Neutron Cross Sections and Resonance Parameters of Yb Isotopes*

S. F. MUGHABGHAB AND R. E. CHRIEN
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 June 1968)

Neutron transmission measurements have been carried out for Yb^{170}, Yb^{171}, Yb^{172}, Yb^{173}, Yb^{174}, and Yb^{176} over a large energy region to determine the s-wave strength functions, level densities, scattering-radius parameters, and thermal cross sections. The values of the latter are found to be 17.0 ± 1.0, 57 ± 3, 10.2 ± 1.5, 28 ± 2, 142 ± 5, and 14.9 ± 1.0 b, respectively. From these, a paramagnetic scattering cross section of 5.2 ± 0.9 b is extracted for Yb^{+++} at 0.0253 eV. By comparing the level densities of Yb^{171} and Yb^{173}, a spin cutoff factor $\sigma=2.8\pm0.8$ b is derived. Average total cross sections are analyzed to determine the strength functions in the 5–15-keV region, which are compared with values derived in the low-energy region.

I. INTRODUCTION

THIS is the second of a series of papers reporting on our studies on the systematics of neutron resonances in the rare-earth region. Summaries of our results appeared in Refs. 1 and 2. A detailed description of the experimental procedure and the analysis of the data can be found in a recent publication by the authors on the Er isotopes,[3] hereafter referred to as I. In this paper, we shall report our results on the Yb isotopes. Natural Yb consists of seven stable isotopes: Yb^{168}, Yb^{170}, Yb^{171}, Yb^{172}, Yb^{173}, Yb^{174}, and Yb^{176}, two of which are even-odd while the rest are even-even isotopes. When this work was initially begun, very little was known about the resonance structure and thermal cross sections of those rare-earth isotopes. Early measurements by Sailor *et al.*[4] using a sample of natural Yb showed that the first positive energy resonance at 0.597 eV belonged to Yb^{168}. Four additional resonances were listed by Sailor *et al.*, but no resonance parameters or isotopic identification was reported for them. In 1962 Carpenter,[5] employing the "sum-coincidence" technique in capture measurements, was able to assign additional resonances to Yb^{171} and Yb^{173}. From his transmission data, he derived resonance parameters for the levels at 4.6, 13.5, and 18.4 eV. By 1965, sufficient amounts of the separated Yb isotopes (with the exception of Yb^{168}) were available from the Isotope Division at the Oak Ridge National Laboratory to make transmission measurements at the fast chopper facility of the Brookhaven Graphite Research Reactor (BGRR) possible. Our preliminary results on the resonance parameters[6] and thermal cross sections[7] of Yb^{170}, Yb^{171}, Yb^{172}, Yb^{173}, Yb^{174}, Yb^{176} were reported previously. Subsequently, Garg *et al.*,[8] using separated Yb isotopes (with the exception of Yb^{168} and Yb^{170}) carried out transmission and self-indication measurements at the Columbia proton-synchrocyclotron, which were repeated by the same group.[9] Shortly afterwards, Wang Nai-Yang *et al.* published their results[10] on the transmission and capture measurements of Yb^{171} and Yb^{173} and natural Yb.

II. EXPERIMENTAL PROCEDURE

The enriched Yb samples used in the present investigation were obtained on a loan basis from the Isotope Division of Oak Ridge National Laboratory. The isotopic compositions and thicknesses of the various Yb isotopes are listed in Table I. Each column refers to a particular sample in terms of its main isotope and the other isotopic impurities. The last row, however, gives the sample thickness in atoms/b calculated on the basis that the chemical composition of these oxide samples is Yb_2O_3. For comparison, we have included in the first column the percentage abundance of the isotopes in natural Yb. It is to be remarked that we did not make any transmission measurements on natural Yb.

The fast chopper facility at the BGRR was used to carry out the transmission measurements on these isotopes. As in our first paper, two sets of measurements were carried out: one with a 29.74-m flight path and a bank of BF_3 detectors; another with a 9.9-m flight path and a He^3 detector for the purpose of determining the thermal cross sections. For the first set of measurements, the following data runs with the indicated energy inter-

* This work was supported by the U. S. Atomic Energy Commission.

[1] R. E. Chrien and S. F. Mughabghab, in *Nuclear Structure Study with Neutrons*, edited by M. Nève de Mévergnies *et al.* (North-Holland Publishing Co., Amsterdam, 1966), p. 524.

[2] R. E. Chrien, S. F. Mughabghab, M. R. Bhat, and A. P. Jain, Phys. Letters **24B**, 573 (1967).

[3] S. F. Mughabghab, R. E. Chrien, and M. R. Bhat, Phys. Rev. **162**, 1130 (1967).

[4] V. L. Sailor, H. H. Landon, and H. L. Foote, Jr., Phys. Rev. **96**, 1014 (1954). See also H. H. Landon, *ibid.* **100**, 1414 (1955).

[5] R. T. Carpenter, Argonne National Laboratory Report No. ANL-6589, 1962 (unpublished). Spin assignments for the Yb^{171} and Yb^{173} resonances were communicated by L. M. Bollinger to Sigma Center, now known as the National Neutron Cross Section Center.

[6] S. F. Mughabghab and R. E. Chrien, Bull. Am. Phys. Soc. **10**, 513 (1965).

[7] S. F. Mughabghab and R. E. Chrien, in Second Conference on Neutron Cross Sections and Technology, 1968, paper E-28 (unpublished).

[8] J. B. Garg, S. Wynchank, H. Liou, W. W. Havens, Jr., H. Ceulemans, and J. Rainwater, Bull. Am. Phys. Soc. **10**, 575 (1965).

[9] S. Wynchank, H. Ceulemans, J. Garg, H. Liou, J. Rainwater, and W. W. Havens, Jr., Columbia University Progress Report, 1967, p. 1 (unpublished); and (private communication).

