Phenomenological Chiral Model for Nonleptonic Hyperon Decays*

JOSEPH SCHECHTER
Physics Department, Syracuse University, Syracuse, New York 13210
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The S- and P-wave nonleptonic hyperon decay amplitudes are calculated using a chiral-invariant effective Lagrangian. This reproduces the current-algebra results in a very simple way, the baryon pole diagram arising without any ambiguity whatsoever. As is well known, this leaves us with the wrong ratio of S- to P-wave amplitudes. It is noted that one way out of this dilemma is to add another term to the effective Lagrangian which vanishes in the limit of zero pion four-momentum, and hence contributes to the physical answer without contributing to the current-algebra result. The addition of the K^* pole diagram leaves us with a rough three-parameter fit to all the amplitudes plus the $K\rightarrow 2\pi$ amplitudes.

In this paper we calculate the nonleptonic hyperon decay amplitudes in an effective Lagrangian model[1] wherein the pseudoscalar mesons are considered to transform nonlinearly under the chiral $SU(3)\times SU(3)$ group. If the Lagrangian only contains baryons and pseudoscalar mesons, we are able to reproduce the usual "current algebra" (CA) results[2] which suffer from the fact that the ratio of S-wave to P-wave amplitudes is roughly twice the experimental value. What were actually calculated in the CA scheme were the values of the amplitudes extrapolated to the unphysical region of zero pion four-momentum and it was not clear which contributions these extrapolations represented. From this work it seems that the CA scheme only took care of the pseudoscalar mesons. By adding an effective interaction involving the vector mesons (K^* pole diagram) we are able to fit all the S- and P-wave amplitudes *roughly*.

Here we follow the approach of Cronin[3] to chiral theories. Our notation is such that when we distinguish between "left- and right-hand" subgroups of $SU(3)\otimes SU(3)$, unprimed tensor indices refer to the left-hand one and primed tensor indices to the right-hand one. Thus in a representation of the Dirac matrices where γ_5 is diagonal we may write the octet $\frac{1}{2}^+$ baryon spinors as

$$N_b{}^a=\begin{pmatrix} L_b{}^a \\ R_{b'}{}^{a'}\end{pmatrix},\quad \bar{N}_b{}^a\equiv(\bar{R}_b{}^{a,\prime}\bar{L}_b{}^a). \tag{1}$$

The meson matrices[3] which satisfy

$$M_{b'}{}^a M_c{}^{b'}=M_b{}^{a'}M_{c'}{}^b=\delta_c{}^a \tag{2}$$

can be expanded in terms of the pseudoscalar-meson octet $\phi_a{}^b$ as

$$M_a{}^{b'}=\delta_a{}^b+2if\phi_a{}^b-2f^2\phi_a{}^c\phi_c{}^b+\cdots, \tag{3a}$$

$$M_{a'}{}^b=\delta_a{}^b-2if\phi_a{}^b-2f^2\phi_a{}^c\phi_c{}^b+\cdots, \tag{3b}$$

where f is the pion decay constant.

We shall assume that the fundamental weak interaction induces an effective interaction among $\frac{1}{2}^+$ baryons and pseudoscalar mesons with the chiral $SU(3)\otimes SU(3)$ transformation property $(T_3{}^2+T_2{}^3)$. This would correspond, for example, to a quark current-current interaction with octet dominance. Of course, there may and probably will be other types of induced weak terms in the effective Lagrangian. The simplest CP-invariant combination with this transformation property is

$$\begin{aligned}H_w=(\delta+\phi)[\bar{L}_a{}^2M_{b'}{}^aR_{c'}{}^{b'}M_3{}^{c'}+\bar{R}_{b'}{}^{c'}M_{c'}{}^3L_2{}^aM_a{}^{b'}]\\+(\delta-\phi)[\bar{L}_3{}^aM_a{}^{b'}R_{b'}{}^{c'}M_{c'}{}^2+\bar{R}_{c'}{}^{b'}M_2{}^{c'}L_a{}^3M_{b'}{}^a]\\+(2\leftrightarrow 3),\end{aligned} \tag{4}$$

