Calculation of the P_{11} πN Phase Shift Using Coupled πN, σN, and εN Channels*

ERNESTOS N. ARGYRES AND AARON ROTSSTEIN
Institute of Theoretical Physics, McGill University, Montreal, Canada
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The matrix ND^{-1} method in the determinantal approximation is applied to the study of the P_{11} πN phase shift using three coupled channels: πN, σN, and ϵN. The ϵ meson (also denoted by S_0) is an S-wave $T=0$ $\pi\pi$ resonance at a mass of about 730 MeV. Good agreement between the results of the model and experimental data is obtained for both the phase shift δ_{11} and the absorprion coefficient η_{11} in the range from threshold to ~600-MeV pion laboratory kinetic energy.

I. INTRODUCTION

RECENTLY a bootstrap calculation involving coupled πN and ϵN channels in the $J^P=\frac{1}{2}^+$, $T=\frac{1}{2}$ state showed that the Roper resonance R can be considered as a self-consistent ϵN bound state.[1] ϵ (also designated by S_0) is a $J^P=0^+$, $T=0$ 2π S-wave resonant state with a mass around 730 MeV,[2] while phase-shift analyses of πN scattering[2–6] and other experimental evidence[7–9] indicate that R is a $J^P=\frac{1}{2}^+$, $T=\frac{1}{2}$ resonance with a mass in the range of 1400–1500 MeV.

In this paper we study the behavior of the real part of the P_{11} πN phase shift (δ_{11}) and the absorption coefficient (η_{11}) using the forces involved in the self-consistent calculation.[1] The basic experimental features of the P_{11} phase shift, which one attempts to reproduce theoretically, are that the real part of the phase shift stays extremely close to zero up to ~180 MeV[10] (pion laboratory kinetic energy) and then rises up through 90° at ~600 MeV, while the absorption coefficient η_{11} starts decreasing rapidly from a value of 1.0 at ~300 MeV. We consider here a coupled three-channel problem ($\pi N\to\pi N$, $\pi N\to\sigma N$, $\pi N\to\epsilon N$, $\sigma N\to\sigma N$, $\sigma N\to\epsilon N$, and $\epsilon N\to\epsilon N$) using the matrix ND^{-1} method in the determinantal approximation. In the $\pi N\to\pi N$ channel we use as input forces N exchange, the N pole, and N^* ($J^P=\frac{3}{2}^+$, $T=\frac{3}{2}$, 1238 MeV) exchange. In the $\pi N\to\sigma N$ we use N and R exchange and in $\pi N\to\epsilon N$, $\sigma N\to\epsilon N$, and $\epsilon N\to\epsilon N$ we use only R exchange. [The effect of including N exchange in the latter three channels (i.e., $g_{\epsilon NN}^2/4\pi\neq 0$) is discussed in Sec. III]. The $R\pi N$ and $R\sigma N$ coupling constants can be determined from the corresponding experimental partial widths of the Roper resonance, while the $R\epsilon N$ and σNN couplings are treated as free parameters as determined from a best fit to the data. In part, this calculation serves to indicate the effect of including the σN channel on the self-consistent value obtained in Ref. 1 for the $R\epsilon N$ coupling constant. It should be noted that the P_{11} phase shift has also been studied with models using various other channels.[11–14]

With regard to the coupled two-channel model of Ref. 1 (πN,ϵN), it is worth noting that although the model can account for the rise in phase shift reaching a value of 90° at around the Roper mass, it fails completely in accounting for the behavior of the absorptive coefficient. The ϵN threshold corresponds to about 864 MeV (pion laboratory kinetic energy) with the result that in such a model η_{11} stays fixed at 1.0 up to that value, while experimentally it starts decreasing rapidly below 1.0 at ~300 MeV. This result clearly indicates the necessity of including the effects of channels with lower-energy thresholds. The channel responsible for this departure of η_{11} from 1.0 is clearly the three-body $\pi\pi N$ channel. As discussed by Schwarz,[12] in the P_{11} state the two pions can be in an S-wave state relative to the nucleon. Therefore, one might try to avoid the complications of a three-body channel by considering this S-wave state of the two pions as a resonance, the σ meson,[2] with a mass of about 410 MeV, since the S-wave $\pi\pi$ forces are very strong whereas the S-wave πN forces are relatively weak.[12] The σN threshold corresponds to a pion laboratory kinetic energy of about 350 MeV. Another candidate for the inelastic channel is πN^* (N^* being the 3, 3 πN resonance).[12,15] Experiment,[2] however, indicates that

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the dominant inelastic decay mode of the Roper resonance is σN and it therefore seems worthwhile as a first step to consider the addition of just the σN channel to the πN, ϵN system.

