Regge Parameters in Crossing-Even KN Scattering*

DAVID J. GEORGE AND ARNOLD TUBIS
Department of Physics, Purdue University, Lafayette, Indiana 47907
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The method of Olsson is used to estimate the Pomeranchuk P and P' parameters in KN scattering. Assuming that the forward crossing-even KN amplitude has the asymptotic form, $-\sum_{s=P,P'}\gamma_s[e^{-i\frac{1}{2}\pi\alpha_s}/\sin\frac{1}{2}\pi\alpha_s](\omega/\omega_0)^{\alpha_s}$, where ω is the kaon lab energy and $\omega_0=1$ BeV, we find that a good fit to the dispersion sum rules is given by $\gamma_P=17.3\pm0.1$ mb BeV, $\alpha_P=1$, $\gamma_{P'}=7.1\pm0.7$ mb BeV, $\alpha_{P'}=0.89\pm0.10$. These parameters give a reasonable fit to the high-energy scattering.

IN a recent paper, Olsson[1] described a method for determining the ρ, ρ' Regge parameters in πN scattering by using modified dispersion relations. In this paper, we apply his method to the forward $K^{\pm}p$ crossing-even amplitude to obtain information on the Pomeranchuk P and P' parameters for KN scattering. We find a set of parameter values which gives an excellent fit to the dispersion sum rules and a reasonable fit to the high-energy scattering (which is presently subject to large uncertainties).

In deriving the Olsson sum rules, we work with the forward amplitude $T(\omega)$ normalized according to the optical theorem,

$$\mathrm{Im}T(\omega)=(\omega^2-m_K^2)^{1/2}\tfrac{1}{2}[\sigma_{K^+p}(\omega)+\sigma_{K^-p}(\omega)], \quad (1)$$

where ω is the kaon laboratory energy, m_K the kaon mass, and $\sigma_{K^{\pm}p}(\omega)$ the total $K^{\pm}p$ cross sections. Natural units, $\hbar=c=1$, are used throughout this note.

We then define

$$F(\omega)=\omega T(\omega)e^{\pi i\epsilon}/(\omega^2-\omega_{\Lambda\pi}^2)^{\epsilon}, \quad (2)$$

where $\omega_{\Lambda\pi}$ is the $\Lambda\pi$ threshold energy and ϵ is a continuous parameter which will be shown to be less than unity.

The asymptotic amplitude is assumed to have the Regge form with Pomeranchuk (P) and Pomeranchuk prime (P') contributions[2]:

$$T_{\mathrm{asym}}(\omega)=-\sum_{i=P,P'}\gamma_i\frac{e^{-i\frac{1}{2}\pi\alpha_i}}{\sin\frac{1}{2}\pi\alpha_i}\left(\frac{\omega}{\omega_0}\right)^{\alpha_i}, \quad (3)$$

where γ is the residue, α the $t=0$ intercept, and ω_0 is taken to be 1 BeV.

Then the function

$$F'(\omega)=\frac{\omega[T(\omega)-T_{\mathrm{asym}}(\omega)]e^{\pi i\epsilon/2}}{(\omega^2-\omega_{\Lambda\pi}^2)^{\epsilon/2}} \quad (4)$$

is superconvergent[3] for $\alpha<\epsilon-2$. However, Dolen, Horn, and Schmid[4] have shown that ϵ can be taken arbitrarily small.

The energy range involved in the sum rules is divided into a "low-energy" part, $\omega_{\Lambda\pi}\leq\omega\leq\bar{\omega}$, and a high-energy part, $\omega>\bar{\omega}$, where $\bar{\omega}$ must be large enough so that $T(\omega>\bar{\omega})\approx T_{\mathrm{asym}}(\omega)$. $\bar{\omega}$ is taken here to be $5m_K$, the highest energy for which $\mathrm{Re}T(\omega)$ data are available. At $5m_K$ there is already a good indication of asymptotic behavior. For example, the K^+p cross section has already reached 17.6 mb compared with the value of 17.3 mb at 20 BeV.[5]

Using the usual methods, we derive the following sum rule:

$$I(\epsilon)=\sum_{i=P,P'}\frac{2\gamma_i}{\pi\sin\frac{1}{2}\pi\alpha}\left(\frac{\bar{\omega}}{\omega_0}\right)^{\alpha_i}\frac{\sin\frac{1}{2}\pi(\alpha_i-\epsilon)}{2+\alpha_i-\epsilon}$$
$$=\sum_{Y=\Lambda,\Sigma}\frac{\omega_Y X_Y}{(\omega_{\Lambda\pi}^2-\omega_Y^2)^{\epsilon/2}\bar{\omega}^{2-\epsilon}}$$
$$+\frac{2}{\pi}\int_{\omega_{\Lambda\pi}}^{\bar{\omega}}\frac{\omega d\omega}{\bar{\omega}^{2-\epsilon}}\frac{\mathrm{Re}T(\omega)\sin\frac{1}{2}\pi\epsilon+\mathrm{Im}T(\omega)\cos\frac{1}{2}\pi\epsilon}{(\omega^2-\omega_{\Lambda\pi}^2)^{\epsilon/2}}, \quad (5)$$

where

$$\omega_Y=(m_Y^2-m_N^2-m_K^2)/2m_N, \quad (6)$$

$$X_Y=g_Y^2[(M_Y-m_N)^2-m_K^2]/4m_N^2. \quad (7)$$

In these expressions m_N, m_Λ, and m_Σ are, respectively, the nucleon, Σ, and Λ masses and $\omega_{\Lambda\pi}$ is given by the expression for ω_Y with $m_Y=m_\Lambda+m_\pi$. We take the following values for g_Λ, g_Σ:

$$g_\Lambda^2/4\pi=13.5, \quad g_\Sigma^2/4\pi=0.2. \quad (8)$$

They are taken from the work of Kim[6] with the modification due to Chan and Meiere.[7]

