# Pole Models for S-Wave Hyperon Decay*

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The $K^*$-pole model and the baryon-pole model are considered as alternatives to the current-algebra calculation of $s$-wave hyperon decay amplitudes. The conditions under which all three of them yield identical results are clarified. A simple dynamical model, based on the $K_1{}^0$ spurion at zero four-momentum, leads to a combination of $K^*$-pole and baryon-pole terms which satisfy these conditions.

## I. INTRODUCTION

THE success of the Sugawara-Suzuki theory[1] in accounting for the empirical properties of $s$-wave hyperon decay has tended to obscure the fact that other dynamical models have been equally successful. Indeed, when octet dominance is assumed to hold, the predictions obtained from the current-current theory by means of the hypothesis of partially conserved axial-vector current (PCAC) and current algebra coincide with those of the $K^*$-dominance model,[2] and also with a particular form of the baryon-pole model.[3] As far as the $K^*$-dominance model is concerned, the result may not be too surprising because the model can be regarded as an "effective representation" of the current-algebra calculation. However, the reason why the requisite conditions (viz., the equality of strong and weak $D/F$ ratios, and their numerical value being $\sqrt{3}$) are realized in the baryon-pole model is not entirely clear. (In our opinion, the dynamical reasons for octet dominance are also not fully understood.[4])

The purpose of this paper is threefold. First, we clarify the conditions under which the $K^*$-dominance model is equivalent to the current-current (employing current algebra) model and show that they are *not*, in general, equivalent.[5] Second, we give a simple algebraic reason for the particular baryon-pole model behaving as it does and inquire if this behavior can be extended under weaker conditions. Finally, we study a simple dynamical model for $s$-wave hyperon decay amplitudes and show how it leads to the expectation that a *sum* of the $K^*$-pole and the baryon-pole terms should give a good description of the $s$-wave hyperon decays. Of course, this leaves the question of relative strength open.[6] We discuss briefly the possibility of extending the model to $p$-wave amplitudes.

There are three appendices: Appendix A contains some properties of $F$ and $D$ matrices which are used in the text and which are interesting in their own right. Appendices B and C contain the complete amplitudes for $s$-wave decays in the $K^*$-pole and the baryon-pole models, respectively.

## II. $K^*$-POLE MODEL

The main assumption in the $K^*$-pole model is that a $K^*\to\pi$ transition dominates the weak amplitude.

The weak Hamiltonian describing the $K^*\to\pi$ transition is given by

$$H_w=g_8K_{\mu i}{}^{*3}\partial_\mu\pi_2{}^i+g_{27}(K_{\mu1}{}^{*3}\partial_\mu\pi_2{}^1+K_{\mu2}{}^{*3}\partial_\mu\pi_1{}^1).+\text{H.c.}\quad(2.1)$$

We have not assumed octet dominance in writing this $H_w$. The coupling of $K^*$ to the baryons is described by

$$H_s=F(\bar{B}_j{}^i\gamma_\mu B_k{}^j-\bar{B}_k{}^j\gamma_\mu B_j{}^i)V_{\mu i}{}^k+D(\bar{B}_j{}^i\gamma_\mu B_k{}^j+\bar{B}_k{}^j\gamma_\mu B_j{}^i)V_{\mu i}{}^k.\quad(2.2)$$

A similar expression involving $\sigma_{\mu\nu}$-type coupling of the baryons can be written down, but the $\sigma_{\mu\nu}$ term vanishes for the $K^*$-pole diagram. Although one expects the coupling of vector mesons to be pure $F$-type, we have included a $D$-type coupling because, at finite momentum transfer, the coupling may not be pure $F$-type and also because of some amusing properties of $D$-type coupling as we shall see later.

Now the decay proceeds through the diagram of Fig. 1. Before we study the detailed form of the amplitudes resulting from Eqs. (2.1), (2.2) and Fig. 1, let

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[1] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997(E) (1965); M. Suzuki, *ibid.* **15**, 986 (1965).

[2] J. Schwinger, Phys. Rev. Letters **12**, 630 (1964); **13**, 355 (1964); B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964).