[10] Wang Nai-Yang, E. N. Kharzhavina, A. B. Popov, Yu S. Yazvitskii, and Yao Chi-Chusng, Yadern. Fiz. **3**, 48 (1966) [English transl: Soviet J. Nucl. Phys. **3**, 34 (1966)].

TABLE I. Isotopic composition and sample thicknesses of the Yb samples used in the present investigation. Each column refers to a particular sample in terms of its main isotope. The rows show the percentage abundance of the indicated isotope, except the last row shows the total sample thickness in atoms/b. For comparison, the first column gives the percentage abundance of a natural Yb sample.

% Isotopic composition	Yb	Yb^{170}	Yb^{171}	Yb^{172}	Yb^{173}	Yb^{174}	Yb^{176}
Yb^{168}	0.135	<0.1	<0.02	<0.01	<0.05	0.01	0.01
Yb^{170}	3.03	85.4 ±0.1	0.27±0.02	0.05±0.02	<0.05	0.02	0.03
Yb^{171}	14.31	5.42±0.05	95.96±0.05	0.75±0.05	0.44±0.05	0.08	0.16±0.02
Yb^{172}	21.82	3.66±0.05	2.03±0.05	97.15±0.05	2.33±0.05	0.20±0.02	0.29±0.02
Yb^{173}	16.13	1.93±0.05	0.63±0.02	1.01±0.05	92.6 ±0.1	0.52±0.05	0.29±0.02
Yb^{174}	31.84	2.86±0.05	0.91±0.02	0.87±0.05	4.3 ±0.1	98.97±0.05	1.45±0.05
Yb^{176}	12.71	0.75±0.05	0.20±0.02	0.19±0.02	0.38±0.05	0.22±0.02	97.77±0.10
$n(10^{-3}$ atom/b)		13.29	18.18	15.48	16.17	12.69	15.64

vals in eV were made: for $Yb^{171,172,173}$ (≈15 000–73.2, 76.4–8.05, and 18.5–0.76); for Yb^{170} (≈87.4–8.4, 18.5–0.71); for $Yb^{174,176}$ (≈15 000–18.0, 18.5–0.71). For the thermal runs, two measurements were made for each isotope covering the energy ranges 2.091–0.222 eV and 0.506–0.0243 eV.

III. ANALYSIS OF THE DATA

The resonance parameters are derived from area analysis using a modified version of the Atta-Harvey code[11] which is described in detail elsewhere.[12,13] A statistical weight factor $g=\frac{1}{2}$ (for the odd isotopes) and a radiation width $\Gamma_\gamma=75\pm7$ meV are assumed in the analysis to obtain the neutron widths. The latter quantity is based on a weighted average value determined in this experiment for the resonances at 7.93, 13.1, and 28.0 eV belonging to Yb^{171} and at 17.7, 31.5, 35.8, and 45.4 eV belonging to Yb^{173}. This average value is in very good agreement with the radiation width of the 0.597-eV resonance in Yb^{168} determined by Sailor *et al.*[4] Since the Yb^{170} sample is not highly enriched, assignment of resonances to this isotope are made in general by a detailed curve fitting of the transmission dips arising from the isotopic impurities. For details of this method, refer to our first paper on this subject.

The neutron strength function is determined by two methods: (a) in the low-energy region from the individual and "average" parameters of resonances, both in the completely resolved and the partially resolved energy regions, and (b) from the slope of the average cross section in the keV region.

IV. RESULTS AND DISCUSSION

A. Resonance Parameters

After the resonance parameters are obtained from area analysis, they are fed into a code[11] which calculates the transmission curves, applying both Doppler and instrumental resolution broadening. Figure 1 illustrates the fitted transmission data for Yb^{170}, Yb^{171}, and Yb^{173} in the low-energy region. The effect of resonances outside the energy region under study can be represented as a sum of contributions of the form $K_0/E^{1/2}+K_1/E$. However, if such "outside" resonances are strong and are close to one side of the energy interval under study, such a procedure is not possible. Under such conditions, the parameters of such "outside" resonances must be incorporated in the calculations. An example of this is illustrated by the Yb^{170} transmission data shown at the top of Fig. 1. In order to obtain the present acceptable fit in the high-energy portion of this run, it was necessary to include the resonance parameters of the 40.0-eV resonance of Yb^{170} which shows up in the next transmission run and is displayed in Fig. 2(a). As shown, the over-all fit to the data in this energy region is very good. It is to be remarked here that the resonance at 8.13 eV assigned by Wang Nai-Yang *et al.*[10] to Yb^{171} obviously belongs to Yb^{170}. Figures 2(a), 2(b), and 2(c) represent the transmission data of Yb^{170}, Yb^{171}, and Yb^{173} in the higher-energy regions that are indicated in the caption of the figure.

The general over-all fit to the data is good except that here one notices slight departures at the low-energy end of the resonances. This was attributed to a tail in the experimental resolution function.[13] An illustration of curve fitting in the highest-energy region ≈15 000–180. eV is shown in Fig. 3 for Yb^{176}. In this figure we have indicated the position of the weak 98.18-eV resonance in Yb^{176} found by the Columbia group. For a list of their parameters, see the BNL compilation of Goldberg *et al.*[14]

For the purpose of obtaining a statistically significant quantity for the neutron strength functions from resonances, we attempted to fit the transmission data in the high-energy region where individual resonances are partially resolved. The neutron reduced widths thus obtained can be considered as average quantities. This approach illustrated here for Yb^{176} is also applied to Yb^{170}, Yb^{171}, and Yb^{173}.

Since these samples are not 100% isotopically enriched, they provide a range of sample thicknesses for

[11] S. E. Atta and J. A. Harvey, Oak Ridge National Laboratory Report No. ORNL 3205, 1961 (unpublished).
[12] R. E. Chrien, Phys. Rev. **141**, 1129 (1966).
[13] M. R. Bhat and R. E. Chrien, Phys. Rev. **155**, 1362 (1967).
[14] M. D. Goldberg, S. F. Mughabghab, S. N. Purohit, B. A. Magurno, and V. M. May, in *Brookhaven National Laboratory Report No. 325* (U. S. Government Printing Office, Washington, D. C., 1967), 2nd ed., Suppl. 2, Vol. IIC.