where δ and ϕ are some arbitrary constants. Expanding (4) using (1) and (3) gives

$$\begin{aligned}H_w=(\delta+\phi)[\bar{N}_a{}^2N_3{}^a+2if\bar{N}_a{}^2\gamma_5N_3{}^b\phi_b{}^a\\+if\bar{N}_a{}^3(1-\gamma_5)N_c{}^a\phi_2{}^c]+(\delta-\phi)[\bar{N}_3{}^aN_a{}^2\\-2if\bar{N}_3{}^a\gamma_5N_b{}^2\phi_a{}^b-if\bar{N}_3{}^a(1-\gamma_5)N_a{}^c\phi_c{}^2]\\+\text{H.c.}+\cdots.\end{aligned} \tag{5}$$

Equation (5) is essentially the CA result.[2] It contains terms corresponding to both S- and P-wave pion emission and "weak mass" or "spurion" terms which give the dominant P-wave contributions in the form of baryon pole diagrams.

The nonpole contributions of (5) to the decay amplitudes are

$$\begin{aligned}\Lambda_-{}^0&:\quad (f/\sqrt{6})(3\phi+\delta)(1+\gamma_5),\\ \Xi_-{}^-&:\quad (f/\sqrt{6})(-3\phi+\delta)(1-\gamma_5),\\ \Sigma_+{}^+&:\quad 2f(\phi-\delta)\gamma_5,\\ \Sigma_-{}^-&:\quad f(\phi-\delta)(1-\gamma_5).\end{aligned} \tag{6}$$

The other amplitudes can be calculated in terms of these by using the $\Delta I=\frac{1}{2}$ relations. The S-wave parts of (6) are identical to the CA case and satisfy the Lee-Sugawara[4] relation. The P-wave parts do not satisfy this relation but are rather small compared to the pole parts and would vanish in the zero pion-four-momentum limit anyway.

To calculate the pole diagrams we need, in addition to (5), the strong meson-baryon interaction. Although we could use the ordinary one, it is not much harder to use

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[1] For reference to recent work, see S. Weinberg, Phys. Rev. **166**, 1568 (1968).

[2] H. Sugawara, Phys. Rev. Letters **15**, 870 (1965); **15**, 997 (1965); M. Suzuki, *ibid.* **15**, 986 (1965); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); S. Badier and C. Bouchiat, Phys. Letters **20**, 529 (1966); L. S. Brown and C. Sommerfield, Phys. Rev. Letters **16**, 751 (1966); S. Okubo, Ann. Phys. (N. Y.) (to be published).

[3] J. A. Cronin, Phys. Rev. **161**, 1483 (1967).

[4] H. Sugawara, Progr. Theoret. Phys. (Kyoto) **31**, 213 (1964); B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).

a broken chiral-invariant interaction. We shall assume the symmetry breaking to transform as $T_3{}^3+T_{3'}{}^{3'}$, giving

$$
\begin{aligned}
H_{\rm st}=\{&M_\Sigma\bar{L}_c{}^aM_{b'}{}^cR_{d'}{}^{b'}M_a{}^{d'}+\tfrac{1}{2}(M_\Xi-M_\Sigma)\\
&\times[\bar{L}_c{}^3M_{b'}{}^cR_{a'}{}^{b'}M_3{}^{d'}+\bar{L}_b{}^dM_{c'}{}^bR_{3'}{}^{c'}M_d{}^{3'}]\\
&+\tfrac{1}{2}(M_\rho-M_\Sigma)[\bar{L}_3{}^aM_{b'}{}^3R_{d'}{}^{b'}M_a{}^{d'}\\
&+\bar{L}_b{}^dM_{3'}{}^bR_{a'}{}^{3'}M_d{}^{a'}]+\text{H.c.}\}\\
&-(\alpha\bar{L}_d{}^c\sigma_\mu L_b{}^d+\beta\bar{L}_b{}^d\sigma_\mu L_d{}^c)(M_{c'}{}^b\;{}_\mu M_c{}^{c'})\\
&-(\alpha\bar{R}_{d'}{}^{c'}\tilde{\sigma}_\mu R_{b'}{}^{d'}+\beta\bar{R}_{b'}{}^{d'}\tilde{\sigma}_\mu R_{d'}{}^{c'})(M_e{}^{b'}\overleftrightarrow{\partial}_\mu M_{c'}{}^e)\,, \quad (7)
\end{aligned}
$$