[3] Riazuddin, Fayyazuddin, and A. H. Zimerman, Phys. Rev. **137**, 1556 (1965).

[4] This is not to say that there have not been many very interesting attempts to explain octet dominance: in a bootstrap model for baryons, R. Dashen, S. Frautschi, and D. H. Sharp, Phys. Rev. Letters **13**, 777 (1964); in current algebra, Y. T. Chiu and J. Schechter, *ibid.* **16**, 1022 (1966).

[5] J. J. Sakurai, Phys. Rev. **156**, 1508 (1967); S. P. Rosen and S. Pakvasa, in *Advances in Particle Physics*, edited by R. E. Marshak and R. L. Cool (John Wiley & Sons, Inc., New York, to be published).

[6] This seems to be a controversial point. It has been argued that the $K^*$-pole model gives too small an amplitude, e.g., D. Loebakka and J. C. Pati, Phys. Rev. **147**, 1047 (1966); and K. Watanabe, *ibid.* **159**, 1369 (1967). On the other hand, K. Nishijima and L. Swank [Phys. Rev. **146**, 1161 (1966)] argue that the baryon-pole model gives too large an amplitude.

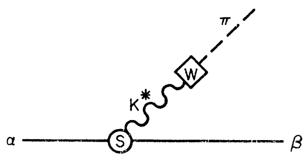


FIG. 1. The $K^*$-pole diagram.

us comment on the relationship of the $K^*$-$\pi$ pole model to the current-current model. It is evident from Eq. (2.1) that whether $H_w$ transforms as **8** or **27**, the decay $\Sigma^+ \rightarrow n+\pi^+$ cannot take place. This is because it would have to go through $\Sigma^+ \rightarrow n+K^{*+}$ which is a superweak ($\Delta S=2$) transition. In the current-current interaction (used in conjunction with PCAC and current algebra) on the other hand, $\Sigma^+ \rightarrow n+\pi^+$ vanishes *only* if the **27** term is absent. This shows that the two approaches are not equivalent in general. They yield similar results only in the limit of octet dominance.

Another way of stating this argument is to observe that, because the $K^*$ is assumed to be pure octet, the $K^*$-$\pi$ model contains no terms in which the baryons are coupled to form a **27**. Thus it might appear as if the introduction of a **27**-plet partner for $K^*$ would make the two models equivalent. This, however, is not true: In such a modified $K^*$-$\pi$ model, couplings of the type $[(\bar{B}B)_8\pi]_{27}$ and $[(\bar{B}B)_{27}\pi]_8$ are allowed, whereas in the current-current model they are not allowed.[7] The first of these couplings vanishes only when octet dominance for $H_w$ is assumed, and the second when $K^*$ is taken to be pure octet. It is only under these conditions that $K^*$ dominance and current algebra are equivalent models.

In the effective coupling due to the $K^*$-pole diagram, the baryon part has the following form:

$$\bar{B}_j\gamma_\mu q_\mu(FF_i+DD_i)_{jk}B_k\,, \tag{2.3}$$

where $q_\mu$ is the momentum transfer and $F_i$ and $D_i$ are the usual $8\times 8$ matrices. Since in this form, the baryons are always coupled to an octet, the derivative coupling amplitudes defined by Eq. (2.3) satisfy the three sum rules of Rosen[8]; i.e.,

$$(\sqrt{\tfrac{3}{2}})\Sigma_-^- - \Lambda_-^0 - 2\Xi_-^- = 0\,, \tag{2.4}$$

$$(\sqrt{\tfrac{3}{2}})\Sigma_0^+ + \Lambda_0^0 - 2\Xi_0^0 = 0\,, \tag{2.5}$$

$$\sqrt{3}\Delta(\Sigma)+\Delta(\Lambda)+2\Delta(\Xi)=0\,, \tag{2.6}$$

and of course $\Sigma_+^+=0$.[9] Here $\Delta(\Sigma)$, $\Delta(\Lambda)$, and $\Delta(\Xi)$ are violations of the $\Delta T=\frac{1}{2}$ rule and are defined by

$$\begin{aligned}\Delta(\Sigma)&=\Sigma_0^+ - (1/\sqrt{2})\Sigma_-^- + (1/\sqrt{2})\Sigma_+^+\,,\\ \Delta(\Lambda)&=\Lambda_-^0+\sqrt{2}\Lambda_0^0\,,\\ \Delta(\Xi)&=\Xi_-^- - \sqrt{2}\Xi_0^0\,.\end{aligned} \tag{2.7}$$

[7] S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. **146**, 1118 (1966).

