treatment of the proposed two-channel model, perhaps with the inclusion of N^* and ρ exchanges, as well as the inclusion of other inelastic channels (in finite number), seems worth further effort. As we have seen in the present paper, the urgent need for more accurate phase-shift analyses at higher energies is apparent. (The latest efforts of Bareyre *et al.*[5] represent a major step toward this knowledge.) With such information on hand, we may then perform the inelastic one-channel calculation with much more confidence in the final results. It will then be of interest to ask the following two questions:

(1) Will the actual inelasticity parameters, when used in conjuction with our one-channel method, be able to generate the correct phase shift?

(2) Will an improved treatment of the two-channel (πN, πN^*) system give the correct inelasticity; if not, will the inclusion of more channels be capable of doing so?

The dilemma, whether the calculated phase shift actually shows the correct behavior or whether we still require CDD poles to produce the right answer even in the presence of the known inelasticity, will be resolvable. In addition, we will then be able to understand better the issue of "equivalence-nonequivalence" of single- and multichannel N/D methods.[34]

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W^+-Meson Production

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The production cross section for weakly interacting W^+ mesons is calculated for the semiweak production process $\gamma+p\rightarrow n+W^+$. The W^+ is assumed to have a magnetic and T-violating electric dipole moment, and the cross section is discussed as a function of these parameters for various possible values of the boson mass.

1. INTRODUCTION

The possibility of there being a weakly interacting meson (W) that transmits the weak interactions was first proposed by Feynman and Gell-Mann[1] in 1958. Since that time the W meson has enjoyed a colorful history, during which various W-production cross sections have been calculated[2–24] and related W-meson searches have been carried out.[25–28] But up to the present time the conjectured W meson has not been detected.

Interest in W mesons has been renewed by some relatively recent work. In particular, the cosmic-ray

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muon experiment of Bergeson *et al.*[29] has posed the possibility of the existence of a short-lived particle (such as a W meson) that has a high probability of decaying into muons.

In the present paper, we calculate the production cross section for W mesons for the reaction $\gamma+p\rightarrow W^+ +n$, first proposed by Charpak and Gourdin.[30] In a previous calculation of this cross section by Reiss and Cha[19] (RC), the magnetic moments of the nucleons were neglected[31] and the W meson was assumed to have both a zero anomalous magnetic moment and a zero electric dipole moment. A related calculation[11] has demonstrated that the inclusion of the latter parameters can alter the computed cross section by as much as an order of magnitude, and therefore we have included the contributions arising from magnetic- and electric-moment terms.

In Sec. 2, we review the interaction between the spinor, vector, and electromagnetic fields, and very briefly discuss some of the alternative ways that have been proposed for producing the W mesons. In Sec. 3, we give our matrix elements and the expression for the differential and total production cross section. Section 4 contains a discussion of the results.

2. INTERACTIONS

The interaction between the spinor and electromagnetic fields is well defined, and we use the standard form of the interaction[32] with the matrix element of the nucleon current given by

$$\langle p'|J_\mu|p\rangle\sim F_1(q^2)\bar{u}(\mathbf{p}')\gamma^\mu u(\mathbf{p})+iF_2(q^2)\bar{u}(\mathbf{p}')\sigma_{\mu\nu}q^\nu u(\mathbf{p}),\quad(1)$$

where

$$q^2=(p'-p)^2,\quad F_{1p}(0)=1,\quad F_{1n}(0)=0,$$
$$F_{2p}(0)=\mu_p/2m,\quad F_{2n}(0)=\mu_n/2m,$$

and $\mu_p=1.79$, $\mu_n=-1.91$, and m is the nucleon mass.