Finally it should be noted that although the coupled two-channel model of πN and σN, using the forces and approximations of the model described in Ref. 1, can account for the experimental behavior of η_{11}, it yields a phase shift that rises too quickly with energy (see Sec. III). A similar phase-shift energy dependence was obtained in the coupled πN, πN^* model of Ref. 12.

II. ND^{-1} EQUATION AND INPUT FORCES

We start by considering the following matrix $T(\omega)$ (in channel space) scattering amplitude:

$$T(\omega)=\begin{bmatrix} f_{1-}^{1/2}(\omega) & h_{\sigma 0,1}^{1/2}(\omega) & h_{\epsilon 0,1}^{1/2}(\omega) \\ h_{\sigma 1,0}^{1/2}(\omega) & g_{\sigma 0+}^{1/2}(\omega) & g_{\sigma\epsilon 0+}^{1/2}(\omega) \\ h_{\epsilon 1,0}^{1/2}(\omega) & g_{\sigma\epsilon 0+}^{1/2}(\omega) & g_{\epsilon 0+}^{1/2}(\omega) \end{bmatrix} \tag{1}$$

as an analytic function in the complex ω plane, where ω is the total center-of-mass energy. The channels πN, σN, ϵN will be labeled by 1, 2, 3, respectively. $T_{11}=f_{1-}^{1/2}$ is the scattering amplitude for $\pi N\to\pi N$ in the state $J=l-s=1-\frac{1}{2}=\frac{1}{2}$, $T=\frac{1}{2}$, and it is normalized so that

$$f_{1-}^{1/2}(\omega)=\frac{1}{q_1(\omega)}\exp[i\delta_{1-}^{1/2}(\omega)]\sin\delta_{1-}^{1/2}(\omega)\,, \tag{2}$$

where $\delta_{1-}^{1/2}$ is the P_{11} πN phase shift, which is real for $m+\mu_\pi<\omega<m+\mu_\sigma$ and complex for $\omega>m+\mu_\sigma$. q_1 is the center-of-mass momentum in the πN system and m is the nucleon mass. $T_{12}\equiv h_{\sigma 0,1}^{1/2}$ $(=T_{21}\equiv h_{\sigma 1,0}^{1/2}$ by time-reversal invariance) is the scattering amplitude for $\pi N\to\sigma N$ $(\sigma N\to\pi N)$ where the final (initial) state has $l=0$, $J=0+\frac{1}{2}$, $T=\frac{1}{2}$ and the initial (final) state has $l=1$,[16] $J=1-\frac{1}{2}=\frac{1}{2}$, $T=\frac{1}{2}$. Similarly $T_{13}\equiv h_{\epsilon 0,1}^{1/2}$ $(=T_{31}\equiv h_{\epsilon 1,0}^{1/2})$ is the amplitude for $\pi N\to\epsilon N$ $(\epsilon N\to\pi N)$. $T_{22}\equiv g_{\sigma 0+}^{1/2}$ is the amplitude for $\sigma N\to\sigma N$ in the state $J=l+s=0+\frac{1}{2}=\frac{1}{2}$, $T=\frac{1}{2}$, and, similarly, $T_{33}\equiv g_{\epsilon 0+}^{1/2}$ is the amplitude for $\epsilon N\to\epsilon N$. $T_{23}\equiv g_{\sigma\epsilon 0+}^{1/2}$ $(=T_{32})$ is the amplitude for $\sigma N\to\sigma N$.

In matrix form the unitarity condition for $T(\omega)$ is given by

$$\mathrm{Im}T(\omega)^{-1}=-\rho(\omega)\,, \tag{3}$$

where

$$\rho(\omega)=\begin{bmatrix} q_1\theta(q_1^2) & 0 & 0 \\ 0 & q_2\theta(q_2^2) & 0 \\ 0 & 0 & q_3\theta(q_3^2) \end{bmatrix}; \tag{4}$$

q_2 and q_3 are the center-of-mass momenta in the σN and ϵN systems, respectively; $\theta(q_i^2)$ are step functions such that $\theta(q_i^2)=1$ for $q_i^2>0$ and $\theta(q_i^2)=0$ for $q_i^2<0$.

