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Dispersion Methods in the $2V$ Sector of the Lee Model*

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The $2V$ sector of the Lee model is examined from the standpoint of dispersion theory. In this approach, vertices of the type pointed out by Blankenbecler and Cook are used for the purpose of establishing the bound state and eigenvalue condition corresponding to the static interaction of two V particles. Following Amado, we also determine the scattering and production amplitudes characteristic of this sector. In relying on his methods, we carry out exact dispersion calculations involving the contraction of a composite particle (the VN bound state). The states found in the V and VN sectors are the only intermediate states used throughout.

I. INTRODUCTION

THE Lee model of a (partly) soluble field theory with a nontrivial renormalization problem continues in the literature to provide a very valuable framework for the discussion of many dynamical questions and techniques of calculation. In recent communications, the Lehmann-Symanzik-Zimmermann (LSZ) formalism[1] and the Tamm-Dancoff (TD) method[2] have been used to obtain a complete solution of the $2V$ sector, thereby extending the solved aspect of the model. This sector is more suggestive than the VN subspace in that both sources undergo renormalization, and is similar to the $V\theta$ sector in that it embraces two elastic scattering amplitudes, a production amplitude, and a bound-state problem. In the present paper, we undertake dispersion calculations of the bound-state parameters and the collision amplitudes, assuming, as in Refs. 1 and 2, that the heavy particles are bosons with zero separation.

The techniques of dispersion theory have previously been used by Blankenbecler and Cook[3] in an attempt to calculate bound-state parameters of physical interest. These authors introduce a vertex function closely related to the Bethe-Salpeter amplitude and show that its corresponding dispersion relation yields bound-state information such as the asymptotic D-S ratio for the deuteron. Since their effort is limited to the one-pion-exchange approximation, it is of interest, even at the static-model level, to apply dispersion methods to composite particle systems involving more intermediate states. It will be shown that the $2V$ system is one such exactly soluble example.

In carrying out our development of the $2V$ eigenvalue condition and its corresponding bound state, we shall accept relevant matrix elements that have already been treated in the literature. We recall that, in their effort to clarify a dispersion analysis of the π-meson lifetime, Goldberger and Treiman[4] (GT) set up a modified Lee theory that led them to consider some matrix elements in the first nontrivial sector of the original model. Of these, we shall require the $V \rightleftarrows N\theta$ vertex function $K(\omega)$ and the $N\theta$ scattering amplitude $\mathfrak{M}(\omega)$. The former is closely related to the function $L(\omega)$ introduced by DeCelles and Feldman,[5] who discussed a dispersion approach to the renormalization constants δm_V and Z. Note that DeCelles has also pursued this idea into the realm of quantum electrodynamics.[6] Matrix elements characteristic of the VN sector will also arise and these are easily found. Our success in achieving a dispersion solution of the $2V$ bound-state problem has its roots in which particles we choose to contract. In accordance with the route adopted here, we need only introduce at various stages the intermediate states characteristic of the V and VN sectors. This is a distinguishing feature over other possibilities that would allow the more complicated states of the $V\theta$ sector to enter. The most involved singular integral equations that we encounter are of the type studied by Blankenbecler and Gartenhaus,[7] examples of which have been solved in the model context by Amado,[8] by Muta,[9] and by Vaughn.[10]

The basic idea behind our treatment of the above-mentioned eigenvalue condition is to generate two simultaneous algebraic relations connecting two vertices of the type pointed out by Blankenbecler and Cook. The same situation exists in the analogous, but much simpler, dispersion calculation of the VN potential energy.[11] That case involves the vertices $\Gamma_1 = \langle N|f_V|B_0\rangle$ and $\Gamma_2 = \langle V|f_N|B_0\rangle$, where f_V (f_N) denotes the V (N) particle current operator at time $t=0$ and $|B_0\rangle$ is called the physical VN bound state with eigenvalue $2m+\omega_0$ and normalization constant Z_0. Both one-particle physical states $|V\rangle$ and $|N\rangle$ are assigned the same energy m. We know that $G(\omega_0)=0$ and $Z_0^{-2}=G'(\omega_0)$, where the prime denotes differentiation and where the inverse of $G^+(W)$ [see Eqs. (19) and (12)] is the Fourier transform of the VN propagator evaluated at the energy $W+2m$. The first condition determines ω_0

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(at zero separation) as a function of the renormalized coupling constant g. When g is below its critical value (no ghost condition), ω_0 is real (negative) and single-valued.[12] It turns out that

$$\Gamma_1 = -\omega_0 Z_0, \tag{1a}$$

$$\Gamma_2 = Z Z_0 \delta m_V. \tag{1b}$$

We shall also accept that δm_V and Z have already been established by dispersion methods.

To obtain the scattering and production amplitudes, we follow the method devised by Amado. In our case, however, this requires the contraction technique for the composite particle (VN bound state). Thus we find an opportunity to gain some model experience with exact dispersion calculations involving the scattering of an elementary particle by a composite one. Of course, it would be even more enlightening to explore the scattering of a θ by the $2V$ system, since we could then investigate the interesting possibility of a three-particle dynamical pole below the $2V+\theta$ elastic threshold. This aspect is presently under investigation.

II. BOUND-STATE PROBLEM

In this section, the methods of dispersion theory are applied to various matrix elements for the purpose of deriving an eigenvalue condition for the static-interaction energy of two V particles. After accomplishing this, without having to calculate state vectors, we go on to construct the TD expansion for the corresponding bound state.

A. Γ Vertex

Our point of departure is the most obvious vertex, namely,

$$\Gamma = \langle V | f_V | B \rangle, \tag{2}$$

where $|B\rangle$ is the physical $2V$ bound state with eigenvalue $2m+\omega_B$ that defines ω_B as the potential energy of static interaction. The V-particle current operator at time t is given by

$$\begin{aligned} f_V(t) &= [-i(d/dt)+m]\psi_V(t) \\ &= -\delta m_V \psi_V(t) - (g/Z)\psi_N(t) A(t), \end{aligned} \tag{3}$$

where $X(\omega)$ and $A(t)$ are abbreviations for $f(\omega)/(2\omega\Omega)^{1/2}$ and $\sum_k X(\omega) a_k(t)$, respectively. As usual, the cutoff factor $f(\omega)$ serves to suppress the ultraviolet divergence, and depends only on the relativistic θ-particle energy $\omega = (k^2+\mu^2)^{1/2}$. The quantization volume is Ω. The Hamiltonian H and the equal-time commutators are given by

$$\begin{aligned} H = Z(m+\delta m_V)\psi_V^\dagger \psi_V + m\psi_N^\dagger \psi_N + \sum_k \omega a_k^\dagger a_k \\ + g\psi_V^\dagger \psi_N A + g\psi_N^\dagger \psi_V A^\dagger \end{aligned} \tag{4}$$

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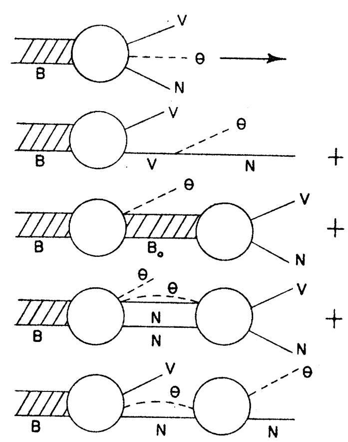

FIG. 1. Dispersion graph for $T(\omega)$ according to Eq. (15).

