

(b) the Hilbert⁹ density, defined by

$$\mathfrak{H}_H^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho},$$

$$T_H^{\mu\nu}(x) = -2\delta S[g]/\delta g_{\mu\nu}(x).$$

In general, neither \mathfrak{H}_c nor \mathfrak{H}_H will separate unambiguously into "particle" and "field" parts. In the case of classical electrodynamics we have

$$S = -\sum_i \int d\tau_i \left[\frac{1}{2} m_i \dot{q}_i^\mu \dot{q}_i^\nu g_{\mu\nu}(q) + e \dot{q}_i^\mu A_\mu(q_i) \right]$$

$$-\frac{1}{16\pi} \int dx (-g)^{1/2} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma},$$

and

$$T_H^{\mu\nu}(x) = \sum_i m_i \int d\tau_i \delta(x - q_i(\tau_i)) \dot{q}_i^\mu \dot{q}_i^\nu$$

$$-\frac{1}{4\pi} (F_{\mu\sigma} F_{\nu\sigma} - \frac{1}{4} F^2 g_{\mu\nu}),$$

⁹ See the treatment of the symmetric stress energy tensor in L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Co., Reading, Mass., 1962), revised 2nd ed.

$$\mathfrak{H}_c^{\mu\nu\rho} = \sum_i m_i \int d\tau_i \delta(x - q_i(\tau_i)) q_i^\mu (\dot{q}_i^\nu + e A^\nu)$$

$$+\frac{1}{4\pi} (A^\mu F^{\nu\rho} - F^{\sigma\rho} A^{\sigma\nu} x^\mu + \frac{1}{4} x^\mu g^{\nu\rho} F^2) - (\mu \leftrightarrow \nu).$$

In this case, the Hilbert definition provides a "natural" particle angular momentum density depending only on the particle variables. In quantum electrodynamics, on the other hand, the *canonical* expression has a part containing only the Dirac field ψ . It is the familiar spin-plus-orbit expression used in (16). To lowest order in β , the interaction angular momentum is the same in both definitions, namely (4).

Whether there is a "natural" separation of angular momentum which (i) reduces to the Hilbert separation for particles and field in the case of zero interaction and (ii) is still asymptotically a covariant separation for interacting particles is unknown.

Does Lorentz-Invariance Imply Causality?

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We investigate whether faster-than-light propagation of particles and wave motions is consistent with Lorentz invariance of the S matrix in the general context of relativistic, off-shell scattering theories. We do three things. First, we show that Lorentz invariance of the Green's functions implies that of certain weight functions; in a field theory, these weight functions are the Wightman functions. Secondly, we show that any off-shell scattering theory with Lorentz-invariant Green's functions arises from a field theory with causal equal-time commutators. Thirdly, we show that in particle theories with normal connectedness structure, the equal-time commutators are necessarily canonical. These results show that there are great difficulties facing theories of faster-than-light motion which aspire to relativistic invariance.

1. INTRODUCTION

THE question of particle and wave motions faster than light has recently been of interest. Feinberg¹ has exhibited a theory of relativistically covariant quantized fields describing noninteracting, faster-than-light, spinless Fermi particles. In this model, neither the particle number nor the no-particle state is Lorentz-invariant. Ruderman and Bludman² have considered, more phenomenologically, faster-than-light motions in

various classical (i.e., nonquantum) contexts. Abers *et al.*³ raise the possibility of particles with spacelike momenta in infinite-component field theories.

We wish to investigate quite generally whether noncausal behavior of this general kind is possible consistently with Lorentz-invariance; specifically, we consider theories with *Lorentz-invariant Green's functions*. We show that noncausal effects are impossible in a large class of theories of this kind. It is to be emphasized that we make no causality assumptions at all

¹ G. Feinberg, Phys. Rev. **159**, 1089 (1967).

² S. Bludman and M. A. Ruderman, Phys. Rev. **170**, 117 (1968).

³ E. Abers, I. T. Grodsky, and R. E. Norton, Phys. Rev. **159**, 1222 (1967).

(in particular, no assumptions about commutation relations).