The interaction-free Lagrangian for the W meson is assumed to have the form

$$L(x)=-T^{\alpha\beta\gamma\delta}\frac{\partial\phi_\alpha{}^\dagger(x)}{\partial x^\gamma}\frac{\partial\phi_\delta(x)}{\partial x^\beta}+M^2g^{\alpha\beta}\phi_\alpha{}^\dagger(x)\phi_\beta(x),\quad(2)$$

where

$$T^{\alpha\beta\gamma\delta}=g^{\alpha\beta}g^{\gamma\delta}+Bg^{\beta\gamma}g^{\alpha\delta}-Cg^{\alpha\gamma}g^{\beta\delta}-D\epsilon^{\alpha\beta\gamma\delta}\quad(3)$$

and $\epsilon^{\alpha\beta\gamma\delta}$ is the Levi-Cevita tensor density. The complete description of massive vector fields using this Lagrangian may be found in the paper by Salzman and Salzman.[33] One may show that $1+B=C$, and in the presence of an electromagnetic field one may make the identifications $B=g_M$, $C=1+g_M$, and $D=g_E$, where g_M and g_E are, respectively, the magnetic and electric g factors of the W meson. Since the discovery of CP, and therefore, presumably, T violation,[34] one should not exclude the possibility of the W's having a T-violating electric dipole moment. Some of the consequences of such a moment are discussed in the references cited at the end of this paper.[35,36]

The weak interaction between the nucleon and the W meson is expressed through the interaction Lagrangian

$$L_I(x)=g\bar{\psi}_p\gamma_\mu\tfrac{1}{2}(1-i\gamma_5)\psi_n(x)\phi^\mu(x)+\text{H.c.},\quad(4)$$

where ψ_p and ψ_n are the field functions associated with the proton and neutron states of the nucleon. The complete interaction Lagrangian includes the coupling term that couples the W meson to the leptons e, μ, and ν:

$$g\bar{\psi}_e(x)\gamma_\mu\tfrac{1}{2}(1-i\gamma_5)\psi_{\nu_e}(x)\phi^\mu(x)+\text{H.c.}\quad(5)$$

In the weak-interaction theory based on the above Lagrangian, ordinary β decay is a second-order process and has the observed strength provided only that the coupling constant g is related to the ordinary Fermi interaction constant G_F by the equation

$$(g/M)^2=(\surd 8)G_F,$$

with $G_F\sim10^{-5}/m^2$.

The mechanisms for the production of W mesons can be classified according to the types of interactions inxolved. By way of motivating the present calculation, we briefly list the merits of certain of the proposed mechanisms.

(*a*) *Production via strong interactions.* Dombey,[4] and later Bernstein and Feinberg,[5] investigated the possibility of producing the W via the mechanism $\pi^++p\rightarrow W^+ +p$. However, the detection of this reaction depends sensitively on the mass of the W and is made difficult by a high reaction background. The lower-mass limit of the W now seems to exclude this possibility.[26]

(*b*) *Production via electromagnetic interactions.* A pair of W's may be produced by the process $\gamma+Z\rightarrow Z+W^+ +W^-$. This process has been investigated by several groups.[11–13] The reaction cross section goes as α^3, but at the present energies it is considerably suppressed by the reduction in the available phase space resulting from the large W mass.

(*c*) *Production via weak interactions.* A single W is produced in the resonance reaction[17,18,21] $N+\bar{p}\rightarrow W^-\rightarrow$ decay products. The process goes as g^4, but the existence of numerous other open channels (available to the initial state) and the high number of background events make detection very difficult.[27,28]

(*d*) *Production via semiweak interactions.* A single W

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is produced with a high-energy neutrino beam,[2,7–9] i.e., $\nu_l+Z\rightarrow Z+l+W$. This cross section goes as $g^2\alpha^2$ and one heavy particle is produced in the final state. The main limitation in this case is the limitation of the neutrino beam, i.e., its intensity and energy. Experiments have been unsuccessful in finding W's using this reaction, but some of the best lower limits on its mass come from these experiments.[26] Another semiweak production process is the one that we have calculated, $\gamma+p\rightarrow n+W^+$. The cross section goes as $g^2\alpha$, which is a factor of α^{-1} better than the above. However, a heavy particle is produced in the reaction and only one charged particle occurs in the final state.

