Off-Mass-Shell Corrections to Current-Algebra Calculation of πN S-Wave Scattering Lengths

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Current-algebra techniques are used to calculate $a^{(+)}$, the even-crossing πN S-wave scattering length. The off-mass-shell extrapolation includes sizable terms ($\sim 0.1 m_\pi^{-1}$) of order q^2 and of all orders in ν. These contributions are found by using a pole model for axial-vector-current–nucleon scattering and on-mass-shell dispersion relations. The experimental result, $a_{\text{exper}}^{(+)} = (-0.001 \pm 0.004) m_\pi^{-1}$, is matched if the σ term is vanishing or very small: $\sigma = (0.06 \pm 0.14) m_\pi$. Alternatively, if we take $\sigma = 0$, then the calculation predicts $a^{(+)} = (-0.011 \pm 0.022) m_\pi^{-1}$, which agrees with experiment. In the case of $a^{(-)}$, the off-shell extrapolation is identical to that used in the derivation of the Adler-Weisberger sum rule. The predicted value of $a^{(-)}$ also agrees with experiment.

1. INTRODUCTION

CURRENT-ALGEBRA calculations of πN S-wave scattering lengths involve the extrapolation of an off-mass-shell scattering amplitude $F(q^2,\nu)$ to the threshold point $(q^2,\nu) = (m_\pi^2, m_\pi)$ from the origin (0,0), where the amplitude is characterized by equal-time commutator and σ terms. Most methods[1,2] of extrapolation ignore terms of order q^2, ν^2, and higher. Other authors[3,4] conjecture a q^2 dependence of the sort considered by Adler.[5]

This paper describes a calculation which includes terms of order q^2 and all terms in ν. These contributions are determined using a pole model for nucleon–axial-vector-current scattering and on-mass-shell dispersion relations. The terms are large in magnitude (on the order of $0.1 m_\pi^{-1}$) but opposite in sign. The resulting predictions for the scattering amplitudes are compatible with a vanishing or very small σ term.

In Sec. 2 we define the off-mass-shell scattering amplitude and derive the current-algebra restrictions on it. Section 3 sketches the calculation of the even-crossing scattering length $a^{(+)}$ in terms of the σ term. In Sec. 4 we compute $a^{(-)}$ from the equal-time commutator contribution. Appendices A and B explain the notation and the pole model for axial-vector-current–nucleon scattering, respectively.

2. CURRENT-ALGEBRA CONDITIONS

Equation (A1) suggests the PCAC (partially conserved axial-vector current) definition of the π^- field[6]:

$$\partial A^{(+)}(x) = m_\pi^2 f_\pi \varphi_{\pi^-}(x). \tag{2.1}$$

Substituting this in the Lehmann-Symanzik-Zimmermann reduction formula for the $\pi^- p$ scattering amplitude gives

$$\langle \pi^-(q'), p(p_f) | S-1 | \pi^-(q) p(p_i) \rangle = \frac{2\pi\delta(p_i+q-p_f-q')}{(4q_0q_0')^{1/2}} \frac{(q^2-m_\pi^2)(q'^2-m_\pi^2)}{(im_\pi^2 f_\pi)^2} \times \int d^4x\, e^{-iqx} \langle p_f | T\{\partial A^{(+)}(0) \partial A^{(-)}(x)\} | p_i \rangle. \tag{2.2}$$

The off-mass-shell forward-scattering amplitude $F(q^2,\nu)$ is defined as

$$F(q^2,\nu) = \left(\frac{q^2-m_\pi^2}{im_\pi^2 f_\pi}\right)^2 R(q^2,\nu), \tag{2.3}$$

where

$$\nu = \frac{p\cdot q}{M_N}, \quad N_p = \frac{1}{(2\pi)^3}\left(\frac{M_N}{p_0}\right),$$

$$R(q^2,\nu) = (iN_p)^{-1} \int d^4x\, e^{-iqx} \times \langle p(p) | T\{\partial A^{(+)}(0) \partial A^{(-)}(x)\} | p(p) \rangle. \tag{2.4}$$

The $\pi^- p$ scattering length is determined by $F(m_\pi^2, m_\pi)$:

$$a^{\pi^- p} = \frac{F(m_\pi^2, m_\pi)}{4\pi(1+m_\pi/M_N)}. \tag{2.5}$$

To analyze $F(q^2,\nu)$ we expand $R(q^2,\nu)$ using the identity[7]

$$R(q^2,\nu) = \mathrm{I} + \mathrm{II} + \mathrm{III}, \tag{2.6}$$

where

$$\mathrm{I}(q^2\,\nu) = (iN_p)^{-1} q^\mu q^\nu \int d^4x\, e^{iqx} \times \langle p(p) | T\{A_\mu^{(+)}(x) A_\nu^{(-)}(0)\} | p(p) \rangle, \tag{2.6$'$}$$

$$\mathrm{II}(q^2,\nu) = (iN_p)^{-1} \int d^4x\, e^{-iqx} \delta(x_0) \times \langle p(p) | [\partial A^{(+)}(0), A_0^{(-)}(x)] | p(p) \rangle, \tag{2.6$''$}$$

* National Science Foundation Graduate Fellow.
[1] Y. Tomozawa, Nuovo Cimento 46, 707 (1966); A. P. Balanchandran, M. G. Gundzik, and F. Nicodemi, *ibid.* 44, 1257 (1966).
[2] S. Weinberg, Phys. Rev. Letters 17, 616 (1966).
[3] K. Kawarabayashi and W. Wada, Phys. Rev. 146, 1209 (1966).
[4] K. Raman, Phys. Rev. 164, 1736 (1967).
[5] S. Adler, Phys. Rev. 140, B736 (1965).
[6] M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); Y. Nambu, Phys. Rev. Letters 4, 380 (1960).
[7] For example, see W. Weisberger, Phys. Rev. 143, 1302 (1966).

$$\mathrm{III}(q^2,\nu) = -(N_p)^{-1}\int d^4x\, e^{-iqx}q^\mu \times\langle p(p)|\,\delta(x_0)[A_0{}^{(+)}(0),A_\mu{}^{(-)}(x)]|p(p)\rangle. \quad (2.6''')$$

The equal-time commutator term $\mathrm{III}(q^2,\nu)$ is determined by the assumption of $SU(2)\otimes SU(2)$ commutation relations[8]:

$$[A_0{}^a(0),A_0{}^b(0,\mathbf{y})] = i\epsilon_{abc}V_0{}^c(0)\delta^3(\mathbf{y}). \quad (2.7)$$

The result is

$$\mathrm{III}(q^2,\nu) = -\nu + \mathrm{III}_{\mathrm{ST}}(q^2,\nu), \quad (2.8)$$

where $\mathrm{III}_{\mathrm{ST}}$ represents Schwinger terms[9] and will be ignored.

If we assume that the current-divergence commutator in (2.6″) is proportional to $\delta^3(\mathbf{x})$, then $\mathrm{II}(q^2,\nu)$ is a q-independent c number, proportional to the σ term ($\sigma \equiv \frac{1}{2}\mathrm{II}$).

$\mathrm{I}(q^2,\nu)$ is the sum of Born and non-Born terms:

$$\mathrm{I}(q^2,\nu) = \mathrm{I}_{\mathrm{B}}(q^2,\nu) + \mathrm{I}_{\mathrm{NB}}(q^2,\nu). \quad (2.9)$$

Using Eqs. (A3) and (A4), the Born contribution can be written

$$\mathrm{I}_{\mathrm{B}}(q^2,\nu) = g_A{}^2\left\{F_1{}^2(\nu+2M_N) - 2F_1D + \frac{D^2\nu}{q^2+2\nu M_N}\right\}, \quad (2.10)$$

where $F_1(q^2)$ and $D(q^2)$ are defined in Appendix A.

Notice that the pole term in (2.10) represents the Born contribution to $F(q^2,\nu)$:

$$F_{\mathrm{B}}(q^2,\nu) = \left(\frac{q^2-m_\pi{}^2}{im_\pi{}^2f_\pi}\right)^2 \frac{g_A{}^2D^2\nu}{q^2+2\nu M_N}. \quad (2.11)$$

Substituting Eqs. (2.8)–(2.10) and (2.6) in Eq. (2.3) gives $\tilde{F}(q^2,\nu)$, the non-Born part of $F(q^2,\nu)$:

$$\tilde{F}(q^2,\nu) = F(q^2,\nu) - F_{\mathrm{B}}(q^2,\nu) = \left(\frac{q^2-m_\pi{}^2}{im_\pi{}^2f_\pi}\right)^2[-\nu(1-g_A{}^2F_1{}^2) + 2g_A{}^2F_1(M_NF_1-D) + \mathrm{I}_{\mathrm{NB}}(q^2,\nu) + \mathrm{II}]. \quad (2.12)$$

This equation contains the two restrictions which current algebra places on $\tilde{F}(q^2,v)$:

$$\tilde{F}(0,0) = \frac{2g_A{}^2M_N}{f_\pi{}^2} - \frac{\mathrm{II}}{f_\pi{}^2}, \quad (2.13')$$

$$\left.\frac{\partial\tilde{F}}{\partial\nu}\right|_{(q^2,\nu)=(0,0)} = \frac{1-g_A{}^2}{f_\pi{}^2}. \quad (2.13'')$$

[8] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[9] J. Schwinger, Phys. Rev. Letters **3**, 296 (1959).

Threshold πN scattering in all charge states is determined by $F^{(\pm)}(m_\pi{}^2,m_\pi)$, the even- and odd-crossing amplitudes on the mass shell,

$$F^{(\pm)}(q^2,\nu) = \tfrac{1}{2}[F(q^2,\nu) \pm F(q^2,-\nu)] \quad (2.14)$$

(analogous definitions apply to Born and non-Born parts of $F^{(\pm)}$). In terms of these amplitudes the current-algebra restrictions, Eqs. (2.13′), and (2.13″), are

$$\tilde{F}^{(+)}(0,0) = \frac{2M_Ng_A{}^2}{f_\pi{}^2} - \frac{\mathrm{II}}{f_\pi{}^2}, \quad \left.\frac{\partial\tilde{F}^{(+)}}{\partial\nu}\right|_{(0,0)} = 0; \quad (2.15')$$

$$\tilde{F}^{(-)}(0,0) = 0, \quad \left.\frac{\partial\tilde{F}^{(-)}}{\partial\nu}\right|_{(0,0)} = \frac{1-g_A{}^2}{f_\pi{}^2}. \quad (2.15'')$$

3. $A^{(+)}$

The even-crossing scattering length $a^{(+)}$ is determined by $F^{(+)}(m_\pi{}^2,m_\pi)$, which is the sum of Born and non-Born terms:

$$F^{(+)}(m_\pi{}^2,m_\pi) = F_{\mathrm{B}}{}^{(+)}(m_\pi{}^2,m_\pi) + \tilde{F}^{(+)}(m_\pi{}^2,m_\pi). \quad (3.1)$$

This can be rewritten as

$$F^{(+)}(m_\pi{}^2,m_\pi) = -F_{\mathrm{B}}{}^{(-)}(m_\pi{}^2,m_\pi) + \Delta\tilde{F}^{(+)}(\nu)|_{\nu=m_\pi} + G(m_\pi{}^2,0), \quad (3.2)$$

where

$$\Delta\tilde{F}^{(+)}(\nu)|_{\nu=m_\pi} = \tilde{F}^{(+)}(m_\pi{}^2,m_\pi) - \tilde{F}^{(+)}(m_\pi{}^2,0), \quad (3.2')$$

$$G(q^2,\nu) = F_{\mathrm{B}}(q^2,m_\pi) + \tilde{F}(q^2,\nu). \quad (3.2'')$$

Using Eqs. (2.11), (A4), and (A5), we can write the first term on the right side of (3.2) as

$$F_{\mathrm{B}}{}^{(-)}(q^2,\nu) = \frac{-2\nu q^2g^2(q^2)}{(q^2)^2-(2\nu M_N)^2}. \quad (3.3)$$

Evaluating this on the mass shell gives

$$F_{\mathrm{B}}{}^{(-)}(m_\pi{}^2,m_\pi) = (2.04\pm0.06)m_\pi{}^{-1}. \quad (3.4)$$

The second term in Eq. (3.2) is determined by on-mass-shell dispersion relations. Using the dispersion integrals in Raman's[4] equation (3.4a) gives

$$\Delta\tilde{F}^{(+)}(\nu)|_{\nu=m_\pi} = (1.33\pm0.20)m_\pi{}^{-1}. \quad (3.5)$$

We must now calculate $G(m_\pi{}^2,0)$ using the current-algebra condition, Eq. (2.13′). First we note the analytic properties of each part of $G(q^2,0)$. Equation (2.11) shows that $F_{\mathrm{B}}(q^2,m_\pi)$ has a branch cut at $q^2=9m_\pi{}^2$ and a pole at $q^2\approx-13.6m_\pi{}^2$. In addition, if we hypothesize (as in Weisberger's article[7]) that matrix elements of ∂A satisfy unsubtracted dispersion relations in q^2, then Eqs. (A4) and (2.11) imply that $F_B(q^2,m_\pi)$ satisfies a once-subtracted dispersion relation in q^2. Weisberger[7] shows that $\tilde{F}(q^2,0)$ is analytic except for a branch cut at $q^2 \approx 8m_\pi^2$. Also, if matrix elements of ∂A satisfy unsubtracted dispersion relations in q^2, then Eqs. (2.3), (2.4), and (2.11) imply that $\tilde{F}(q^2,0)$ obeys a twice-subtracted dispersion relation. The net result is that $G(q^2,0)$ is analytic in the interval $-13.6m_\pi^2 < q^2 < 8m_\pi^2$ and is expected to satisfy a twice-subtracted dispersion relation in q^2. Therefore, $G(q^2,0)$ should have a smooth (largely linear) Taylor expansion in the interval $0 \leq q^2 \leq m_\pi^2$; explicitly,

$$G(m_\pi^2,0)=G(0,0)+\frac{\partial G}{\partial q^2}\bigg|_{(0,0)} m_\pi^2+\text{NLT}$$
$$=\left[F_B(0,m_\pi)+\frac{\partial F_B}{\partial q^2}\bigg|_{(0,m_\pi)} m_\pi^2\right]+\left[\tilde{F}(0,0)+\frac{\partial \tilde{F}}{\partial q^2}\bigg|_{(0,0)} m_\pi^2\right]+\text{NLT}, \quad (3.6)$$

where NLT represents nonlinear terms, expected to be much smaller than the linear ones.

The first bracket in Eq. (3.6) is determined by Eqs. (2.11), (A4), and (A5):

$$\left[F_B(0,m_\pi)+\frac{\partial F_B}{\partial q^2}\bigg|_{(0,m_\pi)} m_\pi^2\right]=\frac{m_\pi}{f_\pi^2}\left[-2g_A^2\frac{M_N}{m_\pi}-\tfrac{2}{3}g_A^2\left(\frac{M_N}{m_\pi}\right)(a_g m_\pi)^2+g_A^2\right], \quad (3.7)$$

where a_g is the rms radius of $g(q^2)$:

$$\frac{\partial g}{\partial q^2}\bigg|_{q^2=0}=g(0)(a_g^2/6). \quad (3.8)$$

The second bracket in Eq. (3.6) is evaluated using the current-algebra condition, Eq. (2.13′), and Eq. (2.12):

$$\left[\tilde{F}(0,0)+\frac{\partial \tilde{F}}{\partial q^2}\bigg|_{(0,0)} m_\pi^2\right]=\frac{m_\pi}{f_\pi^2}\left[2g_A^2\frac{M_N}{m_\pi}+\tfrac{2}{3}g_A^2\left(\frac{M_N}{m_\pi}\right)(a_g m_\pi)^2\right]-\frac{m_\pi^2}{f_\pi^2}\frac{\partial \text{I}_{\text{NB}}}{\partial q^2}\bigg|_{(0,0)}+\frac{\text{II}}{f_\pi^2}. \quad (3.9)$$

Therefore, Eqs. (3.6)–(3.9) give

$$G(m_\pi^2,0)=\frac{m_\pi}{f_\pi^2}[g_A^2]-\frac{m_\pi^2}{f_\pi^2}\frac{\partial \text{I}_{\text{NB}}}{\partial q^2}\bigg|_{(0,0)}+\frac{\text{II}}{f_\pi^2}+\text{NLT}. \quad (3.10)$$

Notice the cancellation of the nucleon "structural" terms (containing a_g) in Eqs. (3.7) and (3.9). This improves the precision of the calculation since experimental and theoretical estimates of a_g are crude.[10] The formal cancellation of the first terms in Eqs. (3.7) and (3.9) is also important for the precision of the result, since these terms are very large ($\sim 21m_\pi^{-1}$). Neither of these cancellations would have occurred if we had determined $G(m_\pi^2,0)$ by evaluating $F_B(m_\pi^2,m_\pi)$ exactly and expanding $\tilde{F}(q^2,0)$ alone.

To find the second term in Eq. (3.10), we adopt a pole model: $\text{I}(q^2,\nu)$ is approximated by the sum of all pole diagrams arising from exchanges in the s, t, and u channels. Then the second term in (3.10) can be written as the sum of contributions from resonances:

$$\frac{-m_\pi^2}{f_\pi^2}\frac{\partial \text{I}_{\text{NB}}}{\partial q^2}\bigg|_{(0,0)}\approx\sum_{\text{res}}\left(\frac{-m_\pi^2}{f_\pi^2}\right)\frac{\partial \text{I}_{\text{NB}}{}^{\text{res}}}{\partial q^2}\bigg|_{(0,0)}. \quad (3.11)$$

A detailed calculation (see Appendix B) shows that the only nonzero s- and u-channel contributions in (3.11) are resonances with spin $\frac{1}{2}$ and $\frac{3}{2}$. These are dominated by $N^*(1236)$. The only t-channel contributions are associated with hypothetical scalar and tensor particles; these are ignored. The result is [Eq. (B4)]

$$\frac{-m_\pi^2}{f_\pi^2}\frac{\partial \text{I}_{\text{NB}}}{\partial q^2}\bigg|_{(0,0)}\approx(-1.03\pm0.12)m_\pi^{-1}. \quad (3.12)$$

The uncertainty in (3.12) is associated with the estimation of the axial-vector-current–NN^* coupling constant.

Equations (3.12) and (3.10) give

$$G(m_\pi^2,0)=(0.56\pm0.16)m_\pi^{-1}+\text{II}/f_\pi^2+\text{NLT}. \quad (3.13)$$

We expect NLT to be much smaller than the linear terms; as a conservative guess we will suppose its magnitude to be less than one-third the magnitude of the linear contributions:

$$-0.19m_\pi^{-1}\leq\text{NLT}\leq0.19m_\pi^{-1}. \quad (3.14)$$

Combining Eqs. (3.2)–(3.5), (3.13), and (3.14) gives

$$F^{(+)}(m_\pi^2,m_\pi)=(-0.15\pm0.32)m_\pi^{-1}+\text{II}/f_\pi^2. \quad (3.15)$$

If we assume that the σ term, $\sigma\equiv\frac{1}{2}\text{II}$, is zero, then Eqs. (3.15) and (2.5) predict

$$a^{(+)}=(-0.011\pm0.022)m_\pi^{-1}. \quad (3.16)$$

This is consistent with the experimental result[11]

$$a_{\text{exper}}{}^{(+)}=(-0.001\pm0.004)m_\pi^{-1}. \quad (3.17)$$

An alternative interpretation of Eq. (3.15) is to use (3.17) to fix $F^{(+)}(m_\pi^2,m_\pi)$; then (3.15) implies that the

[10] For instance, see G. Furlan, R. Jengo, and E. Remiddi, Nuovo Cimento **44A**, 427 (1966); S. Ragusa, *ibid.* **53A**, 855 (1968); E. Kazes, Phys. Rev. **167**, 1543 (1968).
[11] J. Hamilton, Phys. Letters **20**, 687 (1966).

σ term is

$$\sigma=\tfrac{1}{2}\text{II}=(0.06\pm0.14)m_\pi. \tag{3.18}$$

The result, Eq. (3.16) or Eq. (3.18), includes the effect of all terms of order q^2 and ν^2 [for example: $-(m_\pi^2/f_\pi^2)\partial\text{I}_{\text{NB}}/\partial q^2|_{(0,0)}$, $\Delta\tilde{F}^{(+)}(\nu)|_{\nu=m_\pi}$, and the nucleon "structural" terms in a_g]. These contributions are sizable.[12] Most similar calculations[1,2] do not consider such effects and, therefore, yield less precise results. Kawarabayashi and Wada[3] do include an extrapolation in ν and q^2 with the result[13]:

$$\sigma_{\text{KW}}\approx[0.45]m_\pi. \tag{3.19}$$

The discrepancy between Eqs. (3.18) and (3.19) might be due to the Adler-type[5] extrapolation of $F^{(+)}(q^2,\nu)$ in the variable q^2 which is used in Ref. 3 [see Eq. (2.12) of Ref. 3]. It is possible that this extrapolation procedure breaks down, when applied to the interval $(q^2,\nu)=(0,m_\pi)$ to $(q^2,\nu)=(m_\pi^2,m_\pi)$, since $F^{(+)}(q^2,\nu)$ has a branch cut in q^2 and a near-zero at the point $(q^2,\nu)=(m_\pi^2,m_\pi)$. Raman[4] uses a similar extrapolation assumption and finds

$$\sigma_{\text{Raman}}\approx[-0.3]m_\pi. \tag{3.20}$$

The difference between Eqs. (3.18) and (3.20) may again lie in the Adler-type extrapolation, especially since it is applied only to the non-Born nonresonant terms which are very large and nearly cancelled by the Born resonant contributions.

4. $a^{(-)}$

The calculation of $a^{(-)}$ is much more straightforward. The only off-mass-shell extrapolation is identical to the one used to derive the Adler-Weisberger sum rule.[14]

The scattering length $a^{(-)}$ is determined by $F^{(-)}(m_\pi^2,m_\pi)$:

$$F^{(-)}(m_\pi^2,m_\pi)=F_{\text{B}}^{(-)}(m_\pi^2,m_\pi)+\tilde{F}^{(-)}(m_\pi^2,m_\pi). \tag{4.1}$$

The first term is given in Eq. (3.4). The second term can be expanded:

$$\tilde{F}^{(-)}(m_\pi^2,m_\pi)=\frac{\partial\tilde{F}^{(-)}}{\partial\nu}\bigg|_{(m_\pi^2,0)}m_\pi+\Delta\tilde{F}^{(-)}(\nu)\bigg|_{\nu=m_\pi}, \tag{4.2}$$

where $\Delta\tilde{F}^{(-)}$ refers to terms of order ν^3 and higher. To find the first term in (4.2), recall that the current-algebra condition, Eq. (2.15''), gives

$$\frac{\partial\tilde{F}^{(-)}}{\partial\nu}\bigg|_{(0,0)}m_\pi=\frac{m_\pi}{f_\pi^2}(1-g_A^2). \tag{4.3}$$

The accuracy of the Adler-Weisberger relation suggests that we use the off-mass-shell extrapolation required to derive it:

$$\frac{\partial\tilde{F}^{(-)}}{\partial\nu}\bigg|_{(m_\pi^2,0)}m_\pi\approx\frac{\partial\tilde{F}^{(-)}}{\partial\nu}\bigg|_{(0,0)}m_\pi=\frac{m_\pi}{f_\pi^2}(1-g_A^2). \tag{4.4}$$

The second term in (4.2) can be evaluated by using the dispersion integrals in Ref. 4:

$$\Delta\tilde{F}^{(-)}(\nu)|_{\nu=m_\pi}=(-0.08\pm0.01)m_\pi^{-1}. \tag{4.5}$$

Combining Eqs. (4.1), (3.4), (4.2), (4.4), and (4.5) gives

$$F^{(-)}(m_\pi^2,m_\pi)\approx(1.49\pm0.11)m_\pi^{-1}$$

or

$$a^{(-)}\approx(0.103\pm0.008)m_\pi^{-1}. \tag{4.6}$$

Since the uncertainty in (4.6) does not account for the extrapolation error in Eq. (4.4), this result is probably consistent with the experimental data[11]:

$$a_{\text{exper}}^{(-)}=(0.090\pm0.002)m_\pi^{-1}. \tag{4.7}$$

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APPENDIX A: NOTATION

The conventions for the metric, Klein-Gordon equation, and Dirac equation are those of Bjorken and Drell,[15] except that we define $\gamma_5\equiv\gamma_0\gamma_1\gamma_2\gamma_3$.

The currents obey the algebra of $SU(2)\otimes SU(2)$ as in Eq. (2.7). The π-decay constant is

$$\langle 0|\partial A^{(+)}(0)|\pi^-\rangle=\frac{1}{(2\pi)^{3/2}(2q_0)^{1/2}}m_\pi^2 f_\pi, \tag{A1}$$

where

$$A_\mu^{(\pm)}=A_\mu^1\pm A_\mu^2,$$
$$f_\pi=(0.94\pm0.01)m_\pi\quad\text{(experimental value)}. \tag{A2}$$

The matrix element for β decay is

$$\langle N_f|A_\mu^{(+)}(x)|N_i\rangle =iN_p e^{iqx}g_A\bar{U}_{N_f}[\gamma_\mu\gamma_5F_1(q^2)-q_\mu\gamma_5F_2(q^2)]\tau^{(+)}U_{N_i}, \tag{A3}$$

where

$$N_p=\frac{1}{(2\pi)^3}\left(\frac{M_{N_i}M_{N_f}}{p_i^0p_f^0}\right)^{1/2}$$

[12] These higher-order effects should be even more significant in reactions such as KN scattering; a generalization of the above techniques to such amplitudes is presently underway.

[13] The original result of Kawarabayashi and Wada was $\sigma_{\text{KW}}\approx[0.7]m_\pi$. Equation (3.19) represents the result of their method when the more accurate dispersion integrals and scattering lengths of Raman and Hamilton are used (see Refs. 4 and 10).

[14] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965); W. I. Weisberger, *ibid.* **14**, 1047 (1965); also see Refs. 5 and 7.

[15] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill Book Co., New York, 1965).

and $q=p_f-p_i$, $g_A=1.19\pm0.03$, $F_1(0)=1$, and $\tau^{(+)}=\frac{1}{2}(\tau^1+i\tau^2)$. This means that the current-divergence matrix element becomes

$$\langle N_f|\,\partial A^{(+)}(x)\,|N_i\rangle = -N_p e^{iq\cdot x} g_A D(q^2)\bar{U}_{N_f}\gamma_5\tau^{(+)}U_{N_i}, \quad \text{(A4)}$$

where

$$D(q^2)=(M_{N_i}+M_{N_f})F_1(q^2)-q^2F_2(q^2).$$

We define an off-shell πN coupling constant:

$$\left\langle N_f\left|\frac{\partial A^{(+)}(x)}{m_\pi^2 f_\pi}\right|N_i\right\rangle=\sqrt{2}N_p\frac{g(q^2)}{q^2-m_\pi^2}\times e^{iqx}\bar{U}_{N_f}\gamma_5\tau^{(+)}U_{N_i}. \quad \text{(A5)}$$

Experimentally, we have

$$g^2(m_\pi^2)/4\pi=14.6\pm0.4.$$

Comparing (A4) and (A5) gives the usual relation

$$f_\pi=\sqrt{2}M_N g_A/g(0). \quad \text{(A6)}$$

If we define $g(q^2)=g(m_\pi^2)K(q^2)$, then Eq. (A6) implies

$$K(0)\approx 0.88\pm0.03. \quad \text{(A7)}$$

APPENDIX B: POLE MODEL

We examine the contributions to the right side of Eq. (3.11). Exchanges in the s and u channels are associated with baryon resonances N_i^*. The kinematic form[16] of the axial-vector-current–NN_i^* vertex shows that, if the spin of N_i^* is $J_i\geq\frac{5}{2}$, then $\mathrm{I_{NB}}^{N_i^*}(q^2,0)$ is at least of order $(q^2)^2$. Therefore, these terms do not contribute to Eq. (3.11). The baryon resonances[17] of spin $\frac{1}{2}$ and $\frac{3}{2}$ which are considered include $N^*(1236)$, $N^*(1470)$, $N^*(1518)$, $N^*(1550)$, and $N^*(1710)$.

[16] See, for example, Eq. (A1) in H. J. Schnitzer, Phys. Rev. **158**, 1471 (1967).

[17] All masses, coupling constants, and decay constants are taken from A. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

The $N^*(1236)$ contribution is[16]

$$\mathrm{I_{NB}}^{N^*(1236)}(q^2,0)=\frac{4g_A^{*2}(q^2)}{3(M_N^2+q^2-M_{N^*}^2)}\times[(M_{N^*}+M_N)q^{*2}+\tfrac{1}{3}(M_{N^*}-M_N)(E^*+M_N)^2]$$
$$+\frac{g_A^{*2}(q^2)}{9M_{N^*}^2}[2M_{N^*}^3+2(M_{N^*}+M_N)\times(M_{N^*}^2+2M_{N^*}M_N-2M_N^2)+4(M_{N^*}+M_N)q^2+2M_N(M_N^2+q^2)], \quad \text{(B1)}$$

where

$$E^*+M_N=[(M_{N^*}+M_N)^2-q^2]/2M_{N^*},$$
$$q^{*2}=(E^*+M_N)(E^*-M_N).$$

Schnitzer[16] estimates the value of $g_A^{*2}(0)$ (an axial-vector-current–NN^* coupling constant):

$$1.4g_A^2\leq g_A^{*2}(0)\leq 1.7g_A^2. \quad \text{(B2)}$$

Differentiating (B1) and using (B2) gives

$$\frac{-m_\pi^2}{f_\pi^2}\frac{\partial \mathrm{I_{NB}}^{N^*(1236)}}{\partial q^2}\bigg|_{(0,0)}=(-1.03\pm0.12)m_\pi^{-1}. \quad \text{(B3)}$$

An estimate of the other baryon-resonance contributions, using PCAC values for axial-vector-current coupling constants, shows that they are negligible.

The only t-channel contributions to Eq. (3.11) are exchanges of scalar, vector, and tensor mesons. However, covariance arguments imply that the vector-meson terms in $\mathrm{I_{NB}}(q^2,\nu)$ are at least of first order in ν. Therefore, they do not contribute to Eq. (3.11). Hypothetical scalar-meson and tensor-meson terms are expected to be suppressed by weak coupling and high mass; they will be ignored.

Thus, the pole model is dominated by $N^*(1236)$. The net result is

$$\frac{-m_\pi^2}{f_\pi^2}\frac{\partial \mathrm{I_{NB}}}{\partial q^2}\bigg|_{(0,0)}\approx(-1.03\pm0.12)m_\pi^{-1}. \quad \text{(B4)}$$