and

$$[a_k(t), a_{k'}^\dagger(t)] = \delta_{kk'}, \quad [\psi_V(t), \psi_V^\dagger(t)] = Z^{-1}, \quad [\psi_N(t), \psi_N^\dagger(t)] = 1. \tag{5}$$

Applying the LSZ contraction technique to the V particle in Γ, we obtain

$$\Gamma = i\int_{-\infty}^{\infty} e^{imt} \langle 0 | [f_V(t), f_V] \theta(t) | B \rangle dt. \tag{6}$$

If a complete set of intermediate states is introduced, we can use the time translation property

$$f_V(t) = \exp(iHt) f_V \exp(-iHt) \tag{7}$$

to do the integrations, and we find in continuous space that

$$\Gamma = \frac{1}{4\pi^2}\int_\mu^\infty k f^2(\omega) K(\omega) T(\omega) \left(\frac{1}{\omega} + \frac{1}{\omega-\omega_B}\right) d\omega. \tag{8}$$

Only the $N\theta$ scattering states, chosen as in-states, contribute, since $\langle 0 | f_V | V \rangle$ vanishes. The functions $K(\omega)$ and $T(\omega)$ are defined by

$$K(\omega) = X^{-1}(\omega) \langle 0 | f_V | N\theta_\omega, \text{in} \rangle, \tag{9}$$

$$T(\omega) = X^{-1}(\omega) \langle N\theta_\omega, \text{in} | f_V | B \rangle. \tag{10}$$

The expression given by GT for $K(\omega)$ is unaltered by the fact that the sources are being treated as bosons. Hence

$$K(\omega) = -g/[1-\beta(\omega)], \tag{11}$$

where $\beta(W)$ is given as

$$\beta(W) = -\frac{g^2 W}{4\pi^2}\int_\mu^\infty \frac{k f^2(\omega) d\omega}{\omega^2(\omega - W - i\epsilon)}. \tag{12}$$

To make further progress, it is clear that we must now contract a particle in $T(\omega)$. In this regard, it is convenient to select the θ particle, since this does not implicate the complicated states of the $V\theta$ sector. The

N-particle contraction will be considered briefly later on. Therefore we write

$$T(\omega)=-i\int_{-\infty}^{\infty} e^{i\omega t}\langle N|[j(t),f_V]\theta(-t)|B\rangle dt, \quad (13)$$

where the θ-particle current operator $j(t)$ is defined by

$$j(t)=X^{-1}(\omega)[-i(d/dt)+\omega]a_k(t)=-g\psi_N{}^\dagger(t)\psi_V(t). \quad (14)$$

As before, we insert the appropriate intermediate states and do the time integrals to get

$$T(\omega)=\frac{g\Gamma}{\omega}+\Gamma_1\frac{\langle B_0|j|B\rangle}{\omega+\omega_0-\omega_B} +\sum_{k'}\frac{\langle N|f_V|2N\theta_{\omega'},\text{ in}\rangle\langle 2N\theta_{\omega'},\text{ in}|j|B\rangle}{\omega'+\omega-\omega_B} +\sum_{k'}\frac{\langle N|j|N\theta_{\omega'},\text{ in}\rangle\langle N\theta_{\omega'},\text{ in}|f_V|B\rangle}{\omega'-\omega+i\epsilon}. \quad (15)$$

In-states describing the elastic (S-wave) scattering of one θ by two coincident N's are written as $|2N\theta_\omega,\text{ in}\rangle$. This decomposition of $T(\omega)$ is represented by the dispersion graph in Fig. 1. We have noted that $\langle N|j|V\rangle=-g$. Also, in accordance with GT, we have

$$\mathfrak{M}(\omega)=X^{-1}(\omega)\langle N|j|N\theta_\omega,\text{ in}\rangle=-g^2/\omega[1-\beta(\omega)] \quad (16)$$

and

$$e^{i\delta(\omega)}\sin\delta(\omega)=-g^2kf^2(\omega)/4\pi\omega[1-\beta(\omega)], \quad (17)$$

where $\delta(\omega)$ is the phase shift for $N\theta$ scattering. Further, we omit straightforward considerations that yield

$$X^{-1}(\omega)\langle N|f_V|2N\theta_\omega,\text{ in}\rangle=-g\omega\sqrt{2}/G^+(\omega), \quad (18)$$

where

$$G^+(W)\equiv G(W+i\epsilon)=2W[1-\beta(W)]+Z\delta m_V-ZW. \quad (19)$$

Upon setting

$$P(\omega)=X^{-1}(\omega)\langle 2N\theta_\omega,\text{ in}|j|B\rangle \quad (20)$$

and transforming to continuous space, we may now give $T(\omega)$ the form

$$T(\omega)=\frac{g\Gamma}{\omega}-\frac{\omega_0Z_0\Gamma_0}{\omega+\omega_0-\omega_B}-\frac{g\sqrt{2}}{4\pi^2}\times\int_\mu^\infty\frac{k'\omega'f^2(\omega')P(\omega')d\omega'}{(\omega'+\omega-\omega_B)G^+(\omega')} +\frac{1}{\pi}\int_\mu^\infty\frac{e^{i\delta(\omega')}\sin\delta(\omega')T(\omega')d\omega'}{\omega'-\omega+i\epsilon}. \quad (21)$$

Here we have introduced Eq. (1a) and the definition

$$\Gamma_0=\langle B_0|j|B\rangle. \quad (22)$$

This vertex will be analyzed in Sec. II B. At this stage, Γ, Γ_0, and $P(\omega)$ are unknown, but this does not prevent us from solving the above Omnès-type integral equation for $T(\omega)$. Standard methods yield

$$T(\omega)=\frac{1}{1-\beta^*(\omega)}\left(\frac{g\Gamma}{\omega}-\frac{\omega_0Z_0\Gamma_0[1-\beta(\omega_B-\omega_0)]}{\omega+\omega_0-\omega_B} -\frac{g\sqrt{2}}{4\pi^2}\int_\mu^\infty\frac{k'\omega'f^2(\omega')P(\omega')[1-\beta(\omega_B-\omega')]d\omega'}{(\omega'+\omega-\omega_B)G^+(\omega')}\right). \quad (23)$$

Using this result and Eq. (11), we can rewrite Eq. (8) as

$$g\Gamma[1-J(\omega_B)=-\omega_0Z_0\Gamma_0[1-\beta(\omega_B-\omega_0)]J(\omega_0) -\frac{g\sqrt{2}}{4\pi^2}\int_\mu^\infty\frac{k\omega f^2(\omega)P(\omega)[1-\beta(\omega_B-\omega)]J(\omega)d\omega}{G^+(\omega)}, \quad (24)$$

in which

$$J(W)=\frac{1}{\pi}\int_\mu^\infty \operatorname{Im}\left(\frac{1}{1-\beta(\omega)}\right)\frac{(2\omega-\omega_B)d\omega}{(\omega-\omega_B)(\omega+W-\omega_B)}. \quad (25a)$$

This integral is readily evaluated by the methods of GT, and we find

$$J(W)=\frac{\omega_B}{W[1-\beta(\omega_B)]}+\frac{2W-\omega_B}{W[1-\beta(\omega_B-W)]}-\frac{2}{Z}. \quad (25b)$$

The matrix element $P(\omega)$ is the remaining problem in connection with Eq. (24). On contracting the θ particle in this function, we obtain

$$P(\omega)=-i\int_{-\infty}^{\infty} e^{i\omega t}\langle 2N|[j(t),j]\theta(-t)|B\rangle dt. \quad (26)$$