We will actually factor out the threshold behavior of the amplitudes in the usual way, and write the ND^{-1} equations for a matrix amplitude $t(\omega)$ defined as follows:

$$t=\begin{bmatrix} F & H_\sigma & H_\epsilon \\ H_\sigma & G_\sigma & G_{\sigma\epsilon} \\ H_\epsilon & G_{\sigma\epsilon} & G_\epsilon \end{bmatrix}, \tag{5}$$

with[17,18]

$$\begin{aligned} F&=[\omega/(E_1-m)]f_{1-}^{1/2}\,,\\ H_\sigma&=\frac{\omega}{[(E_1-m)(E_2+m)]^{1/2}}h_{\sigma 0,1}^{1/2}\,,\\ H_\epsilon&=\frac{\omega}{[(E_1-m)(E_3+m)]^{1/2}}h_{\epsilon 0,1}^{1/2}\,,\\ G_\sigma&=[\omega/(E_2+m)]g_{\sigma 0+}^{1/2}\,,\\ G_\epsilon&=[\omega/(E_3+m)]g_{\epsilon 0+}^{1/2}\,,\\ G_{\sigma\epsilon}&=\frac{\omega}{[(E_2+m)(E_3+m)]^{1/2}}g_{\sigma\epsilon 0+}^{1/2}\,, \end{aligned} \tag{6}$$

where E_i is the nucleon energy in the center-of-mass system of the ith channel. The t matrix satisfies the unitarity condition

$$\mathrm{Im}t^{-1}(\omega)=-r(\omega)\,, \tag{7}$$

where

$$r(\omega)=\begin{bmatrix} q_1\theta(q_1^2)(E_1-m)/\omega & 0 & 0 \\ 0 & q_2\theta(q_2^2)(E_2+m)/\omega & 0 \\ 0 & 0 & q_3\theta(q_3^2)(E_3+m)/\omega \end{bmatrix}. \tag{8}$$

The ND^{-1} equations for $t(\omega)$ are then given by[17] $t(\omega)=N(\omega)D^{-1}(\omega)$, where

$$N_{ij}(\omega)=B_{ij}(\omega)+\frac{1}{\pi}\int\frac{d\omega'}{\omega'-\omega}\left[B_{ik}(\omega')-\frac{\omega-\omega_0}{\omega'-\omega_0}B_{ik}(\omega)\right]r_{km}(\omega')N_{mj}(\omega') \tag{9}$$

and

$$D_{ij}(\omega)=\delta_{ij}-\frac{\omega-\omega_0}{\pi}\times\int\frac{d\omega'}{(\omega'-\omega)(\omega'-\omega_0)}r_{ik}(\omega')N_{kj}(\omega')\,. \tag{10}$$

[16] Since the intrinsic parity of the π is negative and that of the σ is positive, parity conservation requires that a transition from a state with orbital angular momentum l to one with l' can occur only if $(-1)^{l'}=-(-1)^{l}$, i.e., only if $l'=l\pm 1$.

[17] E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

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The functions $B_{ij}(\omega)$ are the input "forces" in the respective channels. The integration limits in the integrals in Eqs. (9) and (10) are given by the θ functions in the phase-space factor r_{ij}; e.g., $\theta(q_1^2)$ yields the integration ranges of $-\infty<\omega'<-(m+\mu_\pi)$ and $m+\mu_\pi<\omega'<\infty$. The determinantal approximation consists of setting $N_{ij}(\omega)=B_{ij}(\omega)$ and substituting in Eq. (10) to obtain the D matrix.

Specifically, we consider the following input. For $\pi N\to\pi N$ we take

$$B_{11}=[\omega/(E_1-m)][f_{1-}^{1/2}(N\text{ exch.})+f_{1-}^{1/2}(N\text{ pole})]+B_*(N^*\text{ exch.}), \quad (11)$$

where[17]

$$f_{1-}^{1/2}(N\text{ exch.})=-\frac{g_{NN\pi}^2}{4\pi}\frac{1}{4\omega}\left(\frac{\omega-m}{E_1-m}Q_1(x^N)+\frac{\omega+m}{E_1+m}Q_0(x^N)\right), \quad (12)$$

