The unmeasurable cross section for $\pi^0 p \to p\pi^+\pi^-$ in diagram 4 was calculated in terms of the most recent data for $\pi^\pm p \to N\pi\pi$ using isotopic-spin arguments.[48]

The combined form factors and off-shell corrections lead to the following expressions in the different diagrams:

$$\text{diagram 3:}\quad A(\omega_1,\Delta^2)A(\omega_2,\Delta^2)G^{-2}(\Delta^2)\,,$$
$$\text{diagram 4:}\quad A(\omega,\Delta^2)\,,$$

with

$$G^2(\Delta^2)=K^4(\Delta^2)K'^2(\Delta^2)\,,$$
$$A(\omega,\Delta^2)=G^2(\Delta^2)\Lambda^2(\omega,\Delta^2)\,.$$

In diagram 3, for $\omega \lesssim 1.45$ GeV we used

$$\Lambda^2(\omega,\Delta^2)=(q_{\rm off}/q_{\rm on})^2\Gamma^2(\Delta^2)[1+C(\omega,\Delta^2)]^2G^2(\Delta^2)\,,$$

where $G(\Delta^2)$, $\Gamma(\Delta^2)$, and $C(\omega,\Delta^2)$ are the same as in Ref. 39 and $q_{\rm off}$ ($q_{\rm on}$) is the modulus of the three-momentum of an off- (on-) shell π in the c.m. system of a πN state with an invariant mass ω. In all other cases we used

$$A(\omega,\Delta^2)=\left[1+\frac{(\Delta^2+\mu^2)}{\gamma}\right]^{-1}.$$

The additional diagrams obtained from diagrams 3 and 4 by interchanging initial or final-state protons are also included in the calculations neglecting all interference terms. Except where noted, the cutoff parameter [Eq. (4.10) of Ref. 39] was chosen to be $\gamma=30\ \mu^2$. This choice gave approximate agreement in the total cross section between the model and the experiment.

Distributions were calculated using a Monte Carlo program PHYSIK,[49] in which the complete kinematics of each generated event is available. Thus any desired selection criteria may be easily applied in order to compare the model with the corresponding experimental selection.

For the reactions $pp \to pp\pi^+\pi^-\pi^0$ and $pp \to pn\pi^+\pi^+\pi^-$, diagrams 5–7 and 8–10, respectively, were considered. Graphs with all the pions at the same vertex and interference between the diagrams were neglected.

[48] We have avoided using the approximate expression [Eq. (18) in Ref. 46] which neglects the sizable amplitude for isotopic spin $T(\pi\pi)=2$. A detailed discussion of this point is given by J. H. Scharenguivel, Ph.D. thesis, Cambridge University, 1966 (unpublished).

[49] P. Söding (private communication). We used a modified version for the calculation of diagrams (5)–(10) of Fig. 22.

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Measurement of the Branching Ratio and Positron Momentum Spectrum for the Decay $K^0 \to \pi^0+e^++\nu$

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The branching ratio and positron momentum spectrum have been measured for the K_{e3}^+ decay mode. The shape of the momentum spectrum, containing 17 000 events, is consistent with pure vector coupling with a form-factor momentum dependence given by $\lambda^+=0.08\pm0.04$. Upper limits on a possible mixture of scalar or tensor coupling are, respectively, $|f_S/f_+|<0.23$ and $|f_T/f_+|<0.58$. The branching ratio is found to be $(4.92\pm0.21)\%$, based on 960 events. The semileptonic $\Delta I=\frac{1}{2}$ rule is tested by comparing the K_{e3}^+ and K_{e3}^0 rates. We conclude that the present data on K_{e3} decays are in disagreement with this rule.