[8] S. P. Rosen, Phys. Rev. **143**, 138 (1966). This was the way in which Schwinger (see Ref. 2) used the $K^*$-pole model.

[9] Clearly (2.6) is not independent of the other two sum rules; however, it is convenient to express it in this form.

When the expression in Eq. (2.3) is reduced to nonderivative coupling, we note that $\gamma_\mu q_\mu$ acts as a mass-difference operator between the baryons, and this can be expressed by writing the baryon part as

$$\bar{B}_j[M,FF_i+DD_i]_{jk}B_k\,, \tag{2.8}$$

where $M$ is the mass operator. Using the most general form of the mass operator,

$$M=\lambda F_8+\mu D_8+\nu T_{88}\,, \tag{2.9}$$

Eq. (2.8) becomes

$$\bar{B}_j[\lambda F_8+\mu D_8+\nu T_{88},FF_i+DD_i]_{jk}B_k\,. \tag{2.10}$$

From Eq. (2.10) it is easy to see that the amplitudes for nonderivative coupling will satisfy the sum rules of Eqs. (2.4)–(2.6) in the following cases:

(i) $D=0$ and $\nu=0$, i.e., the $\bar{B}BV$ coupling is pure $F$-type and the Gell-Mann–Okubo mass formula is satisfied.[10]

(ii) $\mu=\nu=0$, i.e., the mass splitting is pure $F$-type and the $\Sigma-\Lambda$ mass difference is neglected.[11]

(iii) $F=0$, i.e., the $\bar{B}BV$ coupling is pure $D$-type and $\lambda$, $\mu$, and $\nu$ are such that the global-symmetry (GS) mass formula[12] is satisfied.[13] [This is not obvious from Eq. (2.10) but is shown in Appendix B.]

Both cases (i) and (ii) are close to the accepted values for masses and $F/D$ ratios. Case (iii) is amusing in that it connects pure $D$-type coupling to the global-symmetry mass formula. In the limit of octet dominance the sum rules (2.4)–(2.6) reduce to the Lee-Sugawara sum rule[14] and the $\Delta T=\frac{1}{2}$ rules.

In the above discussion and in the literature, the discussion has been in terms of the $J^p=1^-$, i.e., the vector $K^*$. Now that we know of other low-lying $K^*$'s (e.g., those with $J^p=2^+$), there does not seem to be any reason to single out the vector $K^*$. The tensor ($2^+$) $K^*$ can also contribute to the $s$-wave amplitude and the results are very similar to the above. Again the effective amplitude has derivative coupling, and moreover the $\bar{B}BT$ coupling which contributes (i.e., the non-spin-flip one) is also expected to be predominantly $F$-type.[15] So our remarks apply to a $K^*$-pole model where the $K^*$ can refer to either vector or tensor.

There are other models based on a quark-model Hamiltonian[16] or on the (nonvanishing) divergence of

[10] This was observed by Nishijima and Swank (see Ref. 6) and also by Sakurai (see Ref. 5).

[11] This corresponds to the approximation used by Lee and Swift (see Ref. 2) in their use of the $K^*$-pole model.

[12] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[13] S. P. Rosen and S. Pakvasa, Ref. 5.

[14] H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); B. W. Lee, Phys. Rev. Letters **12**, 83 (1964). With our phase convention this sum rule reads

$$\sqrt{3}\Sigma_0^+ - \Lambda_-^0 - 2\Xi_-^- = 0.$$

[15] V. Barger and M. Olsson, Phys. Rev. Letters **15**, 930 (1965); R. Dashen and S. Frautschi, Phys. Rev. **152**, 1450 (1966).