3. MATRIX ELEMENTS AND CROSS SECTION

The three diagrams relevant to our calculation are shown in Fig. 1. Using the previously described interactions, one may write down the amplitudes corresponding to Figs. 1(a), 1(b), and 1(c), respectively, as

$$M_1=\frac{1}{(2\pi)^2}\left(\frac{m^2}{\epsilon_1\epsilon_2}\right)^{1/2}\frac{eg}{(4\omega_1\omega_2)^{1/2}}\times\left[\frac{1}{\Delta^2-m^2}\bar{u}_2 p\cdot\gamma a_-(\gamma\cdot\Delta+m)V_\alpha{}^p\epsilon^\alpha u_1\right],\quad(6)$$

$$M_2=\frac{1}{(2\pi)^2}\left(\frac{m^2}{\epsilon_1\epsilon_2}\right)^{1/2}\frac{eg}{(4\omega_1\omega_2)^{1/2}}\left[\frac{1}{q^2-M^2}\bar{u}_2\gamma^\alpha a_- u_1\times\left(g_{\alpha\beta}-\frac{q_\alpha q_\beta}{M^2}\right)T^{\sigma\tau\gamma\beta}(q_\tau\epsilon_\sigma+k_{2\sigma}\epsilon_\tau)\rho_\gamma\right],\quad(7)$$

$$M_3=\frac{1}{(2\pi)^2}\left(\frac{m^2}{\epsilon_1\epsilon_2}\right)^{1/2}\frac{eg}{(4\omega_1\omega_2)^{1/2}}\left[-\frac{1}{2k_1\cdot p_2}\times\bar{u}_2 V_\alpha{}^n\epsilon^\alpha(p_2\cdot\gamma-k_1\cdot\gamma+m)\gamma\cdot p a_- u_1\right].\quad(8)$$

We have used the notation

$$k_1=(\omega_1,\mathbf{k}_1),\quad k_2=(\omega_2,\mathbf{k}_2),\quad P_1=(\epsilon_1,\mathbf{p}_1),\quad P_1=(\epsilon_2,\mathbf{p}_2),$$

$\rho_\mu\rightarrow W^+$-polarization vector,

$\epsilon_\mu\rightarrow$ photon-polarization vector,

$$a_-=\tfrac{1}{2}(1-i\gamma_5),\quad V_\alpha{}^p=\gamma_\alpha-iF_{2p}\sigma_{\beta\nu}k_1{}^\nu,$$

$$V_\alpha{}^n=-iF_{2n}\sigma_{\beta\nu}k_1{}^\nu,\quad \sigma_{\mu\nu}=\tfrac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu).$$

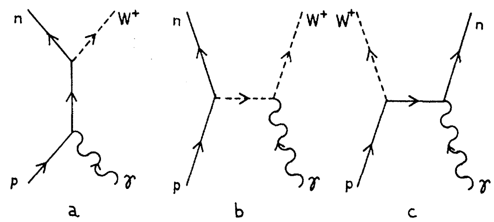

FIG. 1. Diagrams for W^+ production.

The unpolarized differential cross section is given by

$$d\sigma=\left(\frac{eg}{2\pi}\right)^2\frac{m}{4\omega_1}\sum_{\text{spins}}|A_1+A_2+A_3|^2 d\rho,\quad(9)$$

where

$$M_i=[egm/(2\pi)^2(4\epsilon_1\epsilon_2\omega_1\omega_2)^{1/2}]A_i\quad(10)$$

and

$$d\rho=\delta^4(k_1+p_1-p_2-k_2)\frac{d^3p_2}{2\epsilon_1}\frac{d^3k_2}{2\omega_2}.\quad(11)$$

