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Upper Limit on the Decay Rate $K_L{}^0 \to \pi^+\pi^-\gamma$*

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We undertook a search for the decay mode $K_L{}^0 \to \pi^+\pi^-\gamma$. The charged decay products were momentum-analyzed in a magnet and identified in a set of range-shower spark chambers. The position of the γ-ray conversion point in these chambers was compared with the position predicted on the hypothesis of $K_L{}^0 \to \pi^+\pi^-\gamma$. The $K_{\pi 3}$ provided almost the sole background as well as the normalization of our rate measurement. In 962 events with good γ-ray showers we found that one event fitted the $\pi\pi\gamma$ hypothesis within our spatial resolution. However, the expected background was one event. This result corresponds to an upper limit on the decay rate $R(K_L{}^0 \to \pi^+\pi^-\gamma) \leq 7.5\times 10^3$ sec^{-1} at a confidence level of 90%. The result is compared with the predictions of various theoretical models. The possible connections to other decay processes ($K^\pm \to \pi^\pm\pi^0\gamma$, $K_L{}^0 \to \pi^0\pi^0\gamma$, $K_S{}^0 \to \pi^+\pi^-\gamma$) and tests of CP invariance are discussed.

I. INTRODUCTION AND THEORY

THIS article is a report on a spark-chamber experiment performed at the Argonne National Laboratory's ZGS to search for the decay mode $K_L{}^0 \to \pi^+\pi^-\gamma$. There are two CP-violation effects which have been proposed as possibly showing up in this decay.

The decay $K_L{}^0 \to \pi^+\pi^-\gamma$ can proceed by direct emission of the photon or by inner bremsstrahlung. However, inner bremsstrahlung is small because the $K_L{}^\circ \to \pi^+\pi^-$ mode violates CP invariance.[1]

The attempts to calculate the $K_L{}^\circ \to \pi^+\pi^-\gamma$ decay rate should be viewed as estimates rather than precise computations. In all published treatments, the electric quadrupole contribution to the direct emission process is neglected and only the magnetic dipole term is computed. The dipole term is easier to calculate, and since $l_{\pi\pi}=1$ for the $M1$ transition and as $l_{\pi\pi}=2$ for the $E2$ transition the dipole term should be bigger than the quadrupole from angular momentum considerations.

Estimates of the magnetic dipole term to the rate have been made by various authors. The basic ideas of each of these calculations are outlined in the paragraphs which follow.

Cline[2] has calculated an upper limit for the $M1$ direct emission assuming a $|\Delta I|=\frac{1}{2}$ rule in which the photon is ignored in computing the isospin of the final state. There is no *a priori* justification for the assumption of such a rule but it is an interesting speculation. Subtracting the calculated inner bremsstrahlung from the experimental limit for $K^+ \to \pi^+\pi^0\gamma$, he obtains an upper limit on the rate due solely to $M1$ radiation,

$$R_+(K^+ \to \pi^+\pi^-\gamma;\, M1) \leq 10^4\ \text{sec}^{-1}.$$

Using the Wigner-Eckart theorem and neglecting meson mass differences, he finds

$$\frac{R_L(M1)}{R_+(M1)} = \frac{R(K_L{}^0 \to \pi^+\pi^-\gamma;\, M1)}{R(K^+ \to \pi^+\pi^0\gamma;\, M1)} = \left|\frac{\sqrt{2}\Delta_3 - 1}{1+\Delta_3/\sqrt{2}}\right|^2, \quad (1)$$

where Δ_3 is the ratio of $|\Delta I|=\frac{3}{2}$ amplitude to that for $|\Delta I|=\frac{1}{2}$. Cline's $|\Delta I|=\frac{1}{2}$ rule is invoked by setting $\Delta_3=0$ in Eq. (1).

$$\begin{gathered} R_L(M1)=R_+(M1), \\ R_L(M1)/R(\text{all } K_L{}^0 \text{ decays}) \leq 2\times 10^{-3}. \end{gathered} \quad (2)$$

Pepper and Ueda[3] used the pseudoscalar meson pole model.[4–6] They assume that the strangeness-changing vertex can be attributed to an effective two-point kaon pseudoscalar meson $|\Delta I|=\frac{1}{2}$ weak interaction. The diagrams calculated are shown in Fig. 1. The pion pole amplitudes can interfere with the η pole terms so the relative sign of $f_{K^+\pi^+}$ and $f_{K\eta}$ becomes important. A unitary symmetry generalization of the $|\Delta I|=\frac{1}{2}$ rule gives destructive interference. In this way they obtain for the direct process

$$\begin{gathered} \text{Rate}(K_L{}^0 \to \pi^+\pi^-\gamma) = 1.2\times 10^4\ \text{sec}^{-1}, \\ \text{Rate}(K_L{}^0 \to \pi^+\pi^-\gamma)/\text{Rate}(K_L{}^0 \to \text{all}) = 6.7\times 10^{-4}. \end{gathered} \quad (3)$$

This result is rather uncertain since the $f_{K^+\pi^+}$ and $f_{K\eta}$ coupling constants are known only to an order of magnitude.

