Current Commutation Relations in $R(11)$*†

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An earlier investigation of a class of models similar to $SU(6)$ is extended by the study of Adler-Weisberger-type commutation relations within the framework of the $R(11)$ model. The commutators of the $\Delta S=0$ and $|\Delta S|=1$ currents yield $|f_A|\approx 1.5$ and $|f_A|\approx 1.0$, respectively, and $\alpha\equiv(f/d+1)^{-1}\approx 1.0$, when the baryons are assigned to the simplest representation **32**. A sizable mixing with octets in the **320** or some larger representation is required to bring these values into agreement with experiment. We conclude that $SU(6)$ is unique among models of its type in producing interesting results within a framework of reasonable simplicity.

I. INTRODUCTION

ALTHOUGH $SU(6)$ has proven very intriguing as a symmetry of elementary particles, its most straightforward application is marred by several incorrect predictions, such as $|G_A/G_V|=\frac{5}{3}$.[1] Hence, it is of some interest to investigate other groups which might be at least equally consistent with experiment. The assumption that such an alternative group must (locally) contain $SU(3)\otimes R(3)$ and not mix different trialities or statistics in the same irreducible representation leads to $R(11)$ as the next simplest possibility.[2]

An earlier investigation[3] of the weak-interaction predictions of $R(11)$ produced an excellent value for the ratio $|G_A/G_V|$, to be compared with the rather poor result obtained with $SU(6)$ symmetry when the baryons are assigned to a pure 56-dimensional representation. In that investigation, we assumed the structure of the weak currents to be given by the adjoint representation, and assigned the stable spin-$\frac{1}{2}^+$ baryons purely to the 32-dimensional spinor representation of $R(11)$. The remaining states of this representation must then be associated with a second octet of baryons having spin and parity $\frac{1}{2}^+$, presumably including the $N^*(1470)$.[4] Also, it is necessary to assume the existence of as yet unobserved spin-0^- mesons to be associated with the usual 0^- and 1^- states in the lowest meson multiplet. Further investigation into the mass spectrum and two-body decay widths of the second baryon octet[5] was inconclusive, since the masses could not be fixed by any simple assumptions concerning the $R(11)$ mass tensor.

In the present paper we investigate $R(11)$ through commutation relations of the currents, derived from the baryon **32** in analogy to the quark-model derivation for $SU(6)$, and resulting sum rules of the Adler-Weisberger type. For $SU(6)$, this method is independent of assignments of particles to representations; however, as we shall see, such is not the case for $R(11)$.

Thus we are interested in the decomposition of the direct product of **32** with its conjugate representation—that is, since **32** is self-conjugate, in the decomposition[6]

$$\mathbf{32}\times\mathbf{32}=\mathbf{462}_s+\mathbf{330}_a+\mathbf{165}_a+\mathbf{55}_s+\mathbf{11}_s+\mathbf{1}_a\,. \quad (1.1)$$

(Here s and a denote the symmetric and antisymmetric parts of the product, respectively.) Note that the adjoint representation **55** occurs only once [as does the **35** in the product $\mathbf{6}^*\times\mathbf{6}$ for $SU(6)$], so that simple predictions are possible. The decompositions of the $R(11)$ **55** in terms of $R(8)\otimes R(3)$ and $SU(3)\otimes R(3)$ submultiplets are, respectively,

$$\begin{aligned}\mathbf{55}&\rightarrow[0100,0]_{28}+[1000,2]_{24}+[0000,2]_3\\&\rightarrow(\mathbf{8},\mathbf{1})+(\mathbf{10},\mathbf{1})+(\mathbf{10}^*,\mathbf{1})+(\mathbf{8},\mathbf{3})+(\mathbf{1},\mathbf{3})\,,\end{aligned} \quad (1.2)$$

where the pairs of numbers in the brackets are the greatest weights of the $R(8)$ and $R(3)$ representations, respectively [$R(8)$ has four diagonal operators], the subscripts on the brackets give the dimensions of the $R(8)\otimes R(3)$ submultiplets, and the pairs of numbers in the parentheses represent the dimensions of the $SU(3)$ and $R(3)$ representations of the $SU(3)\otimes R(3)$ submultiplets. In analogy with the Gell-Mann scheme[7] we assume that **(8,1)** contains the time components of the vector weak-current and charge-current operators while **(8,3)** contains the space components of the axial-vector weak-current and magnetic-moment operators, in corresponding positions with respect to $SU(3)$. [This is what we expect if the regular representation of $R(11)$ is to contain the nonrelativistic limits of these operators.] We shall then proceed to apply the current commutation relations from $R(11)$ following the methods used by Adler[8] and Weisberger[9] for $SU(6)$.