with

$$x^N=1-[\omega^2+m^2-2(m^2+\mu_\pi^2)]/2q_1^2. \quad (13)$$

$Q_l(z)$ is related to the Legendre polynomial $P_l(z)$ through

$$Q_l(z)=\frac{1}{2}\int_{-1}^{1}dx\frac{P_l(x)}{z-x}. \quad (14)$$

The N-pole contribution is given by[17]

$$f_{1-}^{1/2}(N\text{ pole})=-\frac{g_{NN\pi}^2}{4\pi}\frac{3(E_1-m)}{2\omega(\omega-m)} \quad (15)$$

with $g_{\pi NN}^2/4\pi=14.5$. Since the N^* exchange force diverges like ω as $\omega\to\infty$, it is necessary to introduce some form of cutoff mechanism. This can be achieved, for instance, by a simple upper-limit cutoff in integrals involving the N^* (cutoff mass),[19] or through a smooth damping function of, e.g., the type[17]

$$[1+(\omega^2-\omega_T^2)/Z^2]^{-1}, \quad (16)$$

where ω_T is $m+\mu_\pi$ and Z is a parameter with units of mass. It will be noted that both approaches[17,19] involve a single parameter. Because of the uncertainty as to the exact form of the damping mechanism, we have considered for simplicity in this paper a single-pole expression for the N^* force [in Eq. (11)],

$$B_*=\gamma/(\omega-\omega_*). \quad (17)$$

ω_* is a parameter in the range[20] $-(m+\mu_\pi)<\omega_*<(m+\mu_\pi)$ and for each value of ω_* we determine γ by requiring that B_* equal $[\omega/(E_2-m)]f_{1-}^{1/2}$ (N^* exchange) at threshold $\omega_T=m+\mu_\pi$. For $f_{1-}^{1/2}$ (N^* exchange) we take[19]

$$f_{1-}^{1/2}(N^*\text{ exch.})=\frac{8M^*q_1^{*2}}{9\omega q_1^2}\frac{g_{NN\pi}^2}{4\pi}\Bigg\{(E_1+m)Q_1(x^*)\times\left[\frac{\omega-2m-M^*}{E_1^*+m}3y+\frac{2m-M^*-\omega}{E_1^*-m}\right]+(E_1-m)Q_0(x^*)\times\left[\frac{\omega+2m+M^*}{E_1^*+m}3y+\frac{M^*-2m-\omega}{E_1^*-m}\right]\Bigg\}, \quad (18)$$

where x^* and y are given by

$$x^*=1-[\omega^2+M^{*2}-2(m^2+\mu_\pi^2)]/2q_1^2 \quad (19)$$

and

$$y=1-[\omega^2+M^{*2}-2(m^2+\mu_\pi^2)]/2q_1^{*2}. \quad (20)$$

q_1^* and E_1^* are the momentum and energy of the nucleon in the πN center-of-mass system evaluated at $\omega=M^*$. (M^* is the N^* mass.) The N^* force in Eq. (17) involves therefore a single parameter ω_* and, like the N^* force used in Ref. 17 with the smooth damping function [Eq. (16)], is equal to the Born approximation at threshold and varies as $1/\omega$ for large ω. It is also possible to relate the parameter γ to the cutoff parameter[17] Z [Eq. (16)] by noting that the Born term $[\omega/(E_1-m)]f_{1-}^{1/2}$ in Eq. (18) multiplied by the damping function varies as

$$\frac{4M^*}{3m^2(E_1^*+m)}\frac{g_{NN\pi}^2}{4\pi}\frac{Z^2}{\omega} \quad (21)$$

as $\omega\to\infty$. For ω_* values of, e.g., $5\mu_\pi$, $2\mu_\pi$, and $-5\mu_\pi$ we find that the corresponding values of γ are 27.2, 57.4, and 128, respectively, and using Eq. (21) the corresponding values of Z are 1.4, 2.03, and 3.05 BeV, respectively. The nucleon bootstrap calculations of Abers and Zemach[17] involved Z values in the range 2–4 BeV. One could also consider R and ρ exchange in $\pi N\to\pi N$ but their effect is small compared to the other Born terms.

