling constants) that we do take into account.

We have shown,[1,2] and would like to demonstrate further, that broken $SU(3)$ symmetry, particularly as manifested by the mass splittings of $SU(3)$ multiplets, does not seem to contradict having an essentially zero renormalization of the $SU(3)$ form factors of the vector-current $V_\mu{}^K(x)$ at zero momentum transfer. In Eq. (1), by letting $|\mathbf{p}|=|\mathbf{p}'|=\infty$, we can always deal with the form factors at $q^2=0$, even if $m_K\neq m_\pi$. Therefore, in the actual computation of the diagonal matrix element of V_K, $\langle B|V_K|A\rangle$, we always take an appropriate infinite-momentum limit for the particles appearing in the states A and B in such a way that we deal with the $SU(3)$ form factors of V_K only at zero momentum transfer. Our approximation, therefore, *does not* impose any restriction on the form factors [such as $F_-(q^2)$] multiplied by kinematical factors that vanish in this infinite-momentum limit. (We call this type of form factor an *unrestricted form factor*.) We can therefore still discuss the F_- form factors in our approach.[5] For form factors other than the unrestricted form factors, our approximation uses the $SU(3)$ value for the $SU(3)$ form factor (at zero momentum transfer) and neglects other form factors[6] whose contribution is known to be at least of order ϵ^2. Without contradicting the neglect of the $O(\epsilon^2)$ terms in the diagonal elements of V_K, we neglect the nondiagonal elements of V_K, $\langle b|V_K|A\rangle$, in the same infinite-momentum limit. In the $SU(3)$ limit, $\langle b|V_K|A\rangle=0$. This implies the existence of a relation between the form factors of $\langle b|V_\mu{}^K(x)|A\rangle$ that guarantees $\langle b|V_K|A\rangle=0$ for *any* momentum transfer. In broken $SU(3)$ symmetry, our approximation requires only that the renormalization of this relation at the momentum transfer (usually at $q^2=0$) corresponding to our infinite-momentum limit is small, i.e., $\langle b|V_K|A\rangle$ is still 0 in that limit (effectively, say, to order ϵ), although $\langle b|V_K|A\rangle$ is formally of order ϵ. In summary, we neglect all the nondiagonal elements $\langle b|V_K|A\rangle$ of the vector charge V_K (except for cases when there is a mixing problem for the states under consideration, which will be discussed below) only in the infinite-momentum limit; which, in turn, enables us (effectively to order ϵ) to keep only the $SU(3)$ form factor with the $SU(3)$ value (at zero momentum transfer) for the diagonal elements of V_K.

We now consider the cases where particle mixing takes place. This is the first-order symmetry-breaking effect, which must be taken into account. We express the physical states in terms of $SU(3)$ states (with which the physical states will coincide in the limit $\epsilon\to 0$) to order ϵ whenever these states come into the matrix elements of V_K. We now show that the modified $SU(3)$ form factor of V_K, defined after extracting the first-order mixing effect, also satisfies the Ademollo-Gatto theorem at the zero-momentum-transfer limit. We illustrate this procedure by considering the matrix element $\langle\eta^0(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle$ with $|\mathbf{q}|=\infty$ and the effect of η-X^0 mixing. Let us write, to order ϵ, $\eta=\cos\theta\,\eta_8-\sin\theta\,\eta_1$ and $X=\cos\theta\,\eta_1+\sin\theta\,\eta_8$, where $\eta\to\eta_8$ and $X\to\eta_1$ when $\epsilon\to 0$. Consider now the equation

$$\begin{aligned}\lim_{|\mathbf{q}|\to\infty}&\langle K^+(\mathbf{q}')|[V_{K^+},V_{K^-}]|K^+(\mathbf{q})\rangle\\&=\lim_{|\mathbf{q}|\to\infty}\langle K^+(\mathbf{q}')|V_{\pi^0}+\sqrt{3}V_{\eta^0}|K^+(\mathbf{q})\rangle\\&=2(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')|_{|\mathbf{q}|=\infty}.\end{aligned}\tag{2}$$

On the left-hand side of Eq. (2), we extract, from the complete set of intermediate states sandwiched bebeween the factors V_{K^+} and V_{K^-}, the X^0 state as well as the π^0 and η^0 states. The contribution of all other states is again of order ϵ^2. The π^0 intermediate state gives a contribution $\frac{1}{2}(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')$ to order ϵ from the Ademollo-Gatto theorem. In the limit $|\mathbf{q}|\to\infty$, we need consider only the $SU(3)$ form factors of $\langle\eta^0|V_K|K\rangle$ and $\langle X^0|V_K|K\rangle$. According to our $SU(3)$ approximation, we have[7]

$$\begin{aligned}\lim_{|\mathbf{q}|\to\infty}&\langle\eta^0(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle\\&=\lim_{|\mathbf{q}|\to\infty}\cos\theta\,\langle\eta_8(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle\\&=(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')\cos\theta\,G_+(0)|_{|\mathbf{q}|=\infty}\end{aligned}$$

and

$$\begin{aligned}\lim_{|\mathbf{q}|\to\infty}&\langle X^0(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle\\&=\lim_{|\mathbf{q}|\to\infty}\sin\theta\,\langle\eta_8(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle\\&=(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')\sin\theta\,G_+(0)|_{|\mathbf{q}|=\infty}.\end{aligned}$$

[5] Combined with the use of unsubtracted dispersion relations for F's and the spectral-function-like sum rules (Ref. 1) (to fix the scale) obtained by using our $SU(3)$ approximation, the K_{l3}-decay form factors have been studied. We realize that our expression for the $F_+(q^2)+F_-(q^2)$ evaluated at $q^2=-m_K{}^2$ coincides with the soft-pion result, which holds only at $q^2=-m_K{}^2$. S. Matsuda and S. Oneda, Phys. Rev. **169**, 1172 (1968).