However, we must note one further feature of $R(11)$. On restriction of $R(11)$ to $R(8)\otimes R(3)$, the baryonic

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[6] See Appendix A of Ref. 3 for a discussion of notation and methods.
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representation **32** decomposes into two distinct spin-$\frac{1}{2}$ octets:

$$\mathbf{32}\rightarrow[0010,1]+[0001,1]; \quad (1.3)$$

with respect to $SU(3)$, these octets become indistinguishable. There is no additive quantum number in $R(11)$ which distinguishes between these two octets (which will be denoted by $\mathbf{B}$ and $\mathbf{B}'$); they differ only by belonging to different eigenvalues of the $R(8)$ Casimir operators. Since there seems to be little evidence for $R(8)$ symmetry in particle physics at the present time, we allow the low-lying physical octet of baryons B, together with a second octet of physical spin-$\frac{1}{2}^+$ baryons B', to be linear combinations of the "mathematical" baryons $\mathbf{B}$ and $\mathbf{B}'$. We have previously shown[3] that time-reversal covariance for the baryonic **55** currents restricts this relationship to the form

$$\begin{aligned}\mathbf{B}&=(\eta/\sqrt{2})(B+iB'),\\ \mathbf{B}'&=(\eta\xi/\sqrt{2})(-B+iB'),\end{aligned} \quad (1.4)$$

where both η and ξ have unit magnitude.[10]

II. CURRENT COMMUTATION RELATIONS IN $R(11)$

A. $\Delta S=0$ Axial-Vector Currents

Adler[8] and Weisberger[9] used the quark model for the $\Delta S=0$ time component of the axial-vector current[11]

$$A_0^{(0)}(x)=P^\dagger(x)\gamma_5 N(x), \quad (2.1)$$

where $P(x)$ and $N(x)$ represent protonlike and neutronlike quark fields, respectively, to obtain the equal-time commutation relation

$$\left[A_0^{(0)\dagger}(0),\int d^3y\, A_0^{(0)}(\mathbf{y},0)\right]=-2I_3(0), \quad (2.2)$$

where $I_3(0)$ is the third component of the isospin density at the origin. Evaluation of (2.2) between one-neutron states immediately yields unity for the right-hand side; for the left-hand side, it is necessary to use the hypothesis of partially conserved axial-vector current (PCAC)[12] and standard field-theoretic techniques. Weisberger obtained the result[9,11]

$$[1-X(\pi p)]f_A^2=1, \quad (2.3)$$

where f_A is the axial-vector form factor at zero momentum transfer, and

$$X(\pi p)\equiv\frac{2M^2}{\pi g^2}\int\frac{dq_0|\mathbf{q}|}{q_0^2}[\sigma(\pi^+p,q_0)-\sigma(\pi^-p,q_0)]\approx 0.25. \quad (2.4)$$

The total cross sections $\sigma(\pi^\pm p,q_0)$ are for $\pi^\pm$ on protons at laboratory energy q_0, g is the pion-nucleon coupling constant, and M is the nucleon mass. Adler[8] estimated the corrections to (2.4) due to the fact that the pions are not on the mass shell to be 0.09 ± 0.03. Thus they obtained

$$\begin{aligned}|f_A|&=1.15\pm0.07 \quad \text{(Weisberger)},\\ |f_A|&=1.24\pm0.03 \quad \text{(Adler)},\end{aligned} \quad (2.5)$$

where the errors are intended as reasonable estimates. Höhler and Strauss[13] have reevaluated (2.4) and one of Adler's correction terms using more recent data, with the result

$$|f_A|=1.17 \quad (2.6)$$

and an uncertainty presumably similar to Adler's.

The purpose of the present section is to obtain the corresponding relations when the currents satisfy commutation relations appropriate to $R(11)$.

