Self-Consistent Calculation of Vector- and Pseudoscalar-Meson Mass Splittings*

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It is shown that, within the framework of the model chosen, one can calculate in a completely self-consistent manner the medium-strong (MS) mass splittings within the vector- and pseudoscalar-meson octets. In particular, expressions for the mass splittings may be written down in terms of experimentally observable parameters, one of which, the ρ-π coupling constant $g_{\rho\pi}^2$, is found to be equal to 0.526, in good agreement with the physically observed value 0.60.

I. INTRODUCTION

IN a previous paper,[1] we have shown that the medium-strong (M.S.) mass splittings within the spin-$\frac{1}{2}$ baryon octet could be understood as arising, as a low-energy effect, from the M.S. mass splittings within the pseudoscalar-meson octet. Utilizing the lowest-lying two-particle intermediate state (baryon-meson) approximation for the baryon self-energy as the dynamical model, the observed M.S. mass splittings of the mesons were treated as the driving force[2,3] which generates, in a self-consistent manner, the M.S. mass splittings of the intermediate baryons, the latter effect being referred to as the feedback.[2,3] Within this model, we were able to show that the observed mass splittings could be obtained for reasonable values of the parameters (the πN coupling constant, the f/d ratio, and a cutoff parameter Λ).

An attempt to apply the model to the meson mass spectra by utilizing baryon-antibaryon intermediate states for the meson self-energies failed,[2] a not very surprising result due to the fact that there exist lower-lying states which presumably are more important. In particular, a set of lower-lying two-particle intermediate states can be generated through the interaction of the octets of pseudoscalar mesons (P) and vector mesons (V). Since the self-energies of the vector and pseudoscalar mesons both include the VP intermediate state, it will be convenient to compute the mass splittings of both octets together in a completely self-consistent manner. Thus we assume that the M.S. mass splittings of the vector mesons serve as the driving force for the pseudoscalar mesons and the meson mass splittings serve as the driving force for the vector mesons. In addition, it will be found that baryon-antibaryon states can be neglected.

II. MODEL

Our approach is similar to that used in a paper by Pietschmann[4] in that we use the Zachariasen model[5] approximation in describing the SU_3-symmetric part of the interaction. Thus the SU_3-symmetric interaction Lagrangian for the octet of vector mesons, denoted by the 3×3 matrix V^μ with the octet of pseudoscalar mesons denoted by the 3×3 matrix P, is given by

$$\mathcal{L}_{\rm int}=\epsilon^{\mu\nu\rho\sigma}(h_0/M)\,{\rm Tr}[\partial_\mu V_\nu \partial_\rho V_\sigma P] +ig_0\,{\rm Tr}\{V^\mu[\partial_\mu P,P]\}\,, \quad (1)$$

where charge-conjugation invariance requires the VPP coupling to be F-type and VVP coupling to be D-type.[6] We note that the $I=0$, $Y=0$ component of the vector-meson octet is the ω^8 which is related to the physical ω and ϕ particle by the ω-ϕ mixing theory[6,7]:

$$|\omega^8\rangle=\sin\theta|\omega\rangle+\cos\theta|\varphi\rangle. \quad (2)$$

We have chosen not to include the unitary singlet ω^1, justifying this omission by the results of the calculation. We would tend to expect its contribution to be relatively less important as it is more massive than the other sixteen particles and does not interact with the pseudoscalar mesons through VPP coupling. Thus, the calculated mass splittings for the vector mesons will be $M_{K^*}^2-M_{\omega^8}^2$ and $M_\rho^2-M_{\omega^8}^2$. In this respect our approach differs from Pietschmann's in that he assumes that the symmetry breaking may be described by an ω-ϕ mixing term in the interaction Lagrangian, while we take the point of view that the mass splitting and mixing effects are independent and may be computed separately. Thus we are assuming that the mass splittings can be computed first and the mixing of the ω^1 and ω^8 afterwards, the feedback effect of the latter on the former being assumed negligible.