[16] Riazuddin and K. T. Mahanthappa, Phys. Rev. **147**, 972 (1966); M. K. Gaillard, Phys. Letters **20**, 533 (1966).

the strangeness-changing vector current[17] which are essentially identical to the $K^*$-pole model.

### III. BARYON-POLE MODEL

The baryon-pole model has been used quite often in the past[18,19] to fit the $s$-wave amplitudes. It has been often argued that, since a current-current interaction leads to an effective $TL(1)$-invariant interaction while the effective two-body interaction has to be $TL(2)$-invariant in order to contribute to the $s$-wave pole terms, the pole terms are forbidden in the limit of $SU(3)$.[20] However by the same token $K_1 \to 2\pi$ is also forbidden[7] but its coupling constant is roughly comparable to that for $s$-wave hyperon decay. Hence, even assuming a current-current interaction, it is by no means clear that the baryon poles are forbidden. As long as the transition $K_1 \to$ vacuum is not negligible, the baryon poles will contribute to the $s$-wave amplitudes.

The pole diagrams that contribute are shown in Fig. 2. The structure of the strong and weak vertices is given by

$$H_s = d(\bar{B}_j{}^i\gamma_5 B_k{}^j + \bar{B}_k{}^j\gamma_5 B_j{}^i)P_i{}^k + f(\bar{B}_j{}^i\gamma_5 B_k{}^j - \bar{B}_k{}^j\gamma_5 B_j{}^i)P_i{}^k, \quad (3.1)$$

$$H_w = D_w(\bar{B}_i{}^3\gamma_5 B_2{}^i + \bar{B}_2{}^i\gamma_5 B_i{}^3) + F_w(\bar{B}_i{}^3\gamma_5 B_2{}^i - \bar{B}_2{}^i\gamma_5 B_i{}^3) + \alpha(\bar{B}_1{}^3\gamma_5 B_2{}^1 + \bar{B}_2{}^3\gamma_5 B_1{}^1 + \bar{B}_1{}^1\gamma_5 B_2{}^3 + \bar{B}_2{}^1\gamma_5 B_1{}^3) + \text{H.c.} \quad (3.2)$$

The detailed amplitudes that follow from Fig. 2 and Eqs. (2.2) and (2.3) are given in Appendix C.

Let us first consider the approximation in which we take all the baryon masses to be equal. This is not such a bad approximation since the mass differences are certainly small compared to the sums of the masses which appear in the denominators. Then the $SU(3)$ structure of the amplitudes is

$$\bar{B}_i\{fF_k + dD_k, F_wF_7 + D_wD_7 + \alpha[27]_{7k}\}_{ij}B_jP_k. \quad (3.3)$$

Let us also neglect the contribution from **27** for the moment. Then the baryon part of (3.3) is simply

$$\bar{B}_i\{fF_k + dD_k, F_wF_7 + D_wD_7\}_{ij}B_j. \quad (3.4)$$

Now the anticommutator in (3.4) is

$$fF_w\{F_k,F_7\} + dD_w\{D_k,D_7\} + fD_w\{F_k,D_7\} + dF_w\{D_k,F_7\}. \quad (3.5)$$

The first two terms contain only $\mathbf{8}_D$ and **27** by being symmetric, but the last two terms, in general, contain **10** and **10*** also. However if $D_w/F_w = d/f$, then the last

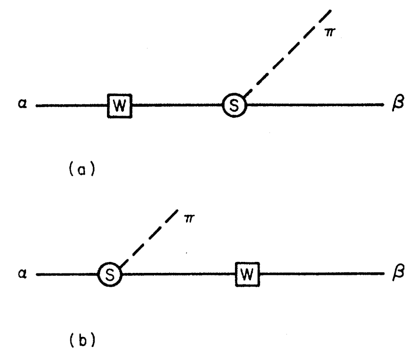


FIG. 2. Baryon-pole diagrams.