[6] Sometimes there appear form factors multiplied by kinematical factors that do not vanish in the infinite-momentum limit, but do vanish in the $SU(3)$ limit. (For example, consider $\langle\rho|V_K|K^*\rangle$.) These may be called "restricted form factors." Thus, in general, the form factors of the diagonal matrix elements of V_K, $\langle B|V_K|A\rangle$, can be divided into three parts: $SU(3)$, restricted, and unrestricted form factors. Consider the equation $\langle A(\mathbf{q})|[V_{K^+},V_{K^-}]|A(\mathbf{q})\rangle=\langle A(\mathbf{q})|V_{\pi^0}+\sqrt{3}V_{\eta^0}|A(\mathbf{q})\rangle$, with $|\mathbf{q}|=\infty$. The unrestricted form factors will not contribute in the limit $|\mathbf{q}|=\infty$. In a way similar to the discussion of Eq. (2), we obtain from the above equation $\lim_{|\mathbf{q}|\to\infty}\langle B(\mathbf{q}')|V_K|A(\mathbf{q})\rangle=[SU(3)\text{ form factor}]+[\text{restricted form factor}]|_{|\mathbf{q}|=\infty}=(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')\alpha[1+O(\epsilon^2)]$, where α is the $SU(3)$ value of $\langle B|V_K|A\rangle$. Therefore we see from the definition of the restricted form factor that it is a quantity of order ϵ^2.

[7] In the limit $|\mathbf{q}|=\infty$ the mass of the η_8 does not enter in the expression. As in the case with the nondiagonal element $\langle b|V_K|A\rangle$, we set $\langle\eta_1(\mathbf{q}')|V_{K^-}|K^+(\mathbf{q})\rangle=0$ for $|\mathbf{q}|=\infty$.

By inserting these expressions in Eq. (2) we obtain

$$G_+(0)=[SU(3)\text{ value}]+O'(\epsilon^2),$$

the $SU(3)$ value being given by $-\sqrt{\frac{3}{2}}$. Therefore, if we extract the mixing effect to order ϵ, the situation becomes similar to the cases without the possibility of mixing discussed previously. Thus, if the possibility of mixing arises, we write the physical states in terms of the $SU(3)$ states (to order ϵ).

If we make this modification, we can summarize our $SU(3)$ approximation in the following way: The operator V_K acts as an $SU(3)$ generator even in broken symmetry in an appropriately chosen infinite-momentum limit.

A direct and rather severe test of our approximation will be provided by measuring the value of $F_+(0)$. Present K_{e3} decay experiments indicate that $F_+(0)=1$ within 2–5%.[8] In view of the large mass difference involved, the effect of symmetry breaking is expected to be more appreciable in the vertex $\langle\pi|V_K|K\rangle$ than in those involving higher-lying multiplets such as $\langle\rho|V_K|K^*\rangle$ and $\langle\Lambda|V_{\bar{K}}|p\rangle$. Therefore the above estimate of $F_+(0)$ based on present experiments seems consistent with our approximation. One may, however, get the impression that the approximation could be too radical to explain the observed broken symmetry. This is certainly a valid question. Our attitude is that one should try and see. We emphasize, however, that we are making this approximation *only* for the $SU(3)$ form factors of the matrix elements of V_K *only* at the zero-momentum-transfer limit. We do not make any assumption at finite momentum transfer. Our conjecture is that $SU(3)$ is broken in such a way that we may still treat the operator V_K as an $SU(3)$ generator in our limit. It is interesting to see to what degree of accuracy broken $SU(3)$ symmetry permits us to make this simplifying approximation.

We now illustrate some of the features of this approximation. Consider a charge commutator $-A_{K^+}=[V_{K^0},A_{\pi^+}]$ inserted, for example, between the $\langle p(\mathbf{q})|$ and $|\Lambda^0(\mathbf{q})\rangle$ states with $|\mathbf{q}|=\infty$. We then use the $SU(3)$ approximation for V_{K^0}. We then obtain a sum rule such as

$$g_{p\Lambda}=g_{\Sigma\Lambda}-(\sqrt{\tfrac{3}{2}})g_{pn}. \tag{3}$$

Here $g_{p\Lambda}$ is, for example, the axial-vector coupling constant (at zero momentum transfer) of the $\Lambda^0\to p+e^-+\bar{\nu}$ decay. In this way we obtain a set of sum rules[1] that, in fact, coincide with those obtained by eliminating the D/F ratio in Cabibbo's original exact $SU(3)$ analysis of semileptonic decays.[9] This indicates that the exact $SU(3)$ sum rules can still be preserved, to a good accuracy, even with broken symmetry to the extent that our $SU(3)$ approximation is justified. (We have assumed here that there are no other states that can mix with the N octet.) We found, however, that this is a rather exceptional case, and we usually obtain with our $SU(3)$ approximation results that manifest more explicitly the effect of symmetry breaking. For example, let us consider the commutator $[V_{K^0},A_{\pi^-}]=0$ taken between the $\langle n(\mathbf{q})|$ and $|\Sigma^+(\mathbf{q})\rangle$ states with $|\mathbf{q}|=\infty$. Using the $SU(3)$ approximation for V_{K^0} and partially conserved axial-vector current hypothesis (PCAC) for A_{π^-}, we obtain a sum rule

$$(\sqrt{\tfrac{1}{2}})g_{\Sigma^0\Sigma^+\pi^-}(1/2m_\Sigma)-(\sqrt{\tfrac{3}{2}})g_{\Sigma^+\Lambda\pi^-}(1/(m_\Sigma+m_\Lambda))+g_{pn\pi^-}(1/2m_p)=0. \tag{4}$$