We assume that the matrix element[1] for K_{e3} decay is

$$M=[m_K f_S\bar{U}_\nu(1+\gamma_5)U_e+\tfrac{1}{2}f_+(p^K+p^\pi)_\alpha\bar{U}_\nu\gamma_\alpha(1+\gamma_5)U_e + (1/m_K)f_T p_\alpha{}^K p_\beta{}^\pi\bar{U}_\nu\sigma_{\alpha\beta}(1+\gamma_6)U_e]\,,$$

where p^K and p^π are the four momenta of the K^+ and π^0, respectively. The form factors f_S, f_+, and f_T for scalar, vector, and tensor coupling are functions of $q^2=(p^K-p^\pi)^2$.[2] We assume that the q^2 dependence for vector coupling is given by the first two terms of a power-series expansion,

$$f_+(q^2)=f_+(0)(1+\lambda^+q^2/m_\pi^2)\,.$$

The experiment was performed at the Rutherford High Energy Laboratory, using a 700-MeV/c separated

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[1] The factors m_K and $1/m_K$ are introduced to give f_S, f_+, and f_T the same dimensions. The matrix element of the hadronic vector current should be written $\frac{1}{2}[f_+(p^K+p^\pi)+f_-(p^K-p^\pi)]$. The contribution from f_- however is proportional to m_e/m_K and has been neglected.

[2] The metric is chosen so that $q^2=m_K{}^2+m_\pi{}^2-2m_KE_\pi$, where E_π is the total energy of the pion.

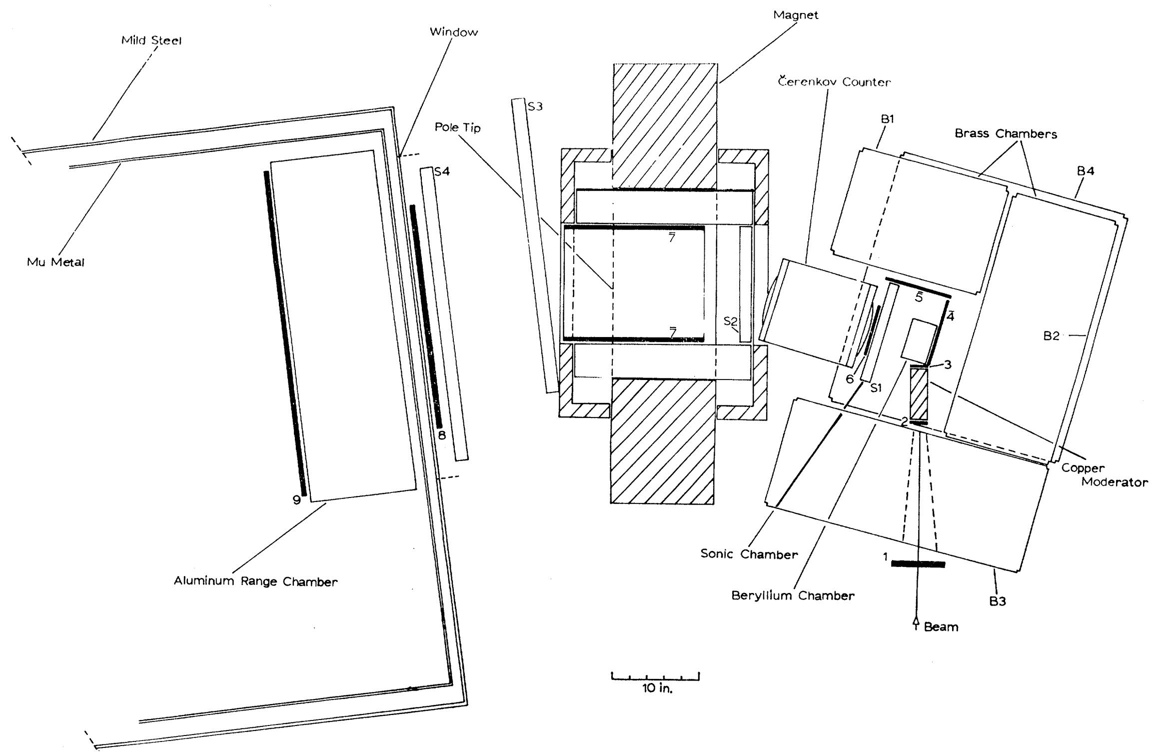

FIG. 1. Plan view of apparatus. Further details concerning the instrumentation, beam, and momentum analysis are presented in a recent paper on K_{e2} decay [D. R. Botterill *et al.*, Phys. Rev. **171**, 1402 (1968)].