where $\sigma_\mu=(-\boldsymbol{\sigma},i)$ and M_Σ, for example, is the mass of the Σ. The derivative coupling terms were added to (7) in order for the axial-vector current as computed in the usual way to have the proper D and F values. This determines[5] α and β as

$$
\begin{aligned}
\alpha&=\tfrac{1}{4}(1-D-F)\simeq-0.05\,,\\
\beta&=\tfrac{1}{4}(-1-D+F)\simeq-0.33\,.
\end{aligned}
\quad (8)
$$

Expanding (7) gives

$$
\begin{aligned}
H_{\rm st}=&M_p\bar{p}p+M_\Lambda\bar{\Lambda}\Lambda+M_\Sigma\bar{\Sigma}\Sigma+M_\Xi\bar{\Xi}\Xi\\
&-2iM_\Sigma f(\bar{N}_c{}^a\gamma_5N_b{}^c-\bar{N}_b{}^c\gamma_5N_c{}^a)\phi_a{}^b\\
&+2if(M_\Xi-M_\Sigma)\bar{N}_c{}^3\gamma_5N_3{}^b\phi_b{}^c\\
&+2if(M_\Sigma-M_p)\bar{N}_3{}^a\gamma_5N_d{}^3\phi_a{}^d-4if(\alpha\bar{N}_e{}^b\gamma_\mu\gamma_5N_a{}^e\\
&\qquad+\beta\bar{N}_a{}^e\gamma_\mu\gamma_5N_e{}^b)\partial_\mu\phi_b{}^a+\cdots. \quad (9)
\end{aligned}
$$

We note that (9) gives corrections to the coupling constants from the mass splitting. Now from (5) and (9) we get the following pole contributions:

$$
\begin{aligned}
\Lambda_-{}^0:&\quad \frac{f\gamma_5}{\sqrt{6}}\left(\frac{(3\phi+\delta)}{(\Lambda-n)}[2n-4\alpha(\Lambda+n)]-\frac{4(\delta-\phi)(\alpha+\beta)(\Lambda+p)}{(\Sigma-p)}\right),\\
\Xi_-{}^-:&\quad \frac{f\gamma_5}{\sqrt{6}}\left(\frac{4(\delta+\phi)(\alpha+\beta)(\Xi+\Lambda)}{(\Xi-\Sigma)}-\frac{(3\phi-\delta)}{(\Xi-\Lambda)}[2\Xi+4\beta(\Xi+\Lambda)]\right),\\
\Sigma_+{}^+:&\quad f\gamma_5\left(\frac{(\delta-\phi)}{(p-\Sigma)}[2n-4\alpha(\Sigma+n)]+\frac{2}{3}\frac{(3\phi+\delta)(\alpha+\beta)(\Lambda+n)}{(\Lambda-n)}+\frac{2(\delta-\phi)}{(\Sigma-n)}[\Sigma+(\beta-\alpha)(\Sigma+n)]\right),\\
\Sigma_-{}^-:&\quad f\gamma_5\left(\frac{2}{3}\frac{(3\phi+\delta)(\alpha+\beta)(n+\Sigma)}{(\Lambda-n)}-\frac{2(\delta-\phi)}{(\Sigma-n)}[\Sigma-(\alpha-\beta)(n+\Sigma)]\right).
\end{aligned}
\quad (10)
$$

[5] J. Schechter, Y. Ueda, and G. Venturi (to be published).