Here the coupling constants g are defined with a pion off the mass shell ($m_\pi\to 0$). In contrast, the pure-$SU(3)$ sum rule is given by

$$(\sqrt{\tfrac{1}{2}})g_{\Sigma^0\Sigma^+\pi^-}-(\sqrt{\tfrac{3}{2}})g_{\Sigma^+\Lambda\pi^-}+g_{pn\pi^-}=0.$$

Thus the effect of symmetry breaking appears as a factor involving the masses of the relevant particles.[10] In this paper, we shall also rederive some of the other examples that we have derived previously[1,2] by using a somewhat more complicated procedure.

III. MODEL OF SYMMETRY BREAKING

We usually assume that the $SU(3)$ symmetry-breaking interaction transforms in the same way as the $I=Y=0$ member of the $SU(3)$ octet. If this is the case, we shall have a commutation relation[11] such as $[\dot{V}_{K^0},V_{K^0}]=0$. Insert this between the $\langle K^0(\mathbf{q})|$ and $|\bar{K}^0(\mathbf{q})\rangle$ states with $|\mathbf{q}|=\infty$. In this limit, because of our $SU(3)$ approximation, we need only to consider the intermediate states π^0, η^0, and X^0. We note that, according to the procedure described in Sec. II, for example,

$$\lim_{|\mathbf{q}|\to\infty}\langle\eta(\mathbf{q}')|V_{K^0}|\bar{K}^0(\mathbf{q})\rangle=(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')(\sqrt{\tfrac{3}{2}})\cos\theta$$

[8] Assuming a form of $f_+(q^2)$, $f_+(q^2)=m_{K^*}^2/(m_{K^*}^2+q^2)$, which is in fact realized in the approach of Ref. 5 and is roughly in agreement with experiment. N. Brene, M. Roos, and A. Sirlin [Nucl. Phys. **B6**, 255 (1968)] obtained, from the present $\Gamma(K_{e3}^+)$ and $\Gamma(K_{2e3}^0)$, $\sin\theta_V=0.220\pm0.003$ and 0.201 ± 0.004, respectively, by assuming $f_+(0)=1$. Comparison of the μ decay and O^{14} β decay gives $\sin\theta_V=0.2095\pm0.0086$. Recently, using the Lagrangian approach, together with the sum rules assured by field algebra, S. L. Glashow and S. Weinberg [Phys. Rev. Letters **20**, 224 (1968)] obtained $f_+(0)\simeq0.85$. In contrast, P. K. Mitter and L. J. Swank [University of Maryland Technical Report No. 829 1968 (unpublished)] have recently found that $f_+(0)\cong1$ by using a similar approach.

[9] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[10] See Refs. 1 and 2. If we use our $SU(3)$ approximation for the charge-charge density commutators taken between a vacuum and an appropriate state, then we obtain sum rules similar to the spectral-function sum rules given, for example, by S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 137 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967). We do derive in our approach the first spectral-function sum rules, but we are not led to the problematical second sum rules.

[11] This type of commutation relation was first utilized by Fubini *et al.* to derive the Gell-Mann–Okubo mass formulas. See Ref. 4.

and

$$\lim_{|\mathbf{q}|\to\infty} \langle\eta(\mathbf{q}')|\dot{V}_{K^0}|\bar{K}^0(\mathbf{q})\rangle = (2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')(\sqrt{\tfrac{3}{2}})\cos\theta\frac{m_\eta^2-m_K^2}{(2q_0 2q_0')^{1/2}}.$$

By inserting these expressions, we obtain the sum rule

$$\sin^2\theta=\frac{3m_\eta^2-4m_K^2+m_\pi^2}{3(m_X^2-m_\eta^2)}. \qquad (5)$$

This is a derivation[12] from our approach of the Gell-Mann–Okubo mass formula including first-order mixing. The extension to other $SU(3)$ multiplets is obvious (see Ref. 2). We note that the spirit of our approximation is not a mere pole approximation in the usual dispersion approach. The infinite-momentum limit is also required by the approximation adopted. We now wish to extend this approach to include also the axial-vector charge. In the following, our arguments are based on the validity of the commutation relations

$$[\dot{V}_{K^0},A_{\pi^-}]=[\dot{V}_{K^{+,0}},A_{K^{+,0}}]=0. \qquad (6)$$

In a quark model, for example, these commutation relations will certainly hold if the symmetry breaking is given, for example, by

$$H'\propto \bar{q}(x)\lambda_8 q(x)$$

or

$$H'\propto \bar{q}(x)\gamma_\mu\lambda_8 q(x)V_\mu{}^0(x)$$

(which may be regarded, for instance, as Ne'eman's fifth interaction[13]), where $V_\mu{}^0(x)$ is a unitary singlet vector meson that breaks $SU(3)$ symmetry. We wish to show that the combined use of the $SU(3)$ approximation and these commutators leads to intermultiplet mass formulas.