The phase-space volume element may be written[37]

$$d\rho=(\pi/\alpha)dx,$$

where $\alpha=(p_1+k_1)^2-m^2$ and $x=k_1\cdot k_2$, with the range of integration $\frac{1}{2}(a\pm b)$, where[38]

$$a=\alpha(\alpha+M^2)/2(\alpha+m^2),\quad(12)$$

$$b=a[(\alpha-M^2)^2-4m^2M^2]^{1/2}/2(\alpha+m^2).\quad(13)$$

Using the above relations, we have

$$d\sigma=2\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{m^2}{\alpha^2}dx\sum_{\text{spins}}|A_1+A_2+A_3|^2.\quad(14)$$

The sum over spins is accomplished using the relations

$$\text{spinor:}\quad\sum_{s=1,2}u_s(p)\otimes\bar{u}_s(p)=\frac{p\cdot\gamma+m}{2m},$$

$$\text{photon:}\quad\sum_{s=1}^{4}\epsilon_\alpha{}^s(k_1)\epsilon_\beta{}^s(k_1)=-g_{\alpha\beta},$$

$$W\text{ meson:}\quad\sum_{s=1}^{3}\rho_\alpha{}^s(k_2)\rho_\beta{}^s(k_2)=-g_{\alpha\beta}+\frac{k_{1\alpha}k_{2\beta}}{M^2}.$$

The calculation of the squared amplitudes $|A_i|^2$ and the cross terms $\mathrm{Re}(A_iA_j{}^\dagger)$ is tedious but straightforward. The differential cross sections are

$$d\sigma_1=\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{dx}{\alpha^2M^2}\left[\alpha^2F_{2p}{}^2+(M^2-2m^2)-3mM^2F_{2p}+2M^2(m^2-M^2)F_{2p}{}^2-\frac{2m^2M^2}{\alpha}-\frac{4m^2M^2(m^2-M^2)}{\alpha^2}-(\alpha-2M^2)\left(2F_{2p}{}^2-\frac{2}{\alpha}(1-3mF_{2p})\right)x\right],\quad(15)$$

[37] W. Williamson, Jr., Am. J. Phys. 33, 987 (1965).
[38] The notation of Ref. 19 is used.

$$d\sigma_2=\tfrac{1}{4}\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{dx}{\alpha^2M^2}\{[(g_M^2+g_E^2+4g_M-4)\tfrac{1}{2}M^2\alpha^2+4M^4(M^2-m^2)](1/x^2)+[(g_M^2+g_E^2-2g_M+2)\alpha^2$$
$$-(g_M^2+g_E^2+4g_M-4)\alpha M^2-4M^2(3M^2-m^2)](1/x)+[-(g_M^2+g_E^2-2g_M+2)2\alpha+(8-g_E^2-g_M^2)M^2$$
$$+2m^2(g_M^2+g_E^2+2g_M-8)]+[(g_E^2+g_M^2)2+4(g_E^2+g_M^2-5g_M+5)m^2/M^2]x\}, \quad (16)$$

$$d\sigma_3=2\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}F_{2n}^2\frac{dx}{\alpha^2M^2}[\tfrac{1}{2}\alpha^2+(m^2-M^2)M^2+(2M^2-\alpha)x], \quad (17)$$

$$d\sigma_{12}=-\tfrac{1}{2}\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{dx}{\alpha^3M^2x}((g_M-1)\alpha^3+2M^2[g_M-(g_M+1)mF_{2p}]\alpha^2-2M^2(m^2+M^2)\alpha+4M^2(M^2-m^2)$$
$$+\{-2[(g_M-1)+(g_M-2)mF_{2p}]\alpha^2+2[(4g_M-1)M^2mF_{2p}-(g_M+2)m^2-M^2(3g_M-2)]\alpha+4M^2(m^2-2M^2)\}x$$
$$+\{2[-(g_M-1)m^2/M^2+2g_M-2(g_M+1)mF_{2p}]\alpha+8(g_M-1)m^2-(g_M-1)8m^3F_{2p}+8mM^2F_{2p}\}x^2), \quad (18)$$