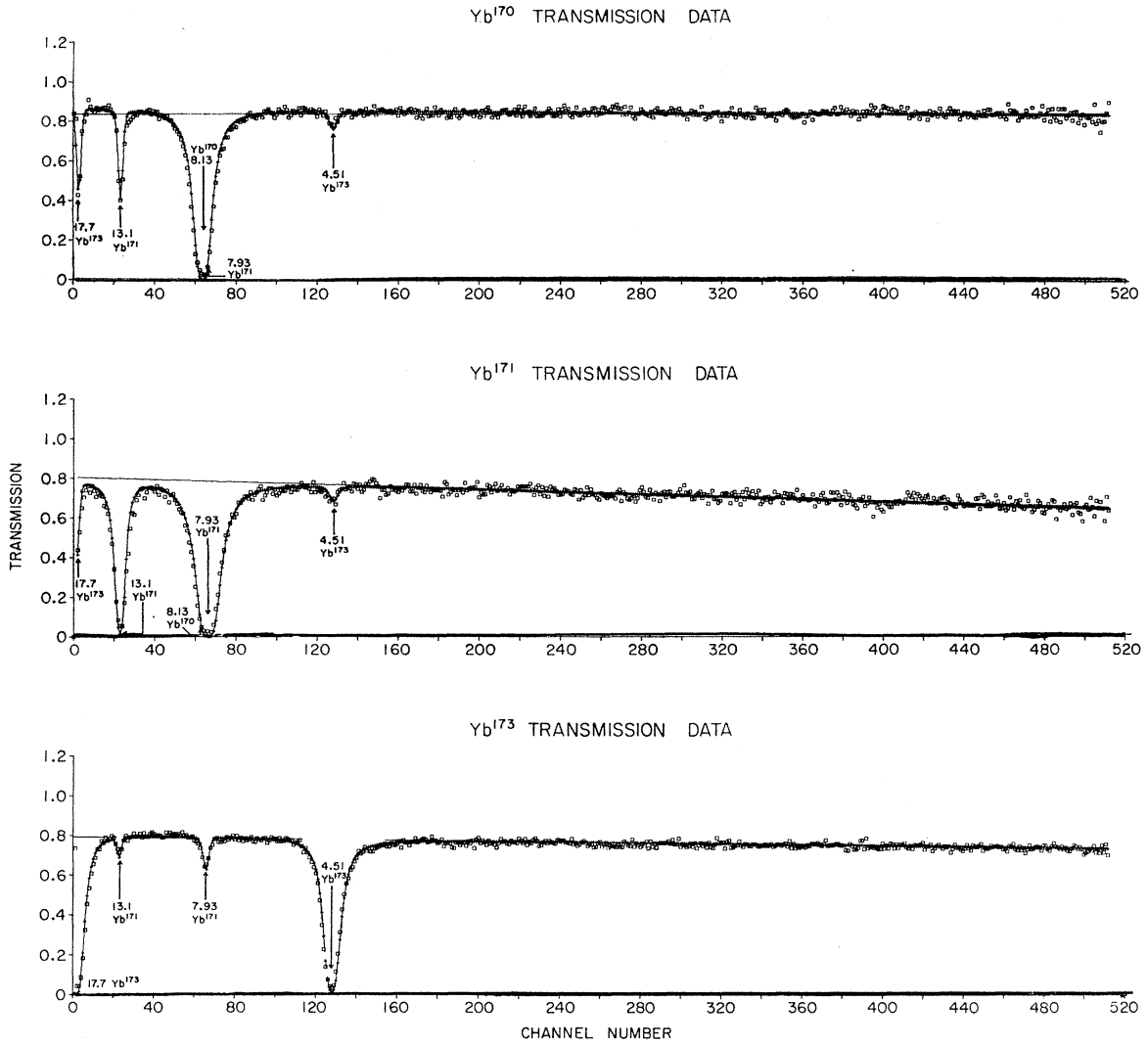

FIG. 1. Fitted transmission data of Yb^{170}, Yb^{171}, and Yb^{173} in the energy interval 18.5–0.71 eV. The squares are the experimental points; the smooth curve is the calculated transmission including Doppler and experimental resolution broadenings.

a given isotope. One then can apply the thin-thick sample method[15] to derive radiation widths for some resonances under favorable cases. Figure 4 illustrates the application of this method to the 7.93- and 13.1-eV resonances in Yb^{171} where the $2g\Gamma_n$ values are plotted against radiation widths, Γ_γ, for the case of three sample thicknesses. Since the spin of the ground state of Yb^{171} is $I=\frac{1}{2}$, s-wave neutron capture by this nucleus will result in highly excited states in Yb^{172} with spins $J=1,0$. The spin of the resonance at 13.1 eV is known[5] to be 1, while that at 7.93 eV is not yet determined. It could possibly be 0. In view of this, the $2g\Gamma_n$ values of the 7.93-eV resonance have been derived for the two possible spin values. As can be noted, the radiation widths of the two resonances at 7.93 and 13.1 eV are 62 ± 6 and 84 ± 6 meV, respectively. The errors on these values represent mainly statistical errors and do not include any systematic errors. However, we feel that this difference in the radiation widths of these two resonances presents some evidence for a spin-0 assignment for the 7.93-eV resonance.

It might be expected that a spin-0 resonance might display a smaller radiation width since the known level scheme of the even-even Yb^{172} exhibits a typical rotational sequence and there are no 1^+ states available at low-excitation energies. It has been observed for the neighboring target nuclei Yb^{168} and Yb^{174} that the thermal capture spectrum is dominated by strong high-energy components.[16,17] In such a case, it would follow that: (1) there would be a considerable fluctuation of

[15] D. J. Hughes, J. Nucl. Energy **1**, 237 (1955).