In general, the relativistically covariant vector and axial-vector baryon currents will be of the form

$$\begin{aligned}V:&\ \bar\psi\gamma_\mu\otimes C_k\psi,\\ A:&\ \bar\psi\gamma_\mu\gamma_5\otimes D_l\psi,\end{aligned} \quad (2.7)$$

where the γ_μ are Dirac gamma matrices and the C_k and D_l are matrices representing internal degrees of freedom such as isospin. In the case of $SU(3)$ internal symmetry, the matrices C_k and D_l both belong to octet representations, F type for C_k, and a mixture of F and D types for D_l.[12,14] It is actually the nonrelativistic limits of the currents (2.7) which contribute to leptonic baryon decays and hence are to be identified with $R(11)$ operators. Using the representation of the gamma matrices

$$\gamma_i=i\sigma_2\otimes\sigma_i(i=1,2,3), \quad \gamma_0=\sigma_3\otimes I, \quad \gamma_5=\sigma_1\otimes I, \quad (2.8)$$

we find in this limit that only the time components of the vector currents and the space components of the axial-vector currents survive, i.e.,

$$V\rightarrow V_0:\ \Phi^\dagger I\otimes C_k\Phi$$

and

$$A\rightarrow\mathbf{A}:\ \Phi^\dagger\boldsymbol{\sigma}\otimes D_l\Phi, \quad (2.9)$$

where Φ denotes the "large" components of ψ. When the fields are quark fields, then V_0 and $\mathbf{A}$ belong, respectively, to the **(8,1)** and **(8,3)** octets of currents in the $SU(6)$ adjoint representation **35**. In the case of $R(11)$, we assume similarly that the currents (2.9) are in the $SU(3)\otimes R(3)$ submultiplets **(8,1)** and **(8,3)** of the adjoint representation **55**.

In order to evaluate the commutator occuring in (2.2) for $R(11)$, we need the time components of the axial-vector currents—not the space components—which are contained in the **(8,3)** submultiplet of the **55**. However,

[10] In Ref. 3 we adopted the convention that B contained the operators which create *particles*, in order that one-particle states and their corresponding fields should have the same transformation properties. We return here to the more usual definition of fields, i.e., $B^*|0\rangle$ transforms similarly to the state $|B\rangle$.

[11] We use the notation and conventions of J. S. Bell, CERN Report No. 66-29, Vol. 1, 1966 (unpublished).

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we note from (2.7) that the commutator of time components of the currents at equal times $t=0$ is

$$[A_0^\dagger(\mathbf{x},0),\,A_0(\mathbf{y},0)] =\psi^\dagger(\mathbf{x},0)[\sigma_1\otimes I\otimes D_l^\dagger,\,\sigma_1\otimes I\otimes D_l] \times\psi(\mathbf{x},0)\delta(\mathbf{x}-\mathbf{y})+\text{S.T.} =\psi^\dagger(\mathbf{x},0)I\otimes I\otimes[D_l^\dagger,D_l]\psi(\mathbf{x},0)\delta(\mathbf{x}-\mathbf{y})+\text{S.T.}, \quad (2.10)$$

while the commutator of space components with the same space direction is

$$[A_j^\dagger(\mathbf{x},0),\,A_j(\mathbf{y},0)] =\psi^\dagger(\mathbf{x},0)[I\otimes\sigma_j\otimes D_l^\dagger,I\otimes\sigma_j\otimes D_l] \times\psi(\mathbf{x},0)\delta(\mathbf{x}-\mathbf{y})+\text{S.T.} =\psi^\dagger(\mathbf{x},0)I\otimes I\otimes[D_l^\dagger,D_l]\psi(\mathbf{x},0)\delta(\mathbf{x}-\mathbf{y})+\text{S.T.}, \quad (2.11)$$

where S.T. denotes possible Schwinger terms. We have used the fact that, for anticommuting fields ψ,

$$[\psi^\dagger(\mathbf{x},0)A\psi(\mathbf{x},0),\,\psi^\dagger(\mathbf{y},0)B\psi(\mathbf{y},0)] =\psi^\dagger(\mathbf{x},0)[A,B]\psi(\mathbf{x},0)\delta(\mathbf{x}-\mathbf{y})+\text{S.T.}$$

Thus,

$$[A_0^\dagger,A_0]=[A_j^\dagger,A_j], \quad (2.12)$$

up to possible Schwinger terms, which presumably do not contribute to the sum rules in question. Since the axial-vector currents A_j are contained in the **55**, we can use the explicit representations of these currents in terms of the baryon fields given in Ref. 3. Finally, we note that the nonrelativistic limit is adequate for evaluation of the commutator since the result, being the time component of a vector-current density evaluated between one-neutron states, is a Lorentz invariant.[11]