A familiar procedure leads to the expansion

$$P(\omega)=gZ_0\Gamma_0\sqrt{2}\left(\frac{1}{\omega-\omega_0}-\frac{1}{\omega+\omega_0-\omega_B}\right) +\sum_{k'}\langle 2N|j|2N\theta_{\omega'},\text{ in}\rangle\langle 2N\theta_{\omega'},\text{ in}|j|B\rangle \times\left(\frac{1}{\omega'+\omega-\omega_B}-\frac{1}{\omega-\omega'+i\epsilon}\right), \quad (27)$$

which is represented graphically in Fig. 2. In arriving at Eq. (27), we have noted that $\langle 2N|j|B_0\rangle=-gZ_0\sqrt{2}$. Inspection of Eq. (27) shows that $P(\omega)=P(\omega_B-\omega)$. This crossing symmetry corresponds to an interchange of the two outgoing θ's in Fig. 2. The matrix element $M(\omega)$ defined by

$$M(\omega)=X^{-1}(\omega)\langle 2N|j|2N\theta_\omega,\text{ in}\rangle \quad (28)$$

is related to $2N\theta$ scattering, and by analogy with the GT calculation of $\mathfrak{M}(\omega)$ we find

$$M(\omega)=-2g^2/G^+(\omega) \quad (29)$$

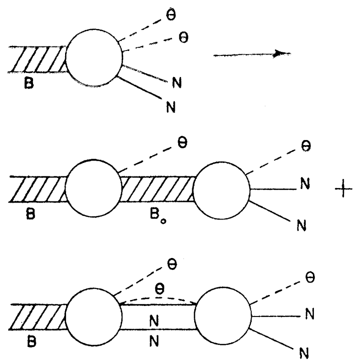

FIG. 2. Dispersion graph for $P(\omega)$ according to Eq. (27). To each of the graphs there corresponds another with the θ particles interchanged.

and

$$e^{i\eta(\omega)}\sin\eta(\omega)=-g^2kf^2(\omega)/2\pi G^+(\omega)\,, \tag{30}$$

where $\eta(\omega)$ is the corresponding phase shift. The combination of Eqs. (27)–(30) yields the integral equation

$$\begin{aligned}P(\omega)=gZ_0\Gamma_0\sqrt{2}\left(\frac{1}{\omega-\omega_0}-\frac{1}{\omega+\omega_0-\omega_B}\right)&\\+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega')}\sin\eta(\omega')P(\omega')&\\\times\left(\frac{1}{\omega'-\omega+i\epsilon}+\frac{1}{\omega'+\omega-\omega_B}\right)d\omega'.&\end{aligned} \tag{31}$$

This is an integral equation of the type discussed in Ref. 7, and its solution is

$$P(\omega)=g\Gamma_0\sqrt{2}G(\omega_B-\omega_0)/Z_0G^{*+}(\omega)G(\omega_B-\omega)\,. \tag{32}$$

Collecting the results, Eqs. (24), (25b), and (32), we have, after some calculation,

$$\begin{aligned}\frac{Z\omega_B}{1-\beta(\omega_B)}\left(\frac{-g\Gamma}{\omega_B}+Z_0\Gamma_0[1-\beta(\omega_B-\omega_0)]\right.&\\\left.-Z_0^{-1}\Gamma_0G(\omega_B-\omega_0)B(\omega_B)\right)+2g\Gamma&\\=Z_0\Gamma_0\{2\omega_0[1-\beta(\omega_B-\omega_0)]+(ZZ_0^2)^{-1}G(\omega_B-\omega_0)\}&\\-2Z_0^{-1}\Gamma_0G(\omega_B-\omega_0)C(\omega_B)\,,&\end{aligned} \tag{33}$$

where the integrals $B(\omega_B)$ and $C(\omega_B)$ are defined by

$$B(\omega_B)=\frac{1}{\pi}\int_\mu^\infty \mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\frac{[1-\beta(\omega_B-\omega)]d\omega}{G(\omega_B-\omega)}\,, \tag{34}$$

$$C(\omega_B)=\frac{1}{\pi}\int_\mu^\infty \mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\frac{\omega[1-\beta(\omega_B-\omega)]d\omega}{G(\omega_B-\omega)}\,. \tag{35}$$

In arriving at Eq. (33), we have incorporated[13]

$$\frac{1}{\pi}\int_\mu^\infty \mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\frac{(2\omega-\omega_B)d\omega}{G(\omega_B-\omega)}=Z^{-2}-\frac{Z_0^2(\omega_B-2\omega_0)}{G(\omega_B-\omega_0)}\,. \tag{36}$$

[13] This integral is involved in Eq. (A6) of Ref. 1.

In Sec. II B, it will be shown that the quantity within large parentheses on the left-hand side of Eq. (33) vanishes. Thus we are led to two independent algebraic relations connecting Γ and Γ_0, the simultaneous solution of which yields the desired eigenvalue condition for ω_B. We do not determine Γ and Γ_0 in this procedure, only their ratio. A separate consideration of the bound-state vector or contraction of B_0 in Γ_0 enables us to obtain expressions for these quantities.

B. Γ_0 Vertex

As in the previous case, our treatment of this vertex also avoids the contraction of a bound state. Instead, we call upon Eq. (14) and a complete set of states to write

$$\begin{aligned}\Gamma_0=-g\langle B_0|\psi_N^\dagger|V\rangle\langle V|\psi_V|B\rangle&\\-g\textstyle\sum_k\langle B_0|\psi_N^\dagger|N\theta_\omega,\,\mathrm{in}\rangle\langle N\theta_\omega,\,\mathrm{in}|\psi_V|B\rangle.&\end{aligned} \tag{37}$$

It is convenient to convert the field operators in these matrix elements into currents. For this purpose we employ

$$f_V=[H,\psi_V]+m\psi_V\,, \tag{38}$$

$$f_N=[H,\psi_N]+m\psi_N\,, \tag{39}$$

and find

$$\langle V|\psi_N|B_0\rangle=-\Gamma_2/\omega_0\,, \tag{40}$$

$$\langle V|\psi_V|B\rangle=-\Gamma/\omega_B\,, \tag{41}$$

$$X^{-1}(\omega)\langle N\theta_\omega,\,\mathrm{in}|\psi_V|B\rangle=T(\omega)/(\omega-\omega_B)\,, \tag{42}$$

$$\begin{aligned}X^{-1}(\omega)\langle N\theta_\omega,\,\mathrm{in}|\psi_N|B_0\rangle&\\=\langle N\theta_\omega,\,\mathrm{in}|f_N|B_0\rangle/X(\omega)(\omega-\omega_0)&\\=\frac{gZ}{(\omega-\omega_0)[1-\beta^*(\omega)]}\left(\frac{\Gamma_2}{\omega Z}+\frac{\Gamma_1}{\omega_0}\right).&\end{aligned} \tag{43}$$

The matrix element $\langle N\theta_\omega,\,\mathrm{in}|f_N|B_0\rangle$ has already been treated via dispersion methods in an earlier paper.[11] Our use of in-states in the present case requires the complex conjugate of $\beta(\omega)$ as shown in Eq. (43). When these expressions are inserted into Eq. (37), we get (in continuous space)

$$\Gamma_0=\frac{-g\Gamma\Gamma_2}{\omega_0\omega_B}-\frac{g^2ZZ_0}{4\pi^2}\times\int_\mu^\infty\frac{kf^2(\omega)(\delta m_V-\omega)T(\omega)d\omega}{\omega(\omega-\omega_0)(\omega-\omega_B)[1-\beta(\omega)]}\,. \tag{44}$$