[16] A. I. Namenson and J. C. Ritter, Bull. Am. Phys. Soc. **13**, 722 (1968).

[17] E. B. Shera, M. E. Bunker, R. K. Sheline, and S. H. Vegors (private communication, 1968); Phys. Rev. **170**, 1108 (1968).

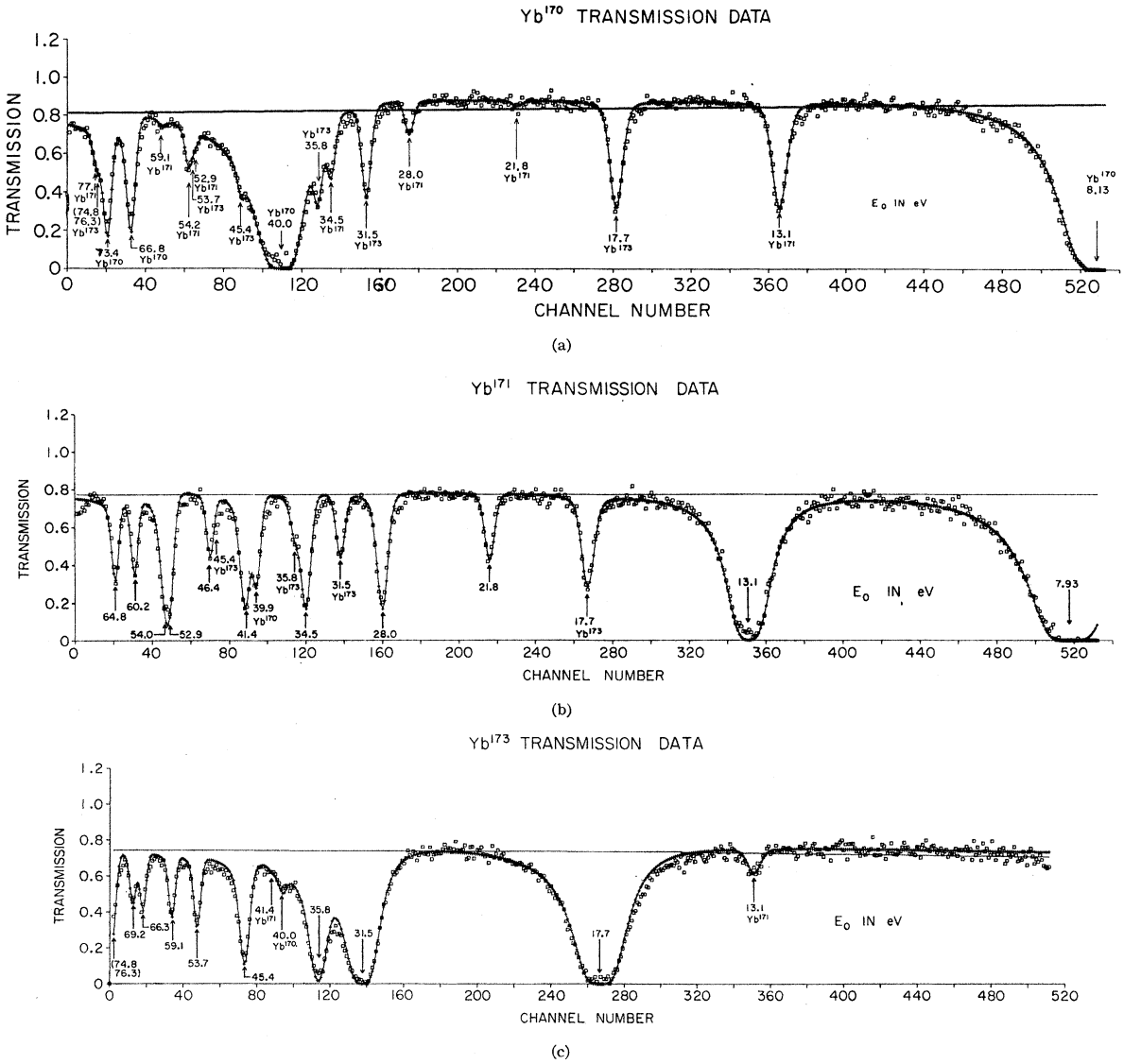

FIG. 2. Fitted transmission data of Yb^{170} in the energy interval 87.4–8.4 eV. Figures 2(b) and 2(c) show the theoretical fits for Yb^{171} and Yb^{173} in energy interval 76.4–8.05 eV, respectively. Below each dip we indicated the resonance energy in eV and the isotope in which it occurs. In Fig. 2(a) the position of the 8.13-eV resonance in Yb^{170} is shown.

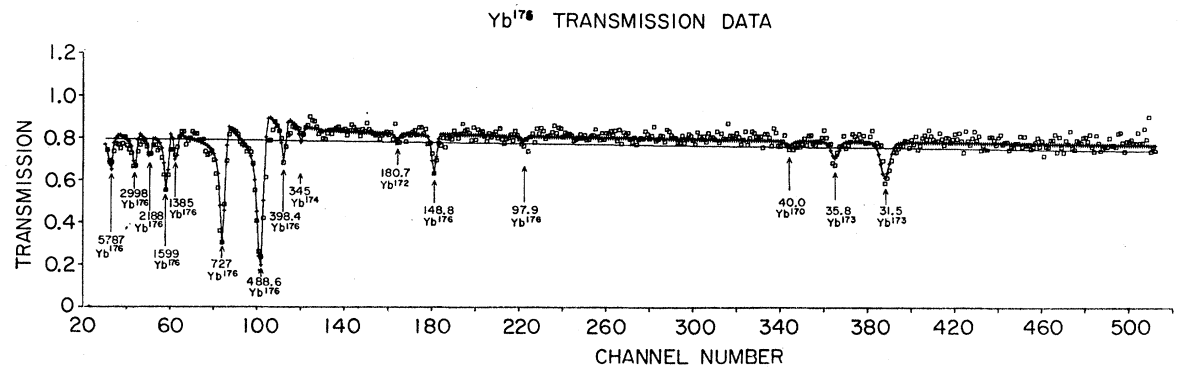

FIG. 3. Fitted transmission data of Yb^{176} in the energy region 5800–18.0 eV. The position of the 97.9-eV resonance found by the Columbia group at 98.18 eV is indicated. Also shown are the strong isotopic impurity resonances.