Oneda, Kim, and Korff[7] estimated the decay rate, using the pseudoscalar meson pole approximation and a current-current type of interaction which transforms like a member of an $SU(3)$ octet. The Feynman dia-

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[1] C. S. Lai and B. L. Young [Nuovo Cimento **52A**, 83 (1967)] calculated the rate for inner bremsstrahlung by itself to be 2.88×10^2 sec^{-1} for $E_\gamma \geq 10$ MeV and 0.68×10^2 sec^{-1} for $E_\gamma \geq 50$ MeV.
[2] D. Cline, Nuovo Cimento **36**, 1055 (1965).
[3] S. V. Pepper and Y. Ueda, Nuovo Cimento **33**, 1614 (1964).
[4] G. Feldman, P. Matthews, and A. Salam, Phys. Rev. **121**, 302 (1961).
[5] S. K. Bose, Phys. Letters **2**, 92 (1962).
[6] S. Oneda, S. Hori, M. Nakagawa, and Toyoda, Phys. Letters **2**, 243 (1962).
[7] S. Oneda, Y. S. Kim, and D. Korff, Phys. Rev. **136**, B1064 (1964).

grams calculated are shown in Fig. 2. Note that Oneda, Kim, and Korff use the same diagrams as Pepper and Ueda plus others involving $K^{\pm}$ and K^{0*}-$\bar{K}^{0*}$. They use $SU(3)$ and a model for $\pi^0 \to \gamma\gamma$ to obtain the appropriate coupling constants. They find that the rate for the direct process is

$$R(K_L{}^0 \to \pi^+\pi^-\gamma) = 0.51 \times (1 \pm 0.5) \times 10^3 \text{ sec}^{-1}$$

and the branching ratio is

$$R(K_L{}^0 \to \pi^+\pi^-\gamma)/R(K_L{}^0 \to \text{all}) = 2.9 \times (1 \pm 0.5) \times 10^{-5}. \quad (4)$$

When Oneda, Kim, and Korff include ω-ϕ mixing, they obtain a larger rate and branching ratio:

$$R(K_L{}^0 \to \pi^+\pi^-\gamma) = (2-3) \times (1 \pm 0.5) \times 10^3 \text{ sec}^{-1},$$

$$R(K_L{}^0 \to \pi^+\pi^-\gamma)/R(K_L{}^0 \to \text{all}) = (1.14-1.70) \times (1 \pm 0.5) \times 10^{-4}. \quad (5)$$

Lai and Young[1] used the method of current algebras to calculate the ratio of the rate for the direct CP-conserving process for $K_L{}^0 \to \pi^+\pi^-\gamma$ to the rate for $K_L{}^0 \to \gamma\gamma$. Since the rate for $K_L{}^0 \to \gamma\gamma$ has been experimentally measured, this gives them a prediction on the absolute rate of $K_L{}^0 \to \pi^+\pi^-\gamma$. They use the partially conserved axial-vector current (PCAC) hypothesis and assume the $|\Delta I| = \frac{1}{2}$ rule to get the CP-conserving direct amplitude. They express the amplitude for $K_L{}^0 \to \gamma\gamma$ in terms of the same matrix element they found for $K_L{}^0 \to \pi^+\pi^-\gamma$. To do this they invoke an $SU(3)$ argument to express the isoscalar part of the photon in terms of the isovector part. In this way they obtain

$$R(K_L{}^0 \to \pi^+\pi^-\gamma)/R(K_L{}^0 \to \text{all modes}) \simeq (10 \pm 2.2) \times 10^{-5}. \quad (6)$$

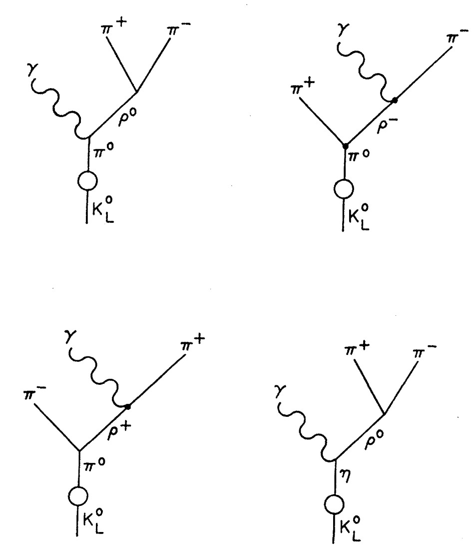

FIG. 1. Feynman diagrams used by Pepper and Ueda in calculating the direct emission contribution to $K_L{}^0 \to \pi^+\pi^-\gamma$.