Proceeding as in Ref. 1, we assume that the pseudoscalar meson masses may be written, to lowest order in the vector meson mass splittings, in the form

$$\mu_i^2=\bar{\mu}^2+\sum_{\beta=1}^{3}\left[\frac{\partial\mu_i^2}{\partial M_\beta^2}\right]_{M,\mu}(M_\beta^2-M^2)\,, \quad (3a)$$

with μ_i ($i=1\cdots3$) denoting the mass of the π, K, and η, M_α ($\alpha=1\cdots3$) denoting the mass of the ρ, K^*, and ω^8, respectively, and where μ and M are the SU_3 central

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[2] S. L. Cohen and C. R. Hagen, Phys. Rev. **157**, 1344 (1967).
[3] G. Barton and D. Dare, Phys. Rev. **150**, 1220 (1966).
[4] H. V. R. Pietschmann, Phys. Rev. **139**, B446 (1965).
[5] F. Zachariasen, Phys. Rev. **121**, 1851 (1961).
[6] S. L. Glashow, Phys. Rev. Letters **11**, 48 (1963).
[7] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

masses for the pseudoscalar and vector mesons, re- spectively. Similarly, one has for the vector mesons,

$$M_\alpha{}^2=\bar{M}^2+\sum_{j=1}^{3}\left[\frac{\partial M_\alpha{}^2}{\partial \mu_j{}^2}\right]_{M,\mu}(\mu_j{}^2-\mu^2). \tag{3b}$$

Expressions for the derivative coefficients may be obtained from the Lehmann mass sum rules which read

$$\mu_i{}^2=\mu_0{}^2-\sum_{j,\alpha}\int_{(\mu_j+M_\alpha)^2}^{\infty}dm^2\frac{\rho_1(m^2,\mu_j{}^2,M_\alpha{}^2)}{m^2-\mu_i{}^2}-\sum_{\alpha,\beta}\int_{(M_\alpha+M_\beta)^2}^{\infty}dm^2\frac{\rho_2(m^2,M_\alpha{}^2,M_\beta{}^2)}{m^2-\mu_i{}^2}, \tag{4a}$$

$$M_\alpha{}^2=M_0{}^2-\sum_{i,\beta}\int_{(M_\beta+\mu_i)^2}^{\infty}dm^2\frac{\tau_1(m^2,M_\beta{}^2,\mu_i{}^2)}{m^2-M_\alpha{}^2}-\sum_{i,j}\int_{(\mu_i+\mu_j)^2}^{\infty}dm^2\frac{\tau_2(m^2,\mu_i{}^2,\mu_j{}^2)}{m^2-M_\alpha{}^2}, \tag{4b}$$

where the leading contributions are VP to ρ_1, VV to ρ_2, VP to τ_1, and PP to τ_2. Taking $\partial/\partial M_\gamma{}^2$ on Eq. (4a) and $\partial/\partial\mu_k{}^2$ on Eq. (4b) gives the result

$$\left(1+\sum_{j,\alpha}\int_{(\mu_j+M_\alpha)^2}^{\infty}dm^2\frac{\rho_1}{(m^2-\mu_i{}^2)^2}+\sum_{\alpha,\beta}\int_{(M_\alpha+M_\beta)^2}^{\infty}dm^2\frac{\rho_2}{(m^2-\mu_i{}^2)^2}\right)\frac{\partial\mu_i{}^2}{\partial M_\gamma{}^2}$$
$$=-\sum_{j}\int_{(\mu_j+M_\gamma)^2}^{\infty}dm^2\frac{\partial\rho_1/\partial M_\gamma{}^2}{m^2-\mu_i{}^2}-\sum_{j,\alpha}\int_{(\mu_j+M_\alpha)^2}^{\infty}dm^2\frac{\partial\rho_1/\partial\mu_j{}^2}{m^2-\mu_i{}^2}\frac{\partial\mu_j{}^2}{\partial M_\gamma{}^2}-2\sum_{\alpha}\int_{(M_\alpha+M_\gamma)^2}^{\infty}dm^2\frac{\partial\rho_2/\partial M_\gamma{}^2}{m^2-\mu_i{}^2}, \tag{5a}$$