Substitution of Eq. (23) into Eq. (44) leads to

$$\begin{aligned}\Gamma_0=-g\Gamma\Gamma_2/\omega_0\omega_B+gZZ_0\Gamma F(\omega_B)&\\-ZZ_0^2\omega_0\Gamma_0[1-\beta(\omega_B-\omega_0)]F(\omega_0)-\frac{gZZ_0\sqrt{2}}{4\pi^2}&\\\times\int_\mu^\infty\frac{k\omega f^2(\omega)P(\omega)[1-\beta(\omega_B-\omega)]F(\omega)d\omega}{G^+(\omega)}\,.&\end{aligned} \tag{45}$$

The integral

$$F(W)=\frac{1}{\pi}\int_{\mu}^{\infty}\mathrm{Im}\left(\frac{1}{1-\beta(\omega)}\right)\times\frac{(\delta m_V-\omega)d\omega}{(\omega-\omega_0)(\omega-\omega_B)(\omega+W-\omega_B)} \quad (46)$$

can be evaluated by the methods of GT, which yield

$$F(W)=(\delta m_V-\omega_0)/(\omega_0-\omega_B)(\omega_0+W-\omega_B)[1-\beta(\omega_0)]+(\delta m_V-\omega_B)/W(\omega_B-\omega_0)[1-\beta(\omega_B)]+(\delta m_V+W-\omega_B)/W(W+\omega_0-\omega_B)[1-\beta(\omega_B-W)]. \quad (47)$$

Finally, by combining Eqs. (32), (45), and (47), we can conclude that

$$g\Gamma/\omega_B=Z_0\Gamma_0[1-\beta(\omega_B-\omega_0)]-Z_0^{-1}\Gamma_0G(\omega_B-\omega_0)B(\omega_B). \quad (48)$$

Thus without further contractions we have found another relation connecting Γ and Γ_0.

If Eqs. (38) and (39) had not been used in deriving Eq. (48), we could still arrive at this result. To see this let us accept Eqs. (40) and (43), which are easy to prove via the contraction technique, and then write Eq. (37) as

$$\Gamma_0=g\frac{\Gamma_2\Lambda}{\omega_0}-\frac{g^2ZZ_0}{4\pi^2}\int_{\mu}^{\infty}\frac{kf^2(\omega)(\delta m_V-\omega)U(\omega)d\omega}{\omega(\omega-\omega_0)[1-\beta(\omega)]}, \quad (49)$$

where Λ and $U(\omega)$ represent the matrix elements $\langle V|\psi_V|B\rangle$ and $X^{-1}(\omega)\langle N\theta_\omega,\mathrm{in}|\psi_V|B\rangle$, respectively. For $U(\omega)$ we have

$$U(\omega)=-i\int_{-\infty}^{\infty}e^{i\omega t}\langle N|[j(t),\psi_V]\theta(-t)|B\rangle dt, \quad (50)$$

which, in the usual way, becomes

$$U(\omega)=\frac{g\Lambda}{\omega}+\frac{Z_0\Gamma_0}{\omega+\omega_0-\omega_B}+\frac{g\sqrt{2}}{4\pi^2}\int_{\mu}^{\infty}\frac{k'f^2(\omega')P(\omega')d\omega'}{(\omega'+\omega-\omega_B)G^+(\omega')}+\frac{1}{\pi}\int_{\mu}^{\infty}\frac{e^{i\delta(\omega')}\sin\delta(\omega')U(\omega')d\omega'}{\omega'-\omega+i\epsilon}. \quad (51)$$

Here we have also accepted the elementary matrix element

$$X^{-1}(\omega)\langle N|\psi_V|2N\theta_\omega,\mathrm{in}\rangle=g\sqrt{2}/G^+(\omega). \quad (52)$$

The solution of Eq. (51) is

$$U(\omega)=\frac{1}{1-\beta^*(\omega)}\left(\frac{g\Lambda}{\omega}+\frac{Z_0\Gamma_0[1-\beta(\omega_B-\omega_0)]}{\omega+\omega_0-\omega_B}+\frac{g\sqrt{2}}{4\pi^2}\int_{\mu}^{\infty}\frac{k'f^2(\omega')P(\omega')[1-\beta(\omega_B-\omega')]d\omega'}{(\omega'+\omega-\omega_B)G^+(\omega')}\right). \quad (53)$$

The result of substituting Eq. (32) into Eq. (53) and then the latter into Eq. (49) is

$$Z_0^2\Gamma_0[1-\beta(\omega_B-\omega_0)]=-gZ_0\Lambda+\Gamma G(\omega_B-\omega_0)B(\omega_B). \quad (54)$$

Turning to Λ and contracting the V particle, we find

$$\Lambda=-\frac{\Gamma}{\omega_B}+\frac{1}{4\pi^2}\int_{\mu}^{\infty}\frac{kf^2(\omega)K(\omega)}{\omega(\omega_B-\omega)}\times[(\omega_B-\omega)U(\omega)+T(\omega)]d\omega. \quad (55)$$

Of course, if Eq. (42) is invoked at this point, the bracketed term in the integrand would vanish and Eq. (48) would follow at once from Eqs. (54) and (55). In the present approach, however, we must put the known expressions for $U(\omega)$ and $T(\omega)$ into Eq. (55) to find, after some calculation,

$$gZ_0\Lambda=\frac{-gZZ_0\Gamma}{\omega_B[1-\beta(\omega_B)]}+\Gamma_0\left(\frac{Z}{1-\beta(\omega_B)}-1\right)\times\{Z_0^2[1-\beta(\omega_B-\omega_0)]-G(\omega_B-\omega_0)B(\omega_B)\}. \quad (56)$$

Lastly, when this result is combined with Eq. (54), we again come up with Eq. (48), which may also be obtained by using Eq. (41) in Eq. (56). In any case, it is clear that our first scheme involving the currents offers a calculational advantage.

C. Eigenvalue Condition

To continue with our derivation of the eigenvalue condition, we rewrite Eq. (33), using Eq. (48), as

$$2gZ\Gamma=Z_0\Gamma_0\{2Z\omega_0[1-\beta(\omega_B-\omega_0)]+Z_0^{-2}G(\omega_B-\omega_0)-2ZZ_0^{-2}G(\omega_B-\omega_0)C(\omega_B)\}. \quad (57)$$

Elimination of Γ and Γ_0 from this relation, and Eq. (48), gives

$$2Z(\omega_B-\omega_0)[1-\beta(\omega_B-\omega_0)]-Z_0^2G(\omega_B-\omega_0)=2ZZ_0^{-2}G(\omega_B-\omega_0)\frac{1}{\pi}\int_{\mu}^{\infty}\mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\times\frac{(\omega_B-\omega)[1-\beta(\omega_B-\omega)]d\omega}{G(\omega_B-\omega)}. \quad (58)$$