The $\Delta S=0$ weak-interaction current $A_0^{(0)}$ which occurs in Eq. (2.2) transforms under $SU(3)$ like the ρ^+ meson (or the Σ^+ baryon). Figure 2 of Ref. 3 shows that this current can be obtained by applying the lowering operator E_{-B} to the current transforming like K^{*+}, which is given in Eq. (2.4) of Ref. 3. We also apply the operator E_{-m} to lower the spin projection from $+1$ to 0. Thus (with a change of phase) we find terms of the form

$$A_3^{(0)}\propto j(\rho_0^+)=\tfrac{1}{4}\{\mathbf{\Xi}_-^{0*}\mathbf{\Xi}_-'^{-}-\mathbf{\Xi}_+^{0*}\mathbf{\Xi}_+'^{-}+\cdots\}, \quad (2.13)$$

with respect to the mathematical baryons in the **32**, where the subscripts denote spin projections. It is convenient to introduce a matrix representation for the currents in terms of a column $\mathfrak{B}$ with components

$$\mathbf{p},\,\mathbf{n},\,\mathbf{\Sigma}^+,\,\mathbf{\Sigma}^0,\,\mathbf{\Sigma}^-,\,\mathbf{\Lambda},\,\mathbf{\Xi}^0,\,\mathbf{\Xi}^-, \quad (2.14)$$

where, for example, $\mathbf{p}$ is a column with components

$$\mathbf{p}_+,\,\mathbf{p}_+',\,\mathbf{p}_-,\,\mathbf{p}_-', \quad (2.15)$$

and the primes denote members of the second mathematical octet in the **32**. In terms of this basis,

$$j(\rho_0^+)=\tfrac{1}{4}\mathfrak{B}^\dagger M^{(0)}{}_4\mathfrak{B}, \quad (2.16)$$

where

$$M^{(0)}=[-x^2(12)+(x/\sqrt{2})(34)-(ix/\sqrt{2})(36) +(x/\sqrt{2})(45)+(ix/\sqrt{2})(65)-(78)]\otimes[(12) -(34)]+[x(12)-(x^2/\sqrt{2})(34)-(ix^2/\sqrt{2})(36) -(x^2/\sqrt{2})(45)+(ix^2/\sqrt{2})(65)+(78)] \otimes[(21)-(43)]. \quad (2.17)$$

The first factor in each direct product is an 8×8 matrix acting on the column given by (2.14), while the second factor is a 4×4 matrix acting on columns such as that given by (2.15); the expression (ab) denotes a matrix with unity in the ath row and bth column, and zeros elsewhere; and $x=\exp(2\pi i/3)$. The scale of the axial-vector weak current is fixed by the requirements that the *vector* weak current give the standard value

$$\langle p|J_\mu|n\rangle=\bar{u}_p\gamma_\mu u_n \quad (2.18)$$

when evaluated between neutron and proton states, and that the axial-vector current have the same "length" as the vector current. By lowering (3.26) of Ref. 3 we find that the normalization factor is $\sqrt{24}$; thus,

$$A_3^{(0)}=(\sqrt{24})j(\rho_0^+)=(\sqrt{\tfrac{3}{2}})\mathfrak{B}^\dagger M^{(0)}\mathfrak{B}. \quad (2.19)$$

Since baryon fields anticommute, the commutator of the two currents takes the form

$$[A_3^{(0)\dagger},A_3^{(0)}]=\tfrac{3}{2}[\mathfrak{B}^\dagger M^{(0)\dagger}\mathfrak{B},\,\mathfrak{B}^\dagger M^{(0)}\mathfrak{B}] =\tfrac{3}{2}\mathfrak{B}^\dagger[M^{(0)\dagger},\,M^{(0)}]\mathfrak{B}. \quad (2.20)$$

