Current Algebra and Double-Pion Photoproduction

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Double-pion photoproduction is studied with the help of the equal-time commutation relations and the hypothesis of the partially conserved axial-vector current. In the present model we disperse both the pions and the photon, and hence the pions are treated symmetrically. Further, we work in the off-mass-shell limit [(pion four-momentum)$^2 \to 0$] in contrast to the soft-pion limits used by earlier authors. Certain difficulties connected with gauge invariance, which arise from this limiting procedure, are noted and discussed; also, the present calculation is expected to be valid only in the low-energy range close to threshold. The results obtained are in good agreement with experiment.

I. INTRODUCTION

In recent years, following the suggestion of Gell-Mann[1] and the successful calculation of the ratio $|G_A/G_V|$,[2] current commutation relations (CCR) and the hypothesis of partial conservation of the axial-vector current (PCAC) have been used by a number of authors to obtain detailed information on low-energy parameters in pionic amplitudes.[3–7] These soft-pion (pion four momentum $\to 0$) calculations proceed mainly in two parts. First, the amplitude for the process is written in a manner such that the pions involved are soft; secondly, to get physically interesting results, smooth extrapolation from the soft-pion point ($q_\mu \to 0$) to the physical point ($q_\mu \neq 0$, $q^2 = m_\pi^2$) is assumed.

If, however, we have more than one pion in the process, then there is an ambiguity as to which pion should be made soft. It was found by Callan and Treiman[5] that form factors (e.g., in K_{l4} decay) depend quite sensitively on the choice of the pion whose four-momentum is allowed to go to zero. Weinberg[6] has shown that, in order to use CCR and PCAC in a consistent manner, all pions involved in the process should be treated symmetrically and dispersed simultaneously.

Recently, Chang[8] has studied the process $\pi N \to \pi\pi N$, contracting all the three pions simultaneously and working in the soft-pion limit for all three pions. He has obtained fair agreement with experiment on both production data and π-π effects. Here we shall study, within the framework of current algebra, the double-pion photoproduction process $\gamma p \to \pi^-\pi^+ p$ close to the threshold. We shall treat both the pions on the same footing; to do so we shall disperse the two pions and the photon and shall work in the off-mass-shell [(pion four-momentum)$^2 \to 0$] limit, in contrast to the existing[9] calculation of the process in which the photon and only one pion are dispersed, the soft-pion limit being taken for the latter. Recently, a number of analogous processes[10] have been studied using CCR and PCAC in this off-mass-shell limit. As will be seen, restrictions on the amplitude imposed by this limiting procedure are less severe than those imposed by the soft-pion limit. For example, the T product of the type $k_{1\lambda}k_{2\nu}\langle N(p_2)|T \times\{A_\lambda, A_\nu, J_\mu\}|N(p_1)\rangle$, which occurs in our formulation and which is quadratic in the pion momenta, either vanishes in the soft-pion limit or reduces to a form in which the limit is ambiguous; hence essentially arbitrary prescriptions must be given[11] to evaluate such terms. On the other hand, in the off-mass-shell limit this T product always gives a finite and unambiguous contribution comparable with other terms comprising the amplitude. It may also be noted that it is this term (rather than the equal-time commutator) that survives in the on-shell limit, and hence some of the dynamical details are present in this term. Further, by making the pions soft, we not only take zero-mass pions, but also take their energy and momenta separately equal to zero. This restriction is quite stringent, as, for example, in the process $\gamma N \to \pi\pi N$, which we shall consider in detail. The soft-pion limit will tend to spoil the energy-momentum conservation unless we make an unphysical assumption that the whole of the incident γ-ray energy

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[1] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[2] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); Phys. Rev. **140**, B736 (1965); W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); Phys. Rev. **143**, 1032 (1966).

[3] For a review, see R. F. Dashen, in *Proceedings of the Eighth International Conference on High-Energy Nuclear Physics* (University of California Press, Berkeley, 1967), p. 51.

[4] For example, see S. Weinberg, Phys. Rev. Letters **17**, 616 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966); A. P. Balachandran, M. G. Gunzik, and F. Nicodemi, Nuovo Cimento **44A**, 1257 (1966); K. Raman, Phys. Rev. Letters **17**, 983 (1966); R. Ramanchandran, Nuovo Cimento **47A**, 669 (1966); Y. Tomozawa, *ibid.* **46A**, 707 (1967); N. Fuchs, Phys. Rev. **150**, 1241 (1966); K. Raman, *ibid.* **159**, 1501 (1967); R. H. Graham, L. O'Raifeartaigh, and S. Pakvasa, Nuovo Cimento **48A**, 830 (1967); H. Abarbanel, Phys. Rev. **153**, 1547 (1967).

[5] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966).

[6] S. Weinberg, Phys. Rev. Letters **16**, 879 (1966); **17**, 336 (1966).

[7] H. J. Schnitzer, Phys. Rev. **158**, 1471 (1967).

[8] Lay-Nam Chang, Phys. Rev. **162**, 1497 (1967).

[9] P. Carruthers and H. W. Huang, Phys. Letters **24B**, 464 (1967).

[10] See, for example, T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 1067 (1967); D. Bondhyopadhyay and K. C. Gupta (to be published); R. Dutt, K. C. Gupta, and J. S. Vaishya, Phys. Rev. (to be published); Debabrata Basu and R. N. Chaudhuri, *ibid.* (to be published).

[11] For example, in the case of nonleptonic hyperon decays the problem of obtaining an unambiguous contribution from the so-called weak-amplitude term in the soft-pion limit is discussed in detail in V. A. Alessandrini, M. A. B. Bég, and L. S. Brown, Phys. Rev. **144**, 1137 (1966).

is transferred to the final nucleon. Thus, to obtain physically interesting results, we shall have to apply corrections not only due to the finite mass of the pions, but also due to their finite four-momentum. On the other hand, if we work in the off-mass-shell limit, we need only apply corrections due to the finite mass of the pions. Lastly, we may remark that the hypothesis of PCAC necessitates the introduction of off-mass-shell pions. Since the $q^2=0$ limit is sufficient for this purpose, we need not consider the restrictive $q_\mu=0$ limit.