This is simplified with Eq. (19) and

$$A(\omega_B-\omega_0)\equiv\frac{1}{\pi}\int_{\mu}^{\infty}\mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\frac{d\omega}{G(\omega_B-\omega)}, \quad (59)$$

$$\frac{1}{\pi}\int_{\mu}^{\infty}\mathrm{Im}\left(\frac{1}{G(\omega)}\right)d\omega=Z_0^2-Z^{-1}, \quad (60)$$

$$\frac{2}{\pi}\int_{\mu}^{\infty}\mathrm{Im}\left(\frac{1}{G^+(\omega)}\right)\frac{\omega d\omega}{G(\omega_B-\omega)}=Z^{-2}+\omega_BA(\omega_B-\omega_0)+Z_0^2(2\omega_0-\omega_B)/G(\omega_B-\omega_0). \quad (61)$$

The final result is

$$D(\omega_B)\equiv Z^2(\omega_B-2\delta m_V)[1-Z_0^{-2}G(\omega_B-\omega_0)\times A(\omega_B-\omega_0)]+Z_0^{-2}G(\omega_B-\omega_0)=0\,, \quad (62)$$

which is exactly the condition originally obtained in the LSZ formalism by setting the denominator function $D(W)$ of the Fourier transform of the $2V$ propagator equal to zero at the bound-state pole. The functions $G(\omega_B-\omega_0)$ and $A(\omega_B-\omega_0)$ are real, since the stability of the $2V$ system requires that $\omega_B<\omega_0+\mu<2\mu$. We can replace δm_V [in Eq. (62)] with m_0-m, where m_0 is the bare mass of the V particle. Note that if x is defined as $\omega_B-\omega_0$, and if the quantity $1-Z_0^{-2}G(x)A(x)$ does not vanish in the seqment $\omega_0<x<\mu$, then there is only one root x_B of Eq. (62) in this segment, the value of which is decided by m_0. The reader is referred to Ref. 2 for further discussion of Eq. (62).

To conclude this subsection, we comment on the situation that arises when the N particle is contracted in $T(\omega)$. In this case, one encounters the graphs shown in Fig. 3, and

$$T(\omega)=\frac{g\Gamma}{\omega}+\frac{1}{\pi}\int_\mu^\infty \frac{e^{i\delta(\omega')}\sin\delta(\omega')T(\omega')d\omega'}{\omega'-\omega+i\epsilon} +X^{-1}(\omega)\sum_{k'}\frac{\langle\theta_\omega|f_V|V\theta_{\omega'}\rangle\langle V\theta_{\omega'}|f_N|B\rangle}{\omega'-\omega_B}+X^{-1}(\omega) \times\sum_{k'k''}\frac{\langle\theta_\omega|f_V|N\theta_{\omega'}\theta_{\omega''}\rangle\langle N\theta_{\omega'}\theta_{\omega''}|f_N|B\rangle}{\omega'+\omega''-\omega_B}. \quad (63)$$

Here we see that intermediate (in- or out-) states characteristic of the $V\theta$ sector make their appearance. If the conditions suitable for a discrete $V\theta$ state are satisfied, we would also have to include it in Eq. (63). In view of Amado's work, the matrix elements $\langle\theta_\omega|f_V|V\theta_{\omega'}\rangle$ and $\langle\theta_\omega|f_V|N\theta_{\omega'}\theta_{\omega''}\rangle$ present no new problems. We do not develop this approach, but there is every expectation that Eq. (62) would again be the final result.

D. Vector $|B\rangle$

To deal with the $2V$ bound-state vector $|B\rangle$, we first note that its normalization constant Z_B may be expressed as

$$Z_B\sqrt{2}=\langle 0|\psi_V\psi_V|B\rangle. \quad (64)$$

The introduction of a complete set of states yields

$$Z_B\sqrt{2}=-\frac{\Gamma}{\omega_B}-\sum_k\frac{X^2(\omega)K(\omega)T(\omega)}{\omega(\omega-\omega_B)}. \quad (65)$$

Since $K(\omega)$ and $T(\omega)$ are known, we find, after some calculation, that Eq. (65) gives the following connection between Γ_0 and Z_B:

$$\Gamma_0=-\frac{gZ_B\sqrt{2}}{Z_0[1-Z_0^{-2}G(\omega_B-\omega_0)A(\omega_B-\omega_0)]}. \quad (66)$$

Equations (48) or (57) could now be used to find a corresponding relation for Γ. Next, we consider the expansion coefficient $\varphi_1(\omega)$ defined by

$$Z_B\varphi_1(\omega)=\langle 0|\psi_N a_k\psi_V|B\rangle. \quad (67)$$

As before, we obtain

$$Z_B\varphi_1(\omega)=X(\omega)\left(\frac{g\Gamma}{\omega\omega_B}+\frac{T(\omega)}{\omega-\omega_B} +g^2\sum_{k'}\frac{X^2(\omega')T(\omega')}{\omega'(\omega'-\omega_B)(\omega'-\omega+i\epsilon)[1-\beta(\omega')]}\right). \quad (68)$$

On substituting the expression for $T(\omega)$, and again after some calculation, this yields

$$Z_B\varphi_1(\omega)=-Z_0\Gamma_0X(\omega)\left(\frac{1}{\omega_B-\omega_0-\omega} +Z_0^{-2}\frac{G(\omega_B-\omega_0)}{\omega-\omega_0}[I_{\omega_B}(\omega_B-\omega)+A(\omega_B-\omega_0)]\right), \quad (69)$$

where

$$I_{\omega_B}(\omega_B-\omega)\equiv\frac{1}{\pi}\int_\mu^\infty I_m\left(\frac{1}{G^+(\omega')}\right) \times\frac{(\omega_B-\omega'-\omega_0)d\omega'}{(\omega'+\omega-\omega_B)G(\omega_B-\omega')}. \quad (70)$$

Hence, in view of Eq. (66), we have

$$\varphi_1(\omega)=\frac{g\sqrt{2}X(\omega)}{1-Z_0^{-2}G(\omega_B-\omega_0)A(\omega_B-\omega_0)}\left(\frac{1}{\omega_B-\omega_0-\omega} +\frac{Z_0^{-2}G(\omega_B-\omega_0)}{\omega-\omega_0}[I_{\omega_B}(\omega_B-\omega)+A(\omega_B-\omega_0)]\right), \quad (71)$$

which is in agreement with Eq. (48) of Ref. 2. The remaining coefficient is

$$2Z_B\varphi_2(\omega,\omega')=\langle 0|\psi_N\psi_N a_k a_{k'}|B\rangle. \quad (72)$$

By inserting a complete set of intermediate states between a_k and $a_{k'}$, we can rewrite this as

$$2Z_B\varphi_2(\omega,\omega')=\langle 0|\psi_N\psi_N a_k|B_0\rangle\langle B_0|a_{k'}|B\rangle +\sum_{k''}\langle 0|\psi_N\psi_N a_k|2N\theta_{\omega''},\text{in}\rangle\langle 2N\theta_{\omega''},\text{in}|a_{k'}|B\rangle. \quad (73)$$

It is convenient to replace $a_{k'}$ with its corresponding current. To do this we use

$$X(\omega')j=[H,a_{k'}]+\omega' a_{k'}. \quad (74)$$

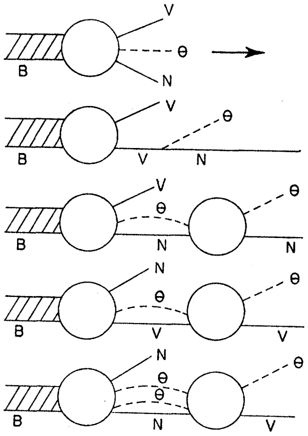

FIG. 3. Dispersion graph of $T(\omega)$ as divided in Eq. (64).