Evaluation of the matrix commutator yields

$$[M^{(0)\dagger},M^{(0)}]=C\otimes[(11)+(33)] +C^*\otimes[(22)+(44)], \quad (2.21)$$

with

$$C=-(11)+(22)-(33)+(55) -(77)+(88)+i(46)-i(64).$$

Under the transformation (1.4) from mathematical $\mathfrak{B}$ to physical baryons $\mathfrak{B}$, (2.20) finally becomes

$$[A_3^{(0)\dagger},A_3^{(0)}] =\tfrac{3}{2}\mathfrak{B}^\dagger\{[-(11)+(22)-(33)+(55)-(77)+(88)] \otimes[(11)+(22)+(33)+(44)]+[(46)-(64)] \otimes[-(12)+(21)-(34)+(43)]\}\mathfrak{B} =\tfrac{3}{2}\{-p_+^*p_++n_+^*n_+-\Sigma_+^{+*}\Sigma_+^++\cdots\}. \quad (2.22)$$

We note that this commutator is no longer equal to minus twice the third component of isospin; in fact, since it connects Σ^0 and Λ', we see that it cannot be expressed as a linear combination of isospin and hypercharge operators. It must be a vector current, and hence it is a combination of terms from the $SU(3)\otimes R(3)$ multiplets (**8**,**1**), (**10**,**1**), and (**10***,**1**) in the **55**. The operators with the correct $SU(3)$ weight [00] in the (**8**,**1**) are just the hypercharge and the third component of isospin; hence it is (**10**,**1**) and (**10***,**1**) which contribute the operator connecting Σ^0 and Λ'. We can write (2.22) in the form

$$[A_0^{(0)\dagger},A_0^{(0)}]=-2I_3+\tfrac{1}{2}Z. \quad (2.23)$$

where

$$Z=[-(11)+(22)+(33)-(55)-(77)+(88)]\otimes[(11)+(22)+(33)+(44)]+3[(46)-(64)]\otimes[-(12)+(21)-(34)+(43)] \quad (2.24)$$

gives the difference between the $R(11)$ and $SU(6)$ results. With respect to this same basis (the physical baryons), the isospin and hypercharge operators are

$$I_3=\tfrac{1}{2}[(11)-(22)+2(33)-2(55)+(77)-(88)]\otimes[(11)+(22)+(33)+(44)] \quad (2.25)$$

and

$$Y=[(11)+(22)-(77)-(88)]\otimes[(11)+(22)+(33)+(44)]. \quad (2.26)$$

Evaluation of (2.23) between one-neutron states yields $\frac{3}{2}$ for the right-hand side. Making this change in (2.3) and using the values $X(\pi p)$ corresponding to (2.4)–(2.6) yields

$$|f_A|=1.41\text{–}1.52 \quad (2.27)$$

for the result from the $\Delta S=0$ commutation relation in $R(11)$.

B. $|\Delta S|=1$ Axial-Vector Currents

Weisberger[9] also obtained an independent evaluation of f_A from the commutator of the time component of the strangeness-changing ($\Delta S=1$) axial-vector weak current

$$A_0^{(1)}(x)=P^\dagger(x)\gamma_5\lambda(x) \quad (2.28)$$

with its Hermitian conjugate, where $\lambda(x)$ is the Λ-like quark field. In $SU(6)$, this commutator is

$$\left[A_0^{(1)\dagger}(0),\int d^3y\,A_0^{(1)}(\mathbf{y},0)\right]=-I_3(0)-\tfrac{3}{2}Y(0), \quad (2.29)$$

where $I_3(x)$ is the third component of the isospin density and $Y(x)$ is the hypercharge density. Evaluation of (2.29) between one-neutron and one-proton states yields, respectively,[9,11]

$$[X(Kn)-(1-2\alpha)^2]f_A^2=-1,$$

and

$$[X(Kp)-(2-4\alpha+8\alpha^2/3)]f_A^2=-2, \quad (2.30)$$

where

$$X(Kn)\equiv\frac{2M^2}{\pi g^2}\int\frac{dq_0|\mathbf{q}|}{q_0^2}[\sigma(K^+n,q_0)-\sigma(K^-n,q_0)]\approx-0.36 \quad (2.31)$$

and

$$X(Kp)\equiv\frac{2M^2}{\pi g^2}\int\frac{dq_0|\mathbf{q}|}{q_0^2}[\sigma(K^+p,q_0)-\sigma(K^-p,q_0)]\approx-0.71, \quad (2.32)$$

including the contributions from the unphysical regions below threshold. The parameter α is related to the f/d ratio:

$$\alpha/(1-\alpha)\equiv d/f. \quad (2.33)$$

The acceptable solution of (2.30) is

$$|f_A|=1.28\pm0.10,\quad \alpha=0.75\pm0.10, \quad (2.34)$$

where the limits given correspond to the estimated uncertainties of about 25% in the evaluation of the integrals (2.31) and (2.32).[9]