$$\left(1+\sum_{i,\beta}\int_{(M_\beta+\mu_i)^2}^{\infty}dm^2\frac{\tau_1}{(m^2-M_\alpha{}^2)^2}+\sum_{i,j}\int_{(\mu_i+\mu_j)^2}^{\infty}dm^2\frac{\tau_2}{(m^2-M_\alpha{}^2)^2}\right)\frac{\partial M_\alpha{}^2}{\partial\mu_k{}^2}$$
$$=-\sum_{\beta}\int_{(M_\beta\mu+k)^2}^{\infty}dm^2\frac{\partial\tau_1/\partial\mu_k{}^2}{m^2-M_\alpha{}^2}-\sum_{i,\beta}\int_{(M_\beta+\mu_i)^2}^{\infty}dm^2\frac{\partial\tau_1/\partial M_\beta{}^2}{m^2-M_\alpha{}^2}\frac{\partial M_\beta{}^2}{\partial\mu_k{}^2}-2\sum_{i}\int_{(\mu_k+\mu_i)^2}^{\infty}dm^2\frac{\partial\tau_2/\partial\mu_k{}^2}{m^2-M_\alpha{}^2}. \tag{5b}$$

The term in brackets on the left multiplying $\partial\mu_i^2/\partial M_\gamma{}^2$ and $\partial M_\alpha^2/\partial\mu_k{}^2$ is the inverse of the wave-function renormalization constant for the pseudoscalar and vector meson, respectively. Thus, when multiplied through to the right-hand side, it changes the bare spectral functions ρ_1, ρ_2, τ_1, and τ_2 to the renormalized spectral functions $\bar\rho_1$, $\bar\rho_2$, $\bar\tau_1$, and $\bar\tau_2$[1] which are given to lowest order by[4]

$$\bar\rho_1=-\frac{a_{ij\alpha}g_{ij\alpha}{}^2}{16\pi^2M^2}m^{-2}[m^4+M_\alpha{}^4+\mu_j{}^4-2(m^2M_\alpha{}^2+m^2\mu_j{}^2+M_\alpha{}^2\mu_j{}^2)]^{3/2}, \tag{6a}$$

$$\bar\rho_2=-\frac{a_{i\alpha\beta}h_{i\alpha\beta}{}^2}{16\pi^2M^2}m^{-2}[m^4+M_\alpha{}^4+M_\beta{}^4-2(m^2M_\alpha{}^2+m^2M_\beta{}^2+M_\alpha{}^2M_\beta{}^2)]^{3/2}, \tag{6b}$$

$$\bar\tau_1=-\frac{a_{\alpha\beta i}h_{\alpha\beta i}{}^2}{16\pi^2M^2}m^{-2}[m^4+M_\beta{}^4+\mu_i{}^4-2(m^2M_\beta{}^2+m^2\mu_i{}^2+M_\beta{}^2\mu_i{}^2)]^{3/2}, \tag{6c}$$

$$\bar\tau_2=-\frac{a_{\alpha ij}g_{\alpha ij}{}^2}{96\pi^2}m^{-1}[m^2-2(\mu_i{}^2+\mu_j{}^2)]^{3/2}. \tag{6d}$$

Here, g and h are the renormalized coupling constants for VPP and VVP coupling,[8] and a triplet of indices (x,y,z) on a coupling constant denotes a particle x dissociating virtually into particles y and z (a_{xyz} is the appropriate isospin factor).