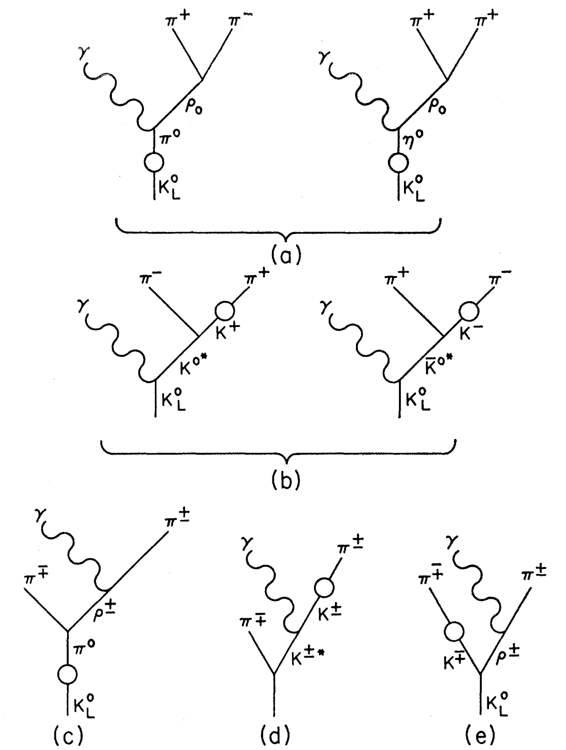

FIG. 2. Feynman diagrams used by Oneda, Kim, and Korff in calculating the direct emission contribution to $K_L{}^0 \to \pi^+\pi^-\gamma$.

Comparing with the experimental branching ratio[8] for $K_L{}^0 \to \pi^+\pi^-$,

$$R(K_L{}^0 \to \pi^+\pi^-)/R(K_L{}^0 \to \text{all}) = (1.53 \pm 0.07) \times 10^{-3},$$

one finds for the inner bremsstrahlung process

$$\frac{R(K_L{}^0 \to \pi^+\pi^-\gamma)}{R(K_L{}^0 \to \text{all modes})} \simeq \begin{Bmatrix} 1.69 \times 10^{-5} & (E_\gamma \geq 10 \text{ MeV}) \\ 0.4 \times 10^{-5} & (E_\gamma \geq 50 \text{ MeV}) \end{Bmatrix}, \quad (7)$$

where E_γ is the energy of the photon in the radiative decay.

II. EXPERIMENTAL DETAILS

The experiment was performed in the 31° neutral beam of the ZGS at the Argonne National Laboratory. See Fig. 3 for a schematic drawing of the beam layout. γ rays in the beam were attenuated by 2 in. (10 radiation lengths) of lead just upstream of the first collimator (see Fig. 3). The electrons and positrons produced in the lead, as well as other charged particles already in the beam, were swept out by a bending magnet (BM-105) between the second and third

[8] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, Matts Roos, Paul Soding, W. J. Willis, and C. G. Wohl, Rev. Mod. Phys. 40, 77 (1968).

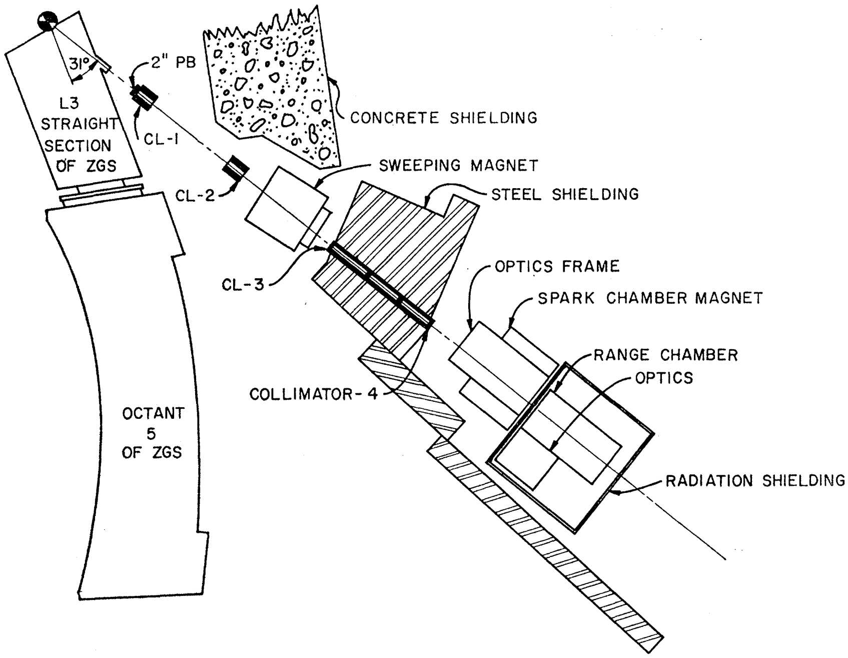

FIG. 3. $K_2{}^0$ beam layout at the Argonne National Laboratory.

collimators. The beam reaching our apparatus consisted mainly of neutrons and $K_L{}^0$ mesons.