It may be mentioned here that many years ago the photoproduction of pion pairs was studied within the context of the static theory by Cutkosky and Zachariasen.[12] In their approximation there are two contributions, one due to the meson current and the other due to the interaction current. The meson current gives rise to two terms that combine to give the so-called Drell term.[13] The interaction current is dominant in the low-energy range and, near 1 BeV, meson-current contributions are comparable with that from the interaction current. The results obtained by them are not in good agreement with experiment close to threshold[14]; in particular, the sudden rise in total cross section near threshold is not well explained. Carruthers and Wong[15] proposed a simple scheme, also based on the static model, in which the interaction current produces the 3-3 pion-nucleon isobar along with an S-wave recoil pion. They produced excellent agreement with experiment near threshold and were able to obtain the sudden rise in total cross section. Carruthers and Huang[9] have also done the current-algebra calculation of the process in the soft-pion limit where they estimated the contribution of the weak-amplitude term relative to that of the commutator term.

In our study of the double-pion photoproduction process, we find additional terms contributing to the amplitude that are not found in earlier investigations. These arise because we disperse both pions and the photon and work in the off-mass-shell limit. We evaluate these matrix elements in the pole model, taking octet and decuplet poles [N and $N^*(1238)$] only. The limiting procedure also results in certain gauge-invariance difficulties that are inherent in such off-mass-shell calculations. In the absence of an unambiguous prescription[16] to remove these difficulties, we assume the validity of PCAC and a very smooth extrapolation from the off-mass-shell to the on-shell limit. We then demand the gauge invariance of the off-mass-shell amplitude and proceed as is shown in the last part of Sec. III. In evaluating the total cross section, the phase-space integration is done exactly and in a covariant manner.[17] The total cross section close to the threshold, i.e., in the energy range 410–750 MeV of the incident γ ray, is evaluated numerically.

In Sec. II, we shall obtain the double-pion photoproduction amplitude. Section III contains the computation of the total cross section, and in Sec. IV we shall compare the results obtained with existing experimental data.

II. DOUBLE-PION PHOTOPRODUCTION

We consider the photoproduction of pion pairs

$$\gamma(k)+N(p_1)\to\pi^\alpha(k_1)+\pi^\beta(k_2)+N(p_2)\qquad(2.1)$$

where α and β are the isospin indices of the pions; p_1 and p_2 are the four-momenta of the nucleons, k_1 and k_2 are those of the pions, and k is that of the photon. To assure Bose symmetry for the pions, we follow Weinberg[6] and reduce both the pions simultaneously.

The matrix element of the above process is given by $\epsilon^\mu M_\mu$, where ϵ^μ is the photon-polarization vector and

$$M_\mu=\int d^4z\, e^{-ik\cdot z}\langle\pi^\alpha(k_1)\pi^\beta(k_2)N(p_2)|J_\mu(z)|N(p_1)\rangle\,,\qquad(2.2)$$

where we have treated the electromagnetic interaction to lowest order. Contracting both pions simultaneously and using PCAC,[18] we get[19]

$$\begin{aligned}M_\mu=&\int d^4x\int d^4y\int d^4z\; e^{ik_1\cdot x}e^{ik_2\cdot y}e^{-ik\cdot z}\\&\times\langle N(p_2)|T\{\partial_\lambda A_\lambda{}^\alpha(x),\partial_\nu A_\nu{}^\beta(y),J_\mu(z)\}|N(p_1)\rangle\\&\times(k_1{}^2-m_\pi{}^2)(k_2{}^2-m_\pi{}^2)(m_\pi{}^2f_\pi)^{-2}1/(4k_{10}k_{20})^{1/2}\,,\qquad(2.3)\end{aligned}$$

where m_π is the mass of the pion, k_{10} and k_{20} are the fourth components of k_1 and k_2, and f_π is the weak π-decay constant defined through (we have suppressed the isospin index)

$$\langle 0|A_\mu(x)|\pi(k_1)\rangle=[e^{-ik_1\cdot x}/(2\pi)^{3/2}]if_\pi k_{1\mu}\,.\qquad(2.4)$$

The value of f_π is given by the Goldberger-Treiman relation[20]

$$f_\pi\simeq m_Ng_A(0)/g_V(0)\,,\qquad(2.5)$$

[12] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956).
[13] S. D. Drell, Phys. Rev. Letters **5**, 278 (1960).
[14] See S. Ferroni, V. G. Gracco, and C. Schaerf, Nuovo Cimento Suppl. **3**, 1051 (1967).
[15] P. Carruthers and How-sen Wong, Phys. Rev. **128**, 2382 (1962).
[16] The gauge-invariance difficulty has been discussed in detail by M. Nauenberg, Phys. Letters **22**, 201 (1966); S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966); S. L. Adler and W. I. Weisberger, *ibid.* **169**, 1392 (1968). However, they have discussed it for the case when the pions are soft. Recently, Balachandran *et al.* [A. P. Balachandran, M. G. Gundzik, P. Narayanswami, and F. Nicodemi, Ann. Phys. (N. Y.) **45**, 339 (1967)] have stated that the prescription of removing gauge invariance is quite ambiguous in soft-pion calculations.
[17] Rajendra Kumar (to be published).
[18] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960). In our notation, $\partial_\mu A_\mu{}^i(x)=m_\pi{}^2f_\pi\phi_\pi{}^i(x)$.
[19] We have adopted here the notation of J. D. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Co., New York, 1964).
[20] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).

where m_N is the mass of the nucleon, $g_A(0)$ is the axial-vector weak-interaction form factor, and $g_r(0)$ is the $N\bar{N}\pi$ coupling constant.