Inserting Eqs. (6) into Eqs. (5), performing the required differentiations, and evaluating at the SU_3 central masses M and μ yields

$$[\partial\mu_i{}^2/\partial M_\gamma{}^2]_{M,\mu}=-K_1\sum_j a_{ij\gamma}g_{ij\gamma}{}^2-2K_1'\sum_\alpha a_{i\alpha\gamma}h_{i\alpha\gamma}{}^2-K_2\sum_{j,\alpha}a_{ij\alpha}g_{ij\alpha}{}^2\left(\frac{\partial\mu_j{}^2}{\partial M_\gamma{}^2}\right)_{M,\mu}, \tag{7a}$$

$$[\partial M_\alpha{}^2/\partial\mu_k{}^2]_{M,\mu}=-L_1\sum_\beta a_{\alpha\beta k}h_{\alpha\beta k}{}^2-2L_1'\sum_i a_{\alpha ik}g_{\alpha ik}{}^2-L_2\sum_{i,\beta}a_{\alpha\beta i}h_{\alpha\beta i}{}^2\left(\frac{\partial M_\beta{}^2}{\partial\mu_k{}^2}\right)_{M,\mu}, \tag{7b}$$

with the integrals $K_1\cdots L_2$ given by

$$\begin{pmatrix}K_1\\K_2\end{pmatrix}=\frac{3}{16\pi^2M^2}\int_{(M+\mu)^2}^{\infty}\frac{dm^2}{m^2}\frac{m^2\pm\mu^2\mp M^2}{m^2-\mu^2}\times[(m^2+M^2-\mu^2)^2-4m^2M^2]^{1/2}, \tag{8a}$$

[8] We are including the factor $1/M^2$ as part of the coupling constants which are assumed to be given by their SU_3 values.

$$K_1'=\frac{6}{16\pi^2M^2}\int_{4M^2}^{\infty}\frac{dm^2}{(m^2-\mu^2)}(m^4-4m^2M^2)^{1/2}, \qquad (8b)$$

$$\binom{L_1}{L_2}=\frac{1}{16\pi^2M^2}\int_{(M+\mu)^2}^{\infty}\frac{dm^2}{m^2}\frac{m^2\mp\mu^2\pm M^2}{m^2-M^2}\times[(m^2+M^2-\mu^2)^2-4m^2M^2]^{1/2}, \qquad (8c)$$

$$L_1'=\frac{1}{16\pi^2}\int_{4\mu^2}^{\infty}\frac{dm^2}{m}\frac{(m^2-4\mu^2)^{1/2}}{m^2-M^2}. \qquad (8d)$$

It is to be noted that all the integrals diverge so that at least one cutoff parameter will be required.

For convenience, we make the substitutions

$$A_{i\gamma}=(\partial\mu_i^2/\partial M_\gamma^2)_{M,\mu},\quad B_{\alpha k}=(\partial M_\alpha^2/\partial\mu_k^2)_{M,\mu},$$

$$S_{i\gamma}=\sum_j a_{ij\gamma}g_{ij\gamma}^2,$$

$$T_{i\gamma}=\sum_\alpha a_{i\alpha\gamma}h_{i\alpha\gamma}^2,\quad U_{ij}=\sum_\alpha a_{ij\alpha}g_{ij\alpha}^2,$$

$$V_{\alpha k}=\sum_\beta a_{\alpha\beta k}h_{\alpha\beta k}^2,$$

$$W_{\alpha k}=\sum_i a_{\alpha ik}g_{\alpha ik}^2,\quad\text{and}\quad X_{\alpha\beta}=\sum_i a_{\alpha\beta i}h_{\alpha\beta i}^2,$$

so that one may write Eqs. (7) in the compact form

$$A_{i\gamma}=-K_1S_{i\gamma}-2K_1'T_{i\gamma}-K_2\sum_j U_{ij}A_{j\gamma}, \qquad (9a)$$

$$B_{\alpha k}=-L_1V_{\alpha k}-2L_1'W_{\alpha k}-L_2\sum_\beta X_{\alpha\beta}B_{\beta k}. \qquad (9b)$$