For $\pi N\to\sigma N$ we consider R and N exchange,

$$B_{12}=B_{21}=\frac{\omega}{[(E_1-m)(E_2+m)]^{1/2}}\times[h_{0,1}^{1/2}(R\text{ exch.})+h_{0,1}^{1/2}(N\text{ exch.})], \quad (22)$$

where $h_{0,1}^{1/2}$ for R exchange is given by

$$h_{0,1}^{1/2}(R\text{ exch.})=-\frac{g_{R\pi N}g_{R\sigma N}}{16\pi\omega q_1q_2}\times\{[(E_1+m)(E_2-m)]^{1/2}(\omega-m_R+m)Q_1(x_1^R)+[(E_1-m)(E_2+m)]^{1/2}(\omega+m_R)Q_0(x_1^R)\} \quad (23)$$

with

$$x_1^R=(m^2-m_R^2+\mu_\sigma^2-2E_1\omega_2)/2q_1q_2, \quad (24)$$

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[19] S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

[20] This range is determined by the condition that B_* be real along the physical cuts in the complex ω plane.

and ω_2 is the energy of the σ in channel 2. The value of $h_{0,1}{}^{1/2}$ for N exchange is obtained from the above expressions by replacing the Roper mass m_R with the nucleon mass m everywhere, and the $g_{R\pi N}$, $g_{R\sigma N}$ couplings by $g_{NN\pi}$, $g_{NN\sigma}$.

For $\pi N \to \epsilon N$ we consider only R exchange,

$$B_{13}=B_{31}=\frac{\omega}{[(E_1-m)(E_3+m)]^{1/2}}h_{\epsilon 0,1}{}^{1/2}(R \text{ exch.}), \quad (25)$$

where $h_{0,1}{}^{1/2}$ (R exch.) is given by Eqs. (23) and (24) with the replacement of the subscript 2 by the subscript 3 and of $g_{R\sigma N}$ by $g_{R\epsilon N}$ and μ_σ by μ_ϵ.

For $\sigma N \to \sigma N$ we consider R and N exchange,

$$B_{22}=[\omega/(E_2+m)][g_{\sigma 0+}{}^{1/2}(R \text{ exch.}) + g_{\sigma 0+}{}^{1/2}(N \text{ exch.})], \quad (26)$$

with

$$g_{\sigma 0+}{}^{1/2}(R \text{ exch.})=\frac{g_{R\sigma N}{}^2}{4\pi}\frac{1}{4\omega}\left(\frac{\omega-2m-m_R}{E_2-m}Q_0(x_2{}^R) + \frac{\omega+2m+m_R}{E_2+m}Q_1(x_2{}^R)\right), \quad (27)$$

and

$$x_2{}^R=1-[\omega^2+m_R{}^2-2(m^2+\mu_\sigma{}^2)]/2q_2{}^2. \quad (28)$$

We proceed similarly for $g_{\sigma 0+}{}^{1/2}$ (N exchange) with the replacements $m_R \to m$ and $g_{R\sigma N} \to g_{NN\sigma}$.

For $\sigma N \to \epsilon N$ we consider only R exchange,

$$B_{32}=B_{23}=\frac{\omega}{[(E_2+m)(E_3+m)]^{1/2}}g_{\sigma\epsilon 0+}{}^{1/2}(R \text{ exch.}), \quad (29)$$

with

$$g_{\sigma\epsilon 0+}{}^{1/2}(R \text{ exch.})=\frac{g_{RN\epsilon}}{2\sqrt{\pi}}\frac{g_{RN\sigma}}{2\sqrt{\pi}}\frac{1}{4\omega q_2 q_3} \times\{[(E_2+m)(E_3+m)]^{1/2}(\omega-2m-m_R)Q_0(x_{23}{}^R) + [(E_2-m)(E_3-m)]^{1/2}(\omega+2m+m_R)Q_1(x_{23}{}^R)\}, \quad (30)$$

and

$$x_{23}{}^R=(m^2-m_R{}^2+\mu_\epsilon{}^2-2E_2\omega_3)/2q_2q_3. \quad (31)$$

Finally, for $\epsilon N \to \epsilon N$ we again consider only R exchange,

$$B_{33}=[\omega/(E_3+m)]g_{\epsilon 0+}{}^{1/2}(R \text{ exch.}) \quad (32)$$

with $g_{\epsilon 0+}{}^{1/2}$ (R exch.) given by Eqs. (27) and (28) with subscript 2 replaced by subscript 3, $g_{R\sigma N} \to g_{R\epsilon N}$, and $\mu_\sigma \to \mu_\epsilon$.