Expanding the T product in (2.3) and making use of the equal-time commutation relations[1,18]

$$\delta(x_0-y_0)[A_0{}^{\alpha}(x),A_\nu{}^{\beta}(y)]=i\epsilon_{\alpha\beta\gamma}V_\nu{}^{\gamma}(y)\delta^4(x-y)\,,$$
$$\delta(x_0-y_0)[A_0{}^{\alpha}(x),V_\nu{}^{\beta}(y)]=i\epsilon_{\alpha\beta\gamma}A_\nu{}^{\gamma}(y)\delta^4(x-y)\,,\quad(2.6)$$
$$\delta(x_0-y_0)[A_0{}^{\alpha}(x),\partial_\nu A_\nu{}^{\beta}(y)]=i\delta_{\alpha\beta}\sigma(y)\delta^4(x-y)\,,$$

we get after a little simplification

$$\begin{aligned}M_\mu=&(k_1{}^2-m_\pi{}^2)(k_2{}^2-m_\pi{}^2)(m_\pi{}^2f_\pi)^{-2}1/(4k_{10}k_{20})^{1/2}\biggl(-\int d^4x\int d^4y\int d^4z\; e^{ik_1\cdot x}e^{ik_2\cdot y}e^{-ik\cdot z}k_{1\lambda}k_{2\nu}\\&\times\langle N(p_2)|T\{A_\lambda{}^{\alpha}(x),A_\nu{}^{\beta}(y),J_\mu(z)\}|N(p_1)\rangle-\int d^4x\int d^4z\; e^{i(k_1+k_2)\cdot x}e^{-ik\cdot z}(i\delta\alpha\beta)\\&\times\langle N(p_2)|T\{\sigma(x),J_\mu(z)\}|N(p_1)\rangle+\tfrac{1}{2}\int d^4x\int d^4z\; e^{i(k_1+k_2)\cdot x}e^{-ik\cdot z}\epsilon_{\alpha\beta\gamma}(k_2-k_1)_\nu\langle N(p_2)|T\{V_\nu{}^{\gamma}(x),J_\mu(z)\}|N(p_1)\rangle\\&-\tfrac{3}{2}\int d^4x\; e^{i(k_1+k_2-k)\cdot x}\Bigl[\epsilon_{3\alpha\gamma}\epsilon_{\beta\gamma\delta}\langle N(p_2)|V_\mu{}^{\delta}(x)|N(p_1)\rangle+\epsilon_{3\beta\gamma'}\epsilon_{\beta\gamma'\delta'}\langle N(p_2)|V_\mu{}^{\delta'}(x)|N(p_1)\rangle\Bigr]\\&-\int d^4x\int d^4y\; e^{ik_1\cdot x}e^{i(k_2-k)\cdot y}\epsilon_{3\beta\gamma}k_{1\lambda}\langle N(p_2)|T\{A_\mu{}^{\gamma}(y),A_\lambda{}^{\alpha}(x)\}|N(p_1)\rangle\,,\\&-\int d^4x\int d^4y\; e^{i(k_1-k)\cdot x}e^{ik_2\cdot y}\epsilon_{3\alpha\gamma}k_{2\nu}\langle N(p_2)|T\{A_\mu{}^{\gamma}(x),A_\nu{}^{\beta}(y)\}|N(p_1)\rangle\biggr)\,.\quad(2.7)\end{aligned}$$

In writing (2.7) we have dropped the term

$$\delta(x_0-y_0)\langle N(p_2)|T\{(\partial/\partial y^\nu+\partial/\partial x^\nu)\times[A_0{}^{\alpha}(x),A_\nu{}^{\beta}(y)],J_\mu(z)\}|N(p_1)\rangle$$

from the expansion of the T product, which vanishes identically because of the conserved vector current (CVC) hypothesis.[21]

Thus we must evaluate matrix elements of the following type:

$$\begin{aligned}&\text{(i)}\quad\langle N(p_2)|T\{A(x),A(y),J(z)\}|N(p_1)\rangle\,,\\&\text{(ii)}\quad\langle N(p_2)|T\{V(x),J(z)\}|N(p_1)\rangle\,,\qquad(2.8)\\&\text{(iii)}\quad\langle N(p_2)|T\{A(x),A(y)\}|N(p_1)\rangle\,.\end{aligned}$$

In addition, we must evaluate the double-commutator terms.

We do not consider the contribution coming from the σ term. Invoking Adler's[22] consistency condition, we have neglected it.

We shall evaluate these matrix elements in the pole model, taking octet and decuplet baryon poles [N and $N^*(1238)$] only. In evaluating these we shall need the weak and electromagnetic form factors of the baryons and baryon isobars, which are discussed below (for convenience, we have dropped the isospin indices).

The weak nucleon–axial-vector vertex (NNA vertex) is given by

$$\langle N(p_2)|A_\mu(0)|N(p_1)\rangle=(m_N{}^2/E_2E_1)^{1/2}[1/(2\pi)^3]\bar{u}(p_2)\times[g_A(q^2)\gamma_5\gamma_\mu+h_A(q^2)\gamma_5q_\mu]u(p_1)\,,\quad(2.9)$$

where $q=p_2-p_1$, and E_2 and E_1 are the energies associated with nucleons p_2 and p_1, respectively. Here we shall only consider the part given by $g_A(q^2)$.

The N^*-N axial-vector vertex[7,23] has four linearly independent form factors when both baryons are on their mass shells. These are given in

$$\begin{aligned}\langle N^*(p')|A_\nu(0)|N(p)\rangle=&(m_NM/EE')^{1/2}[1/(2\pi)^3]\bar{u}_\sigma(p')[g_1(k'^2)k_\sigma'(k'^2p_\nu-k_\nu'p\cdot k')\\&+ig_2(k'^2)k_\sigma'(\gamma_\alpha\epsilon_{\alpha\rho\lambda\nu}p_\rho k_\lambda'\gamma_5)+ig_3(k'^2)\epsilon_{\sigma\alpha\beta\gamma}\epsilon_{\gamma\rho\tau\nu}p_\alpha p_\rho k_\beta'k_\tau'+ig_A{}^*(k'^2)\delta_{\sigma\nu}]u(p)\,,\quad(2.10)\end{aligned}$$

where $k'=p'-p$, $\bar{u}_\sigma(p')$ is the Rarita-Schwinger wave function for a particle of spin $\frac{3}{2}^+$ and mass M, and E and E' are the energies associated with baryons of momentum p and p', respectively. The form factors g_1, g_2, and g_3 are transverse to k' and hence do not contribute in the off-shell limit; only $g_A{}^*(k'^2)$ contributes longitudinally. The projection operator for a particle of spin $\frac{3}{2}$ is given by

$$R_{\mu\nu}=[g_{\mu\nu}-(2/3M^2)p_\mu p_\nu-\tfrac{1}{3}\gamma_\mu\gamma_\nu-(1/3M)(p_\mu\gamma_\nu-p_\nu\gamma_\mu)](\gamma\cdot p+M)/2M\,,\quad(2.11)$$

where p is the momentum of the particle.

