Electromagnetic Properties of Baryons in a Quark-Diquark Model with Broken $SU(6)$*

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An investigation of the electromagnetic properties of baryons is made with a quark-diquark model using broken $SU(6)$ wave functions. The magnetic moments are calculated assuming no orbital contributions and equal gyromagnetic ratios for the quarks. The resulting magnetic-moment sum rules deviate slightly from the $SU(3)$ and $SU(6)$ predictions. The electromagnetic mass splittings are assumed to be composed of intrinsic splitting in the quark and diquark, Coulomb terms, and magnetic terms. We obtain fewer sum rules than with the quark model under analogous assumptions about symmetry breaking.

I. INTRODUCTION

A NUMBER of authors[1–5] have obtained sum rules for the electromagnetic mass splittings of hadrons and the magnetic moments of baryons on the basis of the quark model.[6] In some of these papers, general properties of two-body quark interactions were assumed[1] and in others more explicit assumptions were made about the details of the quark-quark interaction.[2–5]

In one specific dynamical model of the quark-quark interaction,[7] two quarks form a tightly bound state or a diquark, which in turn interacts with a third quark to form a baryon. The diquark was assumed in this model to belong to a six-dimensional representation of $SU(3)$. Using this model, the electromagnetic mass splittings of the hadrons and the baryon magnetic moments have been calculated.[8,9]

One defect of the model as originally proposed is that it is not even approximately invariant under $SU(6)$. For this reason it was not possible previously to obtain the striking prediction[10] of $SU(6)$ that the ratio of the proton to the neutron magnetic moment is $-\frac{3}{2}$. Rather, the gyromagnetic ratios of the quark and diquark were taken as two distinct parameters chosen so as to fit the experimental value of the proton and neutron magnetic moments. Recently this quark-diquark model has been generalized[11] so as to be approximately invariant under $SU(6)$. This is done by taking a quark to belong to a six-dimensional representation of $SU(6)$ and the diquark to belong to a 21-dimensional representation of $SU(6)$. It is the purpose of this paper to calculate the baryon electromagnetic properties using this generalized version of the quark-diquark model.

In some earlier papers[7–9] the quark-diquark model has been called a fermion-boson model or a triplet-sextet model. Both of these terms have defects: the first because the quarks may obey parastatistics, and thus the quark may not be a fermion and the diquark may not be a boson. In the second case we used the term "triplet" to refer to the quark and "sextet" to refer to the diquark. However, according to $SU(6)$, the diquark belongs to a 21-dimensional representation of $SU(6)$ and contains both an $SU(3)$ sextet of spin 1 and an $SU(3)$ triplet of spin 0. Thus it is more convenient to have the terms "sextet" and "triplet" both refer to the diquark and to use the term "quark" to refer to the triplet of half-integral spin.

We shall assume that a diquark has the same quantum numbers as a bound state of two quarks with zero orbital angular momentum. Furthermore, we shall use a two-quark model to infer some other properties of the diquark. Nevertheless, we shall assume the diquark to be essentially elementary in combining it in an S state with a quark to form a baryon.

In making calculations of electromagnetic properties in the quark-diquark model, we shall use perturbation theory. However, the baryon wave functions will not necessarily be $SU(6)$ wave functions but rather wave functions modified to take into account a symmetry

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breaking due to the medium-strong interactions.[11] Thus we obtain the electromagnetic properties by taking the expectation values of the electromagnetic operators between baryon states which are almost but not quite eigenstates of $SU(6)$. We obtain the $SU(6)$ result by letting certain small parameters which measure the difference from the $SU(6)$ wave functions go to zero.

In addition to calculating the baryon electromagnetic mass differences and magnetic moments as in Ref. 9, we shall also calculate certain transition magnetic moments and mass differences. In calculating the magnetic moments of the baryons we shall assume simple additivity of the magnetic moments of the constituent quark and diquark. In calculating the electromagnetic mass splittings of the baryons we shall assume that they arise from two causes: first, from the intrinsic electromagnetic mass splittings of the quark and diquark multiplets, and second, from Coulomb and magnetic interactions between the quark and diquark constituents of the baryons.[5,9]

We do not treat mesons in this paper for the following reason: In the model, a meson can be composed of a quark and antiquark or a diquark and antidiquark. (Other combinations, such as quark-antidiquark do not have the correct quantum numbers, and must be assumed to have more energy if, indeed, they are bound at all.)

It is obviously simpler to choose the quark-antiquark model to describe mesons, in which case we have nothing to add to the usual treatment.[1,5,9] Mesons have already been considered in the $SU(3)$ version of the diquark-antidiquark model,[9] but our present attitude is that such states do not correspond to the ground-state 35-dimensional multiplet of $SU(6)$. If mesons are discovered which belong to a 27-dimensional representation of $SU(3)$, the diquark-antidiquark model of mesons will merit further consideration, since such a multiplet cannot be achieved in the quark-antiquark model.

The plan of the paper is as follows. In Sec. 2 we discuss the electromagnetic properties of the diquark as a bound state of two quarks. In Sec. 3 we obtain the magnetic moment of the baryons including the N^*-N and Σ-Λ transition magnetic moments and the magnetic moment of the Ω on the basis of the quark-diquark model. In Sec. 4 we calculate the electromagnetic mass splittings of the members of the baryon octet and decuplet and obtain a number of sum rules.