IV. DERIVATION OF INTERMULTIPLET MASS FORMULAS

(A) Consider the following matrix elements of $[\dot{V}_{K^0}(V_{K^0}),A_{\pi^-}]=0$, taken between the $K^{*0}(\mathbf{q})$ and $\pi^+(\mathbf{q})$ states, with $|\mathbf{q}|=\infty$, and use our $SU(3)$ approximation:

$$\langle K^{*0}(\mathbf{q})|\dot{V}_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|\pi^+(\mathbf{q})\rangle-\langle K^{*0}(\mathbf{q})|A_{\pi^-}|K^+\rangle \times\langle K^+|\dot{V}_{K^0}|\pi^+(\mathbf{q})\rangle=0, \qquad (7)$$

$$\langle K^{*0}(\mathbf{q})|V_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|\pi^+(\mathbf{q})\rangle-\langle K^{*0}(\mathbf{q})|A_{\pi^-}|K^+\rangle \times\langle K^+|V_{K^0}|\pi^+(\mathbf{q})\rangle=0. \qquad (8)$$

Equation (7) implies that, at $|\mathbf{q}|=\infty$,

$$[E_\rho(\mathbf{q})-E_{K^*}(\mathbf{q})]\langle K^*(\mathbf{q})|V_{K^0}|\rho\rangle\langle\rho|A_{\pi^-}|\pi(\mathbf{q})\rangle -[E_\pi(\mathbf{q})-E_K(\mathbf{q})]\langle K^*(\mathbf{q})|A_{\pi^-}|K\rangle\langle K|V_{K^0}|\pi(\mathbf{q})\rangle=0.$$

Then, compared with Eq. (8), we obtain

$$\lim_{|\mathbf{q}|\to\infty}\frac{E_\rho(\mathbf{q})-E_{K^*}(\mathbf{q})}{E_\pi(\mathbf{q})-E_K(\mathbf{q})}=1.$$

This gives $m_{K^{*0}}{}^2-m_{\rho^0}{}^2=m_{K^+}{}^2-m_{\pi^+}{}^2$. This has been obtained by assuming $SU(6)$-symmetry theory.[14] The present experimental compilation[15] indicates $0.20\pm0.11=0.23$, in GeV,[2] for this relation. If we use the PCAC hypothesis for A_π in Eq. (8), we obtain the broken $SU(3)$ relation

$$\frac{2G_{K^{*+}K^-\pi^0}(m_{K^*}{}^2,\, m_K{}^2,\, m_\pi{}^2=0)}{G_{\rho^+\pi^-\pi^0}(m_\rho{}^2,\, m_\pi{}^2,\, m_\pi{}^2=0)} =\frac{m_\rho{}^2+m_{K^*}{}^2}{2m_\rho{}^2}\simeq\frac{m_{K^*}}{m_\rho}. \qquad (9)$$

In the $SU(3)$ limit, this ratio is unity. This relation has been obtained previously by the present authors in a slightly different way,[1] and agrees with experiments if the ρ width is around 130 MeV.

(B) By replacing the π^+ and K^+ mesons in (7) and (8) with the 2^+ mesons, i.e., $A_2{}^+$ $(I=1)$ and $K^{**}(1400)$ $(I=\frac{1}{2})$ mesons, respectively, we obtain

$$\langle K^{*0}(\mathbf{q})|\dot{V}_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|A_2{}^+(\mathbf{q})\rangle-\langle K^{*0}(\mathbf{q})|A_{\pi^-}|K^{**+}\rangle \times\langle K^{**+}|\dot{V}_{K^0}|A_2{}^+(\mathbf{q})\rangle=0,$$

$$\langle K^{*0}(\mathbf{q})|V_{K^0}|\rho^0\rangle\langle\rho^0|A_{\pi^-}|A_2{}^+(\mathbf{q})\rangle-\langle K^{*0}(\mathbf{q})|A_{\pi^-}|K^{**+}\rangle \times\langle K^{**+}|V_{K^0}|A_2{}^+(\mathbf{q})\rangle=0.$$

With $|\mathbf{q}|=\infty$, we also obtain a similar relation,

$$m_{K^{*0}}{}^2-m_{\rho^0}{}^2=m_{K^{**+}}{}^2-m_{A_2{}^+}{}^2.$$

Corresponding experimental numbers give $0.20\pm0.11=0.29\pm0.17$, in GeV.[2] This seems encouraging. If we use PCAC for A_π in the latter of the above two equations, we obtain, instead of the $SU(3)$ value of 1,

$$\frac{G_{A_2{}^+\rho^-\pi^0}(m_{A_2}{}^2,\, m_\rho{}^2,\, m_\pi{}^2=0)}{2G_{K^{**+}K^{*-}\pi^0}(m_{K^{**}}{}^2,\, m_{K^*}{}^2,\, m_\pi{}^2=0)}=\frac{m_{A_2}{}^2-m_\rho{}^2}{m_{K^{**}}{}^2-m_{K^*}{}^2}. \qquad (10)$$

This sum rule has also been obtained previously by the present authors[2] in a slightly more complicated way, and can be tested by experiments. We now consider the cases where the mixing effect (ω-ϕ and f-f') is known to exist.

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[12] K. Nishijima and J. Swank [Phys. Rev. **146**, 1161 (1966)] also discussed the consequences of this commutator. The difference of our approach from these works (Refs. 11 and 12) is that we use the $SU(3)$ approximation (instead of the pole approximation), which requires us to take an infinite-momentum limit. Nishijima also obtained some of the $SU(6)$-type mass formulas in his approach by using commutators similar to our Eq. (6) [lecture notes at Tokyo Summer School, 1967 (unpublished)]. We thank Dr. L. J. Swank for informative discussions.

[13] Y. Ne'eman, Phys. Rev. **134**, B1355 (1964).