The apparatus can be divided roughly into two parts: (1) scintillation counters for the detection of the decay and thin-foil (0.001-in.-Al) spark chambers in a 10-kG magnetic field for the measurement of the momenta of the charged decay products, and (2) spark chambers with $\frac{1}{8}$-in.-thick aluminum plates for the identification of the decay products including γ rays.

The apparatus is shown in Fig. 4. To make sure that events were decays and not interactions, all decays were required to originate from within a vacuum pipe built into the first of the two magnet spark chambers. The vacuum pipe was an oval tube of thin stainless steel with 0.005-in. Mylar end and side walls. The vacuum pipe was 12 in. long and roughly 57 ft from the target.

About 40 in. downstream from the second magnet chamber were five "range" or "shower" spark chambers made of $\frac{1}{8}$-in. aluminum plates 36 in. high by 48 in.

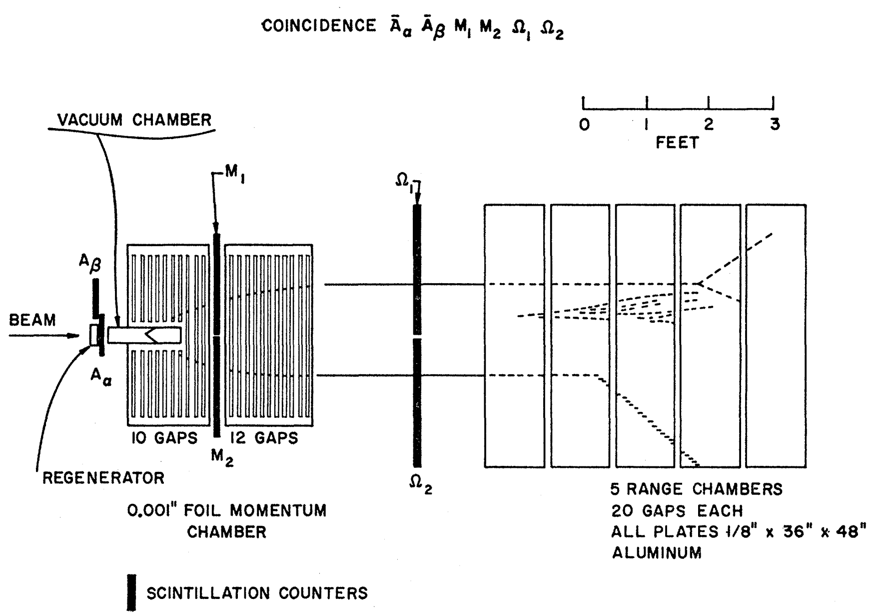

FIG. 4. Arrangement for $K_L{}^0 \to \pi^+\pi^-\gamma$ experiment.

wide. There were twenty $\frac{5}{16}$-in. gaps per chamber. A particle passing through one chamber must traverse 7.62 cm of aluminum or 0.86 radiation lengths. Between the vacuum pipe and the first active gap of the shower chambers there were 4.3 g/cm^2 of Al, 0.7 g/cm^2 of Cu (radiation shield), and 1.9 g/cm^2 of CH (plastic scintillator) for a total of 5.9 g/cm^2 in all.

The triggering mode used was $\bar{A}_\alpha \bar{A}_\beta M_1 M_2 \Omega_1 \Omega_2$. The anticoincidence counters and M_1 and M_2 were of $\frac{1}{8}$-in. scintillator plastic. Ω_1 and Ω_2 were of $\frac{3}{8}$-in. scintillator.

A $\frac{1}{2}$-in.-thick piece of copper regenerator was positioned just upstream of the vacuum pipe during part of the run. See Fig. 4. The possible effects of this regenerator in our experiment will be discussed in Sec. IV.

III. REDUCTION OF DATA AND ANALYSIS

The film from the magnet chambers was scanned for events with good V's originating in the vacuum pipe and with no extra tracks. The selected events were measured on CHLOE,[9] the Argonne National Laboratory's flying-spot digitizer, developed by Hodges and associates. The processing of events was done on the IBM 7094 computer at the University of Illinois. The pattern recognition programs (LINK) developed by Clark of the Argonne National Laboratory[10] were modified to handle more rapidly our particular set of event topologies by Mischke and McBride.[11] If this streamlined set of programs failed to handle an event, then LINK was called.

A scan of the range-shower chamber film was made looking for γ rays. Every frame corresponding to a good V in the magnet was scanned regardless of the analysis of the magnet chamber film for the event. A prior scan of the shower chamber film had been made looking for γ rays from the decay of $K_L{}^0$ into $\pi^+\pi^-\pi^0$. The events found in this scan represented a kinematically selected group since only those events were looked at in the range-shower chambers which were compatible with $K_L{}^0 \to \pi^+\pi^-\pi^0$ kinematics as determined from the momentum measurements in the magnet chambers. An event compatible with $K_{\pi 3}$ kinematics is always also compatible with $K_{\pi\pi\gamma}$ but not vice versa. A comparison of this "3π scan" with the "γ scan" gives a check on the scanning efficiency. The efficiency is discussed later in this section.