K^+ beam. A plan view of the apparatus is shown in Fig. 1. Approximately 1000 K^+ per pulse were stopped in the beryllium plates of a small spark chamber. The decay products were momentum-analyzed by the spectrometer magnet. Particle trajectories through the magnet were determined by the sonic spark chambers S1, S2, S3, and S4.

The spark chambers were basically triggered by a stopping K^+, $(1\ 2\ 3\ \bar{4}\ \bar{5})$, in delayed coincidence with $(6\ 8\ \bar{7})$. Counter 7 suppressed triggers from events in which particles scattered from the coils or pole tips of the magnet. Positrons were identified by a gas Čerenkov counter,[3] i.e., by a $(6\ 8\ \bar{7}C)$ coincidence. The Čerenkov counter was 99% efficient.

For muons from $K_{\mu 3}$ decays the requirement $(\bar{C}\bar{9})$ was introduced to suppress triggers from $K_{\mu 2}$ and $K_{\pi 2}$ decay modes. Information from these triggers, from the brass-plate spark chambers and the aluminum-plate range chamber is to be used in a later stage of our analysis.

Some data were taken with counter 9 disconnected to compare the K_{e3} and $K_{\mu 2}$ rates.

The basic momentum resolution of the spectrometer was $(1.40\pm0.03)\%$. The over-all resolution was $(2.27\pm0.13)\%$, due to the spread in ionization losses in the beryllium chamber. The solid-angle acceptance of the spectrometer was approximately constant at 0.42% of 4π sr for momenta greater than 90 MeV/c.

Figure 2(a) shows the momentum spectrum of delayed $(6\ 8\ \bar{7}\bar{C})$ triggers. Figure 2(b) shows the momentum spectrum of 18 227 events for delayed $(6\ 8\ \bar{7}C)$ triggers. In principle all events in spectrum 2(b) are positrons. It is obvious however that some muons from $K_{\mu 2}$ and pions from $K_{\pi 2}$ decay could trigger the Čerenkov counter, or were in random coincidence with a Čerenkov pulse. Figure 2(c) shows the positron spectrum corrected for the momentum dependence of the spectrometer acceptance and with the $K_{\mu 2}$ and $K_{\pi 2}$ backgrounds subtracted.[4] In addition, a small background $(1\pm1)\%$ has been subtracted arising from electron-positron pairs produced by π^0 photons converting in the beryllium chamber.

The experimental spectrum, Fig. 2(c), has been fitted to theoretical spectra[5] corrected for bremsstrahlung losses, ionization losses, and the finite resolution of the spectrometer. Radiative corrections[6] have been applied to the vector spectrum. Fits were made for vector-

[3] G. Culligan and T. W. Quirk, Nucl. Instr. Methods **53**, 261 (1967).

[4] The background was partly removed by normalizing the $K_{\mu 2}$ peak in Fig. 2(a) to the $K_{\mu 2}$ peak in Fig. 2(b) and then subtracting the remaining normalized spectrum of Fig. 2(a). There was an additional $K_{\pi 2}$ background because the γ rays which accompany the π^+ can trigger the Čerenkov counter. We have calculated the magnitude of this contribution to the background and have allowed for a reasonable error in making the subtraction.

[5] S. W. MacDowell, Ann. Phys. (N. Y.) **18**, 171 (1962).

[6] E. S. Ginsberg, Phys. Rev. **142**, 1035 (1966).