The matrices S, T, U, V, W, and X may be computed using the SU_3 expressions for the coupling constants given in Ref. 1 with $f=1$ for VPP coupling and $f=0$ for VVP coupling. It is to be pointed out that contributions from an intermediate state with two isospin-$\frac{1}{2}$ particles must be counted twice because although there are only three different masses in the octets considered here, there are four different states. In this way, the contributions of both the $I=\frac{1}{2}$, $Y=\pm1$ states are included.[9] The result is

$$S=U=W=\begin{bmatrix}8g^2 & 4g^2 & 0\\ 3g^2 & 6g^2 & 3g^2\\ 0 & 12g^2 & 0\end{bmatrix}, \qquad (10a)$$

$$T=V=X=\begin{bmatrix}\frac{4}{3}h^2 & 4h^2 & \frac{4}{3}h^2\\ 3h^2 & 10/3h^2 & \frac{1}{3}h^2\\ 4h^2 & \frac{4}{3}h^2 & \frac{4}{3}h^2\end{bmatrix}. \qquad (10b)$$

Using Eqs. (10), one may invert Eqs. (9) and solve for A and B. The results, when combined with the numerical values[10] $M_\rho=0.770$ BeV, $M_{K^*}=0.892$ BeV, $\mu_\pi=0.135$ BeV, $\mu_K=0.498$ BeV, and $\mu=\mu_3=\mu_\eta=0.549$ BeV (we postpone choosing the value of $M=M_{\omega^8}$) yield the following expressions for the masses:

$$\mu_1^2=\bar\mu^2+(Z/D)[(4+8\Delta+36\Delta\gamma-288\Delta\gamma^2)\alpha_1+(4+\tfrac{4}{3}\Delta+(32/3)\gamma-4\Delta\gamma-128\gamma^2)\alpha_2], \qquad (11a)$$

$$\mu_2^2=\bar\mu^2+(Z/D)[(6+3\Delta-288\gamma^2)\alpha_1+(10/3+3\Delta+(32/3)\gamma+8\Delta\gamma-32\gamma^2-96\Delta\gamma^2)\alpha_2], \qquad (11b)$$

$$\mu_3^2=\bar\mu^2+(Z/D)[(12+96\gamma-36\Delta\gamma)\alpha_1+(\tfrac{4}{3}+4\Delta-(64/3)\gamma+20\Delta\gamma-128\gamma^2-96\Delta\gamma^2)\alpha_2], \qquad (11c)$$

$$M_1^2=\bar M^2+(Z'/D')[(11.06-55.33\eta-94.22\eta^2)\beta_1+(46.40+11.29\eta+70.40\eta^2)\beta_2], \qquad (11d)$$

$$M_2^2=\bar M^2+(Z'/D')[(19.23+10.57\eta-17.77\eta^2)\beta_1+(21.90-84.80\eta-158.9\eta^2)\beta_2], \qquad (11e)$$

$$M_3^2=\bar M^2+(Z'/D')[(22.53+35.24\eta)\beta_1+(12-158.8\eta-212.2\eta^2)\beta_2], \qquad (11f)$$

where the physically observed value $(\mu_1^2-\mu^2)/(\mu_2^2-\mu^2)=5.3$ has been used. In addition, the convenient substitutions $\gamma=g^2K_2$, $\eta=h^2L_2$, $\alpha_1=g^2K_1$, $\alpha_2=2h^2K_1'$, $\beta_1=h^2L_1$, $\beta_2=2g^2L_1'$, $Z=\mu_2^2-\mu^2$, $Z'=M_2^2-M^2$, and $\Delta=(M_1^2-M^2)/(M_2^2-M^2)$ have been made, and the functions D and D' are given by

$$D=1+14\gamma-288\gamma^3, \qquad (12a)$$

$$D'=1+6\eta-(64/9)\eta^2-(160/9)\eta^3. \qquad (12b)$$

The four mass splittings are then given by

$$\mu_1^2-\mu_3^2=(Z/D)[8(\Delta-1)+24(3\Delta-4)\gamma-288\Delta\gamma^2]\times(\alpha_1-\tfrac{1}{3}\alpha_2), \qquad (13a)$$

$$\mu_2^2-\mu_3^2=(Z/D)[3(\Delta-2)+12(3\Delta-8)\gamma-288\gamma^2]\times(\alpha_1-\tfrac{1}{3}\alpha_2), \qquad (13b)$$

$$M_1^2-M_3^2=(0.053/D')[11.47+90.57\eta+94.22\eta^2]\times(\beta_1-3\beta_2), \qquad (13c)$$

$$M_2^2-M_3^2=(0.053/D')[3.30+24.67\eta+17.77\eta^2]\times(\beta_1-3\beta_2), \qquad (13d)$$

and we require that Eqs. (13) be self-consistently satisfied for the physically observed mass splittings.