Subsequent manipulations then show

$$2Z_B\varphi_2(\omega,\omega')=-\frac{2gZ_0\Gamma_0X(\omega)X(\omega')}{(\omega-\omega_0)(\omega'+\omega_0-\omega_B)}+\frac{\sqrt{2}X(\omega)X(\omega')P(\omega)}{\omega'+\omega-\omega_B}+2g^2\sqrt{2}X(\omega)X(\omega')\times\sum_{k''}\frac{X^2(\omega'')P(\omega'')}{(\omega''+\omega'-\omega_B)(\omega''-\omega+i\epsilon)G^+(\omega'')}. \quad (75)$$

On making the partial fraction decomposition

$$\frac{1}{(\omega''+\omega'-\omega_B)(\omega''-\omega+i\epsilon)}=\frac{1}{\omega'+\omega-\omega_B}\left(\frac{1}{\omega''-\omega+i\epsilon}-\frac{1}{\omega''+\omega'-\omega_B}\right), \quad (76)$$

and using Eq. (31), we obtain

$$2Z_B\varphi_2(\omega,\omega')=\frac{-2gZ_0\Gamma_0X(\omega)X(\omega')}{(\omega-\omega_0)(\omega'+\omega_0-\omega_B)}+\frac{2gZ_0\Gamma_0X(\omega)X(\omega')}{\omega'+\omega-\omega_B}\left(\frac{1}{\omega-\omega_0}-\frac{1}{\omega+\omega_0-\omega_B}\right)+\frac{\sqrt{2}X(\omega)X(\omega')}{\pi(\omega'+\omega-\omega_B)}\int_\mu^\infty e^{i\eta(\omega'')}\sin\eta(\omega'')P(\omega'')\times\left(\frac{1}{\omega''+\omega-\omega_B}+\frac{1}{\omega''+\omega'-\omega_B}\right). \quad (77)$$

With the help of Eqs. (30) and (32) the integral term in Eq. (77) becomes

$$\frac{2gX(\omega)X(\omega')}{\omega'+\omega-\omega_B}\left(\frac{Z_0\Gamma_0}{\omega+\omega_0-\omega_B}-\frac{Z_B\varphi_1(\omega)}{X(\omega)}+\frac{Z_0\Gamma_0}{\omega+\omega_0-\omega_B}-\frac{Z_B\varphi_1(\omega')}{X(\omega')}\right). \quad (78)$$

Thus Eq. (77) simplifies to

$$\varphi_2(\omega,\omega')=\frac{g[X(\omega)\varphi_1(\omega')+X(\omega')\varphi(\omega)]}{\omega_B-\omega-\omega'}, \quad (79)$$

which is exactly the TD expression given for $\varphi_2(\omega,\omega')$ in Ref. 2, Eq. (43c). Having found $\varphi_1(\omega)$, this completes our determination of the expansion coefficients, and $|B\rangle$ is written in terms of normalized bare states as

$$|B\rangle=Z_B[Z|2_V\rangle+(\sqrt{Z})\sum_k\varphi_1(\omega)|1_V,1_N,1_k\rangle+\sum_{kk'}\varphi_2(\omega,\omega')|2_N,1_k,1_k\rangle]. \quad (80)$$

The normalization constant Z_B follows from the requirement $\langle B|B\rangle=1$, or from Eq. (54) of Ref. 2.

III. $VN+\theta$ SCATTERING

We now turn to a dispersive treatment of the elastic scattering of one θ particle by the composite particle B_0 (the VN bound state). To simplify some of the calculation we shall make use of certain knowledge previously gained in the TD method. We begin with the relevant S-matrix element

$$S=\langle B_0\theta_\omega,\ \mathrm{out}|B_0\theta_{\omega'},\ \mathrm{in}\rangle. \quad (81)$$

Contraction of the out-state θ particle leads to

$$S=\delta_{kk'}+2\pi i\delta(\omega-\omega')X^2(\omega)Y(\omega), \quad (82)$$

where the scattering amplitude $Y(\omega)$ is defined by

$$Y(\omega)=X^{-1}(\omega)\langle B_0|j|B_0\theta_\omega,\ \mathrm{in}\rangle. \quad (83)$$

In the spirit of Amado's procedure, we must contract B_0 from the left in $Y(\omega)$. To do this, let us introduce the corresponding field operator

$$\psi_0=Z_0^{-1}\psi_V\psi_N. \quad (84)$$

The current operator $f_0(t)$ associated with B_0 is found to be

$$\begin{aligned}f_0(t)&=[-i(d/dt)+2m+\omega_0]\psi_0(t)\\&=[H,\psi_0(t)]+(2m+\omega_0)\psi_0(t)\\&=(\omega_0-\delta m_V)\psi_0(t)-g(ZZ_0)^{-1}\psi_N(t)\psi_N(t)A(t)\\&\quad-gZ_0^{-1}\psi_V(t)\psi_V(t)A^\dagger(t).\end{aligned} \quad (85)$$

As expected,

$$\langle 0|f_0|B_0\rangle=0. \quad (86)$$

We now use the usual asymptotic definition of a state to write

$$Y(\omega)=-g\sqrt{2}(ZZ_0)^{-1}X^{-1}(\omega)\langle 2_V|B_0\theta_\omega,\ \mathrm{in}\rangle+iX^{-1}(\omega)\times\int_{-\infty}^{\infty}e^{i(2m+\omega_0)t}\langle 0|[f_0(t),j]\theta(t)|B_0\theta_\omega,\ \mathrm{in}\rangle dt. \quad (87)$$

The first term on the right-hand side of this equation is due to the equal-time commutator resulting from the differentiation of the step function. It is convenient to evaluate the scalar product in this term by exploiting Eqs. (1) and (19) of Ref. 2. We find

$$\langle 2_V | B_0\theta_\omega, \text{in}\rangle = 2^{3/2} Z Z_0^{-1} g X(\omega)/D(\omega+\omega_0). \quad (88)$$

Consequently, on substituting intermediate states into Eq. (87), and recalling Eq. (86), we obtain

$$Y(\omega) = \frac{-4g^2}{Z_0^2 D(\omega+\omega_0)} + \sum_{k'} \frac{f^2(\omega')\tilde{K}(\omega')N(\omega',\omega)}{\omega'(\omega'-\omega_0)}, \quad (89)$$

where

$$\tilde{K}(\omega) = X^{-1}(\omega)\langle 0 | f_0 | 2N\theta_\omega, \text{in}\rangle, \quad (90)$$

$$N(\omega',\omega) = [X^{-1}(\omega')X^{-1}(\omega)/2\Omega] \times \langle 2N\theta_{\omega'}, \text{in} | j | B_0\theta_\omega, \text{in}\rangle. \quad (91)$$