In Sec. 2 we consider the relation between Lorentz invariance of Green's functions and Lorentz invariance of Wightman functions. We work mainly in momentum space. Starting from Lorentz-invariant Green's functions and making a very weak assumption about analyticity in energy variables, we establish an integral formula of the Bergmann-Oka-Weil type, and prove that the weight functions are also Lorentz-invariant. These weight functions coincide with the Wightman functions if the Green's functions are those of a field theory; otherwise they may be used to define a field theory (by the Wightman reconstruction). Thus the Lorentz invariance of the Green's functions implies that of the Wightman functions, without causality assumptions. This shows that a theory of interacting particles generalizing Feinberg's¹ can either be replaced by one with a Lorentz-invariant vacuum, or cannot have invariant Green's functions.⁴

In Sec. 3, we go on to consider the support properties of equal-time commutators. We show that these commutators are causal. This is of course sufficient to exclude faster-than-light propagation from this class of theories. We then exploit the connectedness structure of the Green's functions to show that the equal-time commutators are c numbers and, if particles are present, are canonical. We conclude with a discussion of various proposed models and of technical assumptions.

2. INVARIANCE OF GREEN'S FUNCTIONS AND OF WIGHTMAN FUNCTIONS

We suppose first of all that we have a theory with Lorentz-invariant Wightman functions $W^n(x_1 \cdots x_n)$. By using the Wightman reconstruction, we can suppose that they arise from fields $\phi_1(x_1) \cdots \phi_n(x_n)$ and a vacuum state \rangle .⁵ (In the present section, this will be merely a notational convenience.) We shall write out the details only for the case of a single, neutral scalar Bose field $\phi(x)$,⁵ since the extension to several fields and to higher spin types, Fermi statistics, and so forth, will be immediate. Then we have

$$W^n(x_1, \cdots, x_n) = \langle \phi(x_1) \phi(x_2) \cdots \phi(x_n) \rangle. \tag{1}$$

Unless otherwise mentioned, we shall truncate these Wightman functions.⁶ Then the τ products (configuration-space Green's functions) are defined by

$$\tau^n(x_1, \cdots, x_n) = \sum_P \theta_{12}^P \theta_{34}^P \cdots W^n(x_{P(1)} \cdots x_{P(n)}), \tag{2}$$

⁴In Feinberg's original theory, Green's functions are not explicitly covariant, because of the noninvariance of the vacuum; however, our result will apply if the transformation properties of the fields are so chosen as to compensate for the noninvariance of the vacuum, giving covariant Green's functions.

⁵All such expressions involving real arguments are to be understood as distributions in the usual way.

⁶That is, with vacuum intermediate states removed.

where

$$\theta_{12}^P = \theta(x_{P(1)} - x_{P(2)}), \text{ etc.}, \tag{3}$$

and the sum in (2) is over all permutations P of $1 \cdots n$. We wish to perform the Fourier transformation on (2), having regard to the fact that W^n is (by translation invariance) a function of difference variables only. More precisely, we introduce into each term of the sum (2) a separate set of difference variables^{7,8}

$$y_r^P = x_{P(r)} - x_{P(r+1)}, \quad (1 \leq r \leq n-1) \tag{4}$$

so that

$$\theta_{12}^P = \theta(y_1^P), \text{ etc.} \tag{5}$$

Then we associate with each space-time argument x_i the momentum p_i and define

$$q_r^P = \sum_{i=1}^r p_{P(i)}. \tag{6}$$

Momentum conservation will imply

$$\sum_{i=1}^n p_i = 0,$$

so that for any permutation P ,

$$\sum_{j=1}^n p_j x_j = \sum_{j=1}^{n-1} q_j^P y_j^P. \tag{7}$$

Thus changing n to $n+1$ in (2) and using the convolution property of the Fourier transform, we obtain⁷ for the Green's function:

$$G^{n+1}(p_1 \cdots p_{n+1}) = i^n \sum_P \int \rho_P^n(s_1, \mathbf{q}_1; \cdots; s_n, \mathbf{q}_n) \prod_{i=1}^n \frac{ds_i}{s_i - q_{i0} + i\epsilon}. \tag{8}$$

For the moment, we will omit the $i\epsilon$.