[21] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[22] S. L. Adler, Phys. Rev. **137**, B1022 (1965); **139**, B1638 (1965).

[23] J. D. Bjorken and J. D. Walecka, Ann. Phys. (N. Y.) **38**, 35 (1966).

The Goldberger-Treiman relation for N^* gives the value of $g_A{}^*$.[8]

For the $N^*N\gamma$ vertex, we shall follow Gourdin and Salin[24]; thus

$$\langle N^*(p')|V_\mu(0)|N(p)\rangle =\left(\frac{m_N M}{EE'}\right)^{1/2}\frac{1}{(2\pi)^3}\bar{u}_\nu(p')\gamma_5 \times\left(\frac{ic_3(k'^2)}{m_\pi}(\gamma\cdot k'\delta_\mu{}^\nu-k'^\nu\gamma_\mu)-\frac{c_4(k'^2)}{m_\pi{}^2}(p'\cdot k'\delta_\mu{}^\nu-p_\mu' k'^\nu) -\frac{c_5(k'^2)}{m_\pi{}^2}(p\cdot k'\delta_\mu{}^\nu-p_\mu k'^\nu)\right)u(p)\,, \tag{2.12}$$

where the form factors c_3, c_4, and c_5 are taken to be[8]

$$c_3(0)=0.345,\quad c_4(0)=c_5(0)=-0.0035\,. \tag{2.13}$$

Thus the terms proportional to c_3 give the dominant contribution, and hence we do not consider the other two.

The vector currents considered here are generators of the $SU(2)$ isospin group. These matrix elements are taken to be of the following form:

$$\langle N(p_2)|V_\mu(0)|N(p_1)\rangle =\left(\frac{m_N{}^2}{E_2E_1}\right)^{1/2}\frac{1}{(2\pi)^3}\bar{u}(p_2) \times\left(F_1(k'^2)\gamma_\mu+\frac{i\sigma_{\mu\nu}}{2m_N}k'^\nu F_2(k'^2)\right)u(p_1)\,, \tag{2.14}$$

where $k'=p_2-p_1$, $F_1(k'^2)$ and $F_2(k'^2)$ are, respectively, the isovector-charge and magnetic-moment form factors, and[25]

$$\langle N^*(p')|V_\mu(0)|N(p)\rangle =\left(\frac{m_N M}{EE'}\right)^{1/2}\frac{1}{(2\pi)^3}\bar{u}_\nu(p') \times\left\{\gamma_5\left[G_1{}^V(k'^2)\delta_\nu{}^\mu-\frac{ip_\nu\gamma_\mu}{m_N}G_2{}^V(k'^2)\right]\right\}u(p_1)\,, \tag{2.15}$$

where $k'=p'-p$ and $G_1{}^V$ and $G_2{}^V$ are the form factors.

Coming back to the matrix elements of Eq. (2.8), we note the following:

(i) $\langle N(p_2)|T\{A_\lambda{}^\alpha(x)A_\nu{}^\beta(y),J_\mu(z)\}|N(p_1)\rangle$ includes contributions from different diagrams of Fig. 1. We have taken N and $N^*(1238)$ poles wherever possible and have avoided vertices of the type N^*N^*A or

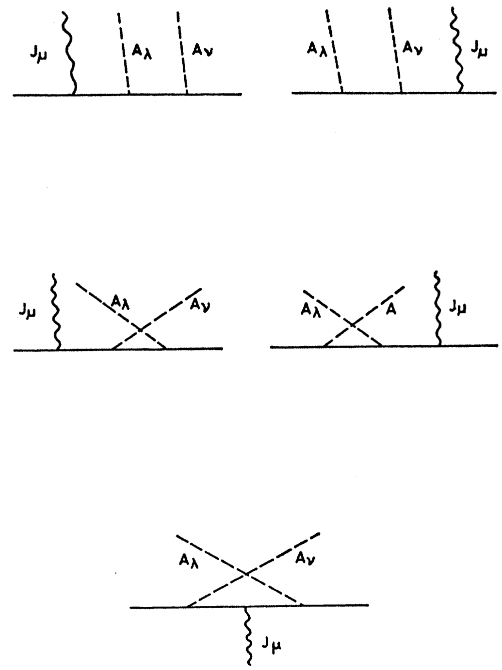

FIG. 1. Diagrams contributing to the matrix element of type (i) in Eq. (2.8). N and N^* poles are taken wherever possible. The A's (— — —) are the axial-vector currents, J (∿∿) is the photon current, and continuous lines represent nucleons or nucleon isobars.

$N^*N^*\gamma$. Evaluation of these diagrams is tedious but straightforward. An idea of the nature of the algebra involved can be had from the evaluation of the last diagram of Fig. 1 as is done in Appendix A. The terms coming from this class of diagrams are gauge-invariant.

(ii) $\langle N(p_2)|T\{V_\nu{}^\gamma(x),J_\mu(z)\}|N(p_1)\rangle$ includes the two diagrams of Fig. 2(a). Here also we take into account both N and N^* poles. These elements are again gauge-invariant and can be evaluated in the pole model.