In Eqs. (10) each baryon symbol stands for its mass. Taking the limit of (10) where the denominators all have equal mass differences Δm and where all the numerator masses are set equal to m we find, using (8),

$$
\begin{aligned}
\Lambda_-{}^0:&\quad \frac{2mf\gamma_5}{(\Delta m)\sqrt{6}}[2(\delta+\phi)(D+F)+(\delta-\phi)(D-F)]\,,\\
\Xi_-{}^-:&\quad \frac{-2mf\gamma_5}{(\Delta m)\sqrt{6}}[(\delta+\phi)(D+F)+(\delta-\phi)(D-F)]\,,\\
\Sigma_+{}^+:&\quad \frac{-2mf\gamma_5}{\Delta m}\,\tfrac{4}{3}\delta D\,,\\
\Sigma_-{}^-:&\quad \frac{2mf\gamma_5}{\Delta m}[-\tfrac{4}{3}\delta D+(\delta-\phi)(D-F)]\,.
\end{aligned}
\quad (11)
$$

Equations (11) are the usual CA ones[2] and satisfy the Lee-Sugawara relation. So far we have essentially reproduced the CA results using only baryons and pseudoscalar mesons in our effective Lagrangian. It is well known[2] that this leaves us with P-wave results that roughly fit the data but S-wave results that are about twice as large as they should be. On the other hand, it is also known[6] that the K^* pole model by itself gives values for the S-wave amplitudes which are very close to the experimental ones. In this scheme we have not yet taken into account an effective K^*-π transition induced by the fundamental weak interaction. If we do so it is evident that by subtracting its contribution from the above we can also fit the S waves. Let us then write for this part of the effective weak interaction

$$
\begin{aligned}
H_{\rm w}'=X\frac{G}{\sqrt{2}}\frac{\sqrt{2}M_V{}^2}{f_V}\frac{1}{f}&\\
\times(\partial_\mu\phi_c{}^2\rho_{3\mu}{}^c&+\partial_\mu\phi_2{}^c\rho_{c\mu}{}^3)+(2\leftrightarrow 3)\,, \quad (12)
\end{aligned}
$$

where $G\simeq10^{-5}/M_p{}^2$, M_V is the vector-meson mass, $f_V{}^2/4\pi\simeq2.5$, and X is a number which would be unity if (12) represented a current-current interaction, with conventionally normalized currents and lacking the Cabibbo suppression factor. The $K_1{}^0\to2\pi$ rate can be fitted[6] with X slightly larger than 1. From (12) and using the ordinary F-type vector-meson–baryon coupling we calculate the K^*-pole-diagram contributions as

$$
\begin{aligned}
\Lambda_-{}^0:&\quad \frac{-GX}{\sqrt{2}f}\frac{3}{\sqrt{6}}(\Lambda-p)\,,\\
\Xi_-{}^-:&\quad \frac{-GX}{\sqrt{2}f}\frac{3}{\sqrt{6}}(\Lambda-\Xi)\,,\\
\Sigma_+{}^+:&\quad 0\,,\\
\Sigma_-{}^-:&\quad \frac{-GX}{\sqrt{2}f}(\Sigma-n)\,.
\end{aligned}
\quad (13)
$$

[6] J. J. Sakuri, Phys. Rev. **156**, 1504 (1967); W. W. Wada, *ibid.* **138**, B1488 (1965); B. W. Lee and A. Swift, *ibid.* **136**, B228 (1964).