III. CALCULATION

Taking the ratios of Eq. (12a)/Eq. (12b) and Eq. (12c)/Eq. (12d) yields expressions for γ and η in

[9] This problem did not arise in Refs. 1 and 2 because the $I=\frac{1}{2}$ baryons (Ξ,N) have different masses.

[10] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

terms of Δ:

$$\gamma=\frac{412.8-118.8\Delta\pm[(412.8-118.8\Delta)^2-1152(\Delta-5.3)(7.9\Delta-23.8)]^{1/2}}{576(\Delta-5.3)}, \quad (14a)$$

$$\eta=\frac{90.57-24.67\Delta\pm[(90.57-24.67\Delta)^2-4(17.77\Delta-94.22)(3.30\Delta-11.47)]^{1/2}}{2(17.77\Delta-94.22)}. \quad (14b)$$

The Gell-Mann–Okubo formula predicts $\Delta=4$ which corresponds to[10] $M=0.928$ BeV, $Z=0.066$ BeV, and an ω-φ mixing angle $\theta=40°$. Thus, insertion of $\Delta=4$ into Eqs. (14) together with the physical requirement that γ and η must be non-negative (for K_2 and $L_2>0$ and one must have g^2 and $h^2>0$) leads to the unique result $\gamma=\frac{1}{4}$ and $\eta=\frac{1}{2}$. Care must be taken, however, before using the triplet of values $\Delta=4$, $\gamma=\frac{1}{4}$, and $\eta=\frac{1}{2}$ in Eqs. (13) to obtain the values of $(\alpha_1-\frac{1}{3}\alpha_2)$ and $(\beta_1-3\beta_2)$ because the functions D and D' vanish there. In the case of Eqs. (13a) and (13b), one sees that the numerators also vanish so that use of L'Hospital's rule gives

$$\lim_{\gamma\to 1/4}\frac{24+192\gamma-1152\gamma^2}{1+14\gamma-288\gamma^3}=9.6. \quad (15)$$

Thus one arrives at $\alpha_1-\frac{1}{3}\alpha_2=0.446$. In the case of Eqs. (13c) and (13d), the numerator does not vanish, hence one is led to the requirement that $\beta_1-3\beta_2=0$. This can, in fact, be explicitly demonstrated by allowing $\Delta\to 4$ through a sequence of values, calculating η for each value Δ in the sequence from Eq. (14b) and then seeing that the resultant value of $\beta_1-3\beta_2\to 0$ as $\Delta\to 4$ $(\eta\to\frac{1}{2})$. Using the definitions of α_1, α_2, β_1, and β_2 together with the results $\gamma=\frac{1}{4}$, $\eta=\frac{1}{2}$, $\alpha_1-\frac{1}{3}\alpha_2=0.446$, $\beta_1-3\beta_2=0$, we find two consistency conditions which the integrals $K_1\cdots L_1'$ must satisfy:

$$0.167K_1L_1L_2+0.298K_2L_1L_2-0.223K_2K_1'L_1=0, \quad (16a)$$

$$L_1'=K_1L_1/K_1'+1.784K_2L_1/K_1'. \quad (16b)$$

Using the expressions for the integrals, Eqs. (8), one finds that Eq. (16a) is satisfied for a cutoff parameter $\Lambda^2=4.65$ BeV^2 which then gives $16\pi^2L_1'=2.771$, corresponding to a cutoff $\Lambda'^2=17.5$ BeV^2. It is our assertion that this rather considerable difference in the values of Λ^2 and Λ'^2 is due to the fact that L_1 is logarithmically divergent while the other five integrals diverge quadratically. This assertion is supported by the fact that L_1' is approximately equal to L_1 and L_2 ($16\pi^2L_1=1.750$ and $16\pi^2L_2=2.430$) as would be expected.