The difficulties connected with gauge invariance arise in the matrix element of type (iii) which includes the diagram of Fig. 2(b) and the double-commutator terms. These difficulties can be traced to the presence of equal-time commutators involving the electromagnetic current, e.g., $[A,J]$ or $[A,[A,J]]$. Upon evaluation using (2.6), the resulting vertex has associated with it the photon momentum. It is then easy to see that gauge invariance is not automatically satisfied in our off-shell limit.[16] In the absence of an unambiguous prescription to remove this difficulty in the off-shell limit, we assume the validity of PCAC and a smooth extrapolation from the off-mass-shell to the on-mass-shell limit. Thus we demand gauge invariance of the off-mass-shell amplitude as well. We then find that matrix elements of type (iii) do not contribute to the process; from the double commutator the term

$$\bar{u}(p_2)[(i\sigma_{\mu\nu}/2m_N)k^\nu(F_1{}^V+F_2{}^V)]u(p_1)$$

alone survives. We may expect that the error involved

[24] M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963); 27, 309 (1963).

[25] C. H. Albright and L. S. Liu, Phys. Rev. Letters 13, 673 (1964).

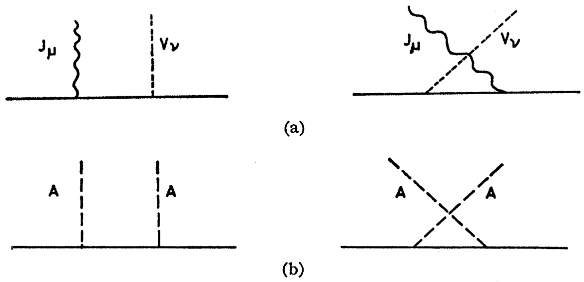

FIG. 2. The upper diagrams (a) contribute to the matrix element of type (ii) and the lower ones (b) to (iii) in Eq. (2.8). Also, V (- --) is the vector current, J (∿) is the photon current, A (— — —) is the axial-vector current, and continuous lines represent nucleons or nucleon isobars.

in omitting the non-gauge-invariant terms is of the same order as that involved in the smooth-extrapolation assumption.

III. NUMERICAL RESULTS

The total amplitude for the process consists of the following gauge-invariant terms:

$$\begin{gathered}(\gamma\cdot\epsilon)(\gamma\cdot k)(\gamma\cdot k_1),\quad (\gamma\cdot\epsilon)(\gamma\cdot k)\,,\\ (k_1\cdot k)(\gamma\cdot\epsilon)(\gamma\cdot k_1)-(k_1\cdot\epsilon)(\gamma\cdot k)(\gamma\cdot k_1)\,,\\ (p_i\cdot k)(\gamma\cdot\epsilon)(\gamma\cdot k_1)-(p_i\cdot\epsilon)(\gamma\cdot k)(\gamma\cdot k_1)\,,\\ (k_1\cdot k)(\gamma\cdot\epsilon)-(k_1\cdot\epsilon)(\gamma\cdot k)\,,\\ (p_i\cdot k)(\gamma\cdot\epsilon)-(p_i\cdot\epsilon)(\gamma\cdot k)\,,\\ (\gamma\cdot k_1)[(k_1\cdot\epsilon)(p_i\cdot k)-(k_1\cdot k)(p_i\cdot\epsilon)]\,,\\ (\gamma\cdot k_1)[(p_1\cdot\epsilon)(p_2\cdot k)-(p_1\cdot k)(p_2\cdot\epsilon)]\,,\\ (\gamma\cdot k)[(k_1\cdot\epsilon)(p_i\cdot k)-(k_1\cdot k)(p_i\cdot\epsilon)]\,,\\ (\gamma\cdot k)[(p_1\cdot\epsilon)(p_2\cdot k)-(p_1\cdot k)(p_2\cdot\epsilon)]\,,\\ (k_1\cdot\epsilon)(p_i\cdot k)-(k_1\cdot k)(p_i\cdot\epsilon)\,,\\ (p_1\cdot\epsilon)(p_2\cdot k)-(p_1\cdot k)(p_2\cdot\epsilon)\,,\end{gathered}\tag{3.1}$$

where $i=1$, 2.

However, for convenience, we shall write the scattering amplitude as

$$\begin{aligned}M_{fi}=\bar{u}(p_2)\{&A_1(\gamma\cdot\epsilon)(\gamma\cdot k)(\gamma\cdot k_1)+A_2(\gamma\cdot\epsilon)(\gamma\cdot k)\\ &+A_3(\gamma\cdot\epsilon)(\gamma\cdot k_1)+[A_{41}p_1\cdot\epsilon+A_{42}p_2\cdot\epsilon+A_{43}k_1\cdot\epsilon]\\ &\times(\gamma\cdot k)(\gamma\cdot k_1)+[A_{51}p_1\cdot\epsilon+A_{52}p_2\cdot\epsilon+A_{53}k_1\cdot\epsilon](\gamma\cdot k_1)\\ &+[A_{61}p_1\cdot\epsilon+A_{62}p_2\cdot\epsilon+A_{63}k_1\cdot\epsilon](\gamma\cdot k)+A_7(\gamma\cdot\epsilon)\\ &\qquad+[A_{81}p_1\cdot\epsilon+A_{82}p_2\cdot\epsilon+A_{83}k_1\cdot\epsilon]\}u(p_1)\,.\end{aligned}\tag{3.2}$$

The coefficients $A_1, A_2, \cdots, A_{83}$ are given in Appendix B. These include the contribution from nucleon poles only. In actual calculation, both N and N^* poles have been included. The resulting expression can be obtained in a straightforward but tedious manner.