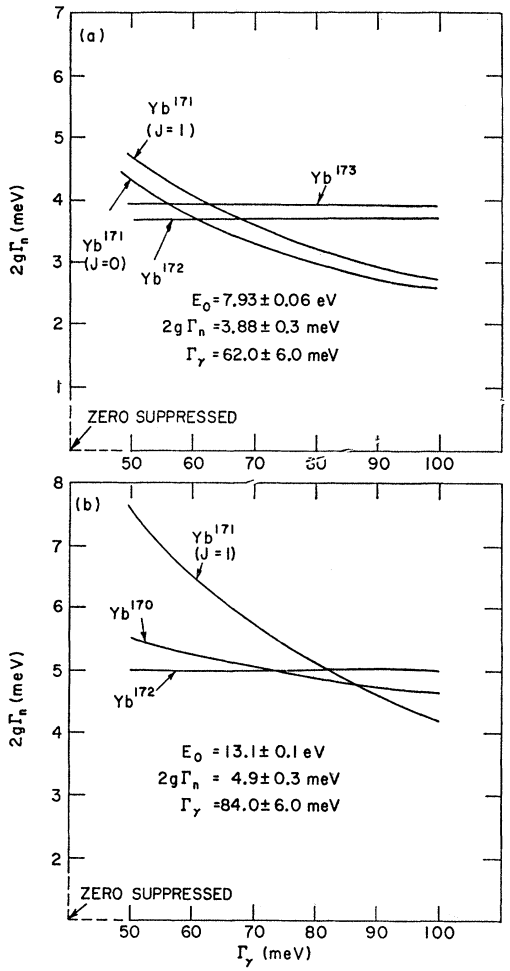

FIG. 4. Radiation widths of the 7.93- and 13.1-eV resonances in Yb^{171} are determined by the thin-thick sample thickness technique. The samples from which the $2g\Gamma_n$-versus-Γ_γ curves are obtained are indicated near each curve.

Γ_γ from resonance to resonance, and (2) the average spin-0 radiation width would be smaller than the spin-1 average width. Differences in Γ_γ for spin-$\frac{1}{2}$ target nuclei have been observed for Se^{77} [18] and Pt^{195}.[19]

The results for the resonance parameters of the even-even and even-odd isotopes are summarized in Tables II and III, respectively. The errors on the neutron widths are derived mainly from counting statistics, uncertainty in determining the base line, and an uncertainty of 7 meV in the radiation width. The resonance at 22.6 eV is observed by Wynchank *et al.*[9] and Wang Nai-Yang *et al.*[10] Both of these groups attributed this resonance either to Yb^{168} or Yb^{170}. Our transmission data on enriched Yb^{170} indicates from Fig. 2(a) that this resonance does not belong to Yb^{170}. Accordingly, we assigned it to

[18] R. E. Coté, L. M. Bollinger, and G. E. Thomas, Phys. Rev. **136**, B703 (1964).
[19] V. D. Huynh, Centre d'Etudes Nucleaires, Saclay, France, Report No. CEA-R 2810, 1965 (unpublished).

TABLE II. Resonance parameters of even-even Yb isotopes. A radiation width 75±7 meV is assumed in the analysis.

E_0(eV)	Γ_n(meV)	Γ_γ(meV)	Γ_n^0(meV)
	Yb^{168}		
0.597	3.3±0.5[a]	70±5[a]	4.3±0.7[a]
22.56[b]	30.4±8.7[b]		6.4±1.8[b]
	Yb^{170}		
8.13±0.06	1.64±0.15		0.583±0.053
40.0±0.2	187±15		29.6±2.4
66.8±0.6	40±7		4.9±0.9
73.4±0.6	55±9		6.4±1.1
95.4±0.8	24±3		2.5±0.3
167.7±2.0	21±4		1.6±0.3
212.4±2.7	225±30		15.4±2.1
269.3±4.1	77±15		4.7±0.9
287.6±4.5	148±22		8.7±1.3
355±6	80±30		4.3±1.1
384±6	65±20		3.3±1.0
394±7	70±20		3.5±1.0
449±8	150±30		6.8±1.4
	Yb^{172}		
139.8±1.5	128±22		10.8±1.9
180.7±2.2	223±32		16.6±2.4
202.2±2.7	16.2±4.9		1.14±0.35
330±6	9±3		0.5±0.2
343±6	12±4		0.65±0.22
465±9	44±11		2.0±0.5
512±11	150±30		6.6±1.3
538±11	53±11		2.3±0.5
630±15	95±30		3.8±1.2
688±17	59±18		2.3±0.7
761±19	730±120		26.5±4.4
818±22	230±60		8.1±2.1
	Yb^{174}		
345±6	440±50		23.7±2.7
885±24	380±95		12.8±3.2
	Yb^{176}		
148.8±1.7	6.95±1.04		0.57±0.09
398.4±7.3	165±40		8.2±2.0
488.6±10.0	2020±400		91.4±18.1
727±18	2700±500		100±19
1385±49	1170±400		31.4±10.1
1599±58	4450±1400		111±35
2188±87	2190±700		46.8±14.9

[a] See Ref. 4.
[b] See Ref. 9.