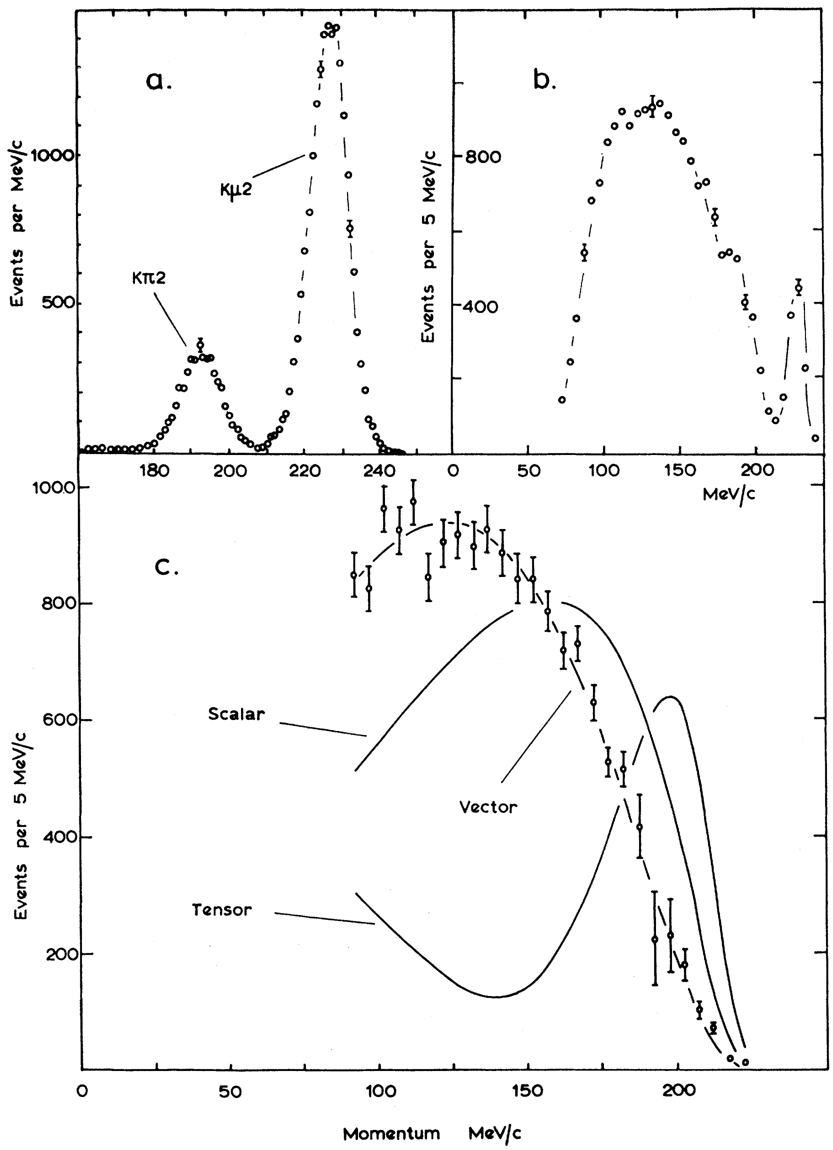

FIG. 2. (a) Momentum spectrum of non-Čerenkov events for the interval 160 to 260 MeV/c showing the $K_{\pi 2}$ and $K_{\mu 2}$ peaks. (b) Momentum spectrum of Čerenkov events below 250 MeV/c. (c) Final K_{e3}^+ spectrum with best fits to pure vector, scalar, and tensor couplings.

scalar and vector-tensor mixtures. The form factors f_S and f_T were assumed constant. The upper limits were, respectively, $|f_S/f_+|<0.23$ and $|f_T/f_+|<0.57$ at the 90% confidence level for $\lambda^+=0$ and $|f_S/f_+|<0.18$ and $|f_T/f_+|<0.58$ for $\lambda^+=0.08$.[7] The limits allow for systematic uncertainties in the bremsstrahlung and ionization losses. The scalar limit is comparable with other experiments. The tensor limit is however lower than previously obtained.[8] A fit was also made for pure vector coupling allowing λ^+ to vary. We find $\lambda^+=0.08\pm0.04$ where the error quoted is partly statistical (±0.03) and partly systematic (±0.03).