The couplings $g_{R\pi N}$ and $g_{R\sigma N}$ can be obtained from the experimental $R \to \pi N$ and $R \to \sigma N$ widths[2] through the equations

$$\Gamma_{R\pi N}=3\frac{g_{R\pi N}{}^2}{4\pi}\frac{q_1(E_1-m)}{m_R}\bigg|_{\omega=m_R}, \quad (33a)$$

$$\Gamma_{R\sigma N}=\frac{g_{R\sigma N}{}^2}{4\pi}\frac{q_2(E_2+m)}{m_R}\bigg|_{\omega=m_R}. \quad (33b)$$

$g_{R\pi N}$ as defined in Eq. (33a) involves the coupling of a neutral pion to a Roper resonance and a nucleon, and $g_{R\sigma N}$ as defined in Eq. (33b) involves the coupling of the σ meson to a Roper resonance and a nucleon. Following the experimental data compiled in Ref. 2, we set $m_R=1470$ MeV with a total Roper width of 210 MeV: 65% πN and 35% $\pi\pi N$. These data then lead to $\Gamma_{R\pi N}=136.5$ MeV, and since σN is the dominant mode[2] in $\pi\pi N$ we have a maximum value for $\Gamma_{R\sigma N}$ of ~ 73.5 MeV. The corresponding coupling strengths are $g_{R\pi N}{}^2/4\pi=1.77$ and $g_{R\sigma N}{}^2/4\pi=0.206$.

In the determinantal approximation, the solutions of the ND^{-1} equations depend on the subtraction point ω_0 which in principle is arbitrary. In the present model, however, since we are explicitly introducing an N pole, the output amplitude will have the correct residue at $\omega=m$ only if we choose $\omega_0=m$. To see this explicitly we note that at $\omega=m$ Eq. (10), with $\omega_0=m$, yields $D_{ij}(m)=\delta_{ij}$. Thus

$$t_{11}(m)=\sum_{j=1}^{3} B_{1j}(D^{-1})_{j1}=B_{11}(m). \quad (34)$$

III. RESULTS AND CONCLUSIONS

The coupled three-channel model as described in the previous section contains three parameters, $g_{R\epsilon N}{}^2/4\pi$,

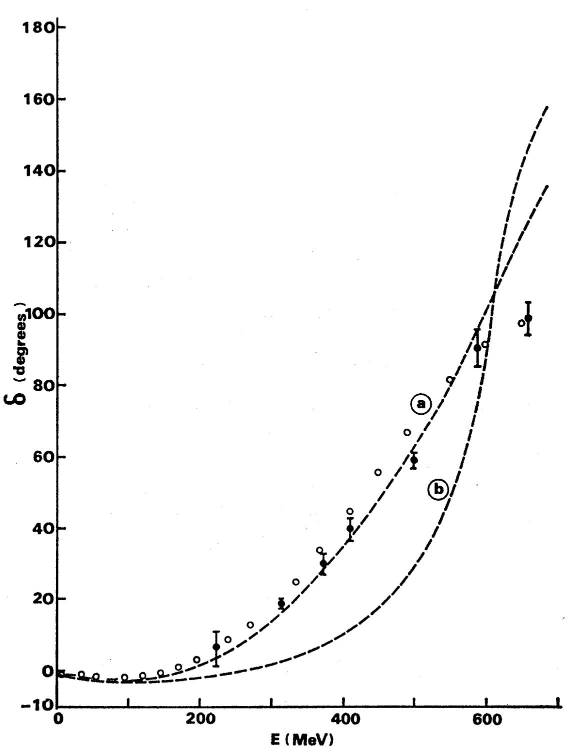

FIG. 1. The calculated P_{11} phase shift (– – –) is plotted versus the pion laboratory kinetic energy E for the coupled πN, σN, ϵN system. (a) $g_{R\epsilon N}{}^2/4\pi=16.0$, $g_{NN\sigma}{}^2/4\pi=1.2$, and $\omega_*=2\mu_\pi$. (b) Same parameters as in (a) except $\omega_*=5\mu_\pi$. The experimental data (●) are from Ref. 6 and the data (○) are from Ref. 4.