For each event in which a γ-ray shower was found, the trajectories of the two charged particles and of the photon (assuming $K \to \pi\pi\gamma$) were computed. Of the two kinematic solutions we found, we could ignore the one corresponding to a backward photon in the center of mass. For that solution the computed photon trajectory either missed the shower chambers or the laboratory energy of the photon was too low to have produced the observed shower. The program rejected those events in which one of the charged particles struck the magnet yoke where interactions with the iron could give rise to spurious γ rays and also to charged secondaries which could falsely trigger the Ω (solid angle) counters.

This program also aided in finding misidentified K_{e3} events and events in which the showers came from a γ ray produced in interactions with the back magnet chamber wall, the solid angle counters, or the front shower chamber wall. All candidates were rescanned by a physicist who measured the shower-origin coordinates to better than 0.5 in. for those events passing his scrutiny.

The events remaining after these checks should be almost entirely $K_{3\pi}$ or $K_{\pi\pi\gamma}$ events. At this stage a comparison of the two scans becomes a significant measure of the scanning efficiency. Since the "3π scan" involved a preselection of events to look at whereas the "γ scan" did not, the expression chosen as a measure of the scanning efficiency for γ rays is $e_\gamma = 1-$ (the number of good events found in the "3π scan" but missed in the "γ scan")/(the number of good events found in both scans), where for this purpose "good event" implies also that the event was kinematically compatible with $K_{3\pi}$ decay. As defined, the scanning efficiency was found to be $e_\gamma = (87\pm 7)\%$.

Each event in which a γ ray was reported was tested for compatibility with the hypothesis of $K_L{}^0 \to \pi^+\pi^-\gamma$. The actual position of the shower origin was compared with that predicted from the direction cosines of the photon which were computed in the program described above.

The "fate" of events is described below.

1. There were 160 000 pictures taken during the runs with 0-in. and $\frac{1}{2}$-in. regenerator.
2. 71 000 of these pictures were found by scanners to have a "good V" in the momentum chambers suitable for digitization on CHLOE.
3. 92.4% of these, or 66 000, were properly digitized. 6% should not have been on the scan list. Only 1.6% were lost because of poor digitization.
4. The pattern recognition programs failed on about 10% of the events which had been properly digitized. This left 59 500 events in the sample.
5. After cutoffs were imposed on the allowed region of decay, 41 000 events remained.
6. From this sample, the scan of the shower chambers yielded 3507 candidates.
7. After the test for kinematic compatibility with the $K \to \pi\pi\gamma$ hypothesis and the test to eliminate events in which one of the charged particles hit the magnet, 1570 candidates remained.
8. 962 of these candidates passed the scrutiny of the

[9] D. Hodges, Technical Memorandum No. 61, Applied Mathematics Division, Argonne National Laboratory, 1963 (unpublished).

[10] R. Clark and W. F. Miller, Methods Comp. Phys. 5, 47 (1966).

[11] R. E. Mischke, Ph.D. thesis, University of Illinois, 1966 (unpublished).

physicist. Most of the events thrown out at this point failed because the γ-ray shower was associated with one of the charged particles entering the chamber. 350 of the candidates were from the no-regenerator run and 612 from the $\frac{1}{2}$-in. regenerator run.

The whole experiment was simulated in a series of Monte Carlo calculations to determine the detection efficiencies of $K_{\pi\pi\gamma}$ and $K_{\pi 3}$ and the effect of spark-measurement error on the predicted photon trajectory. The K spectrum used in these calculations was determined from the regenerated $K_S^0 \rightarrow \pi^+\pi^-$ events. We used the experimentally determined Dalitz plot population density distribution[12] for the $K_{\pi 3}$ efficiency and straight phase space for $K_{\pi\pi\gamma}$. As a check we also tried the spectrum of Lai and Young. This gives a $K_{\pi\pi\gamma}$ detection efficiency that is 9% higher than phase space. In these calculations we required that the γ ray pass through at least two shower chambers (1.72 radiation lengths) and have an energy in the laboratory greater than some minimum value. See Fig. 5 for graphs of the efficiencies $\epsilon_{\pi\pi\gamma}$, $\epsilon_{\pi 3}$, and $\epsilon_{\pi 3}/\epsilon_{\pi\pi\gamma}$ versus the γ-ray energy cutoff. We estimate that the right cutoff is 250 ± 50 MeV. This corresponds to a detection efficiency for $K_{\pi 3}$ of $\epsilon_{\pi 3}=(1.31_{-0.24}{}^{+0.21})\times 10^{-2}$ and for $K_{\pi\pi\gamma}$ of $\epsilon_{\pi\pi\gamma}=(1.12_{-0.12}{}^{+0311})\times 10^{-2}$. The ratio is then $\epsilon_{\pi 3}/\epsilon_{\pi\pi\gamma}=1.17_{-0.04}{}^{+0.05}$. The detection efficiency for $K_{\pi\pi\gamma}$ versus the energy of the photon in the center-of-mass system is shown in Fig. 6.