2. ELECTROMAGNETIC PROPERTIES OF THE DIQUARK

We assume that two quarks combine to form a diquark which belongs approximately to a 21-dimensional representation of $SU(6)$. The $SU(3)$ and $SU(2)$ content of such a 21-dimensional representation is an $SU(3)$ sextet of spin 1 and an $SU(3)$ triplet of spin 0 belonging to the $\bar{\mathbf{3}}$ representation of $SU(3)$:

$$\mathbf{21} \supset {}^{3}\mathbf{6} + {}^{1}\bar{\mathbf{3}} .$$

The group $SU(6)$ is broken by letting the central mass of the sextet diquark be different from the central mass of the triplet diquark. The group $SU(3)$ is also broken. The sextet diquark splits into an isospin triplet of hypercharge $Y=\frac{2}{3}$, an isospin doublet with $Y=-\frac{1}{3}$, and an isospin singlet with $Y=-\frac{4}{3}$. Similarly, the $SU(3)$ triplet splits into an isospin singlet of hypercharge $\frac{2}{3}$ and an isospin doublet with $Y=-\frac{1}{3}$. These isospin multiplets are assumed to be split further by the electromagnetic interaction. We can consider two possibilities: (a) that the electromagnetic splitting is invariant under U-spin transformations, and (b) that because of the indirect effect of the $SU(3)$-breaking medium-strong interaction, there is a small violation of U-spin invariance by the electromagnetic interaction.

Before discussing the properties of the diquark we briefly recapitulate the electromagnetic properties of the quark. The electromagnetic mass-splitting parameter which splits the quark isospin doublet we call ϵ_q. If U-spin invariance holds,[12] then the isospin singlet quark will also have an electromagnetic mass splitting ϵ_q. Any deviation from U-spin invariance can be absorbed as an electromagnetic correction to the medium-strong mass-splitting parameter δ_q which separates the isodoublet from the isosinglet.

If we assume that the quark magnetic moments $\mu(q_i)$ are proportional to their charges Q_i, we obtain

$$\mu(q_i)=\mu_0 Q_i , \tag{1}$$

where μ_0 is a parameter, which will turn out to be the magnetic moment of the proton, and Q_i is the charge of the ith quark. With this assumption, U-spin invariance holds, but Eq. (1) is more restrictive than U-spin invariance. On the other hand, we can assume that quarks all have the same gyromagnetic ratio. Then the quark magnetic moments are as follows:

$$\mu(q_1)=\tfrac{2}{3}\mu_0 , \quad \mu(q_2)=-\tfrac{1}{3}\mu_0 m_q/(m_q+\epsilon_q) ,$$
$$\mu(q_3)=-\tfrac{1}{3}\mu_0 m_q/(m_q+\delta_q+\epsilon_q) ,$$

where m_q is the quark mass. We define the quantity δ by

$$1-\delta=m_q/(m_q+\delta_q) . \tag{2}$$

We assume $m_q \gg \delta_q \gg \epsilon_q$. Then neglecting ϵ_q, the quark moments are

$$\mu(q_1)=\tfrac{2}{3}\mu_0 , \quad \mu(q_2)=-\tfrac{1}{3}\mu_0 , \quad \mu(q_3)=-\tfrac{1}{3}\mu_0(1-\delta) . \tag{3}$$

This violation of U-spin invariance has been previously considered in the quark model in Ref. 1.

We now turn to the properties of the diquark. The quantum numbers are shown in Table I, together with the symbols for the particles and for the mass-splitting

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TABLE I. Quantum numbers and parameters of the diquark.

Symbol	Mass	Isospin I	I_z	Hypercharge Y	Charge Q	Spin	Baryon number
s_1	$m_s+\epsilon_1$	1	1	$\frac{2}{3}$	$\frac{4}{3}$	1	$\frac{2}{3}$
s_2	$m_s+\epsilon_2$	1	0	$\frac{2}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$
s_3	$m_s+\epsilon_3$	1	-1	$\frac{2}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$
s_4	$m_s+\epsilon_4+\delta_s$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	1	$\frac{2}{3}$
s_5	$m_s+\epsilon_5+\delta_s$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$
s_6	$m_s+\epsilon_6+\delta_s'$	0	0	$-\frac{4}{3}$	$-\frac{2}{3}$	1	$\frac{2}{3}$
t_1	$m_t+\epsilon_7$	0	0	$\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$
t_2	$m_t+\epsilon_8+\delta_t$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$
t_3	$m_t+\epsilon_9+\delta_t$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$	0	$\frac{2}{3}$

parameters. First we consider the sextet diquark of spin 1. If U-spin invariance holds, we conclude that the electromagnetic contribution to the mass splitting of s_2 is equal to that for s_4, since s_2 and s_4 belong to the same U-spin multiplet. Similarly, the contribution to s_3, s_5, and s_6 are equal. Thus from U-spin invariance we have

$$\epsilon_2=\epsilon_4\,, \tag{4}$$

$$\epsilon_3=\epsilon_5=\epsilon_6\,. \tag{5}$$

Since the parameter ϵ_1 can be absorbed into the definition of m_s, we can write all the electromagnetic mass splittings of the sextet in terms of two electromagnetic mass-splitting parameters ϵ_2 and ϵ_3, provided that U-spin invariance holds. Subtracting Eq. (4) from Eq. (5), we obtain

$$\epsilon_3-\epsilon_2=\epsilon_5-\epsilon_4\,. \tag{6}$$

But since s_3 and s_2 belong to the same isospin multiplet, we conclude that their entire mass difference arises just from the electromagnetic contribution to the mass splitting, and similarly for s_5 and s_4. We thus have the sum rule

$$m(s_3)-m(s_2)=m(s_5)-m(s_4)\,. \tag{7}$$

The triplet diquark has only one electromagnetic mass-splitting parameter. Thus U-spin invariance gives no new information.