[14] B. Sakita, Phys. Rev. **136**, B1756 (1964); F. Gürsey and L. Radicati, Phys. Rev. Letters **13**, 173 (1964). For complete references, see A. Pais, Rev. Mod. Phys. **38**, 215 (1966). For the quark-model predictions, see, for example, R. J. Dalitz, in *Elementary Particle Physics* (W. A. Benjamin, Inc., New York, 1966), p. 56. Complete references will be found there.

[15] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

(C) Consider the similar equations,

$$\langle\rho^-(\mathbf{q})|\dot{V}_{K^0}(V_{K^0})|K^{*-}\rangle\langle K^{*-}|A_{\pi^-}|\bar{K}^{*0}(\mathbf{q})\rangle - \langle\rho^-(\mathbf{q})|A_{\pi^-}|\omega^0\rangle\langle\omega^0|\dot{V}_{K^0}(V_{K^0})|\bar{K}^{*0}(\mathbf{q})\rangle - \langle\rho^-(\mathbf{q})|A_{\pi^-}|\phi^0\rangle\langle\phi^0|\dot{V}_{K^0}(V_{K^0})|\bar{K}^{*0}(\mathbf{q})\rangle=0,$$

with $|\mathbf{q}|=\infty$. If we use PCAC for A_π, the $\langle\rho^-(\mathbf{q})|A_{\pi^-}|\omega^0(\mathbf{q})\rangle$ and $\langle\rho^-(\mathbf{q})|A_{\pi^-}|\phi(\mathbf{q})\rangle$ at $|\mathbf{q}|=\infty$ become proportional to the $\omega^0\to\rho^-+\pi^+$ and $\phi^0\to\rho^-+\pi^+$ couplings $G_{\omega\rho^+\pi^-}$ and $G_{\phi\rho^+\pi^-}$, respectively, which are defined with a pion off the mass shell ($m_\pi\to 0$). Experimentally, $G_{\omega\rho^+\pi^-}\gg G_{\phi\rho^+\pi^-}$. Therefore let us first neglect the contribution of the ϕ terms. We note that the $\phi\to\rho+\pi$ decay is, in fact, forbidden in some models. We then obtain $m_{K^{*-}}{}^2-m_{\rho^-}{}^2=m_{K^{*0}}{}^2-m_{\omega^0}{}^2$, which implies

$$m_{\rho^-}{}^2\simeq m_{\omega^0}{}^2. \qquad (11)$$

This is indeed very close to experiment[15] ($0.599\pm0.099=0.614\pm0.009$ in GeV^2) and seems to be a very encouraging result of our approach.[16] If we include the ϕ term, we can use the above two equations to obtain an estimate of $\Gamma(\phi\to\rho+\pi)$. Write, to order ϵ, $\omega=\omega_1\cos\theta+\omega_8\sin\theta$ and $\phi=-\omega_1\sin\theta+\omega_8\cos\theta$, where $\omega\to\omega_1$ and $\phi\to\omega_8$ in the limit $\epsilon\to 0$. By using PCAC for A_π and our $SU(3)$ approximation for V_K, we obtain from the above two equations

$$\frac{G_{\phi\rho^+\pi^-}}{G_{\omega\rho^+\pi^-}}=\frac{m_{\rho^-}{}^2-m_\omega{}^2}{m_\phi{}^2-m_\omega{}^2}\tan\theta. \qquad (12)$$

If we take $\theta=40°$ from the vector-meson mass formula, this ratio $\simeq 0.03$. Thus $\Gamma(\phi\to\rho+\pi)$ is small, although the question of how small it is is rather sensitive to the difference $m_{\rho^-}-m_{\omega^0}$. Therefore we have shown from our approach that the smallness (or forbiddenness) of the rate $\Gamma(\phi\to\rho+\pi)$ is intimately connected with the fact that $m_\rho\simeq m_\omega$. Namely, the smallness of $\Gamma(\phi\to\rho+\pi)$ implies $m_\rho\simeq m_\omega$, and vice versa, in our $SU(3)$ approximation and our model of $SU(3)$ breaking manifested by the commutation relations (6).

(D) We next discuss f-f' mixing. Consider the equation (with $|\mathbf{q}|=\infty$)

$$\langle K^{**0}(\mathbf{q})|\dot{V}_{K^0}(V_{K^0})|f^0\rangle\langle f^0|A_{\pi^-}|\pi^+(\mathbf{q})\rangle + \langle K^{**0}(\mathbf{q})|\dot{V}_{K^0}(V_{K^0})|f'\rangle\langle f'|A_{\pi^-}|\pi^+(\mathbf{q})\rangle = \langle K^{**0}(\mathbf{q})|A_{\pi^-}|K^+\rangle\langle K^+|\dot{V}_{K^0}(V_{K^0})|\pi^+(\mathbf{q})\rangle. \qquad (13)$$