$$d\sigma_{13}=-2\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{F_{2n}m}{\alpha^2}\frac{dx}{M^2}\left[\frac{0.75\alpha M^2}{\tfrac{1}{2}\alpha-x}-\frac{\tfrac{1}{2}(3\alpha+5M^2)x}{\tfrac{1}{2}\alpha-x}+\frac{(3+2m^2/\alpha)x^2}{\tfrac{1}{2}\alpha-x}\right.$$
$$\left.-4mF_{2p}\left(\frac{\alpha M^2}{2(\tfrac{1}{2}\alpha-x)}-\frac{(M^2+\tfrac{1}{2}\alpha)x}{\tfrac{1}{2}\alpha-x}+\frac{(1+m^2/\alpha)x^2}{\tfrac{1}{2}\alpha-x}\right)\right], \quad (19)$$

$$d\sigma_{23}=\tfrac{1}{2}\pi\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{F_{2n}dx}{\alpha^2M^2}\left(\frac{(g_M+1)\alpha^2mM^2}{x(\tfrac{1}{2}\alpha-x)}+[(g_M-2)m\alpha^2-5M^2m\alpha]\frac{1}{\tfrac{1}{2}\alpha-x}\right.$$
$$\left.-2m[3(g_M-1)\alpha+(2g_M-1)M^2-2(g_M-1)m^2]\frac{x}{\tfrac{1}{2}\alpha-x}+4(2g_M-1)\frac{mx^2}{\tfrac{1}{2}\alpha-x}\right). \quad (20)$$

Integration of the above equations using the zero-momentum nucleon form factors and the limits of integration given above yields our result,

$$\sigma_1=\frac{\pi}{m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{b'}{M'\alpha'^2}\{\alpha'^2(mF_{2p})^2+(M'-2)-3M'(mF_{2p})+2M'(1-M')(mF_{2p})^2-2M'/\alpha'$$
$$-4M'(1-M')/\alpha'^2-a'(\alpha'-2M')[(mF_{2p})^2-(1/\alpha')(1-3mF_{2p})]\}, \quad (21)$$

$$\sigma_2=\frac{\pi}{4m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{b'}{\alpha'^2M'}\left[[(g_M^2+g_E^2-2g_M+2)\alpha'^2-(g_M^2+g_E^2+4g_M-4)\alpha'M'-4M'(3M'-1)]\right.$$
$$\times\frac{1}{b'}\ln\frac{a'+b'}{a'-b'}+\left(g_E^2+g_M^2+2(g_E^2+g_M^2-5g_M+5)\frac{1}{M'}\right)a'+12\alpha'(g_M-1)+8M'$$
$$\left.-(g_M^2+g_E^2)M'+16(M'-1)(\alpha'+1)\frac{M'}{\alpha'^2}+4(g_E^2+g_M^2+3g_M-6)\right], \quad (22)$$

$$\sigma_3=\frac{\pi}{m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{(mF_{2n})^2b'}{\alpha'^2M'}[(2M'-\alpha')a'+\alpha'^2+2M'(1-M')], \quad (23)$$

$$\sigma_{12}=-\frac{\pi}{2m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{b'}{\alpha'^2M'}\left[\left((g_M-1)\alpha'^2+2[g_M-(g_M+1)mF_{2p}]M'\alpha'-2M'(1+M')+4\frac{M'^2}{\alpha'}(M'-1)\right)\frac{1}{b'}\ln\frac{a'+b'}{a'-b'}\right.$$
$$-2[(g_M-1)+(g_M-2)mF_{2p}]\alpha'+2[(4g_M-1)M'mF_{2p}-(g_M+2)-(3g_M-2)M']+4\frac{M'}{\alpha'}(1-2M')$$
$$\left.+a'\left(-(g_M-1)\frac{1}{M'}+2g_M-2(g_M+1)mF_{2p}+4(g_M-1)\frac{1}{\alpha'}-(g_M-1)\frac{4}{\alpha'}mF_{2p}+4\frac{M'}{\alpha'}mF_{2p}\right)\right], \quad (24)$$