In Eq. (8), the Green's function $G^n(p_1 \cdots p_n)$ is the Fourier transform of $\tau^n(x_1 \cdots x_n)$, and $\rho_P^n(s_1, \mathbf{q}_1; \cdots; s_n, \mathbf{q}_n)$ is the Fourier transform in the q_r^P of $W^{n+1}(y_1^P, \cdots, y_n^P)$, with $(s_i, \mathbf{q}_i) = q_i^P$ and with the other superfaces P omitted for clarity.^{9,10} Equation (8) has the important *spectral property* that the s_i integration is over the range

$$s_i \geq +(\mathbf{q}_i^2 + m^2)^{1/2}, \tag{9}$$

where m is the smallest mass in the theory.¹¹ We may include cases where particles of negative m^2 arise,¹ provided that all values of s and q in (8) are real, and

⁷M. M. Broido and J. G. Taylor, *J. Math. Phys.* (to be published). See also J. G. Taylor, *Boulder Lecture Notes, 1966*, edited by W. E. Brittin, A. O. Barut, and M. Guenin (Gordon and Breach Science Publishers, Inc., New York, 1967), Vol. 9A.

⁸H. Araki, *J. Math. Phys.* **2**, 163 (1961).

⁹Subtractions may be necessary in (8). They are irrelevant here, and we will ignore them.

¹⁰Thus all the functions ρ_P are the same momentum-space Wightman function, only of different arguments.

¹¹We do not need to assume that this smallest mass is nonzero.

that the Wightman function vanishes in a neighborhood of some real point.

Now we assume that the Wightman functions are invariant under proper Lorentz transformations. Then the invariance of each term of (2) under Lorentz transformations is obvious when each argument y_r^P is time-like. When causality is assumed¹² it assures us that for any spacelike y_k^P , a term containing

$$W^n(x \cdots x_{P(k)}, x_{P(k+1)} \cdots)$$

can be combined with a term with

$$W^n(x \cdots x_{P(k+1)}, x_{P(k)} \cdots)$$

to give an invariant result. Since we do not assume causality, we proceed otherwise.

Consider a set of connected Green's functions which are invariant under the proper Lorentz group. If they possess the usual one-particle poles of mass m and two-particle cuts, and only correspond to positive-energy states, then the Bergmann-Oka-Weil representation expressing the corresponding cut-plane maximal analyticity in energy variables is precisely Eq. (8), where the region of integration satisfies (9).⁵ This representation is plainly unique, since the discontinuity in G when the variables q_{i0} pass across the two-particle cuts [i.e., in Eq. (8)] is precisely the weight function ρ . Since the values of G across the cuts are analytic continuations of one another [guaranteed by the form (8)], these analytic continuations will also be invariant (say, by the Bargmann-Hall-Wightman theorem), hence so will be their limiting difference ρ . Q.E.D.

Alternatively, we may show the invariance by operating one Eq. (8) with a Lorentz transformation and averaging both sides with Haar measure over the Lorentz group. We may, under suitable analytic conditions, invert the order of the integration (8) and the one involved in the averaging. The result of this is Eq. (8) with a weight function ρ' which has been averaged over the Lorentz group and so is Lorentz-invariant. By uniqueness, $\rho' = \rho$.¹³

3. CAUSAL COMMUTATORS

We now go over to the use of complete (not truncated) Wightman functions and of complete (not connected) Green's functions. Since this is a linear process, our invariance results will still hold. We shall make little use of the invariance of the Wightman functions, but we will rely heavily on the invariance of the Green's functions.

We start from Eq. (2), where the Wightman functions either are defined from Eq. (1) (if we start from a field theory) or are the configuration-space weight

¹² We note in particular that Araki (Ref. 8) assumes causality both ways—for his Wightman functions (his axiom W2) and for his retarded functions (his axiom R2).

¹³ Because we are using time-ordered rather than more general retarded products, the great complications in Ref. 8 connected with Steinmann relations do not appear here.

functions ρ [given by the Bergmann-Oka-Weil representation (8) (if we start from Green's functions); then the field(s) are those given by the Wightman reconstruction, provided certain additional technical conditions hold].