Since $\Gamma(f^0\to\pi\pi)\gg\Gamma(f'^0\to\pi\pi)$, we first ignore the f' term in the above equations. We then obtain, by applying the same procedure, $m_{K^{**0}}{}^2-m_{f^0}{}^2=m_{K^+}{}^2-m_{\pi^+}{}^2$. Experimentally, this relation implies $0.44\pm0.19=0.23$ in GeV^2. However, we can easily improve this discrepancy (between 0.44 and 0.23, assuming that it is real) by including the f-f' mixing (the mixing angle is denoted θ'). By applying PCAC for A_π, we obtain from the equation involving the $\dot{V}_{K^0}$ in Eq. (13)

$$(m_{K^{**}}{}^2-m_f{}^2)a+(m_{K^{**}}{}^2-m_{f'}{}^2)b=(m_{K^+}{}^2-m_{\pi^+}{}^2)(a+b), \qquad (14)$$

whereas, from the equation involving the V_{K^0},

$$c=a+b, \qquad (15)$$

where a, b, and c are given by

$$a=\sin\theta\, g_{f\pi^0\pi^0}(m_f{}^2-m_\pi{}^2)(m_f{}^4+m_{K^{**}}{}^4+4m_f{}^2m_{K^{**}}{}^2)\times[(\surd 6)m_f{}^4]^{-1}$$

$$b=-\cos\theta\, g_{f'\pi^0\pi^0}(m_{f'}{}^2-m_\pi{}^2)(m_{f'}{}^4+m_{K^{**}}{}^4+4m_{f'}{}^2m_{K^{**}}{}^2)\times[(\surd 6)m_{f'}{}^4]^{-1},$$

$$c=\sqrt{2}g_{K^{**+}K^0\pi^-}(m_{K^{**}}{}^2-m_K{}^2).$$

From these equations we obtain

$$\frac{g_{f'\pi^0\pi^0}}{g_{f\pi^0\pi^0}}=\tan\theta'\frac{m_{K^{**}}{}^2-m_f{}^2-m_K{}^2+m_\pi{}^2}{m_{K^{**}}{}^2-m_{f'}{}^2-m_K{}^2+m_\pi{}^2}. \qquad (16)$$

If we use $\theta'\simeq 30°$,[17] determined from the mass formula of the 2^+ meson, we obtain $g_{f'\pi^0\pi^0}=0.19g_{f\pi^0\pi^0}$, which implies $\Gamma(f'^0\to\pi\pi)\simeq 0.07\Gamma(f^0\to\pi\pi)$. This is consistent with present experiments.[15] We have thus demonstrated the fact that mixing may play an appreciable role in improving the agreement with experiment, and the intricate but interesting interplay between the mass spectrum and the coupling constants.

(E) We now turn to the 1^+ meson. The spin-parity assignment has not yet been firmly established. However, we seem to have two 1^+ multiplets[18]: one with $J^{PC}=1^{++}$ [$A_1(1070)$ is its $I=1$ member] and the other with $J^{PC}=1^{+-}$ [$B(1220)$ is its $I=1$ member]. Then the question arises: What are the $I=\frac{1}{2}$ members of these multiplets? We propose, from the mass formulas that will be derived below, that the $K_A(1230)$ an $K_A'(1320)$ belong to the 1^{++} and 1^{+-} multiplets in the symmetry limit. In the presence of symmetry breaking, the K_A and K_A' can mix, although the A_1 and B cannot. If, for the time being, we neglect this mixing, and replace, in Eqs. (7) and (8), the π^+ and K^+ mesons with the A_1^+ and K_A^+ or the B^+ and $K_A^{+\prime}$ mesons, we obtain, correspondingly, $m_{K^{*0}}{}^2-m_{\rho^0}{}^2=m_{K_A^+}{}^2-m_{A^+}{}^2$ or $m_{K^{*0}}{}^2-m_{\rho^0}{}^2=m_{K_A^{+\prime}}{}^2-m_{B^+}{}^2$. In deriving the latter equation, we have assumed that the B meson decays into $\omega\pi$, but not $\phi\pi$, as indicated by experiment. Namely, present experiments[18] give an upper limit for the partial rate of this decay of $\Gamma(B\to\phi\pi)<1.5\%$. It seems quite possible that the $B\to\phi+\pi$ decay is forbidden for the

[16] We do not encounter the same trouble as S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 137 (1967), which was subsequently criticized by J. J. Sakurai, *ibid.* **19**, 803 (1967).

[17] In Ref. 1, by neglecting the f' term (i.e., from the equation $c=a$) we have determined the value of θ' to be $\simeq 33°$.

[18] We have taken the masses of A_1, B, K_A, and K_A' from the latest compilation by A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968), wallet sheets. Also see G. Goldhaber, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967* (Interscience Publishers, Inc., New York, 1967), p. 57. One of the authors (S. O.) thanks G. Fourez for pointing out the assignment of K_A and K_A' discussed in Sec. V.

same reason as that of the $\phi \to \rho+\pi$ decay. Preliminary data[18] indicate $0.20\pm0.11=0.39\pm0.07$ and $0.20\pm0.11=0.38\pm0.08$, in GeV,[2] for the above mass formulas. Therefore the mass formulas seem to work fairly well and to support our assignment of K_A and K_A'. We may blame the small discrepancy for the possible K_A-K_A' mixing and experimental errors. By considering the K_A-K_A' mixing in the above equations, as we did in parts C and D of this section, we can, in fact, express[19] the K_A-K_A' mixing angle in terms of the masses of $I=1$ and $I=\frac{1}{2}$ 1^{++} and 1^{+-} mesons together with the masses of the ρ and ω mesons. The formula obtained is rather sensitive to the errors in the values of the masses involved. However, so long as the mass of K_A' is greater than that of K_A, the mixing angle θ is less than 45°, which is consistent with the proposed assignment of K_A and K_A' in the symmetry limit. By using present experimental values[18] for the relevant masses, we tentatively obtain $\theta\simeq 26°$. At the moment, the experimentsl error is important, and even $\theta\simeq 0$ is not excluded. Over-all consistency, including the decay branching ratios of 1^+ mesons on the basis of the sum rules between the decay coupling constants obtained in our approach, indicates a value of θ around 12°. It seems to us, at present, that this is a reasonable estimate of the K_A-K_A' mixing angle. Details will be published elsewhere.[19]