We again use the equality (2.12) between the commutator of the time components of the axial-vector currents and that of the space components. The $\Delta S=1$ current corresponding to Eq. (2.29) in $R(11)$ must transform under $SU(3)$ like the K^{*+} meson; thus we obtain the component $j(K_0^{*+})$ with zero-spin projection by merely applying E_{-m} to Eq. (2.4) of Ref. 3. With the normalization factor found earlier, this yields

$$A_3^{(1)}=(\sqrt{24})j(K_0^{*+})=(\sqrt{\tfrac{3}{2}})\mathfrak{B}^\dagger M^{(1)}\mathfrak{B}, \quad (2.35)$$

where

$$M^{(1)}=[-(1/\sqrt{2})(14)+(i/\sqrt{2})(16)-(25)+x^2(37)+(x^2/\sqrt{2})(48)-(ix^2/\sqrt{2})(68)]\otimes[(12)-(34)]+[(1/\sqrt{2})(14)-(i/\sqrt{2})(16)+(25)-x(37)-(x^2/\sqrt{2})(48)-(ix/\sqrt{2})(68)]\otimes[(21)-(43)]. \quad (2.36)$$

As in (2.20), the commutator of $A_3^{(1)}$ with its conjugate is found to be

$$[A_3^{(1)\dagger},A_3^{(1)}]=\tfrac{3}{2}\mathfrak{B}^\dagger[M^{(1)\dagger},M^{(1)}]\mathfrak{B}, \quad (2.37)$$

where

$$[M^{(1)\dagger},M^{(1)}]=D\otimes[(11)+(33)]+D^*\otimes[(22)+(44)]$$

and (2.38)

$$D=-(11)-(22)-(33)+(77)+(88)-i(46)+i(64).$$

With the transformation (1.4) from mathematical to physical states, (2.37) finally becomes

$$[A_3^{(1)\dagger},A_3^{(1)}]=\tfrac{3}{2}\mathfrak{B}^\dagger\{[-(11)-(22)-(33)+(55)+(77)+(88)]\otimes[(11)+(22)+(33)+(44)]+[(46)-(64)]\otimes[(12)-(21)+(34)-(43)]\}\mathfrak{B}=\tfrac{3}{2}\{-p_+^*p_+-n_+^*n_++\cdots\}. \quad (2.39)$$

Note again that (2.39) contains operators which connect Λ' and Σ^0, etc., and thus is not expressible simply in terms of isospin and hypercharge. Using the definitions (2.24)–(2.26), we can express (2.39) in a form analogous to (2.29):

$$[A_3^{(1)\dagger},A_3^{(1)}]=-I_3-\tfrac{3}{2}Y-\tfrac{1}{2}Z. \quad (2.40)$$

Evaluating (2.39) between one-neutron states in one case and one-proton states in the other yields $-\frac{3}{2}$ for the right-hand sides in both cases. Making these changes in (2.30) and using Weisberger's values for the integrals (2.31) and (2.32), we find

$$|f_A|=1.04\pm0.12\,,\quad \alpha=1.01\pm0.10 \tag{2.41}$$

to be the prediction of the $|\Delta S|=1$ commutation relations in $R(11)$, when the baryons are assigned to **32.**

C. Possible Representation Mixing in $R(11)$

In $SU(6)$, if the baryons are assigned to the simplest possible representation **56**, direct evaluation of the axial current matrix element leads to a value of f_A which is in violent disagreement with experiment. This is interpreted to mean that there is considerable mixing with octets from other representations of $SU(6)$. Such mixing does not invalidate the results mentioned above [Eqs. (2.5) or (2.6) and (2.34)], all of which are in reasonably good agreement with the experimental values[15] $|f_A|=1.18\pm0.025$ and $\alpha=0.67\pm0.03$ or 0.63, since the commutators (2.2) and (2.29) involve only the $SU(3)$ diagonal operators I_3 and Y.