$$\sigma_{13}=\frac{2\pi}{m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{(mF_{2n})b'}{a'^2M'}\left\{\left[\tfrac{1}{2}\alpha'(M'-1)+\alpha'(mF_{2p})\right]\frac{1}{b'}\ln\frac{\alpha'-a'+b'}{\alpha'-a'-b'}\right.$$
$$\left.+\tfrac{1}{2}(2-5M')+2mF_{2p}(2M'-1)+\tfrac{1}{2}a'\left[\left(3+2\frac{1}{\alpha'}\right)-4mF_{2p}\left(1+\frac{1}{\alpha'}\right)\right]\right\}, \quad (25)$$

$$\sigma_{23}=\frac{\pi}{m^2}\frac{e^2}{4\pi}\frac{g^2}{4\pi}\frac{b'(mF_{2n})}{M'\alpha'^2}\left((g_M+1)\frac{\alpha'M'}{b'}\ln\frac{a'+b'}{a'-b'}+\frac{\alpha'}{b'}[M'-(g_M-1)]\right.$$
$$\left.\times\ln\frac{\alpha'-a'-b'}{\alpha'-a'+b'}-(1+g_M)\alpha'+(2g_M-1)M'+2(1-g_M)+(2g_M-1)(\alpha'-a')\right), \quad (26)$$

where we have put all energies and masses in units of the nucleon mass, i.e.,

$$\alpha'=\alpha/m^2,\quad M'=M^2/m^2,\quad a'=a/m^2,\quad b'=b/m^2.$$

4. DISCUSSION OF RESULTS

We have computed the cross-section terms given by Eqs. (21)–(26) as a function of the incident photon energy for several values of the physical parameters g_M, g_E, and M. The results for particular values of these parameters are presented in Table I as a function of photon energy measured in units of the threshold energy (X is the photon lab energy/threshold energy). The values of the total cross sections $\sigma_T(g_M,g_E)$ are summarized in Table II. Figure 2 shows a plot of the total cross section $\sigma_T(1,0)$ versus photon energy for three values of the W-meson mass M. Figure 3 shows a similar plot of the contributing terms in the cross section $\sigma_T(-1,\pm1)$ for a value of the W mass equal to twice the nucleon mass.

A study of Table II demonstrates that the total cross section depends most strongly on the parameters g_M and g_E where the M/m ratio is small and the photon energy is in the vicinity of the threshold value. In this case, a change in the values of the parameters g_M and g_E allows a change in the total cross section by as much as a factor of 2 (σ_T assuming its largest value for $g_M=-1$, $g_E=\pm1$). For larger photon energies the cross section, as expected, approaches the same asymptote for all values of g_M and g_E.

The relative sizes of the contributing terms in the total cross section are exhibited in Table I and in

TABLE I. Cross-section terms $\sigma=\sigma(1,0)$ as a function of the ratio X between the photon energy and the threshold energy for $M=2m$, $g_M=1$, and $g_E=0$. (Cross sections are in units of 10^{-10} b.)

X	$\sigma_1(1,0)$	$\sigma_2(1,0)$	$\sigma_3(1,0)$	$\sigma_{12}(1,0)$	$\sigma_{13}(1,0)$	$\sigma_{23}(1,0)$	$\sigma_T(1,0)$
1.1	0.103	0.005	0.159	0.027	−0.108	0.034	0.220
1.2	0.158	0.012	0.237	0.032	−0.136	0.046	0.349
1.3	0.208	0.018	0.304	0.033	−0.149	0.053	0.467
1.4	0.256	0.025	0.367	0.032	−0.156	0.059	0.582
1.5	0.300	0.032	0.424	0.031	−0.159	0.064	0.693
2.0	0.517	0.060	0.688	0.020	−0.151	0.080	1.22
3.0	0.919	0.098	1.16	0.002	−0.119	0.097	2.16
4.0	1.31	0.121	1.61	−0.008	−0.095	0.108	3.04
5.0	1.69	0.138	2.05	−0.016	−0.077	0.115	3.89
6.0	2.06	0.152	2.48	−0.021	−0.064	0.120	4.73