We now turn to the magnetic moments. The sextet diquark has six magnetic moments, and U-spin invariance leads to the following relations among them:

$$\mu(s_2)=\mu(s_4)\,,\quad \mu(s_3)=\mu(s_5)=\mu(s_6)\,. \tag{8}$$

Thus there are three independent magnetic moments among the members of the sextet if we invoke U-spin invariance. Since the triplet diquark has spin 0, its magnetic moment is zero. However, there are possible transition moments which refer to the matrix elements of the magnetic-moment operator between the triplet and sextet states, when the z component of the sextet spin is zero. We call these transition moments $\mu(t_is_j)$, noting that $\mu(s_jt_i)=\mu(t_is_j)$. There are three non-vanishing transition magnetic moments, $\mu(t_1s_2)$, $\mu(t_2s_4)$, and $\mu(t_3s_5)$. From U-spin invariance two of them are equal:

$$\mu(t_1s_2)=\mu(t_2s_4)\,, \tag{9}$$

so that there are two independent parameters describing the transition moments. Thus there are altogether five magnetic-moment parameters describing the diquark. These are too many parameters to enable us to make any meaningful predictions about the magnetic moments of the baryons.

However, we can add the additional assumption that, just as with the quark, the magnetic moments of the diquark sextet are proportional to their charges. This assumption reduces the number of parameters describing the magnetic moments of the diquark from five to three. (In Ref. 9, since a triplet diquark was not assumed to exist, the transition moments were, of course, zero, and only one parameter described the diquark moments.) The number of parameters can be reduced from three to zero from the assumption that the diquark magnetic moments can be calculated from the magnetic moments of the quark constituents, assuming additivity of the quark moments. With this assumption the magnetic-moment operator μ_d of the diquark is given by

$$\mu_d=\mu_1+\mu_2\,, \tag{10}$$

where the subscripts 1 and 2 refer to the first and second quarks in the diquark and not to quarks of type 1 and type 2. Then, assuming that the gyromagnetic ratios are the same for all quarks, i.e., that the quark moments are given by Eq. (3), we obtain the following results for the magnetic moments of the diquarks:

$$\begin{aligned} &\mu(s_1)=\tfrac{4}{3}\mu_0\,, && \mu(s_4)=\tfrac{1}{3}\mu_0(1+\delta)\,,\\ &\mu(s_2)=\tfrac{1}{3}\mu_0\,, && \mu(s_5)=-\tfrac{2}{3}\mu_0(1-\tfrac{1}{2}\delta)\,,\\ &\mu(s_3)=-\tfrac{2}{3}\mu_0\,, && \mu(s_6)=-\tfrac{2}{3}\mu_0(1-\delta)\,,\end{aligned} \tag{11}$$

$$\begin{aligned} &\mu(t_1s_2)=\mu_0\,,\quad \mu(t_2s_4)=\mu_0(1-\tfrac{1}{3}\delta)\,,\\ &\qquad \mu(t_3s_5)=-\tfrac{1}{3}\mu_0\delta\,,\quad \mu(s_jt_i)=\mu(t_is_j)\,.\end{aligned} \tag{12}$$

Thus the diquark moments are given in terms of the quark moments.

We can obtain the magnetic moments of the diquarks assuming U-spin invariance for the magnetic moments rather than for the gyromagnetic ratios by letting the parameter δ go to zero in Eqs. (11) and (12). Then the magnetic moments of the members of the sextet and transition magnetic moments can be simply written

$$\mu(s_i)=\mu_0Q_i\,,\quad i=1,\,\cdots,6 \tag{13}$$

$$\mu(t_is_j)=\mu(s_jt_i)=\mu_0\,,\quad i=1,2\,,\quad j=2i \tag{14}$$

$$\mu(s_jt_i)=0\quad\text{otherwise}.$$

These results are obtained by taking the matrix elements of the diquark operator $\mu_z=\mu_{1z}+\mu_{2z}$ between states of the diquark.

In order to compute the magnetic contribution to the electromagnetic mass splittings of baryons, it is convenient to have the matrix elements of $\boldsymbol{\mu}\cdot\mathbf{A}$, where $\mathbf{A}$ is any vector, between diquark states. These matrix elements are easily computed, but since there are many of them, we shall not write them here.

We now wish to calculate the electromagnetic mass splittings of the diquark under the assumption that this mass splitting arises from the intrinsic electromagnetic mass splitting of the quark isospin doublet and also from Coulomb and magnetic contact interactions between the two quarks. The assumption that mass splittings of bound states of quarks can be calculated from the sum of the electromagnetic mass splittings of the constitutent quarks plus a sum of two-body quark-quark Coulomb and magnetic contact interactions has been previously made in Refs. 4, 5, and 9.