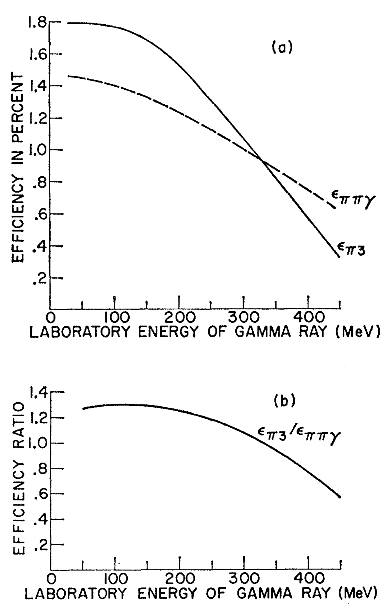

FIG. 5(a) Detection efficiencies for the decay modes $K_L^0 \rightarrow \pi^+\pi^-\gamma$ and $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ versus the cutoff in the laboratory energy of the γ ray. (b) Ratio of the efficiency for $K_L^0 \rightarrow \pi^+\pi^-\pi^0$ to that for $K_L^0 \rightarrow \pi^+\pi^-\gamma$.

[12] B. M. K. Nefkens, A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, and J. H. Smith, Phys. Rev. **157**, 1233 (1967).

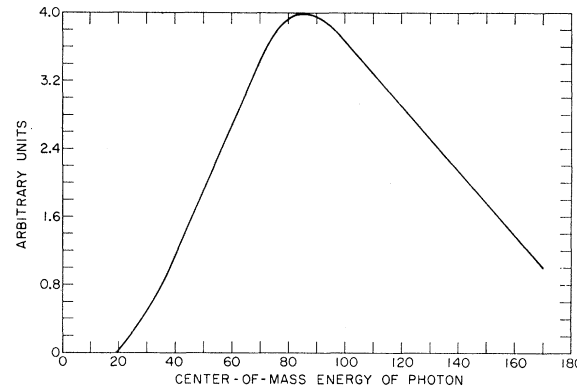

FIG. 6. The detection efficiency (in arbitrary units) for $K_L^0 \rightarrow \pi^+\pi^-\gamma$ versus the energy of the γ ray in the center-of-mass system.

The remaining Monte Carlo program was to determine the effect on the predicted shower origin of measurement errors in the spark locations. $K_{\pi\pi\gamma}$ events were generated exactly as in the $K_{\pi\pi\gamma}$ detection efficiency program. "Spark locations" were assigned to the intersections of each charged-particle (pion) trajectory as determined by the ray-tracing program with the gaps in the magnet chamber. These "sparks" were then shifted randomly with Gaussian-distributed "measurement errors." From the new spark positions the Monte Carlo event was reconstructed in the same way as for an actual event and the entry point of the photon in the shower chambers was calculated. The comparison of the computed entry positions before and after the introduction of "measuring error" tells us how much uncertainty in this position is introduced in the measurement of the magnet chamber sparks. This calculation hinges on the width of the Gaussian error distribution used. In the above calculations the rms value for the error distribution was chosen to be 0.020 in. both vertically and horizontally. In the course of processing many events on this film for this and other experiments, the rms deviation from a fitted helix in the vertical was 0.018 in. and in the horizontal was 0.012 in. Therefore the value 0.020 in. seems rather pessimistic and allows for small uncorrected errors in the optics. With this value the rms "miss" between the computed entry positions for the γ ray before and after "measuring error" was 0.47 in.

IV. RESULTS AND DISCUSSION

One way of displaying the experimental results is in the form of a histogram of the number of events having various values of "miss" for the photon. See Fig. 7. The "miss" plotted is the distance in real space between the actual origin and the predicted origin of the γ-ray shower. The "miss" $\Delta R=(\Delta X^2+\Delta Y^2)^{1/2}$, where ΔX is the difference in the X coordinates between the observed and predicted shower origins and similarly for ΔY. (X and Y are transverse to the beam.)

One expects that γ-ray showers arising from the 3π decay mode of the $K_L{}^0$ meson will be virtually the sole background and that these should be distributed essentially uniformly in X and Y throughout the shower chambers. Under the assumption of uniform background, the data in Fig. 7 were fitted by the least-squares method to a straight line passing through the origin. The χ^2 of this fit is 6.53 for seven degrees of freedom. We note that there is no indication of any peaking above the background near the origin. With our resolution, any real event should be in the first bin.