Yb^{168} and calculated its neutron width from the $ag\Gamma_n$ value given by the Columbia group. Our results are generally in good agreement with those of Wang Nai-Yang[10] and Wynchank *et al.*[9] However, the Columbia data[9] indicates that the resonances at 175.5 eV in Yb^{171} and at 154.8 eV in Yb^{173} are doublets. The resonance at 208.8 eV in Yb^{171} could possibly be a doublet since its neutron reduced width is about six times the average value and since in this energy region we are missing resonances. See Fig. 8.

B. Potential Scattering Radius R'

The technique in extracting a potential scattering radius R' from the data and the corrections necessary to be applied to the transmission data are adequately described in I. It is sufficient to mention here that one

TABLE III. Resonance parameters of even-odd Yb isotopes. For resonances where no radiation width is determined, a value $\Gamma_\gamma=75\pm7$ meV is assumed in the analysis. This is an average value of Γ_γ's determined for Yb^{171} and Yb^{173}.

E_0(eV)	$2g\Gamma_n$(meV)	Γ_γ(meV)	$2g\Gamma_n^0$(meV)
	Yb^{171}		
7.93±0.06	3.88±0.32	62±5	1.38±0.11
13.1±0.1	5.2±0.3	84±6	1.44±0.08
21.8±0.1	0.27±0.03		0.058±0.006
28.0±0.1	3.54±0.24	72±10	0.668±0.045
34.5±0.2	7.5±0.8		1.28±0.14
41.4±0.2	15.1±1.5		2.31±0.23
46.4±0.3	1.70±0.22		0.25±0.03
52.9±0.3	8.5±2.0		1.2±0.3
54.2±0.3	30±5		4.1±0.7
60.2±0.4	9.0±1.0		1.2±0.1
64.8±0.4	16.0±1.5		2.0±0.2
77.1±0.6	22±3		2.5±0.3
81.8±0.6	5.1±1.2		0.56±0.13
84.5±0.7	4.2±0.8		0.46±0.09
95.3±0.8	5.5±0.7		0.56±0.07
107.1±1.0	77±8		7.4±0.8
111.9±1.1	29±3		2.73±0.28
127.0±1.3	32±5		2.85±0.45
139.1±1.5	33±9		2.80±0.76
145.5±1.6	12±4		0.99±0.33
159.9±1.8	90±22		7.1±1.7
165.2±2.0	70±16		5.45±1.25
175.5±2.1	21±6		1.58±0.45
180.0±2.2	23±6		1.72±0.45
194.2±2.5	15±3		1.1±0.2
208.8±2.8	250±30		17.3±2.1
225.2±3.1	30±5		2.0±0.3
	Yb^{173}		
4.51±0.05	0.20±0.02		0.09±0.01
17.7±0.1	31±2	86±15	7.4±0.5
31.5±0.2	79±8	72±15	14.1±1.4
35.8±0.2	48±4	73±10	8.0±0.7
45.4±0.3	32.4±2.4	70±10	4.81±0.36
53.7±0.3	8.9±0.8		1.2±0.1
59.1±0.3	8.1±1.2		1.1±0.2
66.3±0.5	9.6±1.3		1.2±0.2
69.2±0.6	6.3±1.0		0.76±0.12
74.9±0.6	9±2		1.0±0.2
76.3±0.6	36±8		4.1±0.9
95.7±0.8	12±2		1.2±0.2
105.0±1.0	65±10		6.3±1.0
110.2±1.1	12.7±2.0		1.21±0.15
123.4±1.2	22.9±2.5		2.1±0.2
128.1±1.4	73±12		6.5±0.9
133.7±1.4	11±2		0.95±0.17
144.6±1.6	17±3		1.41±0.25
154.8±1.8	7.7±2.8		0.62±0.22
168.1±2.0	49±12		3.78±0.92
179.3±2.2	11.7±2.3		0.87±0.17
196.0±2.6	24±7		1.7±0.5
204.7±2.7	12±4		0.84±0.28
210.3±2.8	29±8		2.0±0.5

must correct for water absorption in the ytterbium oxide samples, which is found to be 0.19% by weight. Values of R' thus obtained are listed in the second column of Table IV.

C. S-Wave Neutron Strength Function

As noted before, two methods were used to derive this quantity: (a) In the low-energy region from the neutron reduced widths of resonances, both in the completely and partially resolved energy regions, and (b) in the 5–15-keV region from the average total cross section.

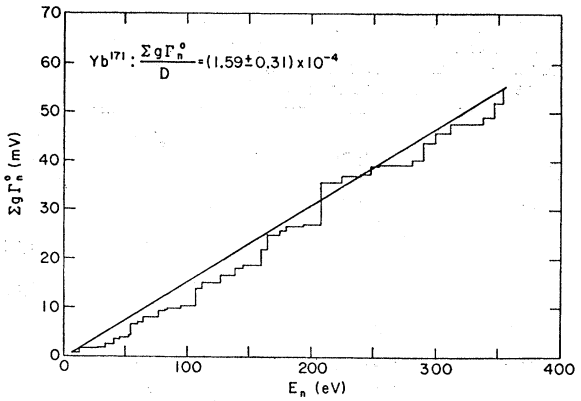

FIG. 5. Cumulative reduced neutron width for Yb^{171} is plotted versus incident neutron energy up to an energy of 354 eV. The slope of the straight line yields a strength function value of $(1.59\pm0.31)\times10^{-4}$ per spin state.

In the first method, the cumulative sum of the neutron reduced widths, weighted by the spin statistical factor $g\Gamma_n^0$, are plotted versus incident neutron energy. The slope of the straight line passing through the points yields the neutron strength function. Figures 5–7 illustrate this method for Yb^{171}, Yb^{173}, and Yb^{170} together with Yb^{172}. The strength function of Yb^{172} as obtained here is rather low when compared with that of Yb^{170}. Such a difference cannot be accounted for in terms of optical-model calculations such as those of Buck and

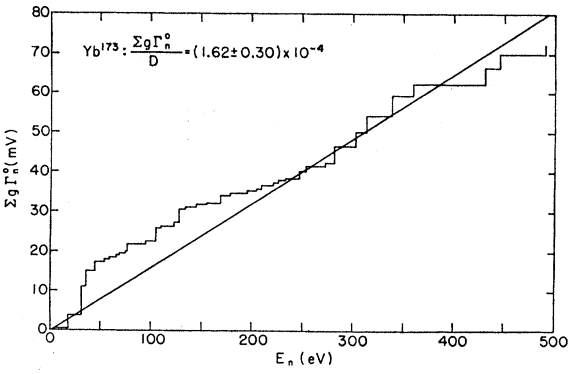

FIG. 6. Strength function per spin state of Yb^{173} as determined in the energy interval 4.51–492 eV is $(1.62\pm0.30)\times10^{-4}$.