We assume that the electromagnetic interaction V_e between two quarks in an S state is given by

$$V_e=aQ_1Q_2-b\boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2/\mu_0^2\,, \tag{15}$$

where again the subscripts 1 and 2 refer to the first and second quark in the diquark and not to quarks of type 1 and type 2. The constant a is the effective inverse distance between the two quarks bound in the diquark, while the constant b is proportional to the absolute square of the quark-quark wave function at the origin. The constants a and b are both positive according to the model. These constants depend on the details of the strong forces between the quarks since these strong forces largely determine the wave function of the two quarks in the bound states. This means that the constants a and b can be expected to be somewhat different for the sextet diquark and for the triplet diquark, since their masses are different because of $SU(6)$ symmetry breaking in the medium-strong forces. Similarly, $SU(3)$ breaking can lead to different constants a and b for diquarks with different isospin. However, we shall neglect these effects as being small. It is now straightforward to obtain the result that the electromagnetic contributions to the masses of the sextet and triplet diquarks are given by to first order in $\delta'=b\delta$:

$$\begin{aligned}
\epsilon_1&=4(a-b)/9\,,\\
\epsilon_2&=\epsilon_q-2(a-b)/9\,,\\
\epsilon_3&=2\epsilon_q+(a-b)/9\,,\\
\epsilon_4&=\epsilon_q-2(a-b+\delta')/9\,,\\
\epsilon_5&=2\epsilon_q+(a-b+\delta')/9\,,\\
\epsilon_6&=2\epsilon_q+(a-b+2\delta')/9\,,\\
\epsilon_7&=\epsilon_q-2(a+3b)/9\,,\\
\epsilon_8&=\epsilon_q-2(a+3b-3\delta')/9\,,\\
\epsilon_9&=2\epsilon_q+(a+3b-3\delta')/9\,.
\end{aligned} \tag{16}$$

Of more interest are the electromagnetic mass differences between members of the same isospin multiplet. We have

$$\begin{aligned}
m(s_2)-m(s_1)&=\epsilon_q-\tfrac{2}{3}(a-b)\,,\\
m(s_3)-m(s_2)&=\epsilon_q+\tfrac{1}{3}(a-b)\,,\\
m(s_5)-m(s_4)&=\epsilon_q+\tfrac{1}{3}(a-b)+\tfrac{1}{3}\delta'\,,\\
m(t_3)-m(t_2)&=\epsilon_q+\tfrac{1}{3}(a+3b)-\delta'\,.
\end{aligned} \tag{17}$$

If we let $\delta\to 0$ in Eq. (17), we obtain the sum rule Eq. (7) which follows from U-spin invariance. We also get an additional sum rule connecting the mass splitting of the quark to the mass splitting of the sextet diquarks. This sum rule is

$$2m(s_3)-m(s_1)-m_s(2)=3m(q_2)-3m(q_1)\,. \tag{18}$$

The electromagnetic properties of the diquark which we have obtained in this section will be useful in calculating the electromagnetic properties of baryons. However, as stated in the Introduction, we shall regard the diquark as essentially elementary in calculating the properties of baryons. This means that we shall not impose any symmetry requirements on the baryon wave function under the interchange of quark and diquark.

3. MAGNETIC MOMENTS OF BARYONS

We can use the properties of the quark and diquark obtained in Sec. 2 to calculate the magnetic moments of the baryons. Our results will depend on whether we assume U-spin invariance for the quark and diquark magnetic moments or for their gyromagnetic ratios. We shall obtain the baryon magnetic moments assuming that U-spin invariance holds for the gyromagnetic ratios rather than for the moments themselves. In other words, we shall use the quark moments of Eq. (3) and the diquark moments of Eqs. (11) and (12). In making the calculation we assume zero orbital angular momentum.

The magnetic-moment operator μ_z for a baryon is given by

$$\mu_z=\mu_{dz}+\mu_{qz}\,. \tag{19}$$

Since the magnetic moments of the baryons are given by expectation values of this magnetic-moment operator with respect to baryon wave functions, the results will depend on details of the wave functions. In Ref. 11 it was shown that the mass splittings of the quark and diquark lead to deviations of the baryon wave functions from those given by $SU(6)$ and $SU(3)$. Therefore, we shall obtain our results for wave functions which differ by a small amount from the $SU(6)$ predictions.[10]

We shall introduce symmetry-breaking parameters η and γ_i. The η is a measure of $SU(6)$ breaking, while γ_i $(i=1,\cdots,4)$ are a measure of $SU(3)$ breaking. The broken-$SU(6)$ wave functions are given in the Appendix and the parameters η and γ_i are defined there. The η and γ_i presumably arise from medium-strong symmetry-breaking interaction. Also defined in the Appendix are additional symmetry-breaking parameters θ_i. However, for simplicity, we let them equal their $SU(3)$ values in the main body of this paper. In terms of the η and γ_i, the magnetic moments of the baryons in units of μ_0 are

$$\mu(p)=1-\tfrac{2}{3}\sin^2\eta\,, \tag{20a}$$

$$\mu(n)=-\tfrac{2}{3}+\tfrac{2}{3}\sin^2\eta\,, \tag{20b}$$

$$\mu(\Lambda)=-\tfrac{1}{3}+\tfrac{1}{3}\delta+\tfrac{1}{3}(1-\tfrac{2}{3}\delta)\sin^2(\eta+\gamma_2)\,, \tag{20c}$$

$$\mu(\Sigma^+)=1-\tfrac{1}{9}\delta-\tfrac{2}{3}(1-\tfrac{1}{3}\delta)\sin^2(\eta+\gamma_3)\,, \tag{20d}$$

$$\mu(\Sigma^0)=\tfrac{1}{3}-\tfrac{1}{9}\delta-\tfrac{1}{3}(1-\tfrac{2}{3}\delta)\sin^2(\eta+\gamma_3)\,, \tag{20e}$$

$$\mu(\Sigma^-)=-\tfrac{1}{3}-\tfrac{1}{9}\delta+(2\delta/9)\sin^2(\eta+\gamma_3)\,, \tag{20f}$$

$$\mu(\Xi^0)=-\tfrac{2}{3}+4\delta/9+\tfrac{2}{3}(1-\tfrac{1}{3}\delta)\sin^2(\eta+\gamma_4)\,, \tag{20g}$$