two terms can be combined to

$$fD_w[\{F_k,D_7\} + \{D_k,F_7\}]. \quad (3.6)$$

This by virtue of Eq. (A8) is just

$$2fD_wd_{k7m}F_m. \quad (3.7)$$

Thus when $D_w/F_w = d/f$, the baryon-antibaryon coupling has only **8** and **27**, and then it follows from the theorem proved by Rosen[21] that the Lee-Sugawara sum rule must be satisfied. Furthermore from Eq. (A11) it is easy to see that when $D_w/F_w = d/f = \sqrt{3}$, the first two terms in (3.5) couple the baryon-antibaryon to a singlet and hence do not contribute to the (strangeness changing) decay amplitudes. Then the only contribution comes from (3.7) which couples the baryon-antibaryon to $8_F$. But this is precisely the coupling obtained from the $K^*$-pole model when $\bar{B}BV$ coupling is pure $F$-type and the masses obey the Gell-Mann–Okubo (GMO) formula.

We have now established the conditions under which the $K^*$ pole and the baryon pole give rise to pure $F$-type octet coupling of the baryons and thus give the same sum rules as the current-algebra calculation does in the limit of octet dominance. We now consider the effects of (a) including the **27**-plet term in the weak Hamiltonian and (b) removing the mass degeneracy of the baryons. First, we make $\alpha \neq 0$, but keep the masses equal; then if $d/f = D_w/F_w = \sqrt{3}$,

$$\Delta(\Sigma) = \Delta(\Lambda) = \Delta(\Xi). \quad (3.8)$$

And for $d/f$ and $D_w/F_w$ arbitrary, we still have

$$\Delta(\Lambda) = \Delta(\Xi), \quad (3.9)$$

$$\Delta(\Lambda) = \sqrt{3}(f/d)\Delta(\Sigma). \quad (3.10)$$

Next, we ignore the **27**-plet term but include the mass splittings. We approximate the masses by the mass formula

$$M = M_0(1 - \delta Y) \quad (3.11)$$

(i.e., $\Sigma$, $\Lambda$ degenerate), where $\delta \sim 0.08$, and work to first order in $\delta$. Then we find that the new solution for $d/f$ and $D_w/F_w$ which satisfies the Lee-Sugawara sum rule

[17] K. Nishijima and L. Swank, Ref. 6; G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Rev. **168**, 1866 (1968); Riazuddin *et al.*, Ref. 3.

[18] H. Sugawara, Nuovo Cimento **31**, 635 (1964); Riazuddin *et al.*, Ref. 3; J. C. Pati and S. Oneda, Phys. Rev. **140**, B1351 (1965).

[19] R. H. Graham and S. Pakvasa, Phys. Rev. **140**, B1144 (1965).

[20] See, e.g., B. W. Lee and A. R. Swift, Ref. 2.

[21] S. P. Rosen, Phys. Rev. **135**, B326 (1965).

and $\Sigma_+^+=0$ is very close to the one before so that $d/f \sim D_w/F_w \sim \sqrt{3}$ to a good approximation. Finally, if we include both the **27**-plet and the mass splittings, we find that the sum rule (3.10) still holds. Of course, if one thinks of the weak vertex as emission of a $K_1^0$ which then undergoes $K_1 \rightarrow$ vacuum transition,[22] then one can not only justify $d/f$ being equal to $D_w/F_w$ but also argue that the **27**-plet should be absent. We have included the **27**-plet since it is not clear that some contribution would not come from current-current interaction. The value $\sqrt{3}$ for $d/f$ seems to be a "magic" number in that it turns up in many different contexts,[23] besides being close to the "observed" value.

## IV. DISCUSSION

In Secs. II and III we have derived the conditions under which the $K^*$-pole model and the baryon-pole model both give rise to almost pure $F$-type coupling of baryons in the $s$-wave decay amplitudes and hence to agreement with experimental values. As these conditions (predominantly $F$-type coupling of vector and tensor mesons, Gell-Mann–Okubo mass formula, $D/F = \sqrt{3}$ for baryon–pseudoscalar-meson coupling) are nearly satisfied in reality, both models are able to fit the data reasonably well.

