Chiral Dynamics for High-Spin Baryons and an Application to Double-Pion Photoproduction*

H. W. HUANG
Columbia University, New York, New York 10027
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An $SU(2)\times SU(2)$ chiral-dynamical Lagrangian for baryons of arbitrary spin and isospin is discussed. Cross sections for low-energy double-pion photoproduction from a nucleon are calculated by assuming that the nucleon and one final pion form an isobar Δ(1236 MeV). The results are compared with those obtained by the soft-pion dispersion-relation approach. It is shown that the static-nucleon model gives satisfactory total cross sections but is inadequate for angular-distribution differential cross sections. Our theoretical predictions are consistent with experiment.

I. INTRODUCTION

THE chiral-dynamical Lagrangian[1–3] represents the starting point for analytic continuation of the current-algebraic soft-pion limit to higher energies. It is hoped that in cooperation with unitarity the Lagrangian may be used to provide the dynamics for the strong interaction. The Lagrangian has since been applied[1–3] to the low-lying members of the particle spectrum. In this paper we study its extension to include baryons of arbitrary spin and isospin.

The general nonlinear realization of $SU(2)\times SU(2)$ chiral symmetry has been given by Weinberg.[3] In order to obtain an explicit interaction Lagrangian, we generalize the method of Wess and Zumino.[2,4] This generalized method reproduces Weinberg's results in a simple way and gives the explicit couplings between baryons and mesons (including photons) immediately. The coupling constants can be measured experimentally. Determination of one coupling constant leads to a number of predictions due to chiral symmetry. Our general formulas explicitly indicate the power (and the limitation) of the prediction.

The method is illustrated by an application to double-pion photoproduction from a nucleon, which is essentially the hard-pion extension of a current-algebraic soft-pion calculation given in a previous paper.[5] In the Lagrangian method we assume that the nucleon and one of the final pions strongly rescatter and form an isobar Δ(1236 MeV). As a result, one must know the coupling constants for $\pi N\Delta$ and $\gamma N\Delta$ to predict the cross sections for double-pion photoproduction. Apparently, knowing these two coupling constants, one can calculate cross sections for many other reactions, including $\pi N\rightarrow\pi\pi N$, $\pi N\rightarrow\pi N\gamma$, etc. These calculations will be presented elsewhere.

In Sec. II we briefly review chiral symmetry and reproduce Weinberg's results[3] in preparation for subsequent discussions. In Sec. III we introduce baryons of arbitrary spin and isospin and construct their chiral gauge-invariant couplings with meson systems. In Sec. IV the $\pi N\Delta$ and $\gamma N\Delta$ coupling constants fitted by Gourdin and Salin[6] are reevaluated by using a new experimental width for Δ(1236). We use the couplings of lowest multipolarity to describe low-energy double-pion photoproduction (from threshold to about 1.5 GeV c.m. total energy). The amplitude mainly consists of a contact term, a pion-current term, and nucleon-pole terms. The A_1 exchange term is shown to be negligible. For comparison, we also use the soft-pion technique, current algebra, and the Chew-Goldberger-Low-Nambu (CGLN)[7] dispersion-relation method to solve the matrix element in Sec. V. It turns out that the two approaches produce essentially coincident solutions for the energy range that we are interested in. This coincidence, however, is not *a priori* obvious, although the two approaches should be identical in the soft-pion limit. It is shown in Sec. VI that the contribution of the nucleon-pole terms to the total cross section is of order $(kq_1/M\omega_1)^2$ compared with the contribution of the dominant contact term, where k is the photon energy, q_1 and ω_1 are the momentum and energy of the pion that does not rescatter with the nucleon (k, q_1, and ω_1 are the quantities in the over-all c.m. system, i.e., the c.m. system of the photon and nucleon), and M is the nucleon mass. This explains why the static-nucleon model due to Cutkosky and Zachariasen,[8] which consists only of the contact term and the pion-current term, gives satisfactory total cross sections. On the other hand, it is also shown that the contribution of the nucleon-pole terms to the angular-distribution differential cross section is of order $kq_1/M\omega_1$ compared with the contribution of the dominant contact term, so that we expect the static-nucleon theory to be inadequate for angular distributions. The numerical figures are

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[1] S. Weinberg, Phys. Rev. Letters **18**, 188 (1967).
[2] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
[3] S. Weinberg, Phys. Rev. **166**, 1568 (1968). A more complete list of references is given in this paper.
[4] H. W. Huang (unpublished).
[5] P. Carruthers and H. W. Huang, Phys. Letters **24B**, 464 (1967).
[6] M. Gourdin and Ph. Salin, Nuovo Cimento, **27**, 193 (1963); Ph. Salin, *ibid.* **28**, 1294 (1963).
[7] G. Chew, M. Goldberger, F. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957); **106**, 1345 (1957).
[8] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956).

given in Sec. VII, where our theoretical predictions are shown to be consistent with experiment.

II. $SU(2)\times SU(2)$ CHIRAL SYMMETRY: REVIEW

To facilitate the discussions in subsequent sections, we shall recapitulate the $SU(2)\times SU(2)$ transformation laws for a system consisting of pions, nucleons, and vector and axial-vector mesons, and also the general results of Ref. 3.[4]

One starts with the following $SU(2)\times SU(2)$ transformations for the nucleons: isospin rotation with parameter $g\boldsymbol{\chi}^\rho$,

$$\psi \to \psi + \tfrac{1}{2} i g \boldsymbol{\chi}^\rho \cdot \boldsymbol{\tau}\psi, \tag{2.1}$$

and chiral transformation with parameter $g\boldsymbol{\chi}^a$,

$$\psi \to \psi + \tfrac{1}{2} i g \boldsymbol{\chi}^a \cdot \boldsymbol{\tau}\gamma^5\psi. \tag{2.2}$$

However, the transformation (2.2) would obviously leave the nucleon-mass term noninvariant. In order to remedy this, we make a canonical transformation

$$\psi \to N \equiv U^{1/2}(\boldsymbol{\pi})\psi, \quad \bar{\psi} \to \bar{N} \equiv \bar{\psi} U^{1/2}(\boldsymbol{\pi}), \tag{2.3}$$

so that $\bar{\psi}U(\boldsymbol{\pi})\psi = \bar{N}N$ is invariant under the $SU(2)\times SU(2)$ transformation. Isospin invariance, Lorentz invariance, and parity conservation then restrict the function $U(\boldsymbol{\pi})$ to the form

$$U(\boldsymbol{\pi}) = \sigma(\pi^2) + i\gamma^5 \Phi(\pi^2)\boldsymbol{\pi}\cdot\boldsymbol{\tau}, \tag{2.4}$$

where $\sigma(\pi^2)$ and $\Phi(\pi^2)$ are two arbitrary real functions of π^2. The reality property is due to the Hermiticity condition on the Lagrangian. The transformation (2.2), on the other hand, implies[9]

$$\sigma^2 + \Phi^2\pi^2 = 1, \tag{2.5}$$

$$\pi_\alpha \to \pi_\alpha + [-(\sigma/\Phi)\delta_{\alpha\beta} - (\Phi'/\sigma')\pi_\alpha\pi_\beta] g\chi_\beta{}^a. \tag{2.6}$$

The primes indicate the derivative with respect to π^2. Because of the self-consistency condition (2.5), one can rewrite (2.4) as

$$U(\boldsymbol{\pi}) = \exp[i\gamma^5\Theta(\pi^2)\hat{\pi}\cdot\boldsymbol{\tau}], \tag{2.7}$$

where $\Theta(\pi^2)$ is a real function of π^2 and is related to σ and Φ by

$$\begin{aligned} &\cos\Theta = \sigma, \quad \sin\Theta = \Phi\pi, \\ &\pi = (\boldsymbol{\pi}^2)^{1/2}, \quad \hat{\pi} = \boldsymbol{\pi}/\pi. \end{aligned} \tag{2.8}$$

The chiral transformation law for the physical nucleon N is then[10]

$$N \to N + \tfrac{1}{2} i g \boldsymbol{\Omega}^a \cdot \boldsymbol{\tau} N, \tag{2.9}$$

where $\boldsymbol{\Omega}^a = (\boldsymbol{\chi}^a \times \hat{\pi})\tan\frac{1}{2}\Theta$.

The covariant derivatives can be easily constructed by using a trick due to Wess and Zumino.[2] One introduces the gauge fields $\boldsymbol{\rho}_\mu$ and $\mathbf{a}_\mu$, which transform as

$$\begin{aligned} \boldsymbol{\rho}_\mu &\to \boldsymbol{\rho}_\mu + \partial_\mu\boldsymbol{\chi}^\rho + g(\boldsymbol{\rho}_\mu \times \boldsymbol{\chi}^\rho), \\ \mathbf{a}_\mu &\to \mathbf{a}_\mu + g(\mathbf{a}_\mu \times \boldsymbol{\chi}^\rho), \\ \boldsymbol{\rho}_\mu &\to \boldsymbol{\rho}_\mu + g(\mathbf{a}_\mu \times \boldsymbol{\chi}^a), \\ \mathbf{a}_\mu &\to \mathbf{a}_\mu + \partial_\mu\boldsymbol{\chi}^a + g(\boldsymbol{\rho}_\mu \times \boldsymbol{\chi}^a), \end{aligned} \tag{2.10}$$

and defines $\boldsymbol{\mathcal{V}}_\mu$ and $\boldsymbol{\mathcal{A}}_\mu$ such that

$$\bar{\psi}\gamma^\mu(i\partial_\mu + \tfrac{1}{2}g\boldsymbol{\rho}_\mu\cdot\boldsymbol{\tau} + \tfrac{1}{2}g\gamma^5\mathbf{a}_\mu\cdot\boldsymbol{\tau})\psi = \bar{N}\gamma^\mu(i\partial_\mu + \tfrac{1}{2}g\boldsymbol{\mathcal{V}}_\mu\cdot\boldsymbol{\tau} + \tfrac{1}{2}g\gamma^5\boldsymbol{\mathcal{A}}_\mu\cdot\boldsymbol{\tau})N. \tag{2.11}$$

From the relations (2.3) it follows that

$$\boldsymbol{\mathcal{V}}_\mu = \boldsymbol{\rho}_\mu + \frac{\sin\Theta}{\pi}(\mathbf{a}_\mu \times \boldsymbol{\pi}) - \frac{2}{g}\frac{\sin^2(\frac{1}{2}\Theta)}{\pi^2}(\boldsymbol{\pi}\times D_\mu\boldsymbol{\pi}),$$

$$\boldsymbol{\mathcal{A}}_\mu = \mathbf{a}_\mu + \frac{1}{g}\frac{\sin\Theta}{\pi}D_\mu\boldsymbol{\pi} - 2\frac{\sin^2(\frac{1}{2}\Theta)}{\pi^2}[(\boldsymbol{\pi}\times\mathbf{a}_\mu)\times\boldsymbol{\pi}] + \frac{1}{g}\left(2\frac{\Theta'}{\pi} - \frac{\sin\Theta}{\pi^3}\right)\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi}\boldsymbol{\pi}, \tag{2.12}$$

where $D_\mu = \partial_\mu + g\boldsymbol{\rho}_\mu\times$. Their transformation laws are

$$\begin{aligned} \boldsymbol{\mathcal{V}}_\mu &\to \boldsymbol{\mathcal{V}}_\mu + g(\boldsymbol{\mathcal{V}}_\mu \times \boldsymbol{\chi}^\rho) + \partial_\mu\boldsymbol{\chi}^\rho, \\ \boldsymbol{\mathcal{A}}_\mu &\to \boldsymbol{\mathcal{A}}_\mu + g(\boldsymbol{\mathcal{A}}_\mu \times \boldsymbol{\chi}^\rho), \\ \boldsymbol{\mathcal{V}}_\mu &\to \boldsymbol{\mathcal{V}}_\mu + g(\boldsymbol{\mathcal{V}}_\mu \times \boldsymbol{\Omega}^a) + \partial_\mu\boldsymbol{\Omega}^a, \\ \boldsymbol{\mathcal{A}}_\mu &\to \boldsymbol{\mathcal{A}}_\mu + g(\boldsymbol{\mathcal{A}}_\mu \times \boldsymbol{\Omega}^a). \end{aligned} \tag{2.13}$$

It is clear that the chiral-guage-covariant derivative is

$$\mathfrak{D}_\mu = \partial_\mu - ig\boldsymbol{\mathcal{V}}_\mu\cdot\mathbf{T}, \tag{2.14}$$

where $\mathbf{T}$ is the isospin operator. We define the covariant curl $\boldsymbol{\mathcal{V}}_{\mu\nu}$ for later use:

$$\boldsymbol{\mathcal{V}}_{\mu\nu} \equiv \partial_\mu\boldsymbol{\mathcal{V}}_\nu - \partial_\nu\boldsymbol{\mathcal{V}}_\mu + g(\boldsymbol{\mathcal{V}}_\mu \times \boldsymbol{\mathcal{V}}_\nu). \tag{2.15}$$

As emphasized in Ref. 3, different choices of the function $\Theta(\pi^2)$ can be transformed to give the same chiral-symmetric Lagrangian by redefining the pion field. The redefinition of the pion field only affects the chiral-symmetry-breaking term, which is unknown beyond the soft-pion limit where PCAC works.[11] One can therefore choose his own convenient $\Theta(\pi^2)$. For our purposes we choose

$$\begin{aligned} &\Phi = (\sin\Theta)/\pi = c, \\ &\sigma = \cos\Theta = (1 - c^2\pi^2)^{1/2}, \end{aligned} \tag{2.16}$$

where c is fixed to be $-1/F_\pi$ by the PCAC relation $\partial^\mu J_\mu{}^{A\alpha} = F_\pi m_\pi{}^2\pi^\alpha$. The axial-vector current of isospin index α, $J_\mu{}^{A\alpha}$, is defined to be $-m_\rho{}^2 a_\mu{}^\alpha/g$.[12] The di-

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[9] Our Eqs. (2.5) and (2.6) are equivalent to Eqs. (2.10) and (2.11) of Ref. 3.

[10] Our Eq. (2.9) is equivalent to Eqs. (3.1) and (3.10) of Ref. 3.

[11] PCAC means the hypothesis of partially conserved axial-vector current: M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); Y. Nambu, Phys. Rev. Letters **4**, 380 (1960); Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. **39**, 703 (1960) [English transl.: Soviet Phys.—JETP **12**, 492 (1961)].

[12] T. D. Lee and B. Zumino, Phys. Rev. **163**, 1667 (1967).

agonalization of $\mathbf{a}_\mu$ and $\partial_\mu\boldsymbol{\pi}$ defines the axial-vector meson $\mathbf{A}_{1\mu}$ as $\mathbf{a}_\mu - D_\mu\boldsymbol{\pi}/2gF_\pi$.

III. CHIRAL DYNAMICS FOR HIGH-SPIN BARYONS

In Sec. II we discussed the chiral dynamics for a system of pions, nucleons, and vector and axial-vector mesons for simplicity. With Eqs. (2.12)–(2.14) one can easily expand the system to include any particle of arbitrary spin and isospin.

We follow Ref. 13 in describing the kinematical properties of the baryons by means of Rarita-Schwinger wave functions.[14] Denote the wave function for spin $S=k+\frac{1}{2}$ by $B^{\mu_1\cdots\mu_k}(x,\lambda)$, where λ denotes the helicity. The wave function is symmetrical in the μ_i. It also satisfies

$$(i\gamma^\mu\partial_\mu - M)B^{\mu_1\cdots\mu_k}(x,\lambda)=0\,, \tag{3.1}$$

$$\gamma_{\mu_1}B^{\mu_1\cdots\mu_k}(x,\lambda)=0\,, \tag{3.2}$$

and therefore

$$\partial_{\mu_1}B^{\mu_1\cdots\mu_k}(x,\lambda)=0 \tag{3.3}$$

when the field is free.

For a baryon of isospin T, we assume its $SU(2)\times SU(2)$ transformation laws to be (omitting the space-time factors)

$$B\rightarrow B+ig\boldsymbol{\chi}^\rho\cdot\mathbf{T}B\,, \tag{3.4}$$

$$B\rightarrow B+ig\boldsymbol{\Omega}^a\cdot\mathbf{T}B\,. \tag{3.5}$$

The chiral gauge-invariant $\mathcal{A}B_aB_b$ vertex of lowest multipolarity is, apart from a coupling constant,

$$\bar{B}_a{}^{\mu_1\cdots\mu_{k'}}\Gamma\mathbf{T}B_{b\mu_1\cdots\mu_k}\mathcal{D}_{\mu_{k+1}}\cdots\mathcal{D}_{\mu_{k'-1}}\mathcal{A}_{\mu_{k'}}+\text{H.c.}\,, \tag{3.6}$$

where B_a is a baryon of spin $S_a=k'+\frac{1}{2}$ and isospin T_a, B_b is a baryon of spin $S_b=k+\frac{1}{2}$ and isospin T_b, and we have taken $S_a>S_b$ for definiteness. $\Gamma=1$ or $i\gamma_5$, depending on the parities of B_a and B_b and the relative orbital angular momentum. The isospin coupling is defined in the standard way[13,15] (omitting space-time factors):

$$B^\dagger\mathbf{T}B\mathcal{A}=\sum_l(-1)^l B_{T_a}{}^{(m)\dagger}T_{(mn)}{}^{(l)}B_{T_b}{}^{(n)}\mathcal{A}^{(-l)}\,, \tag{3.7}$$

$$T_{(mn)}{}^{(l)}=(2T_a+1)^{1/2}C(T_a,1,T_b;m,l,n)\,, \tag{3.8}$$

$$(T^\dagger)_{(mn)}{}^{(l)}=(-1)^lT_{(mn)}{}^{(-l)}\,, \tag{3.9}$$

where the superscripts of the wave functions denote the z components of isospin, and C is the Clebsch-Gordan coefficient.[15]

The chiral guage-invariant $\mathcal{V}B_aB_b$ vertices can be similarly constructed:

$$\bar{B}_a{}^{\mu_1\cdots\mu_{k'}}\gamma^{\mu_{k'+1}}\Gamma\mathbf{T}B_{b\mu_1\cdots\mu_k}\mathcal{D}_{\mu_{k+1}}\cdots\mathcal{D}_{\mu_{k'-1}}\mathcal{V}_{\mu_{k'}\mu_{k'+1}}+\text{H.c.}\,, \tag{3.10}$$

where $\Gamma=1$ or γ_5;

$$\bar{B}_a{}^{\mu_1\cdots\mu_{k'}}\Gamma\mathbf{T}\mathcal{D}^{\mu_{k'+1}}B_{b\mu_1\cdots\mu_k}\mathcal{D}_{\mu_{k+1}}\cdots\mathcal{D}_{\mu_{k'-1}}\mathcal{V}_{\mu_{k'}\mu_{k'+1}}+\text{H.c.}\,, \tag{3.11}$$

where $\Gamma=1$ or $i\gamma_5$;

$$\mathcal{D}^{\mu_{k'+1}}\bar{B}_a{}^{\mu_1\cdots\mu_{k'}}\Gamma\mathbf{T}B_{b\mu_1\cdots\mu_k}\mathcal{D}_{\mu_{k+1}}\cdots\mathcal{D}_{\mu_{k'-1}}\mathcal{V}_{\mu_{k'}\mu_{k'+1}}+\text{H.c.}\,, \tag{3.12}$$

where $\Gamma=1$ or $i\gamma_5$.

The couplings (3.6) and (3.10)–(3.12) are the simplest possible vertices, which reduce to the lowest multipoles, and therefore are most important for low energy. The couplings of higher multipolarities can be similarly constructed.

The coupling constants are determined experimentally, or one can, if one wishes, assume a certain symmetry beyond $SU(2)\times SU(2)$, e.g., $SU(4)$, to relate the coupling constants among different vertices. Since $\mathcal{A}_\mu$, $\mathcal{V}_\mu$, and $\mathcal{D}_\mu$ each contains more than one term, determination of one coupling constant yields many predictions under chiral symmetry. The expressions (2.12) explicitly show the power and the limitation of the prediction. In the subsequent sections we shall concentrate on one such example and leave further applications to other occasions.

IV. PHOTOPRODUCTION OF DOUBLE PION FROM NUCLEON

It has been shown in a previous paper[5] that current algebra[16] and the PCAC hypothesis,[11] in the soft-pion limit, reproduce the total cross sections of the static-nucleon model[8] for double-pion photoproduction. In comparison with experiment,[17,18] the static model seems quantitatively smaller. Therefore we are tempted to see whether the hard-pion approach would improve the static results.

The energy range that we are interested in is from threshold up to about 1.5 GeV c.m. total energy. In this region we expect, and therefore will assume, that one of the final pions strongly rescatters with the nucleon in the total angular momentum $J=\frac{3}{2}$, isospin $I=\frac{3}{2}$, positive-parity state (hereafter abbreviated as the 33 state). The expectation (or the assumption) is based on the experimental observation that in low-energy (from threshold to about 1.35 GeV c.m. total energy) pion-nucleon elastic scattering and single-pion photoproduction, this resonant state strongly dominates the pro-

[13] P. Carruthers, Phys. Rev. **152**, 1345 (1966).
[14] W. Rarita and J. Schwinger, Phys. Rev. **60**, 61 (1941).
[15] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).
[16] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).
[17] J. M. Sellen, G. Cocconi, V. T. Cocconi, and E. L. Hart, Phys. Rev. **113**, 1323 (1959); B. M. Chasan, G. Cocconi, V. T. Cocconi, R. M. Schectman, and D. H. White, *ibid.* **119**, 811 (1960).
[18] J. V. Allaby, H. L. Lynch, and D. M. Ritson, Phys. Rev. **142**, 887 (1966).

cesses. In our phenomenological Lagrangian approach,[19] we shall treat the resonance as an isobar $\Delta(1.236\text{ GeV}, S^p=\frac{3}{2}+)$.

A. Lagrangian for $\pi N\Delta$ and $\gamma N\Delta$ Couplings

The first step is to estimate the coupling constants for the relevant vertices:

$$-\sqrt{2}gF_\pi(\lambda_1/m_\pi)(\bar{N}\mathbf{T}^\dagger B_\mu+\bar{B}_\mu\mathbf{T}N)\mathfrak{A}'^\mu, \quad (4.1)$$

$$-(g/\sqrt{2})(C_1/m_\pi)(-i\bar{N}\gamma_\mu\gamma_5\mathbf{T}^\dagger B_\nu+i\bar{B}_\nu\gamma_\mu\gamma_5\mathbf{T}N)\times\mathfrak{V}'^{\mu\nu}, \quad (4.2)$$

$$-(g/\sqrt{2})(C_2/m_\pi^2)(\bar{N}i\gamma_5\mathbf{T}^\dagger\mathfrak{D}_\mu B_\nu+\mathfrak{D}_\mu\bar{B}_\nu i\gamma_5\mathbf{T}N)\mathfrak{V}'^{\mu\nu}, \quad (4.3)$$

$$(g/\sqrt{2})(C_3/m_\pi^3)(\mathfrak{D}_\mu\bar{N}i\gamma_5\mathbf{T}^\dagger B_\nu+\bar{B}_\nu i\gamma_5\mathbf{T}\mathfrak{D}_\mu N)\mathfrak{V}'^{\mu\nu}, \quad (4.4)$$

where B_μ is the wave function of the isobar $\Delta(1.236\text{ GeV}, S^p=\frac{3}{2}+)$ and we have introduced the electromagnetic interaction by the gauge-invariant substitution[12] $\mathfrak{A}'\equiv\mathfrak{A}(\boldsymbol{\rho}_\mu\to\boldsymbol{\rho}_\mu')$, $\mathfrak{V}'\equiv\mathfrak{V}(\boldsymbol{\rho}_\mu\to\boldsymbol{\rho}_\mu')$; $\rho_\mu'^1\equiv\rho_\mu^1$, $\rho_\mu'^2\equiv\rho_\mu^2$, $\rho_\mu'^3\equiv\rho_\mu^3-(e/g)A_\mu$; A_μ is the electromagnetic field. The isospin transition matrices $\mathbf{T}$ for the couplings (4.1)–(4.4) are[13]

$$T_1=\begin{bmatrix}1 & 0\\ 0 & 1/\sqrt{3}\\ -1/\sqrt{3} & 0\\ 0 & -1\end{bmatrix},\quad T_2=\begin{bmatrix}i & 0\\ 0 & i/\sqrt{3}\\ i/\sqrt{3} & 0\\ 0 & i\end{bmatrix},$$

$$T_3=\begin{bmatrix}0 & 0\\ -2/\sqrt{3} & 0\\ 0 & -2/\sqrt{3}\\ 0 & 0\end{bmatrix}. \quad (4.5)$$

We shall neglect the other couplings of higher multipolarity in our energy region.

In order to estimate the coupling constants λ_1, C_1, C_2, and C_3, we take the $\pi N\Delta$ coupling from (4.1),

$$(1/\sqrt{2})(\lambda_1/m_\pi)(\bar{B}_\mu\mathbf{T}N\partial^\mu\boldsymbol{\pi}+\text{H.c.}), \quad (4.6)$$

and the $\gamma N\Delta$ coupling from (4.2)–(4.3),

$$-(e/\sqrt{2})(C_1/m_\pi)(-i\bar{B}_\nu\gamma_\mu\gamma_5T_3NF^{\mu\nu}+\text{H.c.}), \quad (4.7)$$

$$-(e/\sqrt{2})(C_2/m_\pi^2)(\partial_\mu\bar{B}_\nu i\gamma_5T_3NF^{\mu\nu}+\text{H.c.}), \quad (4.8)$$

$$(e/\sqrt{2})(C_3/m_\pi^2)(\bar{B}_\nu i\gamma_5T_3\partial_\mu NF^{\mu\nu}+\text{H.c.}), \quad (4.9)$$

where $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$. They can be compared with experiments following the isobar-model analysis of Gourdin and Salin,[6,20] which gives a satisfactory description of single-pion photoproduction.

For the pion-nucleon scattering P-wave amplitude in the 33 state, one obtains from the coupling (4.6)

$$\frac{\exp(i\delta_{33})\sin\delta_{33}}{q^3}=\frac{1}{12\pi}\frac{1}{W^2-M_\Delta^2-iM\Gamma_\pi}\times\left(\frac{\lambda_1}{m_\pi}\right)^2(E+M), \quad (4.10)$$

where E and q are the c.m. energy and momentum of the nucleon at resonance, W is the c.m. total energy, and Γ_π is the phenomenological width of the resonance. For $\Gamma_\pi=120$ MeV, one obtains[21]

$$\lambda_1=1.88. \quad (4.11)$$

On the other hand, from the data of single-pion photoproduction, C_2 and C_3 are measured to be negligible ($C_2=C_3=-0.0043$).[20] It follows from (4.6) and (4.7)[22] that

$$\frac{M_{1+}(W)}{qk}=\frac{1}{12\pi}\frac{1}{W^2-M_\Delta^2-iM\Gamma_\gamma}\frac{\lambda_1C_1}{m_\pi^2}e\times[(E_1+M)(E_2+M)]^{1/2}, \quad (4.12)$$

where M_{1+} (CGLN notation[7]) is the amplitude of magnetic-dipole transition to the 33 state, E_1 and E_2 are the initial and final nucleon c.m. energies at resonance, k and q are the initial and final c.m. momenta, W is the c.m. total energy, and Γ_γ is the phenomenological width for photoproduction. What Gourdin and Salin fitted[6] are equivalent to $\Gamma_\gamma=160$ MeV, and $C_1=0.43$ in Eq. (4.12) (with $\lambda_1=1.88$). On the other hand, if one assumes that $\Gamma_\gamma=\Gamma_\pi=120$ MeV and keeps the height of the resonance unchanged, one obtains $C_1=0.32$. In the latter case, we have, from (4.10) and (4.12),

$$\frac{M_{1+}(W)}{qk}=e\frac{C_1}{\lambda_1}\left(\frac{E_1+M}{E_2+M}\right)^{1/2}h_{33}(W)=0.17eh_{33}(W), \quad (4.13)$$

where

$$h_{33}(W)=[\exp(i\delta_{33})\sin\delta_{33}]/q^3.$$

Using dispersion relations, CGLN[7] gave the following solution for the magnetic-dipole amplitude of single-pion photoproduction:

$$\frac{M_{1+}(W)}{qk}=\frac{\mu_p-\mu_n}{2f_p/m_\pi}h_{33}(W), \quad (4.14)$$

where $\mu_p=2.79e/2M$ and $\mu_n=-1.91e/2M$ are, respectively, the total magnetic moments of proton and neutron and $f_p/m_\pi=G_r/2M$ ($G_r^2/4\pi=14.5$) is the strength

[19] The author thanks Professor T. D. Lee for suggesting the examination of this problem with the phenomenological Lagrangian.

[20] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 309 (1963).

[21] The Gourdin-Salin value $\lambda_1=2.07$ was obtained for $\Gamma_\pi=140$ MeV (Ref. 6).

[22] If the reader notes the slight difference between our expression (4.12) and that in Ref. 20, it is because they used free-field equations for N and Δ in converting their interaction Lagrangians. Our expression (4.12) is derived directly from the couplings (4.6) and (4.7).

of πNN gradient coupling. With these experimental numbers, we find

$$M_{1+}(W)/qk=0.175eh_{33}(W)\,, \qquad (4.15)$$

in remarkable agreement with the fitted expression (4.13).

B. Model

Having evaluated the coupling constants λ_1, C_1, C_2, and C_3, we are now ready to calculate the double-pion photoproduction process. We shall consider the following "tree diagrams": (1) contact term, Fig. 1(a); (2) pion-current term, Fig. 1(b); (3) A_1-exchange term, Fig. 1(c); and (4) nucleon-pole terms, Fig. 1(d).

We describe the reaction

$$\gamma(k)+N(p_1)\rightarrow\pi^\alpha(q_1)+\pi^\beta(q_2)+N(p_2) \qquad (4.16)$$

in terms of the S matrix

$$\begin{aligned} S&=-i(2\pi)^4\delta^4(k+p_1-q_1-q_2-p_2)\\ &\qquad\times(M^2/8k_0\omega_1\omega_2E_1E_2)^{1/2}T\,,\\ T&=e\epsilon^\mu M_\mu=e(4\omega_1\omega_2E_1E_2/M^2)^{1/2}\\ &\qquad\times\langle N(p_2)\pi^\beta(q_2)\pi^\alpha(q_1)|J_\mu{}^{\rm em}(0)\epsilon^\mu|N(p_1)\rangle\,, \end{aligned} \qquad (4.17)$$

where ϵ^μ is the photon-polarization vector and we have treated the electromagnetic interaction to lowest order. α, $\beta=1$, 2, 3 represent the isospin indices of the pions. We shall, for definiteness, assume that the pion $\pi^\beta(q_2)$ and nucleon are resonant in the 33 state and shall make the proper symmetrization between the two pions at the end. The projection of the invariant amplitude T in which $\pi^\beta(q_2)$ and $N(p_2)$ are in the 33 state will be denoted by T_{33}.

(1) Contact term: This term arises from the following interaction Lagrangian [from (4.1)]:

$$-(e/\sqrt{2})(\lambda_1/m_\pi)(\epsilon_{3\alpha\beta}\bar{B}_\mu T_\beta NA^\mu\pi_\alpha+\text{H.c.})\,. \qquad (4.18)$$

We calculate its contribution to T and project the $\pi^\beta(q_2)$ $N(p_2)$ system into the 33 state in their c.m. system:

$$ie4\pi\left[\frac{W_{33}+M_\Delta}{2M}\left(\frac{E_1+M}{E_2+M}\right)^{1/2}\right]h_{33}(W_{33})(3\mathbf{q}_2\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})$$
$$\times\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma)\,, \qquad (4.19)$$

where $W_{33}=[(q_2+p_2)^2]^{1/2}$ is the c.m. total energy of $\pi^\beta(q_2)$ and $N(p_2)$.

In the case of single-pion photoproduction, the correspondent contact term would contribute

$$e(f_p/m_\pi)\epsilon_{3\alpha\gamma}\chi_2{}^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\tau^\gamma\chi_1 \qquad (4.20)$$

(where α is the isospin index of the pion and χ_1 and χ_2 are the spinors of the initial and final nucleons) to the invariant amplitude at threshold. The term (4.20) is just the well-known Kroll-Ruderman term.[23]

The significance of the contact term is different in the

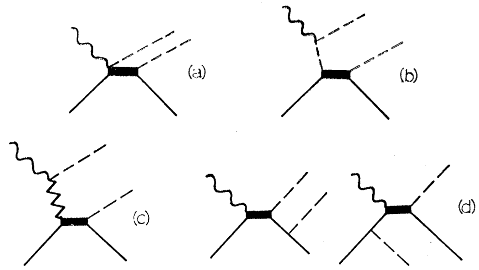

FIG. 1. Various terms in the phenomenological Lagrangian that contribute to the reaction $\gamma N\rightarrow\pi\Delta\rightarrow\pi\pi N$. Solid lines denote nucleons, dashed lines denote pions, wiggly lines denote photons, the zigzag line denotes the axial-vector meson, and rectangular blocks denote the isobar Δ. (a) represents the contact term, (b) the pion-current term, (c) the A_1-exchange term, and (d) the nucleon-pole terms.

two cases. In single-pion photoproduction, (4.20) is the dominant term at threshold, but beyond the threshold it is small compared with the contribution of the 33 resonance. On the other hand, (4.19) dominates the double-pion photoproduction from the threshold ($\sim$1.22 GeV) up to at least 1.5 GeV. This is shown in Fig. 2.

(2) Pion-current term: This term arises from the interaction (4.6) and the pion-current interaction $-e\epsilon_{3\alpha\gamma}\times A^\mu\pi_\alpha\partial_\mu\pi_\gamma$. Its contribution to T_{33} is

$$-ie4\pi\left[\frac{W_{33}+M_\Delta}{2M}\left(\frac{E_1+M}{E_2+M}\right)^{1/2}\right]\frac{2q_1{}^\mu\epsilon_\mu}{m_\pi{}^2-(k-q_1)^2}h_{33}(W_{33})$$
$$\times[3\mathbf{q}_2\cdot(\mathbf{k}-\mathbf{q}_1)-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q}_1)]$$
$$\times\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma) \qquad (4.21)$$

in the c.m. system of $\pi^\beta(q_2)$ and $N(p_2)$.

It turns out that, besides the contact term, this is the only other important term contributing to the total cross section below 1.5 GeV (although it is much smaller than the contact term).

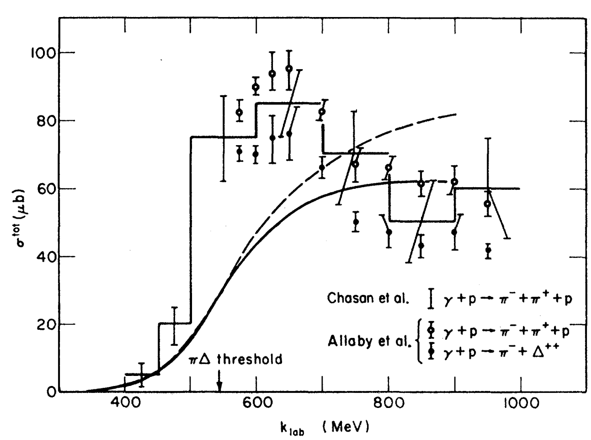

FIG. 2. Total cross section for $\gamma+p\rightarrow\pi^-+\pi^++p$ and $\gamma+p\rightarrow\pi^-+\Delta^{++}\rightarrow\pi^-+\pi^++p$ versus lab photon energy. The solid curve is our total cross section for $\gamma+p\rightarrow\pi^-+\Delta^{++}\rightarrow\pi^-+\pi^++p$. The dashed curve is the contribution of the contact term alone.

[23] N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).

(3) A_1-exchange term: This term arises from the following interactions:

$$\sqrt{2}gF_\pi(\lambda_1/m_\pi)(\bar{B}_\mu \mathbf{T} N \mathbf{A}_1{}^\mu + \text{H.c.}) \qquad (4.22)$$

and

$$(e\kappa/2gF_\pi)\epsilon_{3\alpha\beta}(\partial_\mu A_\nu - \partial_\nu A_\mu) \times(\partial^\nu A_{1\alpha}{}^\mu - \partial^\mu A_{1\alpha}{}^\nu)\pi_\beta . \qquad (4.23)$$

The coupling (4.22) comes from (4.1), while (4.23) is deduced from the chiral gauge-invariant interaction Lagrangian

$$\kappa g(\mathcal{a}_\mu \times \mathcal{a}_\nu)\mathcal{V}^{\mu\nu}, \qquad (4.24)$$

where κ is a constant to be determined by experiment.

The contribution of the A_1-exchange term to T_{33} is

$$ie\kappa 4\pi\left[\frac{W_{33}+M_\Delta}{2M}\left(\frac{E_1+M}{E_2+M}\right)^{1/2}\right]\frac{2}{m_{A1}{}^2-(q_1-k)^2}h_{33}(W_{33}) \times[(3\mathbf{q}_2\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})q_1{}^\mu k_\mu-(3\mathbf{q}_2\cdot\mathbf{k}-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot\mathbf{k})q_1{}^\mu\epsilon_\mu] \times\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma). \qquad (4.25)$$

Its order of magnitude is $\kappa\omega_1 k/m_{A1}{}^2$ times the contact term. If one believes that the value of κ lies in a range $0\lesssim\kappa\lesssim 1$, which was argued on the basis of the widths of ρ and A_1,[2,24] one might neglect the A_1-exchange term ($\lesssim 10\%$ of the contact term).

(4) Nucleon-pole terms: They arise from the following interactions:

$$-(e/\sqrt{2})(C_1/m_\pi)(-i\bar{B}_\nu\gamma_\mu\gamma_5 T_3 N F^{\mu\nu}+\text{H.c.}) \qquad (4.26)$$

[from (4.7)], and the pion-nucleon coupling

$$(f_p/m_\pi)\bar{N}\gamma_\mu\gamma_5\tau^\alpha N\partial^\mu\pi_\alpha+\text{H.c.}, \quad f_p\simeq 1.0. \qquad (4.27)$$

Their contributions to T are given by

$$ie(f_p/m_\pi)\bar{u}(p_2)[\Gamma_5{}^\alpha(q_1)S_F(p_2+q_1)\epsilon^\mu\mathfrak{M}_\mu{}^\beta(p_2+q_1,\, q_2;\, p_1,k) + \epsilon^\mu\mathfrak{M}_\mu{}^\beta(p_2,q_2;\, p_1-q_1,\, k)S_F(p_1-q_1)\Gamma_5{}^\alpha(q_1)]u(p_1), \qquad (4.28)$$

where

$$\Gamma_5{}^\alpha(q)=\not{q}\gamma_5\tau^\alpha,$$
$$S_F{}^{-1}(p+q)=M-(\not{p}+\not{q}),$$

and the invariant single-pion photoproduction amplitude is defined by

$$\epsilon^\mu\bar{u}(p_2)\mathfrak{M}_\mu{}^\beta(p_2,q;\, p,k)u(p_1)\equiv(E_1E_2 2\omega/M^2)^{1/2} \times\langle N(p_2)\pi^\beta(q_2)|\epsilon^\mu J_\mu{}^{\text{em}}(0)|N(p_1)\rangle, \qquad (4.29)$$

which is equal to

$$\tfrac{1}{3}ie\frac{C_1\lambda_1}{m_\pi{}^2}\frac{W_{33}+M_\Delta}{2M}[(E_1+M)(E_2+M)]^{1/2} \times\frac{1}{W_{33}{}^2-M_\Delta{}^2-iM\Gamma_\gamma}[3\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})] \times(\delta_{\beta 3}-\tfrac{1}{3}\tau_\beta\tau_3) \qquad (4.30)$$

when the final-state $\pi^\beta(q_2)$ and $N(p_2)$ are projected into the 33 state in their c.m. system.

The significance of the nucleon-pole terms for the angular-distribution differential cross section will be discussed in Secs. VI and VII.

V. SOFT-PION AND DISPERSION-RELATION APPROACH

The application of current algebra[17] and PCAC[11] to double-pion photoproduction has been described in a previous paper.[5] Here we shall make some remarks regarding its comparison with the Lagrangian approach. In the current-algebra approach, we will take one pion to be soft and use the dispersion-relation method to solve the four-point matrix elements that have πN in the final state. As a result, we need fewer imputs to predict the cross sections in comparison with the Lagrangian approach.

In the expression (4.17), we contract $\pi^\alpha(q_1)$:

$$M_\mu=i\left(\frac{2\omega_1E_1E_2}{M^2}\right)^{1/2}\int d^4x\, e^{iq_1x}(\Box+m_\pi{}^2) \times\langle N(p_2)\pi^\beta(q_2)|T(\pi^\alpha(x)J_\mu{}^{\text{em}}(0))|N(p_1)\rangle, \qquad (5.1)$$

which in turn is reduced to

$$\frac{F_\pi m_\pi{}^2}{m_\pi{}^2-q_1{}^2}M_\mu=\left(\frac{2\omega_2E_1E_2}{M^2}\right)^{1/2}\Bigg(q_1{}^\nu\int d^4x\, e^{iq_1x} \times\langle N(p_2)\pi^\beta(q_2)|T(J_\nu{}^{A\alpha}(x)J_\mu{}^{\text{em}}(0))|N(p_1)\rangle -\epsilon_{3\alpha\gamma}\langle N(p_2)\pi^\beta(q_2)|J_\mu{}^{A\gamma}(0)|N(p_1)\rangle\Bigg) \qquad (5.2)$$

by partial integration and making use of the PCAC relation $\partial^\mu J_\mu{}^{A\alpha}=F_\pi m_\pi{}^2\pi^\alpha$ and the commutation relation[17,25]

$$[J_0{}^{A\alpha}(x),J_\mu{}^{\text{em}}(0)]_{x_0=0}=-i\epsilon_{3\alpha\beta}J_\mu{}^{A\gamma}(0)\delta^3(x). \qquad (5.3)$$

In Eqs. (5.2) and (5.3), we have omitted the contribution of the Schwinger term[26] both because we are treating the electromagnetic interaction to the lowest order and because we are going to take the soft-pion limit $q_1\to 0$. Under such conditions Schwinger terms contribute nothing.[27] From (5.2) we proceed to take the soft-pion limit $q_1\to 0$ and employ the CGLN[7] dispersion-relation method to obtain our approximate solution.

It has been shown[5] that the matrix element of the axial-vector current between N and πN, $\langle N(p_2)\pi^\beta(q_2)| \times J_\mu{}^{A\gamma}(0)|N(p_1)\rangle$, can be decomposed into a pion-pole

[24] H. J. Schnitzer and S. Weinberg, Phys. Rev. 164, 1828 (1967).

[25] M. Veltman, Phys. Rev. Letters 17, 553 (1966); M. Nauenberg, Phys. Rev. 154, 1455 (1967). The connection between the commutation relation and the gauge condition was discussed in these papers.

[26] J. Schwinger, Phys. Rev. Letters 3, 296 (1959).

[27] See Ref. 12 for details.

term

$$-iF_\pi\frac{(k-q_1)_\mu}{m_\pi^2-(k-q_1)^2}\langle N(p_2)\pi^\beta(q_2)|j_\pi{}^\gamma(0)|N(p_1)\rangle \quad (5.4)$$

plus

$$\langle N(p_2)\pi^\beta(q_2)|J_\mu{}^{A\gamma(c)}(0)|N(p_1)\rangle, \quad (5.5)$$

where $j_\pi{}^\gamma=(\Box+m_\pi{}^2)\pi^\gamma$ and $J_\mu{}^{A\gamma(c)}=J_\mu{}^{A\gamma}-\partial^{-2}\partial_\mu\partial^\nu\times J_\nu{}^{A\gamma}$. Using the CGLN expression

$$\langle N(p_2)\pi^\beta(q_2)|j_\pi{}^\gamma(0)|N(p_1)\rangle=(M^2/2\omega_2E_1E_2)^{1/2}\times 4\pi(W_{33}/M)h_{33}(W_{33})[3\mathbf{q}_2\cdot(\mathbf{p}_2+\mathbf{q}_2-\mathbf{p}_1)-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot(\mathbf{p}_2+\mathbf{q}_2-\mathbf{p}_1)](\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma), \quad (5.6)$$

one sees that the contribution of (5.4) to T is one-half the pion-current term (4.21). For the matrix element (5.5), one can solve it by using the technique of CGLN[7,28]:

$$\langle N(p_2)\pi^\beta(q_2)|J_\mu{}^{A\gamma(c)}(0)\epsilon^\mu|N(p_1)\rangle = i\frac{g_AM}{G_\gamma}\left(\frac{M^2}{2\omega_2E_1E_2}\right)^{1/2}4\pi\frac{W_{33}}{M}\left(\frac{(E_1+M)(E_2+M)}{4E_2^2}\right)^{1/2}\times h_{33}(W_{33})(3\mathbf{q}_2\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma) \quad (5.7)$$

in the c.m. system of $\pi^\beta(q_2)$ and $N(p_2)$. The nucleon axial-vector coupling constant is $g_A\simeq 1.18$. Now if one uses the Goldberger-Treiman relation[29] $F_\pi\cong -g_AM/G_\gamma$, one sees that, besides a factor $[2W_{33}/(W_{33}+M_\Delta)]\times(E_2+M)/2E_2$ almost equal to 1, the contribution of (5.7) to T is the same as the contact term (4.19).

We now turn to the integral term in (5.2),

$$q_1{}^\nu\int d^4x\, e^{iq_1x}\langle N(p_2)\pi^\beta(q_2)|\times T(J_\nu{}^{A\alpha}(x)J_\mu{}^{\text{em}}(0))|N(p_1)\rangle. \quad (5.8)$$

In the soft-pion limit $q_1\to 0$, the only important terms are the pion-pole terms, and the nucleon-pole terms in which the axial-vector current $J_\nu{}^{A\alpha}$ attaches to the external nucleon lines.[30] There are two pion-pole terms corresponding to $J_\nu{}^{A\alpha}\sim -F_\pi(\partial_\nu\pi^\alpha-e\epsilon_{3\gamma\alpha}A_\nu\pi^\gamma)$, i.e.,

$$\left(\frac{M^2}{2\omega_2E_1E_2}\right)^{1/2}\frac{F_\pi q_1{}^2}{m_\pi{}^2-q_1{}^2}M_\mu \quad (5.9)$$

and

$$-iF_\pi\frac{\epsilon_{3\alpha\gamma}q_1{}^\mu}{m_\pi{}^2-(k-q_1)^2}\langle N(p_2)\pi^\beta(q_2)|j_\pi{}^\gamma(0)|N(p_1)\rangle. \quad (5.10)$$

The term (5.9) combines with left-hand side of (5.2) to be $F_\pi M_\mu$, while (5.10) yields another one-half of the pion-current term (4.21).[31]

[28] N. Dombey, Phys. Rev. **127**, 653 (1962).
[29] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); **110**, 1478 (1958).
[30] S. L. Adler, Phys. Rev. **139**, B1638 (1965); S. L. Adler and Y. Dothan, *ibid.* **151**, 1267 (1966).
[31] This is also pointed out by S. L. Adler and W. I. Weisberger, Phys. Rev. **169**, 1392 (1968).

The nucleon-pole terms are similar to (4.28):

$$i\left(\frac{M^2}{2\omega_2E_1E_2}\right)^{1/2}\bar{u}(p_2)\left[\mathfrak{M}_\mu{}^\beta(p_2,q_2;p_1-q_1,k)\frac{p_1+M}{-2p_1{}^\nu q_{1\nu}}\times g_A q_1\gamma_5\tfrac{1}{2}\tau^\alpha+g_Aq_1\gamma_5\tfrac{1}{2}\tau^\alpha\frac{p_2+M}{2p_2{}^\nu q_{1\nu}}\times\mathfrak{M}_\mu{}^\beta(p_2+q_1,q_2;p_1,k)\right]u(p_1). \quad (5.11)$$

The solution for the single-pion photoproduction amplitude given by CGLN is

$$\epsilon^\mu\bar{u}(p_2)\mathfrak{M}_\mu{}^\beta(p_2,q;p_1,k)u(p_1)=\frac{\mu_p-\mu_n}{2f_p/m_\pi}4\pi\frac{W_{33}}{M}h_{33}(W_{33})\times[3\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})](\delta_{\beta3}-\tfrac{1}{3}\tau_\beta\tau_3). \quad (5.12)$$

Because of the coincidence between (4.13) and (4.15), the expressions (5.11) and (5.12) essentially reproduce the same nucleon-pole terms (4.28) and (4.30) obtained by our previous method.

We have found that the two methods produce essentially coincident solutions. The first method uses chiral symmetry, PCAC, and the phenomenological Lagrangian in which the coupling constants of $\gamma N\Delta$ as well as of $\pi N\Delta$ must be fitted from the experimental data. The second method takes one pion to be soft ($q_1\to 0$) and employs current algebra, PCAC, and the CGLN dispersion-relation method. It is not obvious from first principles that these two methods should produce numerically coincident solutions over a fairly wide range of energies. Since the two solutions should, in principle, be identical in the soft-pion limit, one might conclude that (a) the soft-pion effect is small and (b) the CGLN method gives good solutions.

VI. RELATION TO STATIC THEORY

The Chew-Low static-nucleon theory[32] for pion-nucleon scattering and single-pion photoproduction was historically the first step toward understanding of the strong interactions. The theory was applied to double-pion photoproduction by Cutkosky and Zachariasen,[8] who gave explicit formulas for the cross sections. It was years later that some data of double-pion photoproduction became available; Carruthers and Wong[33] noted that the rapid rise of the total cross section around 550 MeV (photon lab energy) could be explained by the Cutkosky-Zachariasen theory.

Assuming a P-wave interaction between a pion and a nucleon and imposing the gauge-invariance condition,

[32] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956); **101**, 1579 (1956); G. C. Wick, Rev. Mod. Phys. **27**, 53 (1955); E. Henley and W. Thirring, *Elementary Quantum Field Theory* (McGraw-Hill Book Co., New York, 1962).
[33] P. Carruthers and H.-S. Wong, Phys. Rev. **128**, 2382 (1962).

Cutkosky and Zachariasen obtained, for double-pion photoproduction, a pion-current term and an interaction-current term that are, respectively, equivalent to our pion-current term (4.21) and contact term (4.19) taken in the static limit:

$$-ie4\pi(q_1{}^\mu\epsilon_\mu/k_\mu q_1{}^\mu)h_{33}(W_{33})[3\mathbf{q}_2\cdot(\mathbf{k}-\mathbf{q}_1)-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q}_1)]\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma) \quad (6.1)$$

and

$$ie4\pi h_{33}(W_{33})(3\mathbf{q}_2\cdot\boldsymbol{\varepsilon}-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\tfrac{1}{3}\tau_\beta\tau_\gamma). \quad (6.2)$$

Let us now consider the nucleon-pole terms. If one takes the static limit of (5.11) and (5.12) [or (4.28) and (4.30)], they become

$$-i4\pi(\mu_p-\mu_n)\{-(\delta_{\beta3}-\tfrac{1}{3}\tau_\beta\tau_3)\tfrac{1}{2}\tau_2[3\mathbf{q}_2\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})](\boldsymbol{\sigma}\cdot\mathbf{q}_1/\omega_1)h_{33}(\omega_2)+\tfrac{1}{2}\tau_\alpha(\delta_{\beta3}-\tfrac{1}{3}\tau_\beta\tau_3)\times(\boldsymbol{\sigma}\cdot\mathbf{q}_1/\omega_1)[3\mathbf{q}_2\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-\boldsymbol{\sigma}\cdot\mathbf{q}_2\boldsymbol{\sigma}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})]h_{33}(k)\}. \quad (6.3)$$

For the dominant charge mode $\gamma p\to\pi^-(q_1)\pi^+(q_2)p$, for which experimental data are available, we find the charge factors $\epsilon_{3\alpha\gamma}(\delta_{\beta\gamma}-\frac{1}{3}\tau_\beta\tau_\gamma)=-i$, $(\delta_{\beta3}-\frac{1}{3}\tau_\beta\tau_3)\frac{1}{2}\tau_\alpha=0$, and $\frac{1}{2}\tau_\alpha(\delta_{\beta3}-\frac{1}{3}\tau_\beta\tau_3)=-\frac{1}{3}$. The ratio of the nucleon-pole term (6.3) to the dominant contact term (6.2) is of order kv_1/M, where $\mathbf{v}_1=\mathbf{q}_1/\omega_1$. Although this ratio vanishes in the straight static-nucleon limit $M\to\infty$, it is in fact not small. For a c.m. total energy of 1400 MeV, just above the $\pi\Delta$ threshold, $k/M=0.40$. Nevertheless, the contribution of the nucleon-pole terms to the total cross section is only of order $(kv_1/M)^2$ compared with the contribution of the contact term. The reason is that the $\pi^-(q_1)$ of the contact term is in the S-wave state, while the $\pi^-(q_1)$ of the nucleon-pole terms is in the P-wave state; as a result, the contribution of the interference between these two terms vanishes when one integrates over the angle of $\pi^-(q_1)$. This is why the Cutkosky-Zachariasen static model, which consists only of the contact term and the pion-current term, can describe the total cross section satisfactorily. However, the interference between the contact term and the nucleon-pole terms does contribute to the angular-distribution differential cross section, and we expect its effect to be important (of order kv_1/M compared with the contribution of the contact term). Indeed, in comparison with the experimental data on angular distributions of $\pi^-(q_1)$, the static model is inadequate. The inclusion of the nucleon-pole terms makes the theory consistent with experiment. (See Sec. VII.)

VII. COMPARISON WITH EXPERIMENTAL DATA

The experimental data of low-energy double-pion photoproduction have been published.[7,8] In particular, Allaby *et al.*[18] have measured the angular distribution of π^- for the process $\gamma+p\to\pi^-+\Delta^{++}\to\pi^-+\pi^++p$. To compare our theory with the data, we must transform our (4.19) and (4.21), which are expressed in the c.m. system of $\pi(q_2)$ and $N(p_2)$, to the over-all c.m. system [c.m. system of $\gamma(k)$ and $N(p_1)$]. A complete calculation is straightforward although lengthy. Since we have observed that our total cross section should not differ much from that of the static model, and since we are more interested in the effect of the nucleon-pole terms on angular-distribution differential cross sections, we shall for the present content ourselves with an approximate evaluation and an estimate of possible corrections.

In the following calculations for the cross sections, the polarizations will be summed over final states and averaged over initial states. For convenience, the pion mass will be set equal to 1.

The measured process is $\gamma(k)+p(p_1)\to\pi^-(q_1)+\pi^+(q_2)+p(p_2)$. We calculate the contribution of the contact term to the total cross section to be

$$\int_{M+1}^{W-1}d(W_{33})[1+O(\omega_2q_1/W_{33}q_2)]\times 8\alpha q_1q_2'^3W_{33}^2|h_{33}(W_{33})|^2/kW^2, \quad (7.1)$$

where $W=[(k+p_1)^2]^{1/2}$, $\alpha=e^2/4\pi\simeq1/137$, and $q_2'=[(W_{33}^2+M^2-1)/4W_{33}^2-M^2]^{1/2}$. In the energy range under consideration (from threshold to $W\approx1.5$ GeV), ω_2/W_{33} is smaller than $\frac{1}{4}$, whereas q_1/q_2 is small when the resonance factor $|h_{33}(W_{33})|$ is large, and vice versa. For comparison, we quote the corresponding Cutkosky-Zachariasen static result:

$$\int_1^{W_0-M-1}d(\omega_2)8\alpha q_1q_2^3M|h_{33}(\omega_2)|^2/kW_0, \quad (7.2)$$

where $W_0=M+\omega_1+\omega_2$. In our numerical calculations, the term in (7.1) with the factor $O(\omega_2q_1/W_{33}q_2)$ will be neglected.

For the contributions of the pion-current and nucleon-pole terms, we shall, for simplicity, use their static limits (6.1) and (6.3). Again the ratio of the neglected relativistic corrections for each term to the term itself is of order $\omega_2q_1/W_{33}q_2$.

Our final results—which consist of the contact term, the pion-current term, interference between the contact and pion-current terms, and interference between the contact and the nucleon-pole terms—are the following: total cross section:

$$\sigma^{\text{tot}}=\int_{M+1}^{W-1}d(W_{33})8\alpha\frac{q_1q_2'^3W_{33}^2|h_{33}(W_{33})|^2}{kW^2}+\int_1^{W_0-M-1}d(\omega_2)8\alpha\frac{q_1q_2^3W_{33}^2}{kMW_0}|h_{33}(\omega_2)|^2\times\left[\left(-\frac{\omega_1}{k}+\frac{1}{kq_1}\ln(\omega_1+q_1)\right)+\frac{1}{k^2}\left((-q_2^2+k\omega_1)+\frac{\omega_1q_2^2-k}{q_1}\ln(\omega_1+q_1)\right)\right]; \quad (7.3)$$

angular-distribution differential cross section:

$$\frac{d\sigma}{d\mu_1}=\int_{M+1}^{W-1} d(W_{33})4\alpha\frac{q_1q_2'^3W_{33}^2|h_{33}(W_{33})|^2}{kW^2}+\int_1^{W_0-M-1} d(\omega_2)\alpha\frac{q_1q_2^3W_{33}^2}{kMW_0}|h_{33}(\omega_2)|^2\times\left(\frac{4q_1(\mu_1^2-1)}{k(\omega_1/q_1-\mu_1)}+\frac{2(1-\mu_1^2)(q_1^2+k^2-2q_1k\mu_1)}{k^2(\omega_1/q_1-\mu_1)^2}+\frac{47.1}{9}\frac{q_1k}{\omega_1M}\mu_1\right), \quad (7.4)$$

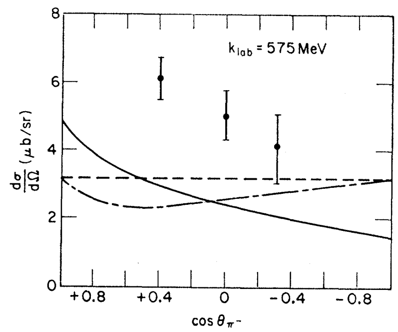

FIG. 3. Angular distribution of π^- in the c.m. system for $\gamma+p\to\pi^-+\Delta^{++}\to\pi^-+\pi^++p$ for a photon lab energy of 575 MeV. $\cos\theta_{\pi^-}=\mathbf{k}\cdot\mathbf{q}_{\pi^-}/kq_{\pi^-}$. The data are measured by Allaby *et al.* (Ref. 19). The solid curve is the result of the present theory. The dashed curve is the contribution of the contact term alone. The dot-dashed curve is the contribution of the contact term plus pion-current term, i.e., the static-nucleon theory.

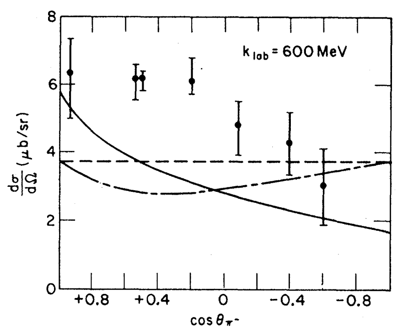

FIG. 4. Same as Fig. 3 for 600 MeV.

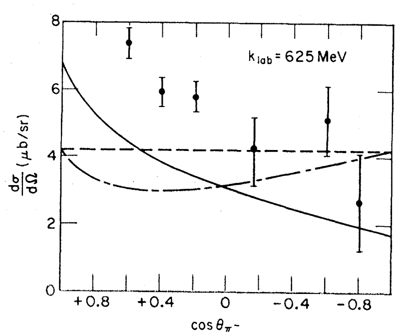

FIG. 5. Same as Fig. 3 for 625 MeV.

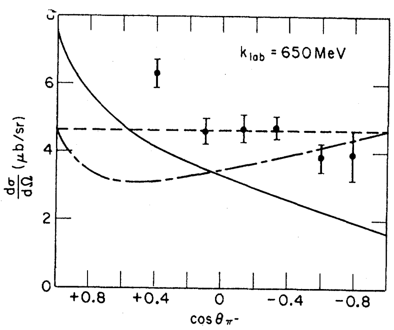

FIG. 6. Same as Fig. 3 for 650 MeV.

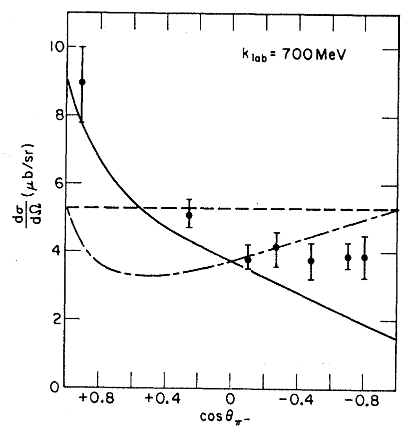

FIG. 7. Same as Fig. 3 for 700 MeV.

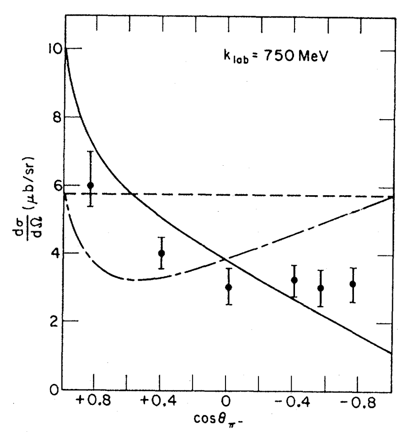

FIG. 8. Same as Fig. 3 for 750 MeV.

where $\mu_1=\mathbf{q}_1\cdot\mathbf{k}/q_1k$ in the over-all c.m. system. In the interference between the contact term and the nucleon-pole term, the factor $h_{33}(\omega_2+M)h_{33}^*(k+M)+$c.c. has been approximated by $2|h_{33}(\omega_2)|^2\equiv 2|h_{33}(\omega_2+M)|^2$. The curves in Figs. 2–8 are obtained by using the experimental Breit-Wigner form[34] for h_{33}. Comparisons

[34] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

between the present theory and the static model for angular distributions are also displayed.

We observe that the static model, although giving a satisfactory total cross section, where the contribution of the nucleon-pole terms is less than 15% $[\sim(kq_1/M\omega_1)^2]$, is not adequate for the angular distributions where the nucleon-pole terms can contribute as much as 40% $[\sim(kq_1/M\omega_1)]$. Our angular distributions have the correct slopes in general. Their quantitative discrepancies reflect the discrepancies in the total cross sections. Apparently more accurate measurements are needed to test the theory.

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Quantum Field Theory of Interacting Tachyons

J. Dhar
Department of Physics and Astrophysics, University of Delhi, Delhi, India

AND

E. C. G. Sudarshan
Department of Physics, Syracuse University, Syracuse, New York, U. S. A. and Centre for Advanced Study in Physics, University of Delhi, Delhi, India.

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A quantum field theory of spin-0 particles traveling with speeds greater than that of light has been constructed. The theory constructed here is explicitly Lorentz-invariant; and the quanta of the field obey Bose statistics. Formalism developed for the free field has been extended to the case of interaction of these particles with nucleons. A new feature of theory is the occurrence of negative-energy particles; this is a necessary consequence of the relativistic invariance of the theory, since the distinction between positive and negative energies is not a relativistically invariant concept for such particles. The occurrence of these negative-energy particles does not, however, prevent the theory from being meaningful; the physical interpretation of the situation is provided by the postulate that *any process* involving negative-energy particles is to be identified with a physical process with *only* positive-energy particles traveling in the opposite direction, with the roles of emission and absorption interchanged. The scattering amplitudes are the same as in the usual theory with m^2 replaced by $-m^2$.

INTRODUCTION

It has generally been believed that no particle can exceed the speed of light.[1] This has meant in turn that in formulating the quantum theory of fields it has been tacitly assumed that all the particles described by such fields belong to one of two classes: those which have a finite rest mass and travel with speeds less than the speed of light; and those which have zero rest mass and hence always travel with the speed of light. We may also consider a third class of particles: those which travel with speeds greater than the speed of light. If we try to ascribe a rest mass to such particles it will be pure imaginary, but this leads to no conceptual difficulties since these particles cannot be brought to rest. The real difficulty with such particles has been that the usual Lorentz transformation properties lead to negative energies in suitable frames. Several years ago it was shown how this difficulty may be overcome[2]; crucial to the resolution of the difficulty is the reinterpretation of "negative-energy particles traveling backward in time" to be positive-energy particles traveling forward in time. All the puzzles and paradoxes that have been put up by various people could be resolved using this basic idea, at least as far as classical theory is concerned. It also motivated two brilliant experiments[3] searching for these faster-than-light particles, which we shall call tachyons.[4] Both these experiments had negative results, but we believe that this should be interpreted to mean that, like particles of vanishing mass, tachyons carry no electric charge.[5]

It is now of interest to consider a quantum theory of

[1] H. Poincaré, Bull. Sci. Math. **28**, 302 (1904); A. Einstein, Ann. Physik (Paris) **17**, 891 (1905).
[2] O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. **30**, 718 (1962).
[3] T. Alväger, P. Erman, Nobel Inst. Report 1966 (unpublished); T. Alväger and N. M. Kreisler, Phys. Rev. **171**, 1357 (1968).
[4] The name "tachyon" is the contribution of G. Feinberg, Phys. Rev. **159**, 1089 (1967).
[5] E. C. G. Sudarshan, Proceedings of the Nobel Symposium, Lerum Sweden, 1968 (to be published). For an exactly solvable Hamiltonian model for the charged scalar theory using both positive- and negative-energy mesons, see E. C. G. Sudarshan, in *Theoretical Physics 1961* (W. A. Benjamin, Inc., New York, 1962).