$$\mu(\Xi^-)=-\tfrac{1}{3}+4\delta/9-(2\delta/9)\sin^2(\eta+\gamma_4)\,, \tag{20h}$$

$$\mu(\Omega)=-1+\delta\,, \tag{20i}$$

where without loss of generality we have set $\gamma_1=0$. We have included the magnetic moment of the Ω^- because in principle it should be measurable. The magnetic moment of the Σ^0 is more remote from experiment, but we have included it anyway. The transition magnetic moments are given in terms of the off-diagonal matrix elements of the magnetic-moment operator. The two transition moments which are most accessible to experiment are $\mu(N^{*+}p)$ and $\mu(\Sigma^0\Lambda)$. In units of μ_0 they are

$$\mu(N^{*+}p)=\tfrac{2}{3}\sqrt{2}\cos\eta\,, \tag{21a}$$

$$\mu(\Sigma^0\Lambda)=-(\surd\tfrac{1}{3})\cos(\eta+\gamma_3)\cos(\eta+\gamma_2)\,. \tag{21b}$$

The above expressions are exact for any value of γ_i and η. It can be seen by expanding Eqs. (20) in powers of η and γ_i that there are no corrections to the magnetic moments to first order in η and γ_i. The first-order corrections are zero because the magnetic-moment operator has no matrix elements between states of the 56-dimensional representation of $SU(6)$ and states belonging to other representations.

Without making further assumptions, we can use Eqs. (20) to derive the following relations:

$$\frac{2\mu(\Xi^-)+\mu(\Xi^0)-\mu(\Omega^-)}{\mu(\Xi^0)-\mu(\Xi^-)}=\frac{2\mu(\Sigma^-)+\mu(\Sigma^+)-\mu(\Omega^-)}{\mu(\Sigma^+)-\mu(\Sigma^-)}=\frac{-\mu(\Omega^-)}{3[\mu(p)+\mu(n)]}=1-\delta \tag{22}$$

and

$$\mu(\Sigma^0)=\tfrac{1}{2}[\mu(\Sigma^+)+\mu(\Sigma^-)]\,. \tag{23}$$

But the last relation just follows from the linear form of the magnetic-moment operator and the fact that isospin is conserved. The above relations would hold for both $SU(3)$ and $SU(6)$ breaking of the quark-diquark wave function [with the quark and diquark individually $SU(3)$ and $SU(6)$ symmetric]. However, it is not likely that they will be tested experimentally in the near future.

It is also possible to break $SU(3)$ symmetry in the wave functions in a more general way than that considered here and still conserve isospin. This was in fact done, but we do not think the results of sufficient interest to present here. This is because the resulting expressions involve several extra parameters (the θ_i of the Appendix) and are considerably more complicated.

From the expressions of Eqs. (20), we obtain the result that the ratio of the proton to neutron magnetic moment is given by (in second order)

$$-\mu(p)/\mu(n)\equiv R=\tfrac{3}{2}+\tfrac{1}{2}\eta^2\,. \tag{24}$$

If the baryon wave functions are $SU(6)$ symmetric, $\eta=\gamma_1=0$ and we get the well-known result $R=\frac{3}{2}$. Experimentally[13] $R=1.46<\frac{3}{2}$, so that any small deviation from $SU(6)$ in the wave function worsens the agreement with experiment, provided that isospin is conserved. The result that $R>\frac{3}{2}$ also holds for a more general small $SU(6)$ breaking in the model in its present nonrelativistic version with no orbital angular momentum, provided that isospin is conserved. This result is also true in the quark model. It seems unlikely that a correction as large as the 3% necessary to obtain agreement with experiment arises from a violation of isospin, since the first-order correction is zero. The correction might arise from exchange currents, relativistic effects, or nonzero orbital angular momentum.

In any case, it is plausible to assume that the effects of $SU(6)$ and $SU(3)$ symmetry breaking are small for the proton and neutron.

For the ratio of the Λ to neutron magnetic moment we obtain

$$\mu(\Lambda)/\mu(n)=\tfrac{1}{2}[(1-\delta)-(1-\tfrac{2}{3}\delta)\sin^2(\eta+\gamma_2)]\sec^2\eta\,. \tag{25}$$

In the event of no $SU(3)$ breaking, that is, $\delta=\gamma_2=0$, the ratio reduces to $\frac{1}{2}$. Expanding Eq. (25) to second order in

[13] A. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967); **40**, 77 (1968).

small parameters, we obtain

$$\mu(\Lambda)/\mu(n)=\tfrac{1}{2}(1-\delta-2\eta\gamma_2-\gamma_2^2). \qquad (26)$$

Experimentally[13]

$$\mu(\Lambda)/\mu(n)=0.38\pm0.08. \qquad (27)$$

From the proton-neutron magnetic-moment ratio, we saw that η^2 had to be small unless relativistic corrections, orbital effects, or other corrections outside the scope of our model are important. If we further assume that the second-order terms in $\eta\gamma_2$ and γ_2^2 are also small, we can get a first approximation for δ from Eqs. (26) and (27). We obtain

$$\delta=0.24\pm0.16. \qquad (28)$$

Further relations may be obtained if the γ_i are negligibly small compared to η. For such a case, we may set $\gamma_i=0$ in our original expressions for the magnetic moments, Eqs. (20), and then expand them to second order in small parameters. From the resulting equations, we obtain

$$\begin{aligned}\mu(p)-\mu(\Sigma^+)&\\ =\tfrac{1}{4}[\mu(\Xi^0)-\mu(n)]&=\tfrac{1}{5}[\mu(\Xi^-)-\mu(\Sigma^-)]\\ =\tfrac{1}{6}[2\mu(\Lambda)-\mu(n)]&=\tfrac{1}{4}[\mu(\Sigma^+)+\mu(\Sigma^-)+2\mu(\Lambda)]\\ &=(0.027\pm0.018)\mu(p),\end{aligned}$$

where we have used Eq. (28). These sum rules differ slightly from the well-known $SU(3)$ and $SU(6)$ predictions.[10,14] At the present time the experimental values of the magnetic moments are not well enough known to check these sum rules.