On the other hand, in $R(11)$ the corresponding commutation relations (2.23) and (2.40) involve the additional operator Z which cannot be expressed in terms of I_3 and Y. Thus representation mixing does affect the results obtained from commutation relations in $R(11)$. We give below an example of representation mixing which will bring the $R(11)$ results (2.27) and (2.41) into agreement with experiment. (However, we make no claim as to the result of a direct evaluation of the axial current matrix elements.)

Comparing the action of Z on the baryonic **32** as given by (2.24) with the action of the diagonal operators $H_\alpha,\ \cdots,\ H_\epsilon$ on the baryonic **32** as given by Figs. 3 and 4 of Ref. 3, we find that

$$Z=2H_\alpha-2H_\gamma-2H_\delta-H_\epsilon\,. \tag{2.42}$$

Using the mappings given in Table II of Ref. 3, we find that Z can also be expressed in terms of $R(8)$ diagonal operators:

$$Z=2H_a-H_c-H_d\,. \tag{2.43}$$

Thus the evaluation of Z will be determined by the $R(8)$ representation into which we map the baryons. The **32** is the smallest $R(11)$ representation in which the baryons could be placed; the next smallest is the **320.** Restriction of **320** to $R(8)\otimes R(3)$ submultiplets gives

$$\begin{aligned}\mathbf{320}=[1000,1]_{320}\to{}&[1001,1]_{112}+[1010,1]_{112}\\&+[0001,3]_{32}+[0010,3]_{32}+[0010,1]_{16}\\&+[0001,1]_{16}\,.\end{aligned} \tag{2.44}$$

Assignment of the baryons to the two $R(8)$ spinor octets would lead to the same results as above. However, the two 56-dimensional $R(8)$ representations both contain $SU(3)$ octets:

$$\begin{aligned}&[1001]\to\mathbf{27}+\mathbf{10}+\mathbf{10}^*+\mathbf{8}+\mathbf{1}\,,\\&[1010]\to\mathbf{27}+\mathbf{10}+\mathbf{10}^*+\mathbf{8}+\mathbf{1}\,.\end{aligned} \tag{2.45}$$

Thus the baryons could be assigned to these, as well as to the two $R(8)$ octets. The state which transforms like the proton, say in the [1001],[16] is given by

$$\begin{aligned}|p\rangle_{56}=(\surd\tfrac{2}{5})\{&\tfrac{1}{3}\sqrt{2}(1-x)|002-1\rangle+\tfrac{1}{6}(x-x^2)|0001D\rangle\\&+|0001B\rangle+\tfrac{1}{6}(x^2-x)|0001A\rangle\\&+\tfrac{1}{3}\sqrt{2}(1-x^2)|200-1\rangle\}\,,\end{aligned} \tag{2.46}$$

where the letters A, B, C, D distinguish between states which are degenerate with respect to the $R(8)$ diagonal operators. We note immediately that $|p\rangle_{56}$ is not an eigenstate of Z since $Z=5$ for the state $|200-1\rangle$, whereas $Z=-1$ for all the others. Let the proton state be given by the combination of $R(8)$ octet and **56** states:

$$|p\rangle=\alpha|p\rangle_8+\beta|p\rangle_{56}\,. \tag{2.47}$$

Then

$$Z|p\rangle=-|p\rangle+(4/\surd5)(1-x^2)\beta|200-1\rangle\,, \tag{2.48}$$

and[16]

$$\langle p|Z|p\rangle=-1+(8/5)|\beta|^2. \tag{2.49}$$

This expression vanishes for

$$|\beta|=(5/8)^{1/2}. \tag{2.50}$$

The commutation relations also involve

$$\begin{aligned}\langle n|Z|n\rangle&=\langle p|E_{+A}ZE_{-A}|p\rangle\\&=\langle p|[E_{+A},Z]E_{-A}|p\rangle+\langle p|Z[E_{+A},E_{-A}]|p\rangle\\&=\langle p|[[E_{+A},Z],E_{-A}]|p\rangle+\langle p|Z|p\rangle\,,\end{aligned} \tag{2.51}$$

where the $E_{\pm A}$ are isospin raising and lowering operators. The double commutator is readily evaluated from (2.43) and the expressions for $E_{\pm A}$ and $H_A=2I_3$ in terms of $R(8)$ operators given by Table II of Ref. 3, with the result

$$[[E_{+A},Z],\,E_{-A}]=-2Z\,. \tag{2.52}$$

Hence for the choice of mixing parameter (2.50) we have

$$\langle n|Z|n\rangle=\langle p|Z|p\rangle=0\,, \tag{2.53}$$

so that the commutation relations (2.23) and (2.40) yield the $SU(6)$ results (which agree with experiment) when evaluated between proton and neutron states.