We observed 960 positron events and 18 540 $K_{\mu 2}$ events when taking data without counter 9. These events were used to determine the K_{e3} branching ratio. We find

$$\Gamma(K_{e3}^+)/\Gamma(\text{all})=(4.92\pm0.21)\%$$

assuming a $K_{\mu 2}$ branching ratio[9] of $(63.46\pm0.38)\%$. Since we observe only that part of the spectrum above 90 MeV/c, the result depends on the spectral shape.

[7] There is no vector-tensor or scalar-tensor interference if terms in m_e/m_K are neglected. Thus, these fits involve two parameters. The limits $|f_S/f_+|<0.23$ and $|f_T/f_+|<0.58$ correspond to intensity limits $I_S/I_V<0.10$ and $I_T/I_V<0.02$, respectively.

[8] (a) G. L. Jensen, F. S. Shaklee, B. P. Roe, and D. Sinclair, Phys. Rev. **136B**, 1431 (1964); (b) R. Cester, P. T. Eschstruth, G. K. O'Neill, B. Quassiati, D. Yount, J. M. Dobbs, A. K. Mann, W. K. McFarlane, and D. H. White, Phys. Letters **21**, 343 (1966); (c) E. Bellotti, E. Fiorini, and A. Pullia, Nuovo Cimento **52A**, 1287 (1967); (d) G. E. Kalmus and A. Kernan, Phys. Rev. **159**, 1187 (1967).

[9] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968).

TABLE I. Summary of K_{e3}^+ branching-ratio measurements.

	Authors[a]	K_{e3}^+ branching ratio	Method
(a)	Birge *et al.*	3.2 ±1.3	Emulsion
(b)	Alexander *et al.*	5.1 ±1.3	Emulsion
(c)	Roe *et al.*	5.0 ±0.5	Xenon B.C.
(d)	Borreani *et al.*	5.12±0.36	Hydrogen B.C.
(e)	Shaklee *et al.*	4.7 ±0.3	Xenon B.C.
(f)	Callahan *et al.*	3.94±0.17	Freon B.C.
(g)	Auerbach *et al.*	4.97±0.16	Spark chambers
(h)	Young *et al.*	5.3 ±0.9	Emulsion
8(c)	Bellotti *et al.*	5.2 ±0.5	Heavy liquid B.C.
	This experiment	4.92±0.21	Spark chambers

[a] See Ref. 11.

The value quoted corresponds to pure vector coupling with $\lambda^+=0.02$.[10] We find that the branching ratio is increased to 4.96% for $\lambda^+=0$ and decreased to 4.78% for $\lambda^+=0.08$. Allowance has been made for this variation in the quoted error.

Previous branching-ratio measurements[11] are summarized in Table I. We see that all measurements are in reasonable agreement except for the measurement of Callahan *et al.*

Averaging all results except that of Callahan *et al.* we find[12]

$$\Gamma(K_{e3}^+)/\Gamma(\text{all})=(4.94\pm0.11)\%$$

and, assuming a K^+ lifetime[9] of $(1.236\pm0.003)\times10^{-8}$ sec, we find

$$\Gamma(K_{e3}^+)=(4.00\pm0.09)\times10^6\ \text{sec}^{-1},$$

corresponding to a Cabibbo angle[13] θ_v given by

$$\sin\theta_v=0.230\pm0.003,\quad \text{for}\quad \lambda=0$$

or

$$=0.224\pm0.003,\quad \text{for}\quad \lambda=0.02.$$

The K_{e3}^0 branching ratio of the K_L^0 is quoted[14] as

$$\Gamma(K_{e3}^0)/\Gamma(\text{all})=(37.2\pm1.0)\%,$$

which, using a measured lifetime[14] for the K_L^0 of $(5.37\pm0.12)\times10^{-8}$ sec leads to