(F) We now discuss the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The extension to other baryons with higher spins is straightforward. We now consider (with $|\mathbf{q}|=\infty$)

$$\langle\Sigma^-(\mathbf{q})|\dot{V}_{K^0}(V_{K^0})|\Xi^-\rangle \times\langle\Xi^-|A_{\pi^-}|\Xi^{*0}(\mathbf{q})\rangle-\langle\Sigma^-(\mathbf{q})|A_{\pi^-}|Y^{*0}\rangle \times\langle Y^{*0}|\dot{V}_{K^0}(V_{K^0})|\Xi^{*0}(\mathbf{q})\rangle=0. \quad (17)$$

Exactly in the same way as for Eqs. (7) and (8), we obtain $E_\Xi-E_\Sigma=E_{\Xi^*}-E_{Y^*}$ (with $|\mathbf{q}|=\infty$), which gives[20] $m_{\Xi^-}{}^2-m_{\Sigma^-}{}^2=m_{\Xi^{*0}}{}^2-m_{Y^{*0}}{}^2$. Experimentally, this relation reads $0.32=0.42\pm0.05$ in GeV^2. We also obtain from the following two equations

$$\langle\Sigma^+|A_{\pi^+}|Y^{*0}\rangle\langle Y^{*0}|\dot{V}_{\bar{K}^0}(V_{\bar{K}^0})|N^{*0}\rangle -\langle\Sigma^+|\dot{V}_{\bar{K}^0}(V_{\bar{K}^0})|p\rangle\langle p|A_{\pi^+}|N^{*0}\rangle=0, \quad (18)$$

$$\langle\Sigma^0|A_{K^+}|\Xi^{*-}\rangle\langle\Xi^{*-}|\dot{V}_{K^0}(V_{K^0})|\Omega^-\rangle -\langle\Sigma^0|\dot{V}_{K^0}(V_{K^0})|\Xi^0\rangle\langle\Xi^0|A_{K^+}|\Omega^-\rangle=0, \quad (19)$$

the results $m_{Y^{*0}}{}^2-m_{N^{*0}}{}^2=m_{\Sigma^+}{}^2-m_p{}^2$ and $m_{\Omega^-}{}^2-m_{\Xi^{*-}}{}^2=m_{\Xi^0}{}^2-m_{\Sigma^0}{}^2$, respectively. Experimentally,[15] these relations read $0.39\pm0.16=0.53$ and $0.46\pm0.01=0.31$, in GeV.[2] Similar relations have also been obtained in $SU(6)$-symmetry theory (except for the fact that we obtain quadratic mass formulas). Agreement with experiment is reasonable but not spectacular. There is, in fact, a reason for this discrepancy. In Eq. (19), we can replace Σ^0 with Λ^0. This equation, together with Eq. (19), then leads to the Σ-Λ mass degeneracy. This was also encountered in the simplest $SU(6)$ mass formula.[14] We claimed that our approximation is good, *effectively*, to order ϵ. We may possibly blame the neglected $O(\epsilon^2)$ effect for this Σ-Λ degeneracy and for the small discrepancy between our mass formulas and experiment. However, we feel that we should wait to draw this conclusion until we are sure that all the mixing effects are considered.[21] (See note added in proof.) Suppose, for example, that there exists another $\frac{1}{2}^+$ baryon multiplet N' beside the usual one N. A possible small Ξ-Ξ' or p-p' mixing in Eqs. (17)–(19) may be able to remove the Λ-Σ degeneracy and also lessen the discrepancy in the above mass formulas. As seen from the discussions in parts C and D of this section, if the couplings $\Xi^*\to\Xi+\pi$ and $\Xi'+\pi$ or $N^*\to N+\pi$ and $N'+\pi$ are comparable (i.e., if the overlap in the same decay channel is large), small mixing angles may still be able to do this job. The small N-N' mixing angle may not have a great effect on the Gell-Mann–Okubo mass formulas for the N octet, since there we deal with the commutator $[V_{K^0},\dot{V}_{K^0}]=0$ and the effect is necessarily of order ϵ^2. We do not go into detail in this paper.

V. CONCLUDING REMARKS

In summary, the discussions presented here seem to indicate further the usefulness of our $SU(3)$ approximation and chiral $SU(3)\otimes SU(3)$ algebra. We have derived not only $SU(6)$-type mass formulas, but also other types of intermultiplet mass formulas. Furthermore, as demonstrated in parts C–E of Sec. IV, our approach is able to derive sum rules that exhibit an intimate connection between the mass spectrum and the coupling constants, if we make a combined use of pion PCAC. By using kaon PCAC, we might be able to obtain more such sum rules. However, since kaon PCAC involves larger off-mass-shell extrapolation ($m_K\to 0$), we did not consider it in this paper. Our results (after removing the effect of mixing) seem especially good for the case of bosons. The method can be easily extended to include higher spin states. For the case of baryons, we are faced, at the moment, with the problem of Σ-Λ degeneracy, which was also met in simple $SU(6)$-symmetry theory. We seem to have three possibilities for this problem.

(i) The most optimistic possibility is to assume that we have not yet taken into account the effect of mixing

[19] G. Fourez and S. Oneda, University of Maryland Technical Report No. 818, 1968 (unpublished); G. Fourez, University of Maryland Technical Report No. 847, 1968 (unpublished).