If we consider a dynamical model[24] in which the $s$-wave decay amplitude is approximated by the off-mass-shell scattering amplitude $K_1^0+\alpha \rightarrow \beta+\pi$ with $K_1^0 \leftrightarrow$ vacuum transition taking place through $H_w$, then the nearest singularities in the amplitude are just the $K^*$ poles (in the $t$ channel) and the baryon poles (in the $s$ and $u$ channel). It seems reasonable to assume that they dominate the amplitude and then the amplitude is given by a *sum* of $K^*$ poles and baryon poles. Since, as we have seen above, $K^*$ poles and baryon poles by themselves give rise to the right sum rules (Lee-Sugawara) and selection rules ($\Sigma_+^+=0$), a sum of the two will continue to do so.

The analog for $p$ waves is, of course, the usual pole model, with baryon poles and $K$ pole. However, the analogy does not extend to the dynamical model above since there does not appear to be a scalar meson analogous to $K_1^0$.

Finally, we would like to stress that, in order to test whether the current-current interaction or $K^*$-pole model or the baryon-pole model gives a dominant contribution to the $s$-wave decay amplitudes, one has to test the sum rules which go beyond[25] the $\Delta T=\frac{1}{2}$ rule and the octet rule. This can only be done when the experimental determination of the amplitudes has greatly improved over the present one.

[22] A. Salam and J. C. Ward, Phys. Rev. Letters **5**, 380 (1960).

[23] W. A. Simmons, Phys. Rev. **164**, 1956 (1967); P. Babu, A. Rangwala, and V. Singh, *ibid.* **157**, 1322 (1967); D. W. Joseph and L. L. Smalley, *ibid.* **150**, 1209 (1966). For the experimental value see, e.g., W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964).

[24] Such a model has been used in a different way by M. C. Li, Nuovo Cimento **53A**, 327 (1968).

[25] For example, the sum rules (2.6) and (3.9,3.10) distinguish between the $K^*$-pole model and the baryon-pole model when the octet rule is relaxed. These are different from the current-current ones (see Ref. 7).

## ACKNOWLEDGMENTS

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## APPENDIX A: SOME PROPERTIES OF $F$ AND $D$ MATRICES

The $F$ and $D$ matrices are defined to be

$$(F_i)_{jk}=-if_{ijk}\,,\qquad (D_i)_{jk}=d_{ijk}\,, \tag{A1}$$

where $f_{ijk}$ are the structure constants of $SU(3)$, and $d_{ijk}$ are determined by anticommutators of the $3\times 3$ matrices $\lambda_i$ $(i=1,\cdots 8)$:

$$[\lambda_i,\lambda_j]=2if_{ijk}\lambda_k\,,\qquad \{\lambda_i,\lambda_j\}=\tfrac{4}{3}\delta_{ij}I+2d_{ijk}\lambda_k\,. \tag{A2}$$

The $\lambda_i$ are traceless and their scale is set by the requirement that

$$\mathrm{Tr}(\lambda_i\lambda_j)=2\delta_{ij}\,. \tag{A3}$$

The symbol $I$ is used to denote the unit matrix.

From various Jacobi-type identities for products of three $\lambda$ matrices, it is possible to derive commutation and anticommutation rules for the $F_i$ and $D_i$. They are

$$[F_i,F_j]=if_{ijk}F_k\,,\qquad [F_i,D_j]=if_{ijk}D_k\,,\qquad [D_i,D_j]=if_{ijk}F_k-\tfrac{2}{3}T_{ij}\,, \tag{A4}$$

$$\{F_i,F_j\}+\{D_i,D_j\}=\tfrac{4}{3}\delta_{ij}I+2d_{ijk}D_k-\tfrac{2}{3}R_{ij}\,, \tag{A5}$$

where $T_{ij}$ is an antisymmetric matrix and $R_{ij}$ is a symmetric one:

$$(T_{ij})_{\alpha\beta}=\delta_{i\alpha}\delta_{j\beta}-\delta_{i\beta}\delta_{j\alpha}\,,\qquad (R_{ij})_{\alpha\beta}=\delta_{i\alpha}\delta_{j\beta}+\delta_{i\beta}\delta_{j\alpha}\,. \tag{A6}$$