$$\Gamma(K_{e3}^0)=(6.92\pm0.23)\times10^6\ \text{sec}^{-1}.$$

It follows therefore that

$$\Gamma(K_{e3}^0)/\Gamma(K_{e3}^+)=1.73\pm0.07.$$

The $\Delta I=\frac{1}{2}$ rule predicts a ratio of 2.05, allowing for phase-space and radiative corrections.[15] Thus the disagreement between the predicted and observed ratios appears highly significant.[16] However, it should be noted that the K_L^0 lifetime quoted is based on one accurate measurement[17] and that the K_{e3}^0 branching ratio or absolute decay rate has not been measured to better than 10% in any one experiment. The error quoted is obtained by an over-all fitting procedure.[18]

We are grateful to the Directorate and staff of the Rutherford High Energy Laboratory for their constant support. We acknowledge the technical assistance of R. D. Downs, J. H. Bibby, and the late P. A. V. Chadd.

[10] The present best estimate of λ^+ is 0.023 ± 0.008. W. J. Willis, Rapporteur's talk at Heidelberg Conference on High Energy Physics, 1967 (unpublished).

[11] (a) R. W. Birge, D. H. Perkins, J. R. Peterson, D. H. Stork, and M. N. Whitehead, Nuovo Cimento 4, 834 (1956); (b) G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, *ibid.* 6, 478 (1957); (c) B. P. Roe, D. Sinclair, J. L. Brown, D. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961); (d) G. Borreani, G. Rinaudo, and A. E. Werbrouck, Phys. Letters **12**, 123 (1964); (e) F. S. Shaklee, G. L. Jensen, B. P. Roe, and D. Sinclair, Phys. Rev. **136B**, 1423 (1964); (f) A. C. Callahan, U. Camerini, R. D. Hantman, R. H. March, D. L. Murphee, G. Gidal, G. E. Kalmus, W. M. Powell, C. L. Sandler, R. T. Pu, S. Natali, and M. Villani, *ibid.* **150**, 1153 (1966); (g) L. B. Auerbach, J. M. Dobbs, A. K. Mann, W. K. McFarlane, D. H. White, R. Cester, P. T. Eschstruth, G. K. O'Neill, and D. Yount, *ibid.* **155**, 1505 (1967); (h) Poh-Shein Young, W. Z. Osborne, and W. H. Barkas, *ibid.* **156**, 1464 (1967).

[12] The average branching ratio, excluding the result of Callahan *et al.*, is $(4.94\pm0.11)\%$, giving a χ^2 of 3.2 for 8 degrees of freedom. The average of all results including those of Callahan *et al.* is $(4.66\pm0.09)\%$ with a χ^2 of 28.6 for 9 degrees of freedom.

[13] N. Cabibbo, in *Proceedings of Thirteenth International Conference on High Energy Physics* (University of California Press, Berkeley, Cal., 1967), p. 30.

[14] W. J. Willis, Rapporteur's talk in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968).

[15] The $\Delta I=\frac{1}{2}$ rule predicts $\Gamma(K_{e3}^0)/\Gamma(K_{e3}^+)=2$. The phase-space correction is $+1.35\%$ for vector coupling with a constant form factor. The radiative correction is $+1\%$ [E. S. Ginsberg (private communication)].

[16] This discrepancy may be explained by the presence of approximately 3% of $\Delta I=\frac{3}{2}$ amplitude relative to $\Delta I=\frac{1}{2}$ amplitude.

[17] T. J. Devlin, J. Solomon, P. Shepard, E. F. Beall, and G. A. Sayer, Phys. Rev. Letters **18**, 54 (1967).

[18] *Note added in proof.* Following recent measurements of K_L^0 branching ratios, $\Gamma(K_{e3}^0)/\Gamma(K_{e3}^+)=1.98\pm0.08$; J. W. Cronin, Rapporteur's talk at Vienna Conference on High-Energy Physics, 1968 (unpublished).