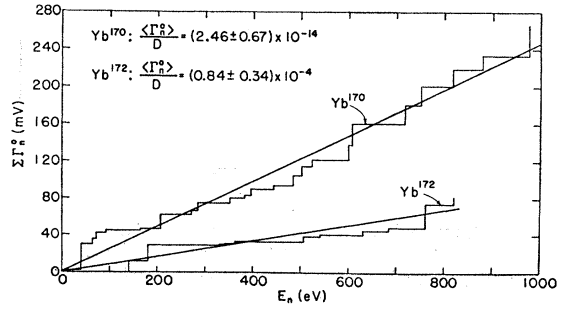

FIG. 7. Strength functions of Yb^{170} and Yb^{172} obtained in the energy regions 8.12–990 eV and 140–818 eV, respectively, are 2.46 ± 0.67 and 0.84 ± 0.34 in units of 10^{-4}.

TABLE IV. Summary of results. R' is the potential scattering radius. The third column gives the strength function determined from individual and "average" resonance parameters determined in the energy region indicated in the fifth column. The strength function as obtained from the average cross section in the 5–15-keV region is represented in the sixth column. The last two columns show the average level spacing and its most probable value, respectively.

Isotope	R'(F)	$10^4\times\langle g\Gamma_n^0\rangle/D$ (from resonances)	$10^4\times\langle g\Gamma_n^0\rangle/D$ (from σ_{av})	Energy range (eV)	D(eV)	D^*(eV)
Yb^{170}	7.3±0.6	2.46±0.67	2.40±0.27	8.13–990	36.8±5.6	$37.5^{+8.5}_{-5.6}$
Yb^{171}	6.3±0.6	1.59±0.31	1.21±0.15	7.93–354	6.5±0.8	$6.4^{+1.1}_{-0.8}$
Yb^{172}	6.3±0.8	0.84±0.34	1.60±0.40	140–818	61.6±9.7	64^{+16}_{-10}
Yb^{173}	7.4±0.6	1.62±0.30	1.86±0.21	4.51–492	8.4±1.0	$8.4^{+1.3}_{-1.0}$
Yb^{174}	6.4±1.0	...	...	...	...	...
Yb^{176}	7.0±0.6	1.70±0.54	...	149–5787	284±53	292^{+79}_{-54}

Perey[20] and Chase *et al.*.[21] which indicate that in this mass region ($A=160$–180) the strength function is a slightly *increasing* function of A. The results thus derived for the strength functions are tabulated in column 3 of Table IV. Also listed in column 5 of the same table is the energy region from which the strength function has been evaluated. The uncertainties attached to the values of the strength functions are derived from the asymptotic relation $(2/N)^{1/2}$, where N is the total number of resonances. An estimate of the number of resonances in the unresolved energy region has been made from a knowledge of the average level spacing D in the resolved energy region. Finally, the results for the strength functions as extracted in the 5–15-keV region from the average total cross section are summarized in column 4 of Table IV. A comparison between the two values of the strength function for each isotope indicates that there is very good agreement between the two methods except for Yb^{172}, where the two values barely overlap within the experimental errors. No definite conclusion about such behavior can be reached within the limitations imposed by the statistical accuracy of these data.

D. Level Spacings and the Spin Cutoff Factor σ

Average level spacings are obtained by the histogram method whereby the integral number of observed resonances is plotted against the incident neutron energy. This is illustrated for Yb^{171} and Yb^{173} in Fig. 8 and for Yb^{170} and Yb^{172} in Fig. 9. Departure from linearity in the plot indicates that levels are being missed because of resolution effects. As noted in Fig. 8, appreciable numbers of levels are being missed in Yb^{171} and Yb^{173} above energies of about 112 and 180 eV, respectively. The average level spacings are collected in column 6 of Table IV. Also listed in the same table is the most probable value of the level spacing, D^*. This is calculated from the Wigner distribution[22] by the method of Muradyan and Adamchuk.[23] The error on D^* is found from the width of the most likely distribution in D at its half-maximum value. As can be noted, the lower error on D approaches the asymptotic value $[(4-\pi)/$

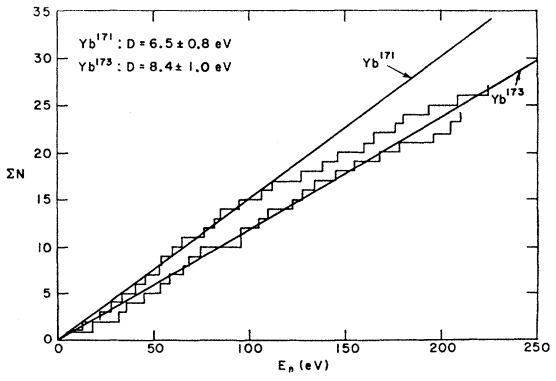

FIG. 8. Cumulative number of observed resonances plotted against incident neutron energy for Yb^{171} and Yb^{173}. Note that appreciable number of resonances are missed above energies of 112 and 180 eV for Yb^{171} and Yb^{173}, respectively.