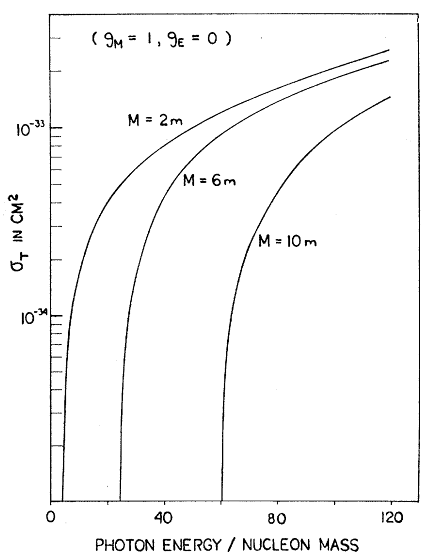

FIG. 2. Total cross section (in cm²) as a function of photon lab energy in nucleon-mass units.

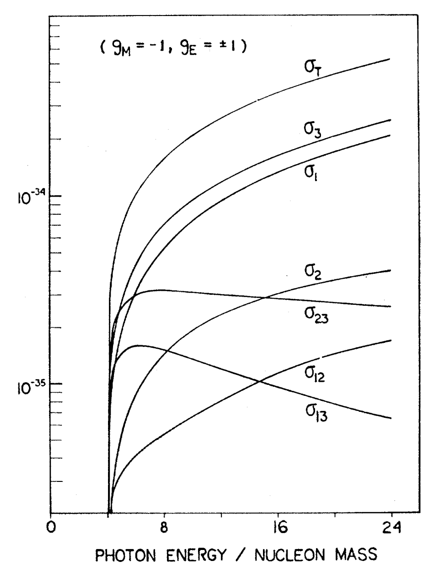

FIG. 3. Cross-section terms as a function of photon lab energy in nucleon-mass units.

TABLE II. Total cross section $\sigma_T = \sigma_T(g_M, g_E)$ as a function of the ratio between the photon lab energy and the threshold energy. The separate columns give σ_T in units of 10^{-10} b for three values of the W mass. X = (photon lab energy)/(threshold energy).

	$\sigma_T(-1, \pm1)$			$\sigma_T(-1, 0)$			$\sigma_T(0, \pm1)$			$\sigma_T(0,0)$			$\sigma_T(1, \pm1)$			$\sigma_T(1,0)$		
X	$2m$	$6m$	$10m$	$2m$	$6m$	$10m$	$2m$	$6m$	$10m$	$2m$	$6m$	$10m$	$2m$	$6m$	$10m$	$2m$	$6m$	$10m$
1.1	0.388	0.940	1.56	0.367	0.933	1.56	0.294	0.876	1.51	0.273	0.870	1.50	0.241	0.825	1.46	0.220	0.818	1.45
1.2	0.568	1.59	2.90	0.538	1.58	2.89	0.444	1.50	2.83	0.414	1.49	2.82	0.379	1.44	2.76	0.349	1.42	2.76
1.3	0.721	2.22	4.32	0.683	2.21	4.31	0.576	2.12	4.23	0.539	2.10	4.22	0.505	2.05	4.16	0.467	2.03	4.15
1.4	0.860	2.85	5.79	0.816	2.83	5.77	0.699	2.74	5.69	0.655	2.72	5.67	0.626	2.67	5.62	0.582	2.65	5.60
1.5	0.986	3.48	7.28	0.936	3.45	7.26	0.815	3.36	7.17	0.766	3.34	7.16	0.743	3.29	7.10	0.693	3.26	7.08
2.0	1.56	6.52	14.8	1.49	6.47	14.7	1.35	6.37	14.6	1.28	6.33	14.6	1.29	6.31	14.6	1.22	6.27	14.5
3.0	2.57	12.2	29.0	2.47	12.6	28.9	2.31	12.0	28.8	2.21	11.9	28.7	2.26	11.9	28.8	2.16	11.9	28.7
4.0	3.51	17.5	42.3	3.39	17.4	42.2	3.23	17.3	42.1	3.09	17.2	42.0	3.16	17.2	42.0	3.04	17.1	42.0
5.0	4.42	22.6	55.1	4.28	22.5	55.0	4.09	22.4	54.9	3.95	22.3	54.8	4.03	22.3	54.9	3.89	22.2	54.8
6.0	5.30	27.6	67.7	5.15	27.5	67.6	4.94	27.4	67.4	4.79	27.2	67.3	4.88	27.3	67.4	4.73	27.2	67.3