Having computed the cutoff Λ^2, one can then compute the coupling constants, using $16\pi^2K_2(\Lambda^2=4.65)=5.969$ and $16\pi^2L_2(\Lambda^2=4.65)=1.750$. The results are $g^2/4\pi=0.526$ and $h^2/4\pi=3.59$ which will be compared with experiment in the next section.

IV. DISCUSSION

Restating the results, we had, for the accepted ω-φ mixing angle $\theta=40°$, the values $\Lambda^2=4.65$, $g^2/4\pi=0.526$, and $h^2/4\pi=3.59$. We first note that in our model, the value of Λ^2 lies below the two-baryon threshold of 5 BeV^2 so that we are completely justified in neglecting the baryon-antibaryon intermediate state (it is to be noted that the baryon-antibaryon intermediate state would also be quadratically divergent in our model). An experimental value for g^2 may be obtained from the decay of the ρ meson. The decay width of ρ into two π's is given by[11]

$$\Gamma_{\rho\to 2\pi}=\frac{2}{3}\frac{g_{\rho\pi\pi}{}^2}{4\pi}\frac{(\frac{1}{4}M_\rho{}^2-\mu_\pi{}^2)^{3/2}}{M_\rho{}^2}, \quad (17)$$

which, upon using $\Gamma_{\rho\to 2\pi}=0.128$ BeV[10] and the SU_3 value $g_{\rho\pi\pi}{}^2=4g^2$, gives $g^2/4\pi=0.60$ so that our computed value is in good agreement with experiment (the experimental errors in the values of Γ and M_ρ are unknown but appear to be at least as large as 10% at this time).

The physical value of h^2 may be obtained from the values of $h_{\omega\rho\pi}$ and $h_{\varphi\rho\pi}$, using the ω-φ mixing formula,[6]

$$h_{\omega^8\rho\pi}=-(h_{\omega\rho\pi}\sin\theta+h_{\varphi\rho\pi}\cos\theta), \quad (18)$$

together with $h_{\omega^8\rho\pi}{}^2=\frac{4}{3}h^2$ from SU_3. One may compute $h_{\omega\rho\pi}{}^2$ from the decay of $\omega\to 3\pi$, provided that it proceeds through an intermediate $\rho\pi$ state,[12] the result for the width being

$$\Gamma_{\omega\to 3\pi}=\frac{3.56}{3^{3/2}}M_\omega{}^{-1}\mu_\pi{}^2\frac{g_{\rho\pi\pi}{}^2}{4\pi}\frac{h_{\omega\rho\pi}{}^2}{4\pi}\frac{(M_\omega-3\mu_\pi)^4}{(\frac{1}{4}M_\rho{}^2-\mu_\pi{}^2)^2}. \quad (19)$$

Using the value computed above for $g_{\rho\pi\pi}{}^2$ and the experimental width $\Gamma=0.011$ BeV, one arrives at $h_{\omega\rho\pi}{}^2=9.27$. A value for $h_{\varphi\rho\pi}{}^2/4\pi$ may also be calculated from Eq. (19), although it would not be entirely correct as the decay $\varphi\to\rho+\pi$ is energetically allowed. Since it is thought that $h_{\varphi\rho\pi}{}^2\ll h_{\omega\rho\pi}{}^2$ anyway,[13] we will use this as an order-of-magnitude estimate. Using $\Gamma_{\omega\to 3\pi}=0.0036$ BeV,[10] one arrives at $h_{\varphi\rho\pi}{}^2/4\pi\cong 0.1$ which is in rough agreement with other estimates. Thus $h_{\omega\rho\pi}/4\pi=\pm 3.05$ and $h_{\varphi\rho\pi}/4\pi=\pm 0.32$ so that using $\theta=40°$, we arrive at $h^2/4\pi=3.6$, 2.2, depending on the relative sign of the couplings. Thus, if the relative sign is positive, we are in excellent agreement, and if negative

[11] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[12] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

[13] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964).

in fair agreement. At the moment there appears to be no conclusive evidence on the relative sign, although there are some indications that it may be positive.[13]

In conclusion, we note that, as in our previous work,[1,2] no singularity such as described by Barton[3] appears in the mass splittings as the feedback is turned off (thus, as γ and $\eta\rightarrow 0$, the functions D and D' remain finite).