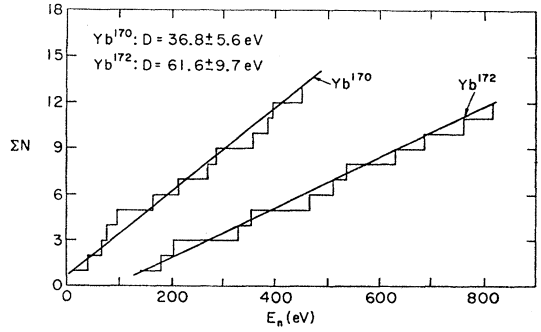

FIG. 9. Average level spacings of Yb^{170} and Yb^{172} as determined from the individual resonances are 36.8±5.6 and 61.6±9.7 eV, respectively.

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TABLE V. Thermal cross sections of Yb isotopes. The total, nuclear scattering, absorption, and paramagnetic scattering cross sections are represented respectively by σ_T, σ_s, σ_a, and σ_{pm}. Each column gives the partial cross sections of a particular isotope. The first column represents the partial cross sections for the natural element, as obtained by compounding it from the isotopes.

	Yb	Yb^{168}	Yb^{170}	Yb^{171}	Yb^{172}	Yb^{173}	Yb^{174}	Yb^{176}
Natural abundance		0.135%	3.03%	14.31%	21.82%	16.13%	31.84%	12.71%
σ_t (b)	67.6±2.0	3799±760[a]	17.0±1.0	57±3	10.2 ±1.5	28±2	142±5	14.9±1.0
σ_s (b)	25.6±3.5	13.8±3.7[a]	2.6±0.5[b]	3.4±0.9[b]	4.3 ±0.9[b]	3.8±0.6[b]	72±5	4.2±0.8[b]
σ_a (b)	36.8±4.1	3780±760[a]	9.4±0.9[b]	48.4±3.3	0.38±0.05[b]	19±2	65±5	5.5±1.0[c]
σ_{pm} (b)	5.2±0.9[d]	5.2±0.9[d]	5.0±1.5	5.2±0.9[d]	5.5 ±1.7	5.2±0.9[d]	5.2±0.9[d]	5.2±1.6

[a] Values calculated from resonance parameters of Refs. 4 and 9.
[b] Values calculated from resonance parameters of present results.
[c] Value adopted from Ref. 36.
[d] Value is an average value found in present experiment.

$\pi N]^{1/2}D$ obtained from the Wigner distribution for level spacings.

It is possible to compare the level spacings of Yb^{171} and Yb^{173} using the level-density formula of Bethe,[24] to deduce the spin cutoff factor σ. Since both of these isotopes are even-odd, two spin states ($J=I\pm\frac{1}{2}$) are formed by s-wave neutron capture. The observed density of states then can be written as

$$\rho_{\text{obs}}=\sum_{J=I-\frac{1}{2}}^{I+\frac{1}{2}}\rho(U,J)$$

or

$$\rho_{\text{obs}}=D_{\text{obs}}{}^{-1}=(2C/U^2)\exp 2(aU)^2 \times\{I\exp[-(I-\tfrac{1}{2})(I+\tfrac{1}{2})/2\sigma^2]+(I+1)\times\exp[-(I+\tfrac{1}{2})(I+\tfrac{3}{2})/2\sigma^2]\},$$

where $U(=S_n+\Delta)$ is the effective excitation energy corrected for neutron or proton pairings (Δ). The neutron separation energies (S_n) are obtained from recent (d,p) and (d,t) measurements[25] on the Yb isotopes. The neutron and proton pairing energies are taken from a table supplied by Gilbert and Cameron.[26] These values are calculated on the basis of a semiempirical mass formula derived recently by Cameron and Elkin.[27] Finally, the only available information on the Fermi gas level-density parameter a of Yb is from the inelastic scattering of high-energy neutrons.[28] The authors of these measurements found $a=15.6$ MeV^{-1} for Yb. However, data on neighboring nuclei[29–33] indicate that this parameter has values ranging from 17–24 MeV^{-1}. In fact, our data on the level spacings of Yb^{170}, Yb^{172}, and Yb^{176} indicate that the value $a=15.6$ MeV^{-1} is low. In view of this, we decided to adopt the value $a=19$ MeV^{-1}, which is predicted by the semiempirical formula of Abdelmalik and Stavisky[34] for Yb^{172}. Another reason for such a choice is for comparison to the result obtained in the consideration of Er^3, in which 19 MeV^{-1} was used. It is of interest to compare the present data with that of I.

With the aid of the above parameters a value of $\sigma=2.8\pm0.8$ is obtained. The error on σ is mainly a reflection of the uncertainty in the a parameter.

This value is in good agreement with a value of $\sigma=2.5\pm0.8$ which was obtained for Er^3 and a value of $\sigma=2.8\pm0.2$ derived by a least-squares fit[2] of the level spacings of the rare-earth isotopes.

E. Thermal Cross Sections, Paramagnetic Scattering, and Absorption Resonance Integrals

At the conclusion of the present experiment dealing with obtaining transmission data for the purpose of resonance parameter analysis, it became clear to us that little was known about the thermal cross sections of the Yb isotopes. For this reason, we decided to extend the energy range of our measurements down to thermal. Accordingly, two transmission runs that overlap in energy were carried out. This supplied us with a convenient method of checking our normalization procedure. The background rate in this energy region was found to be about 1% of the open beam rate. Several corrections were applied to the data before the total cross section of the isotopes were determined. These corrections are due to: (a) isotopic impurities, (b) oxygen scattering since the samples are in oxide form, (c) water absorption by the oxide samples, and (d) dead-time corrections in the electronics and computer. The total cross sections are collected in the second row of Table V. It is possible to decompose σ_T into its partial cross sections:

$$\sigma_T=\sigma_a+\sigma_s+\sigma_{pm},$$

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TABLE VI. Absorption resonance integrals of Yb isotopes. $I_\gamma(+)$ and $I_\gamma(-)$ are the contributions to the absorption resonance integral from positive energy levels and bound levels, respectively. For resonances with undetermined radiation widