$O(4)$ Symmetry and the Regge Trajectory of the Pion*

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An off-shell generalization of $O(4)$ symmetry is developed, which is applicable to unequal-mass reactions involving arbitrary spins. The symmetry breaking at $t\neq 0$ is also investigated. These methods are used to analyze the hypothesis that the pion Regge trajectory has the Toller quantum number $M=1$ at zero momentum transfer, as seems to be implied by high-energy photoproduction data. We find that if this hypothesis is correct, then the pion trajectory is necessarily quite complicated. The $M=1$ trajectory must mix with another trajectory. The $\bar{N}N\pi$ vertex function then shows a zero near $t=0$, in agreement with some fits to high-energy data, but this zero is factorizable. Moreover, a model is exhibited that seems consistent with the hypothesis of partially conserved axial-vector current.

THE nature of the pion's Regge trajectory is one of the outstanding questions of Regge-pole theory. The hypothesis that the pion may have the Toller[1,2] quantum number $M=1$ leads to a very interesting connection with the hypothesis of partially conserved axial-vector current (PCAC),[3] as well as to striking predictions for high-energy reactions, which seem consistent with photoproduction and np charge exchange data.[4,5] There are, however, several difficulties remaining, both with the connection to PCAC and with the high-energy predictions. For example, fits to np charge exchange and to π^+ photoproduction with $M=1$ pion exchange require rapidly varying pion residue functions with zeros near $t=0$.[4,5] Some authors have found this unreasonable, while others have disagreed on the order of these zeros.[6]

The quantum number M arises from the $O(4)$ or $O(3,1)$ symmetry of equal-mass scattering amplitudes at vanishing momentum transfer, $t=0$. In order to cope with the questions raised in the first paragraph, we must generalize the symmetry to unequal masses and investigate its breaking at $t\neq 0$. We do this by adopting an off-shell approach, using the Bethe–Salpeter (BS) equation as a model. Detailed investigations of the $N\bar{N}$ and $\pi\rho$ BS equations in the ladder approximation have been carried out: The results will be published in a lengthier communication.[7] In this paper we present those results that we have been able to infer from the models and that seem to be of a general nature.

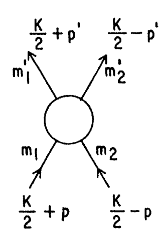

FIG. 1. Kinematics.

We write the BS equation as

$$M_{\alpha'\beta'\alpha\beta}(p',p;K) = B_{\alpha'\beta'\alpha\beta}(p',p;K)+\lambda\int d^4q\, M_{\alpha'\beta'\gamma'\delta'}(p',q;K) \times G_{\gamma'\delta'\gamma\delta}(q,K)B_{\gamma\delta\alpha\beta}(q,p,K)\,, \quad (1)$$

where G is the product of the two single-particle propagators. The momentum assignments are shown in Fig. 1. The M amplitude is related to the physical t-channel helicity amplitudes ($t=K^2$) by

$$T_{\lambda_c\lambda_d,\lambda_a\lambda_b}(s,t) = \bar{u}_{\alpha'}(\lambda_c, \tfrac{1}{2}K+p')\bar{u}_{\beta'}(\lambda_d, \tfrac{1}{2}K-p')M_{\alpha'\beta',\alpha\beta}(p',p;K) \times u_\alpha(\lambda_a, \tfrac{1}{2}K+p)u_\beta(\lambda_b, \tfrac{1}{2}K-p)\,, \quad (2)$$

where the u's are the appropriate external particle wave functions (spinor, polarization vector, etc.). To simplify the BS equation, we define the amplitude (henceforth suppressing subscripts whenever possible)

$$R(p',p,K)=M(p',p,K)G(p,K)\,, \quad (3)$$

which obeys the equation

$$R(p',p,K)F(p,K)=B(p',p,K) +\lambda\int d^4q\, R(p',q,K)B(q,p,K)\,, \quad (4)$$

where $F\equiv G^{-1}$. We make the usual Wick rotation to make the p space Euclidean.

We now expand R, B, and F in a set of basis states which transform irreducibly under $O(4)$ rotations of the relative momenta p and p', with K held fixed. Such a rotation is a symmetry operation for the BS equation at $K=0$, but we shall also be interested in small, nonzero values of t, at which the symmetry is broken.

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[6] D. Amati, G. Cohen-Tannoudji, R. Jengo, and Ph. Salin, Phys. Letters **26B**, 510 (1968).
[7] W. R. Frazer, F. R. Halpern, H. M. Lipinski, and D. R. Snider. U.C.S.D. Report (unpublished); H. M. Lipinski and D. R. Snider, U.C.S.D. Reports (unpublished).

We choose our $O(4)$ reference frame such that $K=(0,0,0,-i\sqrt{t})$. The $O(4)$ basis states are written $|n,M,J,m,\Sigma\cdots\rangle$. The quantum numbers n and M, the eigenvalues of the Casimir operators, are for physical values of J, the maximum and minimum values of J contained in the representation.[2,7] It is frequently convenient to label the representations by j_1 and j_2, where $n=j_1+j_2$ and $M=j_1-j_2$. Then j_1 and j_2 are the eigenvalues of the Casimir operators of the two $SU(2)$ subgroups of $O(4)\sim SU(2)\times SU(2)$. The quantum number Σ is the total spin, not in the rest frame but in the frame in which the relative momentum points in the timelike direction. The three dots indicate other quantum numbers, such as intrinsic parities, which may be needed to complete the description of the state.

The amplitudes $M_{\alpha'\beta'\alpha\beta}(p',p,K)$ are matrix elements between states $|\alpha,\beta,\Omega_p\rangle$, where the direction of p is specified by three angles $\Omega_p=\{\varphi,\theta,\psi\}$, and where α and β are the spinor indices. The transformation coefficients between such states and the $O(4)$ basis states

$$Z_{Jm\Sigma\cdots}{}^{nM}(\alpha,\beta,\Omega)=\langle\alpha,\beta,\Omega|nMJm\Sigma\cdots\rangle\,,\quad |M|\leq J\leq n \text{ and } |M|\leq\Sigma\leq n \quad (5)$$

have been calculated in Ref. 7. They vanish unless $|M|\leq J\leq n$ and $|M|\leq J\ \Sigma\leq n$. By way of illustration, the transformation coefficient for a two-particle state with spins 0 and S is given by

$$Z_{Jm\Sigma}{}^{nM}(\alpha,\Omega)=N_\Sigma{}^{nM}\sum_{m'}D_{Jm,\Sigma m'}{}^{nM*}(\Omega)D_{\alpha,\Sigma m'}{}^{j_1j_2}(\Omega)\,,\quad (6)$$

where the D's are the $O(4)$ representation matrices [see Ref. 7 or Ref. 2, Eqs. (6) and (7)], and where j_1 and j_2 specify the way the field of the spin-S particle transforms under homogeneous Lorentz transformations. For example, the vector field has $j_1=j_2=\frac{1}{2}$. The normalization factor is

$$N_\Sigma{}^{nM}=\frac{1}{\pi}\left[\frac{(n+M+1)(n-M+1)}{2(2\Sigma+1)}\right]^{1/2}.\quad (7)$$

After projecting Eq. (4) onto our $O(4)$ basis states, we find that the BS equation takes the following form:

$$\sum_{\Upsilon'}R_J{}^{\Upsilon''\Upsilon'}(p',p,t)F_J{}^{\Upsilon'\Upsilon}(p,t)=B_J{}^{\Upsilon''\Upsilon}(p',p,t)+\lambda\sum_{\Upsilon'}\int q^3dq\,R_J{}^{\Upsilon''\Upsilon'}(p',q,t)B_J{}^{\Upsilon'\Upsilon}(q,p,t)\,,\quad (8)$$

where we have introduced the symbol $\Upsilon\equiv\{n,M,\Sigma\}$ to stand for the set of $O(4)$ quantum numbers. Henceforth we specialize to simple interactions for which B is independent of K, and is therefore diagonal in the $O(4)$ quantum numbers: $B_J{}^{\Upsilon'\Upsilon}=\delta_{\Upsilon'\Upsilon}B^\Upsilon$.[8] No interesting features of the problem are changed by this simplification. The symmetry breaking is now confined to the inverse propagator F, which takes the form

$$F_J{}^{\Upsilon'\Upsilon}(p,t)=F^{(0)}(p)\delta_{\Upsilon'\Upsilon}+(\sqrt{t})F_{\Upsilon'\Upsilon J}{}^{(1)}(p)+tF_{\Upsilon\Upsilon'J}{}^{(2)}(p)+\cdots.\quad (9)$$

If we take G to be the simple unrenormalized propagator, then Eq. (9) is a finite polynomial in $\sqrt{t}$. At $t=0$, the equations decouple and $R_J{}^{\Upsilon'\Upsilon}=0$ unless $\Upsilon=\Upsilon'$.[9] There is no problem with unequal masses in this formalism, since we are working with off-shell amplitudes. When we discuss the construction of the physical helicity amplitudes from the R amplitudes, we shall find the peculiarities of the unequal-mass kinematics appearing explicitly.

Our formalism is also well suited to discussion of the $O(4)$ symmetry breaking at $t\neq0$ as a perturbation about the symmetry point.[10] We are interested in Regge poles and their residues near $t=0$. To find the Regge trajectories it is necessary to continue the amplitude $R_J{}^{\Upsilon'\Upsilon}(p',p,t)$ in J, keeping $\kappa=n-J$ and $\kappa'=n'-J$ integrals.[2] At $t=0$, $\kappa=\kappa'$ and $M=M'$, so that it is possible to label trajectories by their M and κ values ($\kappa=0$ means a parent trajectory, $\kappa=1$ means first daughter, etc.). In order to investigate the $M=1$ hypothesis for the pion, we shall concentrate our attention on a trajectory having $M=1$ at $t=0$. We assume that our interaction B is sufficiently well behaved that the BS equation is of Fredholm type. If B is not sufficiently cooperative, we impose a cutoff. This will be discussed in more detail in forthcoming papers. Moreover, we assume that B_J is regular in J, permitting the continuation of the scattering amplitude in J, and we further assume that there exists a Regge trajectory $J=\alpha(t)$, with $M=1$ at $t=0$. This can be investigated numerically in specific BS models, and indeed we have found that, in the BS equation for $N\bar{N}$ scattering in the ladder approximation through scalar meson exchange, the highest-ranking trajectory at $t=0$ with the quantum numbers of the pion has $M=1$. A numerical investigation of the behavior of the trajectories for small finite t is in progress. Such a calculation has already been carried out by Chung and Snider for the spinless case.[11]

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[8] More correctly, the kernel $B_J{}^{\Upsilon'\Upsilon}$ of the integral equation is diagonal only in the $O(4)$ Casimir quantum numbers n and M, at $t=0$.

[9] Since $F^{(1)}$ involves only one power of K, it obeys a selection rule: Either $\Delta n=\pm1$ and $\Delta M=0$ or $\Delta n=0$ and $\Delta M=\pm1$. Similarly, $F^{(2)}$ involves only $|\Delta n|\leq2$, $|\Delta M|\leq2$, so that only a finite set of coupled equations need be solved to obtain a solution to a given order in $\sqrt{t}$.

[10] Off-shell generalizations of $O(4)$ have also been made by G. Domokos, Phys. Rev. **159**, 1387 (1967); R. Sawyer, *ibid.* **167**, 1372 (1968); G. Domokos and P. Suranyi, Hungarian Academy of Sciences Central Research Institute for Physics Report (to be published). R. Delbourgo, A. Salam, and J. Strathdee [Phys. Letters **25B**, 230 (1967)] have constructed a generalization to finite t, but it is not the same as the one presented here. In particular, their formalism necessarily involves daughter trajectories.

[11] V. Chung and D. R. Snider, Phys. Rev. **162**, 1639 (1967).

TABLE I. Kinematical behavior of transformation coefficients $X(\lambda,t)$ connecting $O(4)$ basis states to helicity states; $N_\Sigma{}^{nM}d_{J\Sigma\lambda}{}^{nM}$ factor. Here $\Delta=m_1{}^2-m_2{}^2$ and $p=NP/2\sqrt{t}$, where $N^2=-t+(m_1+m_2)^2$ and $P^2=-t+(m_1-m_2)^2$.

Υ n	M	Σ	σ or ν at $J=0$	$N_\Sigma{}^{nM}d_{J\Sigma0}{}^{nM}(\psi_p)$ $(\lambda=0)$	$N_\Sigma{}^{nM}d_{J\Sigma1}{}^{nM}(\psi_p)$ $(\lambda=1)+(\lambda=-1)$	$(\lambda=1)-(\lambda=-1)$
J	1	1	ν	$(\sqrt{J})p^{J-1}$	$\Delta p^{J-1}/\sqrt{t}$	p^J
$J+1$	1	1	ν	$(\sqrt{J})\Delta p^{J-1}/\sqrt{t}$	$a\Delta^2p^{J-1}t^{-1}+bp^{J+1}$	$\Delta p^J/\sqrt{t}$
J	0	1	ν	$(\sqrt{J})\Delta p^{J-1}/\sqrt{t}$	p^{J-1}	0
$J+1$	0	1	σ	$aJ\Delta^2p^{J-1}t^{-1}+bp^{J+1}$	$\Delta\sqrt{J}p^{J-1}/\sqrt{t}$	0
J	0	0	σ	p^J	0	0
$J+1$	0	0	σ	$\Delta p^J/\sqrt{t}$	0	0

In this paper, we are concerned primarily with the general nature of the pion residue function, not with numerical results. When we apply perturbation theory about $t=0$, we find a result that is, at first sight, quite discouraging: We find that as long as nondegenerate perturbation theory is valid, the physical pion cannot lie on an $M=1$ trajectory. This has been guessed by several authors,[12,13] who made qualitative arguments similar to the following: If the pion pole were at $t=0$ with $M=1$, its residue would have to vanish, since $J\geq M$ for integral values. To be more precise, an $M=1$ "pion" would choose nonsense at $t=0$. Then if it is possible to continue smoothly to $t=m_\pi{}^2$, this pion would continue to choose nonsense. In the context of the BS equation it is possible to prove these statements. If, however, the uncoupled trajectories cross, then perturbation theory is no longer valid. In order to make these remarks more quantitative, we must return to another technical matter: the projection of physical helicity amplitudes.

We wish to obtain the transformation coefficient between a physical helicity state and an $O(4)$ basis state,

$$X^{\Upsilon*}(\lambda_1,\lambda_2,t)=\langle Jm\lambda_1\lambda_2|nMJm\Sigma\cdots\rangle. \tag{10}$$

We can obtain this by combining Eqs. (2) and (5), and by observing that the external wave function $u_\alpha(q)$ of a particle of spin S is given simply, in terms of representation matrices of $O(3,1)$, by

$$u_\alpha(\lambda,q)=D_{\alpha,S\lambda}{}^{j_1j_2}(\Omega_q). \tag{11}$$

For the spin-zero–spin-S scattering example of Eq. (6),

TABLE II. The $d_{\Sigma S\lambda}{}^{j_1j_2}$ factor in $X^\Upsilon(\lambda,t)$. To form X^Υ's, multiply coefficient from Table I by this factor.

Spins	Σ	Helicity state	Factor
0 1	0	$\lambda=0$	NP
0 1	1	$\lambda=\pm1$	1
0 1	1	$\lambda=0$	1
$\frac{1}{2}$ $\frac{1}{2}$	1	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle-\lvert-\frac{1}{2}\,-\frac{1}{2}\rangle$	N
$\frac{1}{2}$ $\frac{1}{2}$	0	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle-\lvert-\frac{1}{2}\,-\frac{1}{2}\rangle$	P
$\frac{1}{2}$ $\frac{1}{2}$	1	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle+\lvert-\frac{1}{2}\,-\frac{1}{2}\rangle$	P
$\frac{1}{2}$ $\frac{1}{2}$	0	$\lvert\frac{1}{2}\,\frac{1}{2}\rangle+\lvert-\frac{1}{2}\,-\frac{1}{2}\rangle$	N

[12] G. F. Chew (private communication).

[13] R. Sawyer, Phys. Rev. Letters 19, 137 (1967).

we find that

$$X^\Upsilon(\lambda,t)=N_\Sigma{}^{nM}d_{J\Sigma\lambda}{}^{nM}(\psi_p)d_{\Sigma S\lambda}{}^{j_1j_2}(\psi_1-\psi_p), \tag{12a}$$

where

$$\cos\psi_1=\frac{t+m_1{}^2-m_2{}^2}{2m_1\sqrt{t}},\qquad \cos\psi_p=\frac{m_1{}^2-m_2{}^2}{[t(2m_1{}^2+2m_2{}^2-t)]^{1/2}}. \tag{12b}$$

These angles are not necessarily real, so that d's in Eq. (12a) are defined by analytic continuation.

Tables I and II list the important features of some of the X's, which we shall need later. Note that Eqs. (12) exhibit the well-known singular nature of the limits of equal masses and $t\to0$. One case is especially simple: First take equal masses, then go to $t=0$. At this point, $\psi_p=\psi_1=\frac{1}{2}\pi$, and Eq. (12a) reduces to essentially the form derived by Freedman and Wang.[2] A virtue of our formalism is that this limit appears as a special case of our more general formulas, and all the appropriate kinematical factors are exhibited.

Now let us return to the question of an $M=1$ trajectory in the vicinity of $t=0$. We see from Tables I and II that the residue of such a pole vanishes at $J=0$ in the $\lambda=0$ (sense) state, but remains finite in the $\lambda=1$ (nonsense) state, as we mentioned above. But the question remains: Away from $t=0$, where the $O(4)$ symmetry is broken and $M\neq1$ contributions are mixed in, does the residue of a trajectory which is $M=1$ at $t=0$ continue to choose nonsense at $J=0$? We can answer this by analyzing "sense and nonsense" in the $O(4)$ basis states $|n,M,J,m,\Sigma\rangle$. The transformation coeffi-

TABLE III. The t and J dependence near $t=0$ or $J=0$ of $O(4)$ matrix elements of the inverse propagators. The matrix is symmetric.

n	M	Σ	$J,1,1$	$J,0,1$	$J+1,0,1$	$J,0,0$	$J+1,0,0$
J	1	1	1				
J	0	1	$\sqrt{t}$	1			
$J+1$	0	1	$t\sqrt{J}$	$\sqrt{(tJ)}$	1		
J	0	0	$\sqrt{(tJ)}$	$\sqrt{J}$	$\sqrt{t}$	1	
$J+1$	0	0	$t\sqrt{J}$	$\sqrt{(tJ)}$	1	$\sqrt{t}$	1

cients Z^{T} vanish unless, for physical J, $|M|\leq J\leq n$ and $|M|\leq\Sigma\leq n$. We label the corresponding states σ or ν at a given physical J according to whether or not these restrictions are met, while we reserve for the terms sense and nonsense their usual meaning in terms of helicities. These labels are given in Table I for $J=0$. One finds the result from Table I that σ states are sense choosing at $J=0$, while ν states are nonsense choosing. Thus if the $t\neq 0$ perturbation is to succeed in making the $M=1$ trajectory choose sense at $J=0$, it must involve mixing with an σ state. But we have found that matrix elements for σ-ν transitions vanish like $\sqrt{J}$, just as is well known to be the case for sense-nonsense transitions. (See Table III.) Then as long as nondegenerate perturbation theory is applicable, the residue of a trajectory which is $M=1$ at $t=0$ continues to choose nonsense even in the presence of the finite-t perturbation.

This reasoning fails to apply to a case where there is a crossing of the uncoupled trajectories.[14] To exhibit this situation most clearly, we consider a model version of Eq. (8), which can be derived as an approximation to Eq. (8) by expanding in eigenfunctions of the kernels. In this model we couple one ν state (i.e., the $M=1$, $J=n$ state which we hope to associate with the pion) with one σ state by means of the algebraic equation[7]

$$\begin{pmatrix} a[J-\alpha_\nu(t)] & ct^\omega\sqrt{J} \\ dt^\omega\sqrt{J} & b[J-\alpha_\sigma(t)]\end{pmatrix}\begin{pmatrix} R_{\nu\sigma} \\ R_{\sigma\sigma}\end{pmatrix}=\begin{pmatrix} 0 \\ B_\sigma\end{pmatrix}, \quad (13)$$

where a, b, c, and d are constants; $\alpha_i(t)$ is the i trajectory in the absence of coupling; and ω depends on the particular trajectory chosen (see Table III). There is a similar equation in which the final subscript is ν. The solution of these equations is

$$\begin{aligned} R_{\nu\nu}&=b(J-\alpha_\sigma)B_\nu/\det, & R_{\nu\sigma}&=-ct^\omega(\sqrt{J})B_\sigma/\det, \\ R_{\sigma\sigma}&=a(J-\alpha_\nu)B_\sigma/\det, & R_{\sigma\nu}&=-dt^\omega(\sqrt{J})B_\nu/\det, \end{aligned} \quad (14)$$

where

$$\det=ab(J-\alpha_\nu)(J-\alpha_\sigma)-cdt^{2\omega}J. \quad (15)$$

At $t=0$ the trajectories decouple, and we shall be interested in the one which is type ν at $t=0$, which we shall call the π trajectory. Again at $J=0$ the trajectories decouple, and either $J=\alpha_\sigma$ or $J=\alpha_\nu$ at this point. That is, the π trajectory coincides with either the σ or ν trajectory at $J=0$. If it coincides with α_ν, nondegenerate perturbation theory applies, and indeed we see from Eq. (14) that $R_{\sigma\sigma}$ vanishes at this point. However, the π trajectory may coincide instead with α_σ at $J=0$, by virtue of a nearby crossing of the uncoupled trajectories, such as shown in Fig. 2. We shall call this situation *trajectory mixing*. In this case, $R_{\sigma\sigma}$ is finite at $J=0$, and $R_{\nu\nu}=0$ instead. This seems to be the only possible way in which the pion could lie on a trajectory which has $M=1$ at $t=0$. The rest of this paper will be devoted to

[14] R. Sugar and R. Blankenbecler, Phys. Rev. Letters **20**, 1014 (1968).

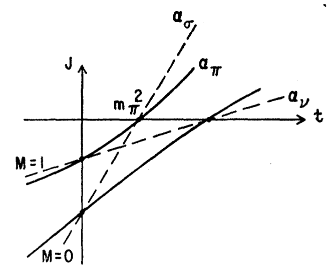

FIG. 2. A Chew-Frautschi plot showing a level-crossing situation; - - - - uncoupled trajectories, —— actual trajectories. Since the lower trajectory is pure $M=1$ at $J=0$, it chooses nonsense at that point.

exploring the consequences of such a trajectory-mixing model of the pion.[15]

The residues of the poles in Eq. (14) factor, as they also do in any Fredholm-type BS equation, yielding a pole of the form $M_{ij}=\Gamma_i\Gamma_j/(J-\alpha_\pi)$, where $i,j=\nu,\sigma$. In the trajectory-mixing model the vertex functions of the π trajectory can be seen from Eq. (14) to behave as follows: $\Gamma_\nu\propto\sqrt{\alpha_\pi}$ and $\Gamma_\sigma\propto t^\omega$. The vertex functions in helicity amplitudes are then given by

$$\gamma_\pi(\lambda,t)=\sum_{\mathrm{T}} X^{\mathrm{T}}(\lambda,t)\Gamma_{\mathrm{T}}(t). \quad (16)$$

In the following, we shall consider only one trajectory of each type, ν and σ, although in general there may be several trajectories of each type involved in the mixing, as well as others whose contributions can be obtained from simple nondegenerate perturbation theory. These add no new features to the problem.

There are several possible trajectory-mixing models for the pion, depending on which of the σ trajectories participates in the mixing with the ν trajectory. This is a question of detailed dynamics, to which we cannot give a reliable answer, but we shall exhibit the form of the pion residue function in two models. Perhaps the most natural choice for the trajectory is $M=0$, $\Sigma=0$, $\kappa=0$ (parent). With this choice $\omega=\frac{1}{2}$, and one finds from Tables I and II and Eqs. (14) and (16) that some important $\lambda=0$ (sense) pion residues are: (see Tables I and II for definitions of the kinematic factors)

(spin 0)-(spin 1),

$$\gamma_\pi(t)=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma N^2P^2](p^J/NP); \quad (17a)$$

(spin $\frac{1}{2}$)-(spin $\frac{1}{2}$), singlet,

$$\gamma_\pi(t)=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma P^2](p^J/P); \quad (17b)$$

(spin $\frac{1}{2}$)-(spin $\frac{1}{2}$), triplet,

$$\gamma_\pi(t)=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma N^2](p^J/N); \quad (17c)$$

[15] At $t=0$, the trajectory of the pion's conspirator coincides with the trajectory at the pion. In contrast with the pion's trajectory, we assume that the pion's conspirator does not mix with any other trajectory. If the trajectory is rising, it will choose nonsense at $J=0$ by virtue of the above arguments.

where the g's are functions, presumably slowly varying, of the external masses and of t. The factors in parentheses agree with the well-known kinematical factors for unequal-mass scattering.[16]

Note that the $\bar{N}N\pi$ vertex, given by Eq. (17b), probably has a zero in the small-t region (since $P^2=t$ for this case), as found necessary if the photoproduction data is to be fit with $M=1$ pion exchange.[4] If the masses of the spin-$\frac{1}{2}$ particles are made unequal, the zero moves away from the small-t region. The zero in the $\bar{N}N\pi$ vertex implies a double zero in the pion residue in np charge exchange, whereas the published fits use a single zero.[5] These fits should be reexamined.

The spin-0–spin-1 residue also indicates a nearby zero, but not at the same place as the $\bar{N}N\pi$ vertex zero. On the other hand, consider the $\bar{N}N^*\pi$ vertex, where N^* denotes a "pseudonucleon" of spin $\frac{1}{2}$ and negative parity—for example, a pion and nucleon in a relative S state. This vertex (evaluated at $t=m_\pi^2$) is just the s-wave π-N scattering amplitude with the π-N center-of-mass energy equal to m_2. Then the pion will occur in the triplet state, so Eq. (17c) is relevant. Since this vertex involves N^2 rather than P^2, it does not behave very differently for equal or unequal external masses. This does not conflict with anything currently known from study of high-energy reactions, but it is relevant to Mandelstam's remarks about the possible connection of PCAC with the pion trajectory having $M=1$ at $t=0$.

Mandelstam argued that if the pion has $M=1$ and if it had zero mass then its coupling to all equal-mass channels would have to vanish.[3] This is Adler's self-consistency condition, deduced from PCAC. This is indeed consistent with the limit $t=m_\pi^2\rightarrow 0$ in Eq. (17). But there is the difficulty, already noted in other contexts,[17] that the $\bar{N}N^*\pi$ vertex vanishes only by virtue of the factor $\sqrt{t}$. This factor is the same for equal mass N and N^* (soft-pion emission) as it is for unequal mass (hard-pion emission), whereas according to PCAC, the π-N s-wave scattering amplitude should be small at threshold compared to its value well above threshold.[18]

While we have concluded that the hypothesis of an $M=1$ pion trajectory does not unambiguously imply PCAC, we find it possible to construct models in which this hypothesis is not obviously inconsistent with PCAC. One such model has the $M=1$ trajectory mix with an $M=0$, $\Sigma=1$, $\kappa=1$ (i.e., first daughter) trajectory. This would require a very steep slope of the uncoupled daughter trajectory $\alpha(t)$ in order to avoid having its parent lie unacceptably high.[19] Such a model leads to the following predictions:

(spin 0)-(spin 1),

$$\gamma_\pi=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma N^2P^2](p^J/NP)\,; \qquad (18a)$$

$\frac{1}{2}$-$\frac{1}{2}$, singlet,

$$\gamma_\pi=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma N^2P^2](p^J/P)\,; \qquad (18b)$$

$\frac{1}{2}$-$\frac{1}{2}$, triplet,

$$\gamma_\pi=(\sqrt{t})[g_\nu\alpha_\pi(t)+g_\sigma N^2P^2](p^J/N)\,. \qquad (18c)$$

The contributions of other trajectories not involved in the mixing can be calculated from perturbation theory. The additional terms which these trajectories contribute to Eqs. (17) and (18) are proportional to powers of t or to $(m_1^2-m_2^2)$.

All three vertex functions in Eq. (18) have zeros near $t=0$ in the equal-mass case. Moreover, all three predict that the ratio of soft- to hard-pion emission will be small, of the order of $m_\pi^2/(m_1-m_2)^2$.

To summarize, we have found that if the pion lies on a Regge trajectory having $M=1$ at $t=0$, as seems to be implied by the high-energy data, then the pion trajectory is necessarily quite complicated. The $M=1$ trajectory must mix with another trajectory. The resulting $N\bar{N}$ residue function shows a zero near $t=0$, in agreement with some fits to high-energy data, but this zero is factorizable. Finally, a model has been exhibited which seems consistent with PCAC.

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[19] The parent $M=0$, $\Sigma=1$ trajectory might be identified with the A_1.