The S -matrix element (in momentum space) is obtained by multiplying with $\prod_i (p_i^2 - m^2)$ and taking the limit $p_i^2 \rightarrow m^2$. We assume that we start with invariant Green's functions, so this is an invariant process and must give invariant results. We shall carry it out in configuration space so as to investigate questions of causality. It will be sufficient to consider the difference variables $x_1 - x_k$ ($k = 2 \cdots n$), since the additional effects for other $x_j - x_k$ will add linearly; thus we operate with the one operator $\square_{x_1}^2 + m^2$. We shall write

$$(\square^2 + m^2)\phi(x) = j(x), \tag{10}$$

so that

$$\begin{aligned} & (\square_{x_1}^2 + m^2)\tau^n(x_1, \cdots, x_n) \\ &= \sum_P \theta_{12}^P \theta_{34}^P \cdots \langle \phi(x_{P(1)}) \cdots j(x_1) \cdots \phi(x_{P(n)}) \rangle \\ &+ \sum_i \sum_{\substack{P \text{ not} \\ 1, i}} \theta_{12}^P \theta_{34}^P \cdots \\ & \quad \times \langle \phi(x_{P(1)}) \cdots C[x_1, x_i] \cdots \phi(x_{P(n)}) \rangle, \end{aligned} \tag{11}$$

where

$$\begin{aligned} C[x_1, x_i] &= \delta(x_{i0} - x_{10})[\phi(x_1), \phi(x_i)] \\ &+ \delta(x_{i0} - x_{10})[\phi(x_1), \phi(x_i)]. \end{aligned} \tag{12}$$

On the right-hand side of Eq. (11) the two terms must be separately invariant because of their different support properties. This assumes that certain Schwinger-like terms are absent from the first of these terms. We shall show this in the note added in proof. Now consider the second term. We see at once that its invariance is equivalent to that of

$$\sum_i C[x_1, x_i]. \tag{13}$$

Each term in (12) is a function of the difference variable $y_i = x_1 - x_i$. Consider the effect of a Lorentz transformation on the function (13). The support of the transformed function will be contained in the union (on i) of the planes $(\Lambda y_i)_0 = 0$. If (13) is invariant, it must have an invariant support, which latter must be contained in the set

$$\bigcap_{\Lambda} \bigcup_i \{y_i : (\Lambda y_i)_0 = 0\}.$$

Since the union is finite, we may commute it with the intersection, and obtain easily the set $\bigcup_i \{y_i = 0\}$. Without loss of generality we can assume that $y_i \neq y_j$ for $i \neq j$ [otherwise the fields $\phi(x_i)$ and $\phi(x_j)$ always appear together in the τ product and can be lumped together, etc.]. Thus each term (12) has support at $y_i = 0$ only and must be separately invariant, so that its y_i dependence (as a distribution) is of the form $P(\square^2)\delta^4(y_i)$, where P is some polynomial. Such a

distribution cannot have a factor $\delta(y_i)$; thus the second term in (12) must vanish for all Green's functions, and we have

$$[\phi(x), \phi(x')]_{-} |_{x_0=x_0'} = 0. \quad (14)$$

As for the first term in (12), its form shows that we must have $P = \text{const.}$ Thus

$$[\phi(x), \phi(x')]_{-} |_{x_0=x_0'} = A \delta^3(\mathbf{x} - \mathbf{x}'), \quad (15)$$

where A is some operator. Equations (14) and (15) show that *Lorentz-invariant Green's functions imply causal commutators.*

This proof will be complete and will hold quite generally, provided that all quantities are well-defined and do not exhibit extraordinary pathologies. We will now argue somewhat more heuristically that the operator A in (15) is actually a c number. We do this by considering the connectedness structure in the variable x_1 of the terms in Eq. (11.) A term will be called *disconnected* in x_1 if it is of the form $F(x_1 - x_i)G(x_2 \cdots x_n)$, where G is not a function of x_i . It is clear that the x_1 -disconnected parts of the term in (11) containing $j(x_1)$ will be of the form

$$\langle T(j(x_1)\phi(x_i)) \rangle \tau^{n-2}(x_2 \cdots x_n),$$

and one easily sees that the first factor represents *self-energy corrections* to the disconnected scattering of x_1 into x_i , with the other particles scattering among themselves in an arbitrary fashion. The surviving term (11) will have the factor $\delta^4(x_1 - x_i)$. This δ function is precisely what is required to give the free propagator for disconnected x_1 -to- x_i scattering. We define our requirement of a *normal connectedness structure* by saying that these two terms must combine together to form the full propagator for x_1 to x_i scattering, as given by the Dyson equation for local polynomial interactions, multiplied by the full Green's function for the scattering of the remaining particles among themselves. It is well known that this normal connectedness structure holds in the usual local field theories (where, however, canonical commutation relations are used). If we assume that it holds in general, the second factor in the δ -function term from (11) must again be $\tau^{n-2}(x_2 \cdots x_n)$. This can hold for Green's functions of all orders only if $C[x_1, x_i]$, defined by Eq. (12), is actually a c number α , so that Eq. (15) simplifies to