III. CONCLUSION

When the baryons are given the simplest possible assignment in $R(11)$, the Adler-Weisberger sum rules derived from current commutation relations for $\Delta S=0$

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[16] Exactly parallel statements can be made concerning the protonlike state in [1010], because of the symmetry of Z and of $R(8)$ under interchange of c and d operators. In particular, (2.49) is unchanged if $|p\rangle_{56}$ is taken to be a linear combination of states from the [1001] and the [1010].

and $|\Delta S|=1$ currents lead to inconsistent values of f_A; and neither of these values, nor the value of α, agrees well with experiment. By contrast, the corresponding values derived from $SU(6)$ commutation relations are entirely consistent with experiment. Thus when baryons are given a "simple" assignment, $R(11)$ might appear to be similar to $SU(6)$ in that it gives mixed good (the value for f_A obtained in Ref. 3) and bad results.

However, as Gell-Mann has pointed out,[7] results depending only on commutation relations should be considered more fundamental than symmetry predictions which depend on assignments to representations, since the former may still hold in the presence of symmetry breaking which obscures the latter. From this viewpoint, $SU(6)$ is quite satisfactory since the correct Adler-Weisberger predictions are independent of the assignment of baryons to $SU(6)$ representations. On the other hand, the corresponding predictions from $R(11)$ disappear when representation mixing is allowed. Furthermore, mixing with a representation of dimension at least 320 is required to bring the results into agreement with experiment, so that little can be claimed on the grounds of simplicity. These same criticisms apply also to the possibility that a modification of PCAC might bring the $R(11)$ results into agreement with experiment.

Thus we conclude that $SU(6)$ provides by far the most promising model of its kind [in the sense of Ref. 3, of a simple group which incorporates both $SU(3)$ and rotational symmetry], at least unless one is willing to go to a complexity an order of magnitude greater.

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$O(3,1)$ Symmetry and Conspiracy in Photoproduction of Pions at High Energy*

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Photoproduction of pions at high energy and small angles has been studied by using the Lorentz group including space reflection and the PCT transformation to project the "crossed partial waves." The dominant contribution to the differential cross section for photoproduction of pions at high energy comes from Toller poles in the complex λ plane with $M=1$ and $\tau=\pm1$, where M and λ are the Casimir operators of the group $O(3,1)$ and τ is the signature of the Toller pole. These poles in the complex λ plane have been decomposed and classified into different families of Regge poles with alternating signature and parity. The residues of the principal Regge trajectories with $\tau'=1$, $\sigma'=\pm1$, where τ' and σ' are, respectively, the signature and parity of the Regge trajectories, have been calculated explicitly in terms of the residue of the Toller pole with $M=1$ and $\tau=1$. These calculations allow us to derive a relation between the residues of the two conspiring Regge trajectories $\alpha_\pi(t)$ and $\alpha_\pi^{(c)}(t)$ at $t=0$, which is consistent with the kinematic constraints on the helicity amplitudes in photoproduction of pions along the forward direction.

I. INTRODUCTION

FOLLOWING the successful application of $O(3,1)$ symmetry by Toller[1] in the study of forward elastic scattering, Salam, Delbourgo, and Strathdee[2] have extended the use of the Lorentz group to nonforward and inelastic processes. The purpose of this paper is to study the photoproduction of pions at high energy and along the forward direction by using the principal series of unitary irreducible representation of the Lorentz group including space reflection and the PCT transformation to expand the helicity amplitudes in the crossed s channel. (For the meaning of "crossed partial waves" see Refs. 1 and 2.)

Recent experimental results[3] on photoproduction of pions showing a sharp peak along the forward direction have aroused considerable interest in this subject.[4] The large slope of the forward peak can be explained in terms of the exchange of a pion Regge trajectory, but the ordinary one-pion-exchange model does not give a peak. In fact, all single-particle-exchange models will predict a vanishing cross section along the forward direction in photoproduction of pions.[5] However, it has been possible to explain the existence of this sharp peak along the forward direction by assuming the exchange of a pion trajectory together with a second Regge trajectory, called the conspirator, which has opposite parity to and is degenerate with the former at $t=0$.[4]

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