In the numerical computation of the total cross section near threshold, we ignore the momentum dependence of the form factors used here. We take the following values of the different parameters:

$$\begin{aligned}m_N&=939, & g_A(0)&=1.17\,,\\ m_\pi&=139.6, & g_\gamma^2/4\pi&=14.6\,,\\ M&=1236, & g_A^*(0)&=1.44\,,\\ F_1^{pp\gamma}&=1, & c_3(0)&=0.345\,,\\ F_2^{pp\gamma}&=1.79, & G_1^V(0)&=3.5\,,\\ F_1^{nn\gamma}&=0, & G_2^V(0)&=-1.5\,,\\ F_2^{nn\gamma}&=-1.91, & F_1^{NNV}&=0.5\,,\\ e^2/4\pi&=1/137, & F_2^{NNV}&=1.85\,.\end{aligned}$$

The total cross section for the process is given by

$$\begin{aligned}\sigma=e^2\frac{m_N^2}{p_1\cdot k}\left(\frac{g_\gamma}{m_Ng_A}\right)^4\int\int\int\frac{d^3p_2}{2E_2(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3}\\ \times\frac{d^3k_1}{2\omega_1(2\pi)^3}(2\pi)^4\delta^4(p_1+k-k_1-k_2-p_2)|\bar{M}_{fi}|^2\,,\end{aligned}\tag{3.3}$$

where

$$\omega_i=(\mathbf{k}_i^2+m_\pi^2)^{1/2}\quad\text{and}\quad E_2=(\mathbf{p}_2^2+m_N^2)^{1/2}.$$

The bar over M_{fi} means that we average over the initial spin states and sum over the final spin states.

For covariant integration, we introduce the following variables:

$$\begin{aligned}x_1&=(k_2+k_1)^2\,,\\ x_2&=(p_1-p_2)^2\,,\\ x_3&=(p_2+k_1)^2\,,\\ x_4&=(p_1-k_2)^2\,,\end{aligned}\tag{3.4}$$

in terms of which the (3.3) will become

$$\begin{aligned}\sigma=e^2\left(\frac{g_\gamma}{m_Ng_A}\right)^4\frac{m_N^2\pi s}{2(2\pi)^5(p_1\cdot k)\lambda(s,m_N^2,0)}\\ \times\int_{x_{1-}}^{x_{1+}}\int_{x_{2-}}^{x_{2+}}\int_{x_{3-}}^{x_{3+}}\int_{x_{4-}}^{x_{4+}}\frac{dx_1dx_2dx_3dx_4F(s,x_1,x_2,x_3,x_4)}{[\lambda(s,m_N^2,x_1)\lambda(s,m_\pi^2,x_3)]^{1/2}[(1-Z_1^2)(1-Z_2^2)(1-Z_3^2)]^{1/2}}\,,\end{aligned}\tag{3.5}$$

where $F(s,x_1,x_2,x_3,x_4)$ is the new $|\bar{M}_{fi}|^2$ obtained after changing the variables. Here

$$Z_1=\frac{2s(2m_N^2-x_2)-(s+m_N^2)(s+m_N^2-x_1)}{[\lambda(s,m_N^2,0)\lambda(s,m_N^2,x_1)]^{1/2}},$$

$$Z_2=\frac{-2sx_1+(s+x_1-m_N^2)(s+m_\pi^2-x_3)}{[\lambda(s,x_1,m_N^2)\lambda(s,m_\pi^2,x_3)]^{1/2}}$$

$$Z_3=\frac{Z_4-Z_1Z_2}{[(1-Z_1^2)(1-Z_2^2)]^{1/2}},$$

with

$$Z_4=\frac{2s(x_4-m_N^2-m_\pi^2)+(s+m_N^2)(s+m_\pi^2-x_3)}{[\lambda(s,m_N^2,0)\lambda(s,m_\pi^2,x_3)]^{1/2}}.$$

s is the c.m. energy squared and $\lambda(a,b,c)$ is equivalent to $a^2+b^2+c^2-2ab-2bc-2ac$.

The limits of integration $x_{1\pm}$, $x_{2\pm}$, $x_{3\pm}$, and $x_{4\pm}$ are found to be[17]

$$x_{1-}=(2m_\pi)^2,\quad x_{1+}=[(\sqrt{s})-m_N]^2,$$

$$(x_2)_\pm=2m_N^2-\frac{(s+m_N^2)(s+m_N^2-x_1)}{2s}\pm\frac{[\lambda(s,m_N^2,0)\lambda(s,m_N^2,x_1)]^{1/2}}{2s},$$

$$(x_3)_\pm=s+m_\pi^2-\tfrac{1}{2}(s+x_1-m_N^2)\pm\frac{[\lambda(s,m_N^2,x_1)\lambda(x_1,m_\pi^2,m_\pi^2)]^{1/2}}{2x_1},$$

$$(x_4)_\pm=m_N^2+m_\pi^2-\frac{(s+m_N^2)(s+m_\pi^2-x_3)}{2s}+\frac{[\lambda(s,m_N^2,0)\lambda(s,m_\pi^2,x_3)]^{1/2}}{2s}\times\{Z_1Z_2\pm[(1-Z_1^2)(1-Z_2^2)]^{1/2}\}.$$

The curve of Fig. 3 summarizes the results obtained for the total cross section in the energy range 410–750 MeV of the incident γ ray (E_γ).

IV. CONCLUSIONS

We have calculated the total cross section for the process $\gamma+p\rightarrow\pi+\pi+p$ in the framework of current algebra. The results are in good agreement with experiment. We would like to point out that the calculations have been done in the limit $q^2=0$, which is less restrictive than the soft-pion limit. We also disperse both pions simultaneously. By doing so we have taken into account the earlier suggestion of Carruthers and Wong,[15] that a more precise treatment of the Bose symmetry of the final pions may be required than that given by Cutkosky and Zachariasen,[12] who treated the pions differently and reinstated symmetry by symmetrizing their results.