4. ELECTROMAGNETIC MASS SPLITTING OF BARYONS

We assume that the electromagnetic mass splitting of a baryon is the sum of three contributions. These are: (a) an intrinsic contribution arising from the electromagnetic mass splittings among the different members of the quark and diquark, (b) a Coulomb interaction between the quark and diquark, and (c) a contact magnetic-moment interaction between the quark and diquark.

We write the electromagnetic mass operator ΔM so as to exhibit these three contributions explicitly:

$$\Delta M=I+V_c+V_m, \qquad (29)$$

where

$$I=\Delta m_d+\Delta m_q, \qquad (30)$$

$$V_c=AQ_dQ_q, \qquad (31)$$

$$V_m=-B\boldsymbol{\mu}_d\cdot\boldsymbol{\mu}_q/\mu_0^2. \qquad (32)$$

Here A is the average inverse distance between the quark and diquark bound in a baryon and B is proportional to the square of the quark-diquark wave function at the origin. We assume that A and B are the same for all baryons. Without this assumption, the expressions for the mass differences would involve too many parameters to be useful. With this operator we have calculated the electromagnetic mass shifts using the wave functions given in the Appendix with the approximation of carrying the small parameters η and γ_i to first order. The results are given in Table II. We shall examine these results first for the case when $\gamma_i=\eta=0$, then for $\eta\neq0$ while $\gamma_i=0$, and finally for $\gamma_i\neq0$ and $\eta\neq0$.

For the case when $\eta=\gamma_i=0$, the following relations are obtained:

$$(n-p)+(\Xi^--\Xi^0)-(\Sigma^--\Sigma^+)=0, \qquad (33a)$$

$$(\Xi^{*-}-\Xi^{*0})-2(Y^{*-}-Y^{*0})+(N^{*-}-N^{*0})=0, \qquad (33b)$$

$$(N^{*0}-N^{*+})-(Y^{*0}-Y^{*+})-(N^{*-}-N^{*0})+(Y^{*-}-Y^{*0})=0, \qquad (33c)$$

$$3(N^{*0}-N^{*+})-(N^{*-}-N^{*++})=0, \qquad (33d)$$

$$(n-p)-(N^{*0}-N^{*+})=0, \qquad (33e)$$

$$N^{*-}+N^{*+}-2N^{*0}-\Sigma^--\Sigma^++2\Sigma^0=0, \qquad (33f)$$

$$(\Xi^--\Xi^0)-(\Xi^{*-}-\Xi^{*0})+(Y^{*-}-Y^{*+})-(\Sigma^--\Sigma^+)=0. \qquad (33g)$$

Equation (33a) is the Coleman-Glashow[14] relationship. It has been obtained without assuming U-spin invariance since we have kept δ different from zero.

The last three equations related mass differences between members of the octet to mass differences between members of the decuplet. They differ from the corresponding relations obtained in Refs. 8 and 9 in which $SU(6)$ was maximally violated. Unfortunately there is no experimental evidence to check Eq. (33f). But Eqs. (33d) and (33e) may be combined to form

$$3(n-p)=N^{*-}-N^{*++}. \qquad (34)$$

Experimentally[13]

$$3(n-p)=3.9\text{ MeV},\quad N^{*-}-N^{*++}=7.9\pm6.8\text{ MeV}.$$

Experiment must be improved considerably to check this. The members of (33g) also satisfy the following equations:

$$(\Xi^--\Xi^0)-(\Xi^{*-}-\Xi^{*0})=\tfrac{2}{3}(b+2B)(1-\delta), \qquad (35a)$$

$$(\Sigma^--\Sigma^+)-(Y^{*-}-Y^{*+})=\tfrac{2}{3}(b+2B)(1-\delta). \qquad (35b)$$

Since b and B are positive quantities and $\delta<1$, we may write the following inequalities:

$$\Xi^--\Xi^0>\Xi^{*-}-\Xi^{*0}, \qquad (36a)$$

$$\Sigma^--\Sigma^+>Y^{*-}-Y^{*+}. \qquad (36b)$$

It is easy to see that the size of the inequality depends on how large the magnetic contribution is, that is, on the magnitudes of the parameters b and B. If the magnetic contribution is appreciably smaller than the Coulomb contribution, we expect that $\Xi^--\Xi^0$ and $\Sigma^--\Sigma^+$ will be

[14] S. Coleman and S. Glashow, Phys. Rev. Letters 6, 423 (1961).

TABLE II. Contributions to the electromagnetic mass differences of baryons. The symbol for a particle stands for its mass. Each mass difference is equal to a sum of certain terms (given in the row at the top of the table) times the appropriate coefficients shown in the table. The parameter ϵ_q is an intrinsic electromagnetic mass-splitting parameter of the quark, a and b are electromagnetic mass-splitting parameters of the diquark, δ is a measure of the departure from $SU(3)$ symmetry of the magnetic moments of the quark and diquark, and A and B are measures of the strength of the Coulomb and magnetic interactions between quark and diquark. For exact definitions, see the text. The parameter η is a measure of the departure from $SU(6)$ symmetry of the baryon wave functions arising from medium-strong interactions, and γ_3 and γ_4 are measures of the departure from $SU(3)$ symmetry.