$g_{NN\sigma}^2/4\pi$, and the parameter ω_* related to the cutoff mechanism for N^* exchange. We obtained good agreement with experiment for δ_{11} and η_{11}, as indicated in Figs. 1 and 2, with $g_{R\epsilon N}^2/4\pi=16.0$ (which is to be compared with a coupling strength of 14.0 as found in Ref. 1 corresponding to the most reasonable N^* cutoff), $g_{NN\sigma}^2/4\pi=1.2$, and $\omega_*=2\mu_\pi$. The position of the Roper resonance agrees very well with the experimental value of 1470 MeV, which is also the Roper mass used in the input forces. From the residues of the t_{11} and t_{22} amplitudes at the resonance it is possible to calculate the output widths for which we find that $\Gamma_{R\pi N}=139$ MeV and $\Gamma_{R\sigma N}=85$ MeV. These last two widths are to be compared with the corresponding experimental input widths of 136.5 and 73.5 MeV. In the above we have taken the entir $\pi\pi N$ deecay mode

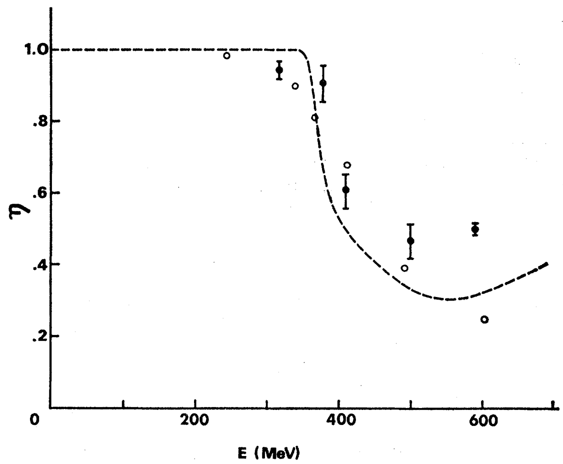

FIG. 2. The absorption coefficient η_{11} (– – – –) as calculated in the coupled three-channel model, with the same parameters as in curve (a) of Fig. 1, is plotted against the pion laboratory kinetic energy. The references for the experimental data are the same as in Fig. 1.

to be σN. The residue of the t_{33} amplitude at the resonance yields an output $R\epsilon N$ coupling constant of 6.1, and, since the input value was 16.0, we find that crossing symmetry has not been completely maintained in the third channel ($\epsilon N \to \epsilon N$).

It is found that both the $R\epsilon N$ coupling constant and ω_* strongly affect the energy dependence of the phase shift and the absorptive coefficient. Larger values of $g_{R\epsilon N}$ lead to resonances at lower masses, while variation of ω_* affects the shape of δ_{11} but has little effect on the position of the resonance [Fig. 1, curve (b)]. This feature of the ω_* dependence is similar to the results obtained by Abers and Zemach,[17] in which the position of the bound state is weakly dependent on the parameter associated with the cutoff mechanism of the N^* exchange force. It should also be noted that our values for $g_{NN\sigma}$ are comparable to those obtained from N-N

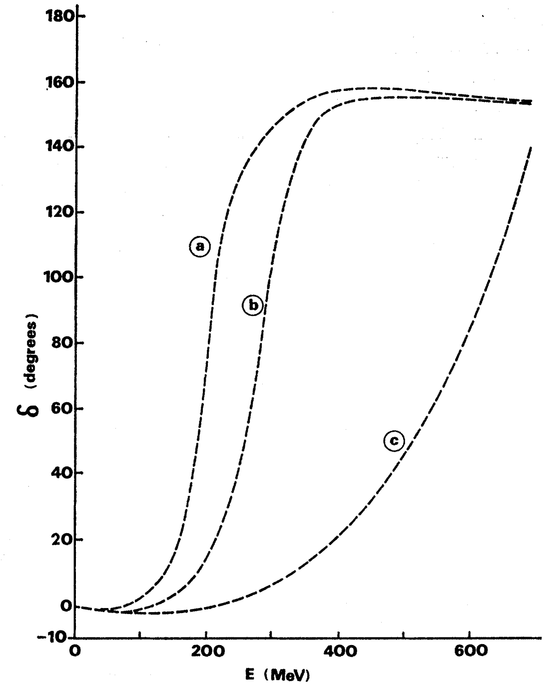

FIG. 3. Curves (a) and (b) are phase shifts as calculated in the coupled πN, σN model with $g_{NN\sigma}^2/4\pi=5.0$ and 3.0, respectively. (c) δ_{11} computed from the coupled πN, ϵN model with $g_{R\epsilon N}/4\pi=16.0$. For all curves $\omega_*=2\mu_\pi$.