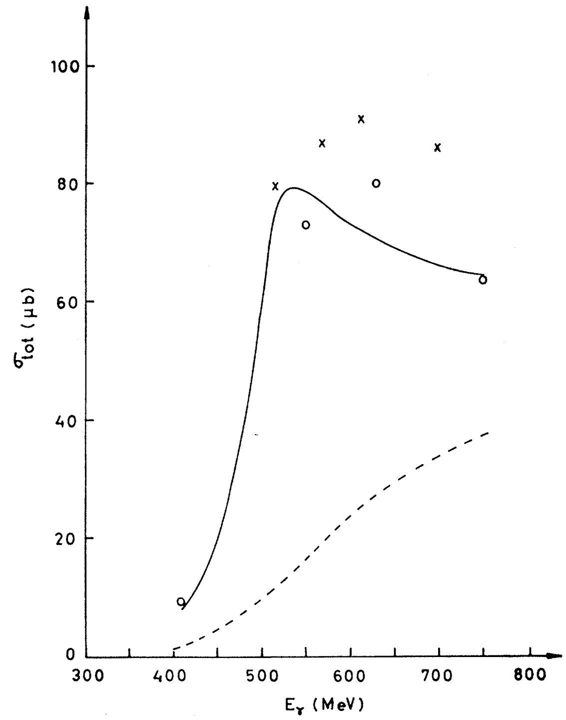

FIG. 3. Total cross section plotted as a function of the incident γ-ray lab energy. The solid curves show our results and the dashed curves are the results of Cutkosky and Zachariasen (Ref. 12). Experimental points are shown: (×) results of Stanford group (Ref. 26) and (○) those of Chasan *et al.* (Ref. 27).

Our numerical results are in better agreement with recent experimental data than those of Cutkosky and Zachariasen,[14] as shown in Fig. 3. However, they are quite close to the results of Carruthers and Wong[15] obtained by using the static model. Our total cross section reaches its maximum at an incident γ-ray energy that is lower than that corresponding to the experimental value. Also, at higher energy ($E_\gamma>500$ MeV), the cross section is lower than that given by the recent experiment of the Stanford group[26] and agrees well with the earlier measurement of Chasan *et al.*[27] This may be because we have not included the Roper resonance and the effect of final-state interactions. We may expect their inclusion to bring the results into better agreement with experiments.

We make use of Adler's consistency condition,[22] and neglect the contribution of the σ term. Whether its inclusion, which has been strongly suggested by many authors,[28,8] will change the results appreciably needs further analysis. In view of the close agreement of our calculated cross section with the experimental results, one would expect its contribution to be small.

[26] J. V. Allaby, H. L. Lynch, and D. M. Ritson, Phys. Rev. **142**, 887 (1966).
[27] B. M. Chasan, G. Cocconi, V. T. Cocconi, R. M. Schechtman, and D. H. White, Phys. Rev. **119**, 811 (1960).
[28] See, for example, K. Raman, Phys. Rev. **164**, 1736 (1967).

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APPENDIX A

Here we evaluate the last diagram of Fig. 1. Nucleon (N) and nucleon isobar (N^*) poles can be included in three different ways, taking (i) N-N poles, (ii) N^*-N poles, and (iii) N-N^* poles. We shall consider in this Appendix the case (iii) and thus write the matrix element as

$$[ig_A{}^*(0)]\left(\frac{ic_3(0)}{m_\pi}\right)[g_A(0)]k_{1\lambda}k_{2\nu}\epsilon^\mu\bar{u}(p_2)\Bigg[\delta_{\nu_1}{}^\lambda\left(g_{\nu_1\nu_2}-\tfrac{1}{3}\gamma_{\nu_1}\gamma_{\nu_2}-\frac{2(p_2+k_1)_{\nu_1}(p_2+k_1)_{\nu_2}}{3M^2}-\frac{(p_2+k_1)_{\nu_1}\gamma_{\nu_2}}{3M}+\frac{(p_2+k_1)_{\nu_2}\gamma_{\nu_1}}{3M}\right)$$
$$\times\frac{\gamma\cdot p_2+\gamma\cdot k_1+M}{(p_2+k_1)^2-M^2}\gamma_5(\gamma\cdot k\delta_\mu{}^{\nu_2}-k^{\nu_2}\gamma_\mu)\frac{\gamma\cdot p_1-\gamma\cdot k_2+m_N}{(p_1-k_2)^2-m_N{}^2}\tfrac{1}{4}\tau^\alpha\tau^\beta\Bigg]u(p_1)\,,$$

which we put in the following form after some numerical simplification involving γ matrices:

$$\left(\frac{g_A{}^*(0)c_3(0)g_A(0)}{4m_\pi}\right)(2p_2\cdot k_1+k_1{}^2-M^2+m_N{}^2)^{-1}(k_2{}^2-2p_1\cdot k_2)^{-1}$$
$$\times\bar{u}(p_2)\Bigg[(\gamma\cdot\epsilon)(\gamma\cdot k)(\gamma\cdot k_1)\bigg(\frac{2}{3M}(M^2+Mm_N-p_2\cdot k_1)(p_1\cdot k_2-2m_N{}^2)-\frac{2m_N}{3M}(M+m_N)p_2\cdot k_1$$
$$+\frac{m_N}{3M^2}[k_1\cdot k(Mm_N+2p_2\cdot k_1-2M^2)+p_2\cdot k(2p_2\cdot k_1+M^2+Mm_N)]\bigg)-(\gamma\cdot\epsilon)(\gamma\cdot k)\bigg(\frac{2}{3M}(M+m_N)(p_2\cdot k_1)(p_1\cdot k_2)$$
$$+\frac{4m_N}{3M}(k\cdot k_1-p_2\cdot k_1)(M^2+Mm_N-p_2\cdot k_1)-\tfrac{1}{3}(M+m_N)m_Nk_1\cdot k+\frac{2m_N}{3M^2}p_2\cdot k_1[k_1\cdot k(M-m_N)-p_2\cdot k(M+m_N)]\bigg)$$
$$+(\gamma\cdot\epsilon)(\gamma\cdot k_1)\bigg(\frac{k_1\cdot k}{3M^2}[p_1\cdot k_2(Mm_N-2M^2+2p_2\cdot k_1)+2m_N(M-m_N)p_2\cdot k_1-m_NM^2(M+m_N)]$$
$$+\frac{p_2\cdot k}{3M^2}[p_1\cdot k_2(M^2+Mm_N+2p_2\cdot k_1)-2m_Np_2\cdot k_1(M+m_N)-4m_NM(M^2+Mm_N-2p_2\cdot k_1)]\bigg)$$
$$-(\gamma\cdot k)(\gamma\cdot k_1)\bigg(\frac{k_1\cdot\epsilon}{3M^2}[p_1\cdot k_2(Mm_N-2M^2+2p_2\cdot k_1)+2m_Np_2\cdot k_1(M-m_N)-m_NM^2(M+m_N)]$$
$$+\frac{p_2\cdot\epsilon}{3M^2}[p_1\cdot k_2(M^2+Mm_N+2p_2\cdot k_1)-2m_Np_2\cdot k_1(M+m_N)-4m_NM(M^2+Mm_N-2p_2\cdot k_1)]\bigg)$$
$$+(\gamma\cdot\epsilon)\bigg(\frac{k_1\cdot k}{3M^2}\{2p_2\cdot k_1[p_1\cdot k_2(M-m_N)+m_N(Mm_N-M^2+2p_2\cdot k_1)-2m_N{}^2(M-m_N)]-M^2(M+m_N)(p_1\cdot k_2-2m_N{}^2)\}$$
$$-\frac{p_2\cdot k}{3M^2}\{2p_2\cdot k_1[(M+m_N)(p_1\cdot k_2+2m_N{}^2-2m_NM)+m_N(2p_2\cdot k_1+M^2+Mm_N)]\}\bigg)$$
$$-(\gamma\cdot k)\bigg(\frac{k_1\cdot\epsilon}{3M^2}\{2p_2\cdot k_1[p_1\cdot k_2(M-m_N)+m_N(Mm_N-M^2+2p_2\cdot k_1)-2m_N{}^2(M-m_N)]$$
$$-M^2(M+m_N)(p_1\cdot k_2-2m_N{}^2)\}-\frac{p_2\cdot\epsilon}{3M^2}2p_2\cdot k_1[(M+m_N)(p_1\cdot k_2+2m_N{}^2-2m_NM)+m_N(2p_2\cdot k_1+M^2+Mm_N)]\bigg)$$
$$-(\gamma\cdot k_1)[2m_N(p_2\cdot\epsilon k_1\cdot k-k_1\cdot\epsilon p_2\cdot k)]+(p_2\cdot\epsilon k_1\cdot k-p_2\cdot kk_1\cdot\epsilon)$$
$$\times\left(\frac{2}{3M^2}p_1\cdot k_2(2p_2\cdot k_1+M^2+Mm_N)+\tfrac{2}{3}m_N(M+m_N)-\frac{4m_N}{3M^2}(M-m_N)p_2\cdot k_1\right)\Bigg]\tau^\alpha\tau^\beta u(p_1)\,.$$

APPENDIX B

In the following, we have replaced the form factors with their numerical values given in Sec. III:

$$A_1=1.225\left(\frac{p_1\cdot k_2+p_2\cdot k_1+2m_N^2}{p_1\cdot k(2p_2\cdot k_1+k_1^2)}-\frac{p_1\cdot k_2+p_2\cdot k-2m_N^2}{p_2\cdot k(2p_1\cdot k_2-k_2^2)}\right)+2.61\frac{p_1\cdot k_2-p_2\cdot k_1-2m_N^2}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_2=1.225\left(\frac{2m_N(k_1\cdot k-p_2\cdot k_1)}{p_2\cdot k(2p_1\cdot k_2-k_2^2)}-\frac{p_2\cdot k_1(2m_N^2+p_1\cdot k)}{m_Np_1\cdot k(2p_2\cdot k_1+k_1^2)}\right)+2.61\frac{p_2\cdot k_1(2m_N^2-p_1\cdot k_2)}{m_N(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)}-\frac{0.8112}{m_N},$$

$$A_3=\frac{1.225}{m_N}\left(\frac{2m_N^2+p_2\cdot k_1}{2p_2\cdot k_1+k_1^2}-\frac{p_1\cdot k_2-2m_N^2}{2p_1\cdot k_2-k_2^2}\right)-5.22\frac{m_N(p_2\cdot k+k_1\cdot k)}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{41}=-\frac{1.225}{m_N}\frac{2m_N^2+p_2\cdot k_1}{p_1\cdot k(2p_2\cdot k_1+k_1^2)},$$

$$A_{42}=\frac{1.225}{m_N}\frac{p_1\cdot k_2-2m_N^2}{p_2\cdot k(2p_1\cdot k_2-k_2^2)}+\frac{5.22m_N}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{43}=\frac{5.22m_N}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{51}=A_{52}=A_{53}=0,$$

$$A_{61}=\frac{2.45p_2\cdot k_1}{p_1\cdot k(2p_2\cdot k_1+k_1^2)},$$

$$A_{62}=\frac{2.45(p_2\cdot k_1-k_1\cdot k)}{p_2\cdot k(2p_1\cdot k_2-k_2^2)}+\frac{5.22p_2\cdot k_1}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{63}=\frac{2.45(2m_N^2+p_2\cdot k_1-p_1\cdot k)}{p_1\cdot k(2p_2\cdot k_1+k_1^2)}+\frac{5.22(p_1\cdot k_2-2m_N^2)}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_7=2.45\left(\frac{k_1\cdot k-p_2\cdot k_1}{2p_1\cdot k_2-k_2^2}+\frac{p_2\cdot k_1}{2p_2\cdot k_1+k_1^2}+\frac{k_1\cdot k}{p_1\cdot k}\frac{2m_N^2+p_2\cdot k_1-p_1\cdot k}{2p_2\cdot k_1+k_1^2}\right)-5.22\frac{p_2\cdot k_1p_2\cdot k+k_1\cdot k(p_1\cdot k_2-2m_N^2)}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{81}=2.45\frac{k_1\cdot k(p_2\cdot k_1+2m_N^2)}{m_Np_1\cdot k(2p_2\cdot k_1+k_2^2)},$$

$$A_{82}=-10.44\frac{m_Nk_1\cdot k}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)},$$

$$A_{83}=10.44\frac{m_Np_2\cdot k}{(2p_2\cdot k_1+k_1^2)(-2p_1\cdot k_2+k_2^2)}.$$