$$[\phi(\mathbf{x}), \phi(\mathbf{x}')]_{-} |_{x_0=x_0'} = \alpha \delta^3(\mathbf{x} - \mathbf{x}'). \quad (16)$$

Our assumptions so far have not strictly excluded the possibility that $\alpha = 0$; but if we require further a correct particle interpretation, with the propagator having unit residue at $p^2 = m^2$, we get at once $\alpha = iZ_3^{-1}$, with $Z_3^{-1} \geq 1$; Z_3 is given in the usual way by the Lehmann representation [which latter is a special case of Eq. (8)]. Thus we conclude that *Lorentz-invariant Green's functions imply canonical equal-time commutation relations* for theories with particles having correctly normalized

propagators and a normal connectedness structure. Concerning the possibility of Schwinger-like contributions and consequent difficulties in passing from Eq. (15) to Eq. (16), however, see the Note added in proof.

We note that this result extends immediately to a generally covariant theory, since such a theory is also Lorentz-covariant.

4. DISCUSSION

The theory of Feinberg¹ is a theory of a covariant field, having a noninvariant vacuum. Thus the Wightman functions will not be invariant, and the work of Sec. 2 shows that a theory with interactions of this type cannot have invariant Green's functions either.

Faster-than-light propagation is forbidden in any theory with causal commutators. Hence the work of Sec. 3 shows that even noninteracting theories of the type proposed by Feinberg are inconsistent with invariant Green's functions. In the noninteracting case, this means that faster-than-light propagation is inconsistent with the Feynman propagators for the free fields, a result which is really not surprising. The same argument shows that in the infinite spin-multiplet theories,³ spacelike particle momenta are inconsistent with invariant Green's functions. Other theories with invariant vacuum and Green's functions, such as that recently proposed by Korff and Fried,¹⁴ are similarly shown to be impossible.

We cannot make any such assertions about the more phenomenological proposals of Ref. 2, since these are nonrelativistic. Considerations of relativistic invariance are of course fundamental in our argument. Reference 2 is concerned *inter alia* with the propagation of vibrations in crystals and similar problems in which translation invariance may not hold. We consider it unlikely that our methods can be applied even to relativistic theories lacking in translation invariance, since all the arguments of Sec. 2, based on the consequent use of difference variables, break down.

We may ask ourselves whether our technical assumptions are reasonable. The most important of these concerns the passage from (2) to (8) and back by Fourier transform. The terms of (2) and the integrands of (8) are, formally, products of distributions, and may contain ambiguities. There is a large class of theories in which the Wightman functions are⁵ boundary values of genuine analytic functions in x space,¹⁵ and then we can distort the contour of x -space integration to make the integrals in (8) well-defined. Similarly, when starting from (8) we may expect to be able to distort contours once Lorentz invariance is established. Beyond this class of theories, it is very difficult to make any general statement about the validity of such operations.

¹⁴ D. Korff and Z. Fried, *Nuovo Cimento* **52**, 173 (1967).

¹⁵ G. Källén and A. S. Wightman, *Kgl. Danske Videnskab. Selskab Mat. Fys. Skrifter* **1**, 6 (1958); G. Källén and H. Wilhelmsson, *ibid.* **1**, 9 (1959).