scattering data.[21,22] The analysis of the N-N data leads to $g_{NN\sigma}^2/4\pi=3.0$–5.0. Larger values of $g_{NN\sigma}$ in the three-channel calculation cause η_{11} to decrease too rapidly with energy and lead to phase shifts which rise too rapidly at low energies. We also find that including N exchange in those channels where only R exchange had been used ($g_{NN\epsilon}^2/4\pi\neq 0$) results in output $R\pi N$ and $R\sigma N$ widths which are exceedingly large. Also worth noting is that the dip in η_{11} at around 550 MeV (Fig. 2) seems consistent with recent data for the absorption coefficient over a larger energy range.[23] If we study the present model beyond $\sim$650 MeV, we find that both δ_{11} and η_{11} increase too rapidly with energy when compared with the experimental data.[23]

In Fig. 3 we have presented the phase shifts resulting from the coupled two-channel models. For the coupled πN, σN system [curves (a) and (b) of Fig. 3], we find that with $g_{NN\sigma}^2/4\pi=3.0$ and 5.0 ($\omega_*=2\mu_\pi$) as obtained from N-N scattering data,[21,22] the model leads to bound σN states, corresponding to πN resonances at energies below 350 MeV. For no value of $g_{NN\sigma}$ was it found possible to obtain a resonant σN state. Variation

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of ω_* does not change this result. The δ_{11} obtained from the coupled πN, ϵN model with $g_{R\epsilon N}^2/4\pi=16.0$ and $\omega_*=2\mu_\pi$ is presented in curve (c) of Fig. 3. As pointed out in Sec. I, $\eta_{11}=1.0$ up to $\sim$864 MeV for the coupled πN, ϵN model, in complete disagreement with experiment. As in the three-channel case, inclusion of N exchange in $\pi N \to \epsilon_N$ and $\epsilon N \to \epsilon N$ leads to an excessively large Roper width.

In summary, we see that the coupled three-channel model of πN, σN, and ϵN presented here, using only Born input forces in the determinantal approximation, can, with relatively few parameters, account quantitatively for the energy dependence of δ_{11} and η_{11} in the range from threshold to $\sim$600 MeV. This approach also yields output partial widths for the Roper resonance which are in good agreement with the experimental data.[24]

[24] The calculations involved in this paper were carried out on the IBM 7044 Computer at the McGill Computing Centre.

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Weak Parity-Nonconserving Potentials*

D. Tadić†
Brookhaven National Laboratory, Upton, New York 11973
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The weak parity-nonconserving potential due to the exchange of vector bosons and pions is calculated for several weak-interaction Lagrangians. Experimental tests using few-nucleon reactions are discussed. Effects in the reaction $n+p\to d+\gamma$ are estimated in detail. A simple approximate determination of the effective potential for the complex nuclei is included. For reliable information on the weak-interaction Lagrangians, comparison of several processes is needed.

I. INTRODUCTION

Although the idea that weak interactions are described by Lagrangians that are certain combinations of the bilinear products of currents is a very attractive one, it has not yet been exhaustively tested. While very successful in explaining the semileptonic decays, this theory is less directly testable in the case of nonleptonic ones. Some successes of the current algebra concerning them[1] seem to support it, but there are still open questions about the nonleptonic decays of the hyperons.[2–4] On the other hand, the nonrenormalizability of the theory and violent divergencies encountered already in the second approximation seem to raise some doubt in the basic soundness of the whole scheme. Thus, it seems worthwhile to look at the weak-interaction problem from a somewhat different angle, by exploiting the weak parity-nonconserving nuclear effects.[5] Up to now these effects have been measured using complex nuclei only,[6–9] where theoretical interpretation is complicated. Moreover, the experimental results themselves are not quite compatible or conclusive. Fortunately, this situation is very likely to improve in the near future, when few-nucleon reactions are performed. One of these reactions, $n+d\to \mathrm{H}^3+\gamma$, is favored[10–12] because its normal transition matrix element is hindered, so that parity-nonconserving effects should be more pronounced. The other one, $n+p\to d+\gamma$, offers the great advantage of a relatively easy theoretical analysis,[13] and seems also to be feasible in the near future.[14] In view of this, it seems useful to make a systematic study of some consequences and predictions that follow from the conventional current-current

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† On leave of absence from University of Zagreb and Institute "R. Bošković," Zagreb, Yugoslavia.

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