	ϵ_q	$\frac{1}{9}a$	$\frac{1}{9}b$	$\frac{1}{9}A$	$\frac{1}{9}B$	$\frac{1}{9}\delta(b+2B)$
$n-p$	1	$-1-2\eta$	$1+2\eta$	$-2(1-\eta)$	$2(1-\eta)$	0
$\Sigma^0-\Sigma^+$	1	$-1-5(\eta+\gamma_3)$	$4-\eta-\gamma_3$	$-2+5(\eta+\gamma_3)$	$8+\eta+\gamma_3$	-2
$\Sigma^--\Sigma^0$	1	$2+\eta+\gamma_3$	$1-7(\eta+\gamma_3)$	$4-\eta-\gamma_3$	$2+7(\eta+\gamma_3)$	-2
$\Xi^--\Xi^0$	1	$2(1-\eta-\gamma_4)$	$2(2-5\eta-5\gamma_4)$	$2(2+\eta+\gamma_4)$	$2(4+5\eta+5\gamma_4)$	-4
$N^{*+}-N^{*++}$	1	-4	4	-8	8	0
$N^{*0}-N^{*+}$	1	-1	1	-2	2	0
$Y^{*0}-Y^{*+}$	1	-1	1	-2	2	1
$N^{*-}-N^{*0}$	1	2	-2	4	-4	0
$Y^{*-}-Y^{*0}$	1	2	-2	4	-4	1
$\Xi^{*-}\Xi^{*0}$	1	2	-2	4	-4	2

only slightly larger than $\Xi^{*-}-\Xi^{*0}$ and $Y^{*-}-Y^{*+}$, respectively. The experimental results are[13]

$$\Xi^--\Xi^0=6.5\text{ MeV},$$
$$\Xi^{*-}-\Xi^{*0}=4.9\pm2.2\text{ MeV},$$
$$\Sigma^--\Sigma^+=7.9\text{ MeV},$$
$$Y^{*-}-Y^{*+}=5.8\pm3.2\text{ MeV}.$$

The present experimental errors are too large to confirm the prediction.

If it were true that the magnetic contribution is appreciably smaller than the intrinsic and Coulomb terms, then it is legitimate to neglect δ since it is a small correction to the magnetic term. But the intrinsic and Coulomb terms in the mass operator are U-spin invariant, and the magnetic term will be U-spin invariant with the δ neglected. Under this assumption we may supplement our equations by

$$N^{*0}-N^{*+}=Y^{*0}-Y^{*+},\tag{37a}$$

$$N^{*-}-N^{*0}=Y^{*-}-Y^{*0}=\Xi^{*-}-\Xi^{*0}.\tag{37b}$$

The inequalities (36) still hold.

For the case $\gamma_i=0$, while $\eta\neq0$, the octet Coleman-Glashow relation and equations among the members of the decuplet are unaffected. In other words, $SU(6)$ breaking does not change relationships within an irreducible $SU(3)$ representation. Only Eqs. (33e)–(33g), which relate mass differences between the octet and decuplet, change. They become

$$(n-p)-(N^{*0}-N^{*+})=2\eta(A-a+b-B)/9,\tag{38a}$$

$$(\Sigma^0-\Sigma^+)-(\Sigma^--\Sigma^0)-(N^{*+}-N^{*++})+(N^{*0}-N^{*+})=\tfrac{2}{3}\eta(A-a+b-B),\tag{38b}$$

$$(\Xi^--\Xi^0)-(\Xi^{*-}-\Xi^{*0})+(Y^{*-}-Y^{*+})-(\Sigma^--\Sigma^+)=2\eta(a-A+B-b)/9.\tag{38c}$$

Finally, for $\eta\neq0$ and $\gamma_i\neq0$, the Coleman-Glashow relation no longer holds. A correction term is added of the form

$$2(\gamma_4-2\gamma_3)(A-a)/9+2(5\gamma_4-4\gamma_3)(B-b)/9.\tag{39}$$

This must be less than or equal to 1 MeV, which is about the total error in the measurement of mass in the Coleman-Glashow relation. This puts an upper limit on the values that γ_i can assume.

This version of the model, incorporating approximate $SU(6)$ invariance, leads to several different sum rules for the mass differences than the previous version, which strongly violates $SU(6)$ invariance. However, since present experiments are not good enough to distinguish between these sum rules, the present evidence for $SU(6)$ does not seem compelling as far as the electromagnetic mass splittings are concerned.

The model contains more parameters than the quark model, and therefore gives fewer predictions. In particular, according to the quark model with $SU(6)$ breaking, we would not get Eqs. (38) but rather the equations that would follow if

$$A-a+b-B=0.$$

Thus, whereas the quark model predicts equalities for certain linear combinations of the masses, the quark-diquark model predicts inequalities. Better experiments will be necessary to distinguish between the two cases.