In order to evaluate these sum rules, we need both the real and imaginary parts of $T(\omega)$. Kim has calculated the K^-p real and imaginary parts for $\omega_{\Lambda\pi}<\omega<1.4m_K$ which we use in a slightly modified form. The modification only applies to $\omega_{\Lambda\pi}<\omega<0.7m_K$ and has been described elsewhere.[8] We note that this modification is not sensitive to the assumed high-energy behavior

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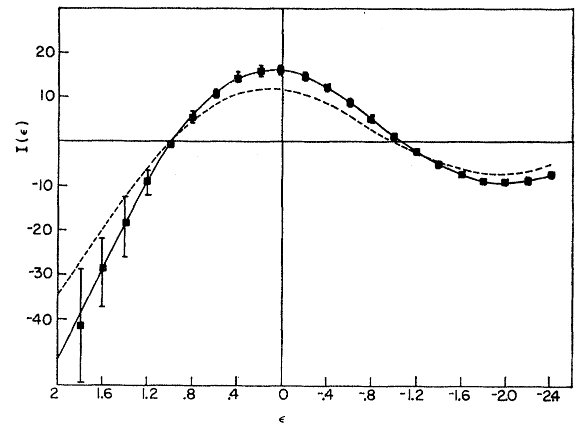

FIG. 1. Plot of $I(\epsilon)$, as calculated from (5), compared with experiment. The broken line is the prediction of $I(\epsilon)$ given by the Pomeranchon contribution alone.

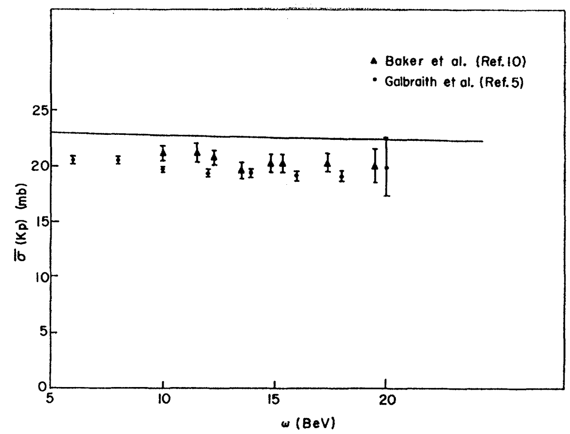

FIG. 2. The predicted high-energy Kp total cross section, compared with experiment.

of $T(\omega)$, and in fact comes almost entirely from the pole terms.

For the K^+p contribution we use the effective range of Goldhaber *et al.* for the S-wave scattering,[9]

$$p\cot\delta = (A_1{}^+)^{-1} + \tfrac{1}{2}R_1{}^+p^2\,, \tag{9}$$

where p is the c.m. momentum and

$$A_1{}^+ = 0.29, \quad R_1{}^+ = 0.5\text{ F}. \tag{10}$$

For energies greater than $1.4m_K$ (500 MeV/c laboratory momentum), we use experimental values of $\mathrm{Re}T(\omega)$ and $\mathrm{Im}T(\omega)$.[5,10]

Using the data described above, we have calculated $I(\epsilon)$ for $2>\epsilon>-2$ and the results are shown in Fig. 1. As Olsson has pointed out,[1] the sum rules, as $\epsilon\to-2$, approach trivial identities and consequently are of little practical use in determining high-energy parameters.

[9] S. Goldhaber *et al.*, Phys. Rev. Letters **9**, 135 (1967).

[10] The $\mathrm{Im}T(\omega)$ were taken from: W. F. Baker *et al.*, Phys. Rev. **129**, 2285 (1963); R. J. Abrams *et al.*, Phys. Rev. Letters **19**, 259 (1967), **19**, 678 (1967); J. D. Davies *et al.*, *ibid.* **18**, 62 (1967); R. L. Cool *et al.*, *ibid.* **16**, 1228 (1966); **17**, 102 (1966). The $\mathrm{Re}T(\omega)$ results are summarized in N. Zovko, Z. Physik **196**, 16 (1966).

Our best fit gives the following Regge parameters:

$$\gamma_P = 17.3\pm0.1\text{ mb BeV}, \quad \alpha_P = 1\,, \tag{11}$$

$$\gamma_{P'} = 7.1\pm0.7\text{ mb BeV}, \quad \alpha_{P'} = 0.89\pm0.10\,. \tag{12}$$

Our value of $\alpha_{P'}$ is somewhat larger than that indicated from the analysis of πN scattering.[11] This might be due to the inaccuracy of the data used, to the smallness of $\bar{\omega}$, to the limited validity of the concept of a P' trajectory, and perhaps to other factors such as the presence of Regge cuts.

The fit to the high-energy cross sections using these parameters is shown in Fig. 2. The data is taken from Baker *et al.*[10] and from Galbraith *et al.*[5] The prediction is consistently higher but the experimental points lie within the range of $\gamma_{P'}$ expressed in Eq. (12).

In conclusion, we believe we have found a consistent set of P and P' parameters for KN scattering which agree reasonably well with both dispersion theory and high-energy cross sections.

[11] Y.-C. Liu and S. Okubo, Phys. Rev. **168**, 1712 (1968); earlier references may also be found in this paper.