The only other handle on this type of technical problem is to investigate model theories. Since Lorentz covariance is clearly a basic requirement, the only available model would appear to be that of Thirring.¹⁶ The Green's functions of this model are indeed invariant, but contain infinite constants which would appear to invalidate the equivalence between (2) and (8). We have a mass divergence, an infrared "problem," and divergences associated with vacuum fluctuations.¹⁷

Since the model is nonperturbative, these divergences must be taken seriously. The question is whether they have anything to do with the relativistic invariance properties. We consider the infrared problem to be normal in any theory with massless particles, since there will be a cut superimposed on the pole (compare also the Bloch-Nordsieck model and its generalizations). The vacuum fluctuations are satisfactorily dealt with in Ref. 17. The mass infinities are removed in Ref. 17 by a cutoff procedure, and then (equivalently) by introducing a measure of nonlocality into the theory. There are then difficulties with relativistic covariance. Essential though such steps may be in a "computation" (and in quantum electrodynamics, for instance) they seem to us to be at variance with the structure of the model. This model contains local products of fields which may be expected to give (possibly divergent) ambiguities as a matter of course; in connection with investigations of the general structure of the theory, we feel that these should be dealt with by methods adapted from the theory of distributions.¹⁸

This is all we can say about soluble models. For more general theories, the question of whether the Fourier transform is 'allowed' or not appears to be open. Particularly great difficulties occur with "nonrenormalizable" local field equations, though again, the exact extent of these (i.e., outside perturbation theory) is unknown.¹⁹ It seems possible that this type of question must be dealt with separately in each theory.

Finally, we remark that the reinterpretation of the indefinite metric solution of Heisenberg's "master equation" by means of nonlocal interactions²⁰ most

probably rules out both these possibilities as giving covariant results, especially since these theories are constructed so that divergences are absent. It may be possible that a nonlocal theory does, indeed, satisfy microscopic causality, so that the nonlocality is only apparent. This appears to us to be rather unlikely; we hope to give a fuller discussion of this elsewhere.

Note added in proof. In the text we argued that the Lorentz invariance of the right-hand side of Eq. (11) implied that each of the two terms is separately invariant. The basis of this argument is the observation that where the first term depends in x_1 like, say, $\theta(x_1-x_i)$, the second term contains $\delta(x_1-x_i)$ or $\dot{\delta}(x_1-x_i)$. The θ -like terms will be independent of the δ -like terms only if the coefficients of the θ functions are themselves not too singular. Thus we must consider the possibility that the Wightman-like function

$$\langle \phi(x_{P(1)}) \cdots j(x_1) \cdots \phi(x_{P(n)}) \rangle \quad (\text{A1})$$

may contain singular terms of the type $\delta(x_{1,0}-x_{i,0})$ or $\dot{\delta}(x_{1,0}-x_{i,0})$; these are the possible terms which we called Schwinger-like in the discussion following Eq.(12).

The only known situation in which Schwinger terms are believed to exist is in theories with conserved vector currents, and then only in time-ordered products involving at least *two* current operators. Thus we are not forced to take such terms into account. Nevertheless, we shall now show that they cannot affect our main results.

The most important point here is that since (A1) is certainly covariant, $\dot{\delta}(x_{1,0}-x_{i,0})$ contributions are absent. Hence nothing from (A1) can cancel the manifestly noninvariant last term of (12), which must therefore vanish. This was the most important result in the text.

The possibility remains that (A1) contains a singularity of the type $\delta(x_{1,0}-x_{i,0})$. Covariance of (A1) implies that the singularity must be even $\delta^4(x_1-x_i)$. The resulting contribution to (11) is poorly defined [having a factor $\theta(x_{1,0}-x_{i,0})\delta^4(x_1-x_i)$], but will not interfere with the support property of the commutator $[\phi(x_1), \phi(x_i)]_-$. Equation (15) will still hold, but the connectedness-structure arguments leading to Eq. (16) appear to break down. A term like $\delta^4(x_1-x_i)$ corresponds to disconnected scattering and seems rather unlikely to appear in the term containing the currents, but it is difficult to give decisive arguments for its exclusion.

Symposium on Nonlocal Quantum Field Theory, Dubna, 1967 (unpublished).

¹⁶ W. Thirring, Ann. Phys. (N. Y.) **3**, 91 (1958).

¹⁷ We have in mind the treatment of C. M. Sommerfield, Ann. Phys. (N. Y.) **26**, 1 (1964).

¹⁸ J. G. Taylor, Nuovo Cimento **17**, 695 (1960).

¹⁹ J. G. Taylor, Nuovo Cimento Suppl. **1**, 857 (1963), paper IV; M. B. Halpern, Ann. Phys. (N. Y.) **39**, 351 (1966).

²⁰ W. Heisenberg, in Solvay Conference, Brussels, 1967 (unpublished). See also H. P. Dürr, Report at the International