The manipulations leading to the evaluation of $\tilde{K}(\omega)$ and $N(\omega',\omega)$ are direct extensions of the methods used by GT and Amado, respectively. In the former case we find

$$\tilde{K}(\omega) = -g\sqrt{2}(\omega-\omega_0)/Z_0 G^+(\omega). \quad (92)$$

At $\omega=\omega_0$, we note that this vertex function is equal in magnitude to $gZ_0\sqrt{2}$. The integral equation for $N(\omega',\omega)$ turns out to be

$$\begin{aligned} N(\omega',\omega) = & -g\sqrt{2} Z_0 \omega \delta_{kk'}/f^2(\omega) \\ & + \frac{g\sqrt{2}Z_0}{2\Omega} Y(\omega)\left(\frac{1}{\omega'-\omega_0} - \frac{1}{\omega'-\omega-i\epsilon}\right) \\ & + \frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega_1)} \sin\eta(\omega_1)\, N(\omega_1,\omega) \\ & \times \left(\frac{1}{\omega_1+\omega'-\omega-\omega_0-i\epsilon} + \frac{1}{\omega_1-\omega'+i\epsilon}\right) d\omega_1. \end{aligned} \quad (93)$$

This equation has the solution

$$\begin{aligned} N(\omega',\omega) = & -g\sqrt{2} Z_0 \omega \delta_{kk'}/f^2(\omega) \\ & + 2\sqrt{2} g^3/2\Omega Z_0 G^{*+}(\omega') G(\omega+\omega_0-\omega') \\ & + g\sqrt{2} Y(\omega) G^+(\omega)/2\Omega Z_0 G^{*+}(\omega') G(\omega+\omega_0-\omega'). \end{aligned} \quad (94)$$

On combining Eqs. (89), (92), and (94), we obtain an expression for $Y(\omega)$ that leads us to express Eq. (82) in the form

$$S = \delta_{kk'} + 4\pi i g^2 \delta(\omega-\omega') X^2(\omega) \times \frac{Z^2(\omega+\omega_0-2\delta m_V)[1+Z_0^{-2}G^+(\omega)A(\omega)] - Z_0^{-2}G^+(\omega)}{G^+(\omega)D(\omega+\omega_0)}. \quad (95)$$

When $\omega \to \infty$, we find that $Z_0^{-2}G^+(\omega)A(\omega)$ approaches the negative number $1-(ZZ_0^2)^{-1}$. Recall that ZZ_0^2 is the probability of finding the bare VN component in $|B_0\rangle$. We also have that $G^+(\omega) \to \omega Z$ in this limit. Thus, at high energy, it follows that $Y(\omega)$ behaves like $-2g_0^2\omega^{-1}(1-ZZ_0^2)$, where g_0 is the unrenormalized coupling constant. This may be interpreted as the Born approximation for $2N+\theta$ scattering multiplied by the probability $1-ZZ_0^2$ of finding $|B_0\rangle$ virtually dissociated into $2N+\theta$. In accordance with the $2V$ bound-state conditions stated earlier and exhibited in Fig. 1(a) of Ref. 2, one can verify that $Y(\omega)$ is also negative in the low-energy limit $\omega \to \mu$.

IV. PRODUCTION

Again, as in Amado's paper, we can now calculate the S-matrix element for the production process $B_0\theta \to 2N+2\theta$. We do this by first contracting a θ particle from the left in

$$S = \langle 2N\theta_{\omega'}\theta_{\omega''}, \text{out} | B_0\theta_\omega, \text{in}\rangle. \quad (96)$$

The result may be expressed in the form

$$S = (4\pi i\Omega/\sqrt{2}) \times \delta(\omega+\omega_0-\omega'-\omega'')X(\omega)X(\omega')X(\omega'')\mathcal{P}(\omega',\omega), \quad (97)$$

where the amplitude $\mathcal{P}(\omega',\omega)$ is defined by

$$\mathcal{P}(\omega',\omega) = [X^{-1}(\omega)X^{-1}(\omega')/2\Omega] \langle 2N\theta_{\omega'}, \text{out} | j | B_0\theta_\omega, \text{in}\rangle. \quad (98)$$

A simple calculation involving a complete set of intermediate states and the S-matrix element for $2N+\theta$ scattering leads to

$$\mathcal{P}(\omega',\omega) = e^{2i\eta(\omega')}N(\omega',\omega) = G^{*+}(\omega')N(\omega',\omega)/G^+(\omega'). \quad (99)$$

Keeping in mind the energy conservation due to the δ function in Eq. (97), and using Eq. (94) along with the expression for $Y(\omega)$, which may be read from Eq. (95), we write Eq. (97) as

$$S = \frac{8\pi i Z^2 g^3 X(\omega)X(\omega')X(\omega'')\delta(\omega_0+\omega-\omega'-\omega'')(\omega+\omega_0-2\delta m_V)}{Z_0 G^+(\omega')G^+(\omega'')D(\omega+\omega_0)}. \quad (100)$$

An amusing feature about this amplitude is that its numerator vanishes when the energy $\omega+2m+\omega_0$ is equal to twice the bare V-particle mass. In the segment $\omega_0 \leq \omega \leq \mu$, the functions $\omega+\omega_0-2\delta m_V$ and $Z_0^{-2}G^+(\omega)/Z^2[1+Z_0^{-2}G^+(\omega)A(\omega)]$ are negative and positive, respectively. Hence the numerator of the $B_0\theta$ scattering amplitude will not have a zero in this segment. Note that $\mu+\omega_0-2\delta m_V$ has been chosen negative in Fig. 1(a) of Ref. 2.

V. $2N2\theta$ SCATTERING

This collision process, unlike the previous one, requires a separate treatment, both here in the dispersion approach and in the TD method. We begin with the S-matrix element

$$S=\langle 2N\theta_{\omega'''}\theta_{\omega''},\ \text{out}|2N\theta_{\omega'}\theta_{\omega},\ \text{in}\rangle. \tag{101}$$

If a complete set of in-states is inserted into the matrix element $\langle 2N\theta_{\omega''},\ \text{out}|2N\theta_{\omega'}\theta_{\omega},\ \text{in}\rangle$ resulting from the contraction of $\theta_{\omega'''}$, we find

$$\begin{aligned}S=&\tfrac{1}{2}(\delta_{k'k'''}S_{kk''}{}^{2N\theta}+\delta_{kk'''}S_{k'k''}{}^{2N\theta}\\&+(2\pi i/\sqrt{2})\delta(\omega+\omega'-\omega''')X(\omega''')X(\omega')X(\omega)\\&\times\sum_{k_1}X(\omega_1)S_{k_1k''}{}^{2N\theta}\mathcal{A}(\omega_1,\omega'\omega).\end{aligned} \tag{102}$$

In this expression, $\mathcal{A}(\omega_1,\omega'\omega)$ is defined by

$$\begin{aligned}\mathcal{A}(\omega_1,\omega'\omega)=&X^{-1}(\omega_1)X^{-1}(\omega')X^{-1}(\omega)\\&\times\langle 2N\theta_{\omega_1},\ \text{in}|j|2N\theta_{\omega'}\theta_{\omega},\ \text{in}\rangle\end{aligned} \tag{103}$$

and $S_{kk'}{}^{2N\theta}$ is the S-matrix element for $2N+\theta$ scattering given by

$$S_{kk'}{}^{2N\theta}=\delta_{kk'}-4\pi iX^2(\omega)\delta(\omega-\omega')g^2/G^+(\omega). \tag{104}$$

Introducing the definition

$$R(\omega',\omega)=X^{-1}(\omega')X^{-1}(\omega)\langle B_0|j|2N\theta_{\omega'}\theta_{\omega},\ \text{in}\rangle, \tag{105}$$

we are led to the following coupled equations:

$$R(\omega',\omega)=-g\sqrt{2}[Z_0X(\omega)X(\omega')]^{-1}C(\omega',\omega)-\frac{g\sqrt{2}}{4\pi^2Z_0}\int_\mu^\infty\frac{k_1f^2(\omega_1)\mathcal{A}(\omega_1,\omega'\omega)d\omega_1}{G^+(\omega_1)} \tag{106}$$

and

$$\begin{aligned}\mathcal{A}(\omega_1,\omega'\omega)=&-\frac{g^2\sqrt{2}}{X^2(\omega')}\left(\frac{\delta_{k_1k'}}{G^+(\omega)}+\frac{\delta_{k_1k}}{G^+(\omega')}\right)\\&-g\sqrt{2}Z_0R(\omega',\omega)\left(\frac{1}{\omega_0-\omega_1}+\frac{1}{\omega_1+\omega_0-\omega-\omega'}\right)\\&+\frac{1}{\pi}\int_\mu^\infty e^{i\eta(\omega_2)}\sin\eta(\omega_2)\ \mathcal{A}(\omega_2,\omega'\omega)\\&\times\left(\frac{1}{\omega_2-\omega_1+i\epsilon}+\frac{1}{\omega_1+\omega_2-\omega-\omega'-i\epsilon}\right)d\omega_2.\end{aligned} \tag{107}$$

These equations are analogous to those developed by Srivastava[14] for the $N+2\theta$ amplitude. The extra term in Eq. (106) accounts for the fact that $|2N\theta_{\omega'}\theta_{\omega},\ \text{in}\rangle$ contains a component consisting of two bare V particles, as shown by

$$C(\omega',\omega)=Z^{-1}\langle 2_V|2N\theta_{\omega'},\theta_{\omega},\ \text{in}\rangle. \tag{108}$$

By appealing to Eq. (35) of Ref. 2 and Eqs. (24) and (25) of Ref. 1, we find

$$C(\omega',\omega)=\frac{2^{3/2}g^2X(\omega)X(\omega')G(\omega+\omega'-\omega_0)}{Z_0{}^2D(\omega+\omega')G^+(\omega)G^+(\omega')}. \tag{109}$$

The result that we obtain for $\mathcal{A}(\omega_1,\omega'\omega)$ is

$$\begin{aligned}\mathcal{A}(\omega_1,\omega'\omega)=&\frac{-g^2\sqrt{2}}{X^2(\omega_1)}\left(\frac{\delta_{k_1k'}}{G^+(\omega)}+\frac{\delta_{k_1k}}{G^+(\omega')}\right)\\&+\frac{4g^4\sqrt{2}G(\omega+\omega'-\omega_0)}{Z_0{}^2G^{*+}(\omega_1)G^+(\omega+\omega'-\omega_1)G^+(\omega)G^+(\omega')[1-Z_0{}^{-2}G(\omega+\omega'-\omega_0)A(\omega+\omega'-\omega_0)]}\\&-\frac{4g^4\sqrt{2}G(\omega+\omega'-\omega_0)}{Z_0{}^4G^{*+}(\omega_1)G^+(\omega+\omega'-\omega_1)G^+(\omega)G^+(\omega')D(\omega+\omega')[1-Z_0{}^{-2}G(\omega+\omega'-\omega_0)A(\omega+\omega'-\omega_0)]}.\end{aligned} \tag{110}$$

Finally, the combination of Eqs. (102), (104), and (110) yields

$$\begin{aligned}S=&\tfrac{1}{2}(S_{k'k'''}{}^{2N\theta}S_{kk''}{}^{2N\theta}+S_{kk'''}{}^{2N\theta}S_{k'k''}{}^{2N\theta})+8\pi ig^4\delta(\omega+\omega'-\omega''-\omega''')X(\omega''')X(\omega'')X(\omega')X(\omega)\\&\times\frac{Z^2(\omega+\omega'-2\delta m_V)Z_0{}^{-2}G^+(\omega+\omega'-\omega_0)}{G^+(\omega)G^+(\omega')G^+(\omega'')G^+(\omega''')D(\omega+\omega')}.\end{aligned} \tag{111}$$

The $S^{2N\theta}$-matrix elements in this expression have an obvious interpretation, while the last term incorporates the effect of the coupling to the two V particles. Note that the connected amplitude vanishes when the energy $2m+\omega+\omega'$ equals twice the bare V-particle mass.

VI. CONCLUDING REMARKS

We have shown, in Sec. II, that the $2V$ bound state provides another exactly soluble problem in dispersion theory. As a convenient illustration, we have considered both V and N as Bose particles with zero separation. It was found that there are essentially two ways in which one can approach the solution of this problem. The one that we have presented includes the intermediate states characteristic of the V and VN sectors, but circumvents those found in the $V\theta$ sector. For this

[14] P. K. Srivastava, Phys. Rev. 131, 461 (1963).

reason, we are inclined to state that this way is the path of least mathematical resistance. We leave it as a future exercise to discuss the exact solution for the case in which the N particle is contracted in $T(\omega)$.

In dealing with the bound-state eigenvalue condition, we encountered the vertex $\Gamma_0=\langle B_0|j|B\rangle$. On inserting the known structure for j, we maintained a course that ultimately led to the eigenvalue condition. On the other hand, if we had contracted B_0 at this point, we would have found the connection between Γ_0 and Z_B as given in Eq. (66). Similar contractions in Γ_1 and Γ_2 lead to Eqs. (1a) and (1b). For example, in the case of Γ_2 we have

$$\Gamma_2=Z_0^{-1}\delta m_V+i\int_{-\infty}^{\infty} e^{i(2m+\omega_0)t}\langle 0|[f_0(t),f_N^\dagger]\theta(t)|V\rangle dt, \quad (112a)$$

where $Z_0^{-1}\delta m_V$ has come from an equal-time commutator. Proceeding in the usual way, we find

$$\Gamma_2=Z_0^{-1}\delta m_V\left[1+\frac{Z}{\pi}\int_\mu^\infty \operatorname{Im}\left(\frac{1}{G^+(\omega)}\right)d\omega\right], \quad (112b)$$

which reduces [with Eq. (60)] to $\Gamma_2=ZZ_0\delta m_V$.

Since all previous dispersive treatments of the ordinary Lee model have dealt with questions involving only one V particle, we have also found it interesting to extend this method of solution to the collision processes in the $2V$ sector. In particular, these calculations have the merit of allowing us to illustrate fully, in an exact manner, the mechanism for the contraction of a composite particle. Of course, B_0 is an elementary example of such a particle, but we can also think of applying similar techniques to the B particle. For this case, we would introduce the field operator $\psi_B=Z_B^{-1}\psi_V\psi_V$ and its corresponding current $f_B=[H,\psi_B]+(2m+\omega_B)\psi_B$. The contraction of B could be useful in the calculation of the S-matrix element $\langle B\theta_\omega, \text{out}|B\theta_{\omega'}, \text{in}\rangle$.

Finally, we wish to include a remark concerning an alternative approach to the $2V$ bound-state problem. It has already been shown by Mugibayashi[15] that the Bethe-Salpeter equation for the VN system is an exactly soluble example with no redundant states. Therefore it seems worthwhile that, in a future investigation, we give similar consideration to the Bethe-Salpeter amplitude

$$\Phi(t_1,t_2)\equiv\langle 0|T(\psi_V(t_1)\psi_V(t_2))|B\rangle \quad (113)$$

and the integral equation that it satisfies.

[15] N. Mugibayashi, Progr. Theoret. Phys. (Kyoto) **25**, 803 (1961).