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APPENDIX

We let the symbol for a particle denote its wave function in this Appendix. We use the symbols q_i, s_i, and t_i for the $SU(3)$ wave functions of a quark, sextet diquark, and triplet diquark, respectively. Also, we denote the spin wave functions of a quark by α (spin up) and β (spin down), and the three spin wave functions of the sextet diquark by a, b, and c. The triplet diquark has spin 0, and we omit its wave function. We also define the spin wave function χ:

$$\chi=(\sqrt{\tfrac{2}{3}})a\beta-(\sqrt{\tfrac{1}{3}})b\alpha. \qquad \text{(A1)}$$

We write the wave functions for the baryon octet and decuplet in terms of these quark and diquark wave functions. For the decuplet, we cannot break $SU(6)$ without also breaking $SU(3)$. The most general wave functions for a broken decuplet of spin $\frac{3}{2}$, assuming isospin conservation, are (omitting the spin variables)

$$\begin{aligned}
N^{*++}&=s_1q_1,\\
N^{*+}&=(\sqrt{\tfrac{1}{3}})s_1q_2+(\sqrt{\tfrac{2}{3}})s_2q_1,\\
N^{*0}&=(\sqrt{\tfrac{2}{3}})s_2q_2+(\sqrt{\tfrac{1}{3}})s_3q_1,\\
N^{*-}&=s_3q_2,\\
Y^{*+}&=s_1q_3\sin\theta_1+s_4q_1\cos\theta_1, \qquad \text{(A2)}\\
Y^{*0}&=s_2q_3\sin\theta_1+(\sqrt{\tfrac{1}{2}})(s_4q_2+s_5q_1)\cos\theta_1,\\
Y^{*-}&=s_3q_3\sin\theta_1+s_5q_2\cos\theta_1,\\
\Xi^{*0}&=s_4q_3\cos\theta_2+s_6q_1\sin\theta_2,\\
\Xi^{*-}&=s_5q_3\cos\theta_2+s_6q_2\sin\theta_2,\\
\Omega&=s_6q_3,
\end{aligned}$$

where θ_1 and θ_2 are parameters. If $SU(3)$ is a good symmetry,

$$\sin\theta_1=\sin\theta_2=\sqrt{\tfrac{1}{3}}. \qquad \text{(A3)}$$

Then $SU(6)$ also holds, and the members of the decuplet belong to a 56-dimensional representation of $SU(6)$.

Assuming isospin holds, the broken octet wave functions are given by

$$\begin{aligned}
p&=\sin\Gamma_1\,[(\sqrt{\tfrac{2}{3}})s_1q_2-(\sqrt{\tfrac{1}{3}})s_2q_1]\chi+\cos\Gamma_1\,t_1q_1\alpha,\\
n&=\sin\Gamma_1\,[(\sqrt{\tfrac{1}{3}})s_2q_2-(\sqrt{\tfrac{2}{3}})s_3q_1]\chi+\cos\Gamma_1\,t_1q_2\alpha,\\
\Lambda&=\sin\Gamma_2\,(\sqrt{\tfrac{1}{2}})(s_4q_2-s_5q_1)\chi\\
&\quad+\cos\Gamma_2\,[(\sqrt{\tfrac{1}{2}})(t_2q_2-t_3q_1)\sin\theta_3+t_1q_3\cos\theta_3]\alpha,\\
\Sigma^+&=\sin\Gamma_3\,(s_1q_3\cos\theta_4-s_4q_1\sin\theta_4)\chi+\cos\Gamma_3\,t_2q_1\alpha, \qquad \text{(A4)}\\
\Sigma^0&=\sin\Gamma_3\,[s_2q_3\cos\theta_4-(\sqrt{\tfrac{1}{2}})(s_4q_2+s_5q_1)\sin\theta_4]\chi\\
&\quad+\cos\Gamma_3\,(\sqrt{\tfrac{1}{2}})(t_2q_2+t_3q_1)\alpha,\\
\Sigma^-&=\sin\Gamma_3\,(s_3q_3\cos\theta_4-s_5q_2\sin\theta_4)\chi+\cos\Gamma_3\,t_3q_2\alpha,\\
\Xi^0&=\sin\Gamma_4\,(s_4q_3\sin\theta_5-s_6q_1\cos\theta_5)\chi+\cos\Gamma_4\,t_2q_3\alpha,\\
\Xi^-&=\sin\Gamma_4\,(s_5q_3\sin\theta_5-s_6q_2\cos\theta_5)\chi+\cos\Gamma_4\,t_3q_3\alpha.
\end{aligned}$$

If $SU(3)$ holds, then

$$\Gamma_1=\Gamma_2=\Gamma_3=\Gamma_4 \qquad \text{(A5)}$$

and

$$\sin\theta_3=\sin\theta_4=\sin\theta_5=\sqrt{\tfrac{1}{3}}. \qquad \text{(A6)}$$

If, in addition, the members of the baryon octet belong to the 56-dimensional representation of $SU(6)$, then

$$\Gamma_i=\tfrac{1}{4}\pi. \qquad \text{(A7)}$$

We can introduce $SU(6)$- and $SU(3)$-breaking parameters explicitly by writing

$$\Gamma_i=\tfrac{1}{4}\pi+\eta+\gamma_i,\quad i=1,\cdots,4 \qquad \text{(A8)}$$

where η is a measure of $SU(6)$ breaking and the γ_i are a measure of $SU(3)$ breaking. Without loss of generality, we have taken $\gamma_1=0$.

In the main body of this work, we have for simplicitly restricted ourselves to the case

$$\sin\theta_i=\sqrt{\tfrac{1}{3}},\quad i=1,\cdots,5. \qquad \text{(A9)}$$

This means that the sextet and triplet conserve $SU(3)$ separately in their coupling to the quark, with $SU(3)$ or $SU(6)$ being broken by differences in the sextet or triplet coupling. However, the statement that $-\mu(p)/\mu(n)>\frac{3}{2}$ is true in the general case for small $SU(6)$ and $SU(3)$ symmetry-breaking parameters.