Because $f_{ijk}$ is antisymmetric and $d_{ijk}$ is symmetric under the exchange of neighboring indices, the second rule of Eq. (A4) can be rearranged to give

$$F_kD_l+F_lD_k=D_kF_l+D_lF_k=d_{klm}F_m\,. \tag{A7}$$

In turn this leads to the relation

$$\{F_k,D_l\}+\{F_l,D_k\}=2d_{klm}F_m\,. \tag{A8}$$

With suitable modifications of the numerical coefficients, the above relations are valid for the $F$ and $D$ matrices of any $SU(n)$ algebra. There is, however, one more relation which is valid only for $SU(3)$. It arises from a particular property of traceless $3\times3$ matrices, namely,

$$\mathrm{Tr}(\lambda_i\lambda_j)\,\mathrm{Tr}(\lambda_k\lambda_l)+\mathrm{Tr}(\lambda_i\lambda_k)\,\mathrm{Tr}(\lambda_j\lambda_l)+\mathrm{Tr}(\lambda_i\lambda_l)\,\mathrm{Tr}(\lambda_j\lambda_k)=\mathrm{Tr}[\lambda_i(\textstyle\sum\lambda_j\lambda_k\lambda_l)], \tag{A9}$$

where $\sum$ denotes the sum over all permutations of $j$, $k$, $l$, and it takes the form

$$\{D_i,D_j\}=\tfrac{1}{3}\delta_{ij}I-d_{ijk}D_k+\tfrac{1}{3}R_{ij}. \tag{A10}$$

From the above and Eq. (A5) it follows that

$$3\{D_i,D_j\}+\{F_i,F_j\}=2\delta_{ij}I. \tag{A11}$$

It is now possible to evaluate the traces of products of $F$ and $D$ matrices. From Eq. (A4) it follows that

$$\mathrm{Tr}(F_i)=\mathrm{Tr}(D_i)=0. \tag{A12}$$

Because $F_i$ is antisymmetric and $D_j$ is symmetric, the trace of $F_iD_j$ must vanish:

$$\mathrm{Tr}(F_iD_j)=0. \tag{A13}$$

Equations (A5) and (A10) then imply that

$$\begin{aligned}\mathrm{Tr}(F_iF_j)&=3\delta_{ij},\\ \mathrm{Tr}(D_iD_j)&=(5/3)\delta_{ij}\end{aligned} \tag{A14}$$

and

$$\begin{aligned}\mathrm{Tr}(F_iF_jD_k)&=\tfrac{3}{2}d_{ijk},\\ \mathrm{Tr}(D_iD_jF_k)&=\tfrac{5}{6}if_{ijk}.\end{aligned} \tag{A15}$$

Further results can be found in the work of Kaplan and Resnikoff.[26]

## APPENDIX B: $K^*$-POLE AMPLITUDES

In this Appendix we write down the explicit form of the amplitudes for the nonderivative coupling in the $K^*$-pole model:

$$\Lambda_-^0=(1/\sqrt{6})(M_\Lambda-M_N)(g_8+g_{27})(3F+D), \tag{B1}$$

$$\Lambda_0^0=(1/\sqrt{6})(M_\Lambda-M_N)(g_8-g_{27})(3F+D), \tag{B2}$$

$$\Xi_-^-=(1/\sqrt{6})(M_\Xi-M_\Lambda)(g_8+g_{27})(D-3F), \tag{B3}$$

$$\Xi_0^0=(1/\sqrt{6})(M_\Xi-M_\Lambda)(g_8-g_{27})(D-3F), \tag{B4}$$

$$\Sigma_-^-=(M_\Sigma-M_N)(g_8+g_{27})(D-F), \tag{B5}$$

$$\Sigma_0^+=(1/\sqrt{2})(M_\Sigma-M_N)(g_8-g_{27})(D-F), \tag{B6}$$

$$\Sigma_+^+=0. \tag{B7}$$

From the above expressions it is easy to verify that the following is true:

$$(\sqrt{\tfrac{3}{2}})\Sigma_-^- -\Lambda_-^0-2\Xi_-^- =(1/\sqrt{6})(g_8+g_{27})[D\Delta(\mathrm{GS})+3F\Delta(\mathrm{GMO})], \tag{B8}$$

$$(\sqrt{\tfrac{3}{2}})\Sigma_0^+ +\Lambda_0^0-2\Xi_0^0 =(1/\sqrt{6})(g_8-g_{27})[D\Delta(\mathrm{GS})+3F\Delta(\mathrm{GMO})], \tag{B9}$$

$$\sqrt{3}\Delta(\Sigma)+\Delta(\Lambda)+2\Delta(\Xi) =-(2/\sqrt{6})g_{27}[D\Delta(\mathrm{GS})+3F\Delta(\mathrm{GMO})], \tag{B10}$$

where $\Delta(\mathrm{GMO})=2M_N+2M_\Xi-3M_\Lambda-M_\Sigma$ and vanishes if the Gell-Mann–Okubo mass formula is satisfied, and $\Delta(\mathrm{GS})=3M_\Sigma+M_\Lambda-2M_N-2M_\Xi$ and vanishes if the global-symmetry mass formula is satisfied. This verifies the statements made in Sec. II.

## APPENDIX C: BARYON POLE AMPLITUDES

Here we write the detailed form of the amplitudes in the baryon-pole model without any approximations, i.e., including a **27**-plet term in the Hamiltonian and with exact masses:

$$\Lambda_-^0=\frac{1}{\sqrt{3}}\left[\frac{-2d(D_W+F_W+\alpha)}{M_\Sigma+M_N}-\frac{(f+d)(3F_w-D_w+\alpha)}{M_\Lambda+M_N}\right], \tag{C1}$$

$$\Lambda_0^0=-\frac{1}{\sqrt{6}}\left[\frac{-2d(D_w+F_w-\alpha)}{M_\Sigma+M_N}-\frac{(f+d)(3F_w-D_w+\alpha)}{M_\Lambda+M_N}\right], \tag{C2}$$

$$\Xi_-^-=\frac{1}{\sqrt{3}}\left[\frac{(d-f)(3F_w+D_w-\alpha)}{M_\Xi+M_\Lambda}-\frac{2d(D_w-F_w+\alpha)}{M_\Xi+M_\Sigma}\right], \tag{C3}$$

$$\Xi_0^0=\frac{1}{\sqrt{6}}\left[\frac{(d-f)(3F_w+D_w-\alpha)}{M_\Xi+M_\Lambda}-\frac{2d(D_w-F_w-\alpha)}{M_\Xi+M_\Sigma}\right], \tag{C4}$$

$$\Sigma_-^-=\sqrt{2}\left[\frac{f(-F_w-D_w+\alpha)}{M_\Sigma+M_N}+\frac{d(3F_w-D_w+\alpha)}{3(M_\Lambda+M_N)}\right], \tag{C5}$$

$$\Sigma_0^+=-\frac{(d+3f)(F_w+D_w+\alpha)}{M_\Sigma+M_N}, \tag{C6}$$

$$\Sigma_+^+=\sqrt{2}\left[\frac{(d+2f)(D_w+F_w)+d\alpha}{M_\Sigma+M_N}+\frac{d(3F_w-D_w+\alpha)}{3(M_\Lambda+M_N)}\right]. \tag{C7}$$

The statements made in Sec. III can be checked explicitly using these amplitudes.

[26] L. M. Kaplan and M. Resnikoff, J. Math. Phys. **8**, 2194 (1967); some of the results were also derived independently by V. I. Ogievetskii and I. V. Polybarinov, Yadern. Fiz. **4**, 853 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 605 (1967)].