An alternative way of displaying the experimental results is to plot the X and Y coordinates of the miss. This is shown in Fig. 8. It also allows us to check that there is no peak shifted in some systematic way from the origin. The plot was made of the X and Y components of the "shower origin miss" ($+X$ up, $+Y$ to the right facing downstream) of each event with "miss" less than 5 in. However, more important to note is that the change produced in the histogram of "miss" (Fig. 7) is completely negligible and does not produce a peak near the origin.

In part of the run, a $\frac{1}{2}$-in. copper regenerator was present. One needs to consider if there are any possible effects on the observed data due to this regenerator. The presence of the $\frac{1}{2}$-in. copper regenerator in front of the vacuum pipe can have two effects. First, it could contribute $K_S{}^0 \to \pi^+\pi^-\gamma$ events which we could not distinguish from $K_L{}^0 \to \pi^+\pi^-\gamma$. A calculation of this sets an upper limit of 0.5 event of this type in our sample. Secondly, some of the K mesons scatter in the regenerator and then decay. For such events we will get wrong answers in the kinematics analysis. In particular, a $K_L{}^0 \to \pi^+\pi^-\gamma$ event following such a scatter would result in a bad prediction for the γ conversion point. Such events would be lost. If a $K_L{}^0 \to \pi^+\pi^-\pi^0$ decay occurred after the K scattered in the copper, a fraction of these would appear kinematically incompatible with $K_{\pi 3}$ and would not be used in the normalization. If one estimates the diffraction cross section to be 600 mb, one finds that less than 6.5% of the K's should undergo diffraction scattering in the regenerator. Therefore this effect does not give a significant correction to our upper limit.

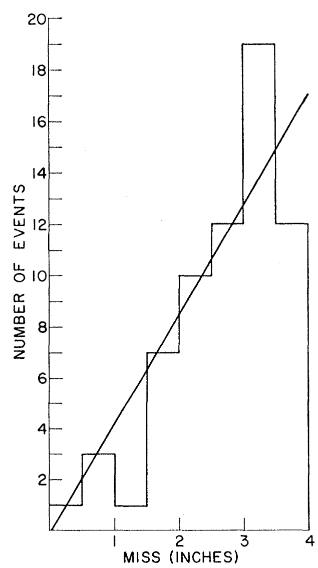

FIG. 7. Histogram of the "miss" between expected and actual shower-origin positions. The straight line fitted to the data has a χ^2 value of 6.53.

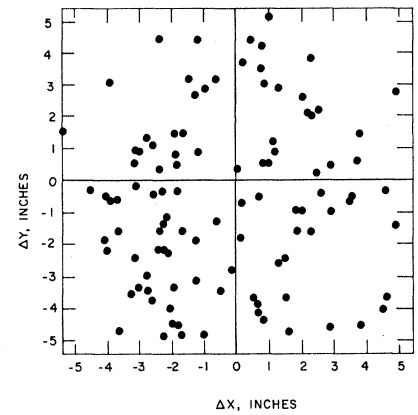

FIG. 8. The distribution of shower-origin miss. The coordinates are in inches in real space.

As one can see from the background fit, one background event is expected in the first bin. Since this is the case, the only thing we can do is set an upper limit on the decay rate and evaluate the confidence level corresponding to this limit.

If we assume there are no more than $N_{\pi\pi\gamma}(\max)$ real $K_{\pi\pi\gamma}$ events in our sample, then

$$\frac{R(K_L{}^0 \to \pi^+\pi^-\gamma)}{R(K_L{}^0 \to \text{all})} \leq \frac{N_{\pi\pi\gamma}(\max)}{N_{\pi 3}} \times \frac{\epsilon_{\pi 3}}{\epsilon_{\pi\pi\gamma}} \times \frac{R(K_L{}^0 \to \pi^+\pi^-\pi^0)}{R(K_L{}^0 \to \text{all})}. \quad (8)$$

In a situation in which an event can come from either of two random and indistinguishable processes, the distribution of events follows a single Poisson distribution in which the mean is just the sum of the two distributions.[13] Thus if μ_r is the mean of distribution for real $K_{\pi\pi\gamma}$ events, and μ_b is the mean of distribution of background events, then the probability of observing k events, either real or background, is

$$P(k;\mu_r,\mu_b) = (\mu_r+\mu_b)^k e^{-(\mu_r+\mu_b)}/k!. \quad (9)$$

From Fig. 7 we see that the expected number of background events within our resolution is $\mu_b = 1$. The confidence level for an upper limit corresponding to

[13] Arthur Sard and R. D. Sard, Rev. Sci. Instr. **20**, 526 (1949).

TABLE I. Summary of experimental results and comparison with theoretical predictions.