[20] We always obtain quadratic mass formulas by this method of computation. For the Gell-Mann–Okubo mass formulas for the same $SU(3)$ multiplet, we can always rewrite the quadratic mass formula as a linear one, since the difference of these mass formulas is of order ϵ^2, which we neglect. In Ref. 2, we obtained linear formulas for baryons by neglecting the ϵ^2 effect.

[21] For 0^-, 1^-, and 2^+ mesons, mixing effects (X^0-η, ω-ϕ, and f-f') seem to bring the Gell-Mann–Okubo mass formulas into excellent agreement with experiment. In this sense, it seems quite natural to expect similar mixing effects for the mass formulas under consideration.

to the fullest extent, as mentioned in part F of Sec. IV. In this connection, the study of the existence of $\frac{1}{2}^+$ or $\frac{3}{2}^+$ baryons other than the known ones is extremely enlightening. We note that there is some indication that such baryons do exist [e.g.,[15] $N'(1470)$]. We also note that the existence of another $\frac{1}{2}^+$ baryon multiplet N' will modify the sum rules [Eq. (3)] for the axial-vector coupling constants of semileptonic weak interactions through the N-N' mixing. This is certainly relevant to the question of whether or not we have $\theta_A = \theta_V$.

(ii) The second possibility is that our model of $SU(3)$ breaking manifested by the commutators involving time derivatives, i.e., Eqs. (6), is too simple and that in the realistic models we need to have some correction terms. (See note added in proof.)

(iii) The third possibility is that this Σ-Λ degeneracy is indeed due to our $SU(3)$ approximation for the matrix elements of V_K involving baryons, and that the approximation is better for the boson cases (probably because of the existence of G-parity selection rules). The masses of resonances that appear in our intermultiplet mass formulas involve sizeable experimental errors. Therefore it is also possible that the over-all agreement with experiment is, after all, of the same order as we have obtained in the case of $\frac{1}{2}^+$ and $\frac{3}{2}^+$ $SU(6)$-type mass formulas. We note, however, that the Gell-Mann–Okubo mass formulas (which are the quadratic ones) for the $N_{1/2}{}^+$ and $N_{3/2}{}^+$ baryons derived along the lines discussed in Sec. III agree very well with experiment. This indeed seems to justify the neglect of the $O(\epsilon^2)$ term. Therefore, *at the moment*, we rather prefer possibilities i or ii to possibility iii. We also note that if the Σ-Λ degeneracy should really be blamed for our $SU(3)$ approximation, then Cabibbo's analysis of semileptonic decays based on the hypothesis of small renormalization of the $V_\mu{}^K(x)$ current at zero momentum transfer must also be modified.

Note added in proof. We are now more inclined to believe that possibility (ii) is the case. In this paper we have considered only the simplest model of $SU(3)$ breaking given by (for example, in a quark model)

$$H_1' \propto \bar{q}(x)\lambda_8 q(x). \qquad (20)$$

In this model all the commutators used in this paper are valid. However, we may add a more general $SU(3)$-breaking interaction

$$H_2' \propto \sum_{ij} \sum_{\alpha} d_{8ij}\bar{q}(x)\Omega_\alpha\lambda_i q(x)\bar{q}(x)\Omega_\alpha\lambda_j q(x), \qquad (21)$$

where the Dirac matrix Ω_α corresponds to a choice of S, V, T, A, and P interactions. Under this model of $SU(3)$ breaking, $H' = H_1' + H_2'$, only the following commutators involving the $\dot{V}_K$ are valid:

$$[\dot{V}_{K^+}, A_{K^+}] = [\dot{V}_{K^0}, A_{K^0}] = 0,$$
$$[\dot{V}_{K^+}, V_{K^+}] = [\dot{V}_{K^0}, V_{K^0}] = [\dot{V}_{K^+}, V_{\pi^+}] = 0: \qquad (22)$$

If we use one of the above commutators, $[\dot{V}_{K^0}, A_{K^0}] = 0$, taken between the states $\langle \bar{\Sigma}(\mathbf{q})|$ and $|\Omega^-(\mathbf{q})\rangle$, with $(\mathbf{q}) = \infty$, we obtain with the $SU(3)$ approximation the Gürsey-Radicat $SU(6)$ mass formula

$$m_{\Sigma}{}^{-2} - m_{\Xi}{}^{-2} = m_{\Xi^*}{}^{-2} - m_{\Omega}{}^{-2} = m_{Y^*}{}^{-2} - m_{\Delta}{}^{-2}. \qquad (23)$$

As long as we stick to the commutators (22), we shall not encounter the problem of Σ-Λ degeneracy which, however, can take place in Eq. (19). This was pointed out to us by C. A. Nelson (private communication). We wish now to point out that the commutators with the $\dot{V}_K$ used in this paper are also valid if we add a slightly stronger (but still very reasonable) assumption that the H_2' satisfies a sort of chiral invariance. Namely, let us assume, for example, that the H_2' has a V and A form given by

$$H_2' \propto \sum_{ij} d_{8ij}[A_\mu{}^{(i)}(x)A_\mu{}^{(j)}(x) + V_\mu{}^{(i)}(x)V_\mu{}^{(j)}(x)]. \qquad (24)$$

We then obtain commutators used in this paper, such as

$$[\dot{V}_{K^0}, A_{\pi^-}] = [\dot{V}_{K^0}, A_{K^+}] = [\dot{V}_{K^+}, A_{\pi^+}] = 0, \qquad (25)$$

in addition to the ones given by (22). These commutation relations seem approximately valid, since they give rise to the sum rules discussed in this paper that are reasonably in agreement with experiments.

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