Fig. 3. The dominant terms arise from Feynman diagrams a and c involving internal nucleon lines. The contribution from the Feynman diagram b involving an internal W-meson line is suppressed by a factor roughly equal to the square of the nucleon to the W-meson mass ratio m/M. Diagram c is not included in the RC calculation.

The present calculation reduces to that of RC in the limit in which $F_{2p} = F_{2n} = 0$, $g_M = 1$, and $g_E = 0$. The latter case is summarized in Table III for a particular value of the mass ratio m/M. A comparison of Table III with the results presented by RC shows an order-of-magnitude disagreement between the present results and those of RC. The main disagreement stems from an apparently spurious factor of 4π that RC include in their cross-section formula as a consequence of their definition of the W-meson coupling constant. An additional disagreement arises from a discrepancy in the over-all sign of the cross term σ_{12}. The sign of the latter cross term reflects the relative phase between the matrix elements M_1 and M_2. In the present calculation, this phase is chosen to be consistent with the required gauge invariance of the total amplitude

$$M = \epsilon_\mu (M_1{}^\mu + M_2{}^\mu + M_3{}^\mu)$$

expressed via the relation

$$k_{1\mu}(M_1{}^\mu + M_2{}^\mu + M_3{}^\mu) = 0\,.$$

The apparently spurious factor of 4π in the result of RC is also pointed out in a recent paper by Weis and Kabir on the electroproduction of W mesons.[39]

TABLE III. Cross-section terms $\sigma = \sigma(1,0)$ as a function of the ratio X between the photon energy and the threshold energy for $M = 2m$, $g_M = 1$, $g_E = 0$, and $F_{2p} = F_{2n} = 0$. (Cross sections are in units of 10^{-12} b.)

X	$\sigma_1(1,0)$	$\sigma_{12}(1,0)$	$\sigma_2(1,0)$	$\sigma_T(1,0)$
1.1	0.787	−0.565	0.501	0.723
1.2	1.20	−1.24	1.15	1.11
1.3	1.54	−1.92	1.85	1.47
1.4	1.84	−2.55	2.54	1.83
1.5	2.10	−3.13	3.21	2.18
2.0	3.04	−5.23	6.04	3.85
3.0	4.01	−7.05	9.77	6.73
4.0	4.47	−7.68	12.14	8.93
5.0	4.75	−7.89	13.8	10.7
6.0	4.92	−7.95	15.2	12.2

A comparison of the magnitudes of the cross-section terms in Tables I and III shows the importance of the magnetic-form-factor terms. The dominance of the latter terms in the present calculation is somewhat surprising, but one should note that we have used the static form factors because the photons are on the light cone. The size of the cross sections do not look overly encouraging for experiments in the immediate future.

[39] A. Weis and P. K. Kabir, Nucl. Phys. B4, 643 (1968).