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Application of Modified Dispersion Relations to the Forward KN Crossing-Even Scattering Amplitude*

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It has recently been shown that a knowledge of the zeros of a forward elastic scattering amplitude could be used to derive new modified dispersion relations. Using the phase representation, we show that the forward crossing-even KN amplitude probably has six zeros in the complex ω (kaon lab energy) plane. Two of these zeros can be very accurately determined from low-energy scattering data. The modified dispersion relations derived using the knowledge of these zeros yield information on the high-energy parameters, and in general provide a consistency test of the presently available data. The infinite-energy total cross section estimated from a dispersion sum rule is about 15.5 mb, in fair agreement with the experimental total cross section of about 17.3 mb at 20 BeV.

I. INTRODUCTION

In a recent paper,[1] we derived a new class of modified dispersion relations which depended on a knowledge of the zeros of the forward elastic scattering amplitude. In particular, we derived an expression for the infinite-energy cross section for πN scattering. We have now applied this method to KN scattering in order to test the data of Kim[2] and to gain some information on the real part of the scattering amplitude at high energies.

We show in this paper that, according to the phase representation,[3] the forward crossing-even KN amplitude probably has six zeros in the complex ω (kaon laboratory energy) plane. There seem to be two possible arrangements for the zeros, with the presently available data being not precise enough to distinguish between the two possibilities.

In order to test the data on the real part of the scattering amplitude, we have calculated the infinite-energy cross section using the two accurately determined zeros and find fairly good agreement with experiment. We also have calculated the infinite-energy cross section using subtractions at the points on the imaginary axis where the scattering amplitude is a minimum. These points may or may not turn out to correspond to zeros when more accurate data become available.

II. ZEROS OF $T(\omega)$

We normalize the KN forward scattering amplitude $T(\omega)$ by writing the optical theorem in the form

$$\mathrm{Im}T(\omega)=\tfrac{1}{2}(\omega^2-m_K{}^2)^{1/2}[\sigma_{K^+p}(\omega)+\sigma_{K^-p}(\omega)]\,,\quad (1)$$

where ω is the kaon lab energy, m_K is the kaon mass, and $\sigma_{K^\pm p}(\omega)$ is the $K^\pm p$ total cross sections. Natural units ($\hbar=c=1$) are used throughout this work.

From the phase representation[3] we find that, for large ω,

$$T(\omega)\propto\omega^{N-M-2\delta(\infty)/\pi}\,,\quad (2)$$

where N and M are, respectively, the number of zeros and poles of $T(\omega)$, and $\delta(\infty)$ is the phase of $T(\omega)$ at infinity. Now, from the available data, we find $\mathrm{Im}T(\omega)>0$ for ω on the real axis above the anomalous $(\Lambda\pi)$ threshold and $\mathrm{Re}T(\omega)>0$ at the $\Lambda\pi$ threshold. We must therefore have $0<\delta(\omega)<\pi$. If we assume that $T(\omega)$ becomes pure imaginary in the infinite-energy limit, we have $\delta(\infty)=\frac{1}{2}\pi$ and thus

$$T(\omega)\propto\omega^{N-M-1}.\quad (3)$$

Since $T(\omega)$ has a Pomeranchon-exchange contribution, it has the high-energy behavior

$$T(\omega)\propto\omega\,,\quad (4)$$

and so we deduce

$$N=M+2\,.\quad (5)$$

Finally, since $T(\omega)$ has two sets of poles (Λ and Σ) on the real axis, it has six zeros.

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