Rate($K_L{}^\circ \to \pi^+\pi^-\gamma$) (sec^{-1})	Rate($K_L{}^\circ \to \pi^+\pi^-\gamma$)/Rate($K_L{}^\circ \to$ all)	Source
7.5×10^3	4.2×10^{-4}	This experiment
1.2×10^4	6.7×10^{-4}	Pepper and Ueda
$(2-3)\times(1\pm0.5)\times10^3$	$(1.14-1.70)\times(1\pm0.5)\times10^{-4}$	Oneda, Kim, and Korff (with ω-ϕ mixing)
$0.51\times(1\pm0.5)\times10^3$	$2.9\times(1\pm0.5)\times10^{-5}$	Oneda, Kim, and Korff (no ω-ϕ mixing)
$(1.7\pm0.4)\times10^3$	$(10\pm2.2)\times10^{-5}$	Lai and Young
2.88×10^2	1.69×10^{-5}	Inner bremsstrahlung $E_\gamma\geq10$ MeV
0.68×10^2	0.4×10^{-5}	Inner bremsstrahlung $E_\gamma\geq50$ MeV

$N_{\pi\pi\gamma}(\max)$ is

$$\text{C.L.}=1-\int_{N_{\pi\pi\gamma}(\max)}^{\infty} P(k=1;\,\mu_r,\,\mu_b=1)d\mu_r.$$

Our result at the 90% confidence level is

$$\text{Rate}(K_L{}^0\to\pi^+\pi^-\gamma)\leq7.5\times10^3 \text{ sec},$$
$$\text{Rate}(K_L{}^0\to\pi^+\pi^-\gamma)/\text{Rate}(K_L{}^0\to\text{all})=4.2\times10^{-4}.$$

The previous upper limit on the branching ratio was 3.0×10^{-3}.[14]

Table I summarizes our result and the predictions of theory. Our upper limit on the decay rate is larger than all estimates of the rate except that of Pepper and Ueda. We see that their estimate (1.2×10^4 sec^{-1}) is too high by about a factor of 2. That Pepper and Ueda obtain such a large answer is interesting since Oneda, Kim, and Korff get 1/20 of this rate when they compute the same diagrams plus some others. However, Pepper and Ueda have a very large destructive interference effect in their calculation so that their rate is the difference between two much larger rates. For this reason, their calculation is very sensitive to the choice of the coupling constants. Since Pepper and Ueda claim they only know the coupling constants to an order of magnitude, they can probably fix up their answer without resorting to any new mechanisms.

We must stress that our upper limit for $K_L{}^0\to\pi^+\pi^-\gamma$ is a full order of magnitude above the rate for inner bremsstrahlung alone. Therefore it is of interest to improve our measurement by a factor of at least 5 to test the other theoretical predictions of the direct process.

We would like to point out that the small upper limit set in this experiment makes the proposed tests of CP invariance in this decay[15,16] experimentally unfeasible.

We cannot relate the rate for $K_L{}^0\to\pi^+\pi^-\gamma$ to that for $K_S{}^0\to\pi^+\pi^-\gamma$ or to that for $K_L{}^0\to\pi^0\pi^0\gamma$. The $K_S{}^0\to\pi^+\pi^-\gamma$ almost certainly goes predominantly by inner bremsstrahlung which is CP-conserving for this decay. The $K_L{}^0\to\pi^0\pi^0\gamma$ has no inner bremsstrahlung and no dipole transition. The latter is forbidden since conservation of angular momentum demands that the two pions be in an $l=1$ state, and this is forbidden by Bose statistics. The dominant mechanism for $K_L\to\pi^0\pi^0\gamma$ is thus expected to be the electric quadrupole transition.

However, it is possible to relate our measurement to the K^+ and K^- radiative decays. The Cline extension of the $\Delta I=\frac{1}{2}$ rule does give us the possibility of making a prediction from our result. According to Eq. (2) in Sec. I, $R_L(M1)=R_+(M1)$, where $R_L(M1)$ is the magnetic dipole contribution to $K_L{}^0\to\pi^+\pi^-\gamma$. Using our upper limit at the 90% confidence level, one obtains

$$R_+(M1)\leq7.5\times10^3 \text{ sec}^{-1}.$$

Since the magnetic dipole contribution to the K^+ decay is incoherent with the electric dipole and inner bremsstrahlung, the prediction above is for the total $M1$ contribution to the rate. Cline and Fry[17] report the rate $(K^+\to\pi^+\pi^0\gamma)=(1.8\pm0.6)\times10^4$ sec^{-1} for the π^+ energy between 55 and 80 MeV. Our predicted upper limit for magnetic dipole emission in this energy region is

$$R_+(M1)\leq3.0\times10^3 \text{ sec}^{-1}.$$

The K^+ and K^- radiative decays are of particular interest at the moment because a large CP violation could manifest itself either as an asymmetry between K^+ and K^- Dalitz plots[18] or as a difference in the partial rates.[19] These effects, if they exist, result from interference between the inner bremsstrahlung and $E1$ amplitudes. The presence of magnetic dipole emission will only tend to mask the effects. Our estimate from Cline's hypothesis and our experiment is rather encouraging to a search for CP violation in the K^+ and K^- radiative decay modes since it predicts an upper limit on the $M1$ contribution of less than $\frac{1}{6}$ of the observed rate.

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