From (12) we see that the K^* pole diagrams vanish for zero pion four-momentum. Adding (6), (10), and (13) and putting in numbers for whatever is known gives the final results

$$\begin{aligned}
\Lambda_-{}^0:\quad & [1.2\phi+0.4\delta-(2,4\times10^{-7})X] \\
& \quad +\gamma_5[12.3\phi+10.7\delta+(16.5\times10^{-7})Y], \\
\Xi_-{}^-:\quad & [-1.2\phi+0.4\delta+(2.9\times10^{-7})X] \\
& \quad +\gamma_5[-7.6\phi-13.7\delta-(5.4\times10^{-7})Y], \\
\Sigma_+{}^+:\quad & 0 \quad +\gamma_5[-2.4\phi-9.5\delta], \\
\Sigma_-{}^-:\quad & [\phi-\delta-(2.9\times10^{-7})X] \\
& \quad +\gamma_5[-5.7\phi-6.7\delta-(6.3\times10^{-7})Y].
\end{aligned} \tag{14}$$

In (14) we have put in for good measure the contribution from an axial-vector pole diagram where the interaction is taken to be analogous to (12) and where Y replaces X. The D/F ratio of the axial-vector meson-baryon coupling was taken to be 1.7. A possible K-meson pole diagram is expected to be negligible since if the K-π effective transition is written analogously to (12), its contribution will be reduced by the factor $(\mu/M_K)^2\simeq\frac{1}{12}$ compared to the other P-wave diagrams.

The experimental values[7] are

$$\begin{aligned}
10^7\Lambda_-{}^0:\quad & (3.36\pm0.04)+\gamma_5(23.9\pm1.0), \\
10^7\Xi_-{}^-:\quad & (-4.38\pm0.06)+\gamma_5(14.3\pm1.2), \\
10^7\Sigma_+{}^+:\quad & (0.02\pm0.09)+\gamma_5(41.3\pm0.8), \\
10^7\Sigma_-{}^-:\quad & (4.03\pm0.04)+\gamma_5(-0.3\pm0.9).
\end{aligned} \tag{15}$$

Comparing (14) and (15) shows that we may achieve a very rough fit by choosing $X=1.3$, which is about what we expect, $\phi=-\frac{4}{3}\delta=6.0\times10^{-7}\mu$, and $Y=0$. This gives

$$\begin{aligned}
10^{+7}\Lambda_-{}^0:\quad & 2.4+26.3\gamma_5, \\
10^{+7}\Xi_-{}^-:\quad & -5.5+16.1\gamma_5, \\
10^{+7}\Sigma_+{}^+:\quad & 0+28.3\gamma_5, \\
10^{+7}\Sigma_-{}^-:\quad & 6.7-4.2\gamma_5.
\end{aligned} \tag{16}$$

In this fit both the S and P waves have the correct order of magnitude. We note that the axial-vector pole contribution does not seem to be needed here.[8]

Attempts have been made[9] to calculate the weak spurion parameters ϕ and δ by saturating a current-current product with some single-particle intermediate states and using whatever experimental form-factor information is available. In the present scheme, the values of ϕ and δ with which we compare are changed. The somewhat lower value of $|\phi/\delta|$ might indicate a more sizable decuplet intermediate state contribution to the current-current product. This might also explain the doubling of ϕ and δ if the Cabibbo suppression factor is retained for the baryon part but, on the other hand, might let us get away without $\sin\theta$ at all. It is clear that this and many other points demand further investigation.

Note added in proof. A similar model has been proposed by B. W. Lee, Phys. Rev. **170**, 1359 (1968). Lee shows that if, in addition to a term which is the analog of (4), we allow derivative coupling terms, we can introduce many arbitrary parameters into the theory.

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[7] J. P. Berge, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

[8] A calculation by R. H. Graham and S. K. Yun [Phys. Rev. **171**, 1550 (1968)] gives the $K\rightarrow3\pi$ decays in terms of the axial-pseudoscalar transition. This might be a reflection of an induced four-pseudoscalar-meson weak vertex.

[9] Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1966); S. Biswas, A. Kumar, and R. Saxena, Phys. Rev. Letters **17**, 264 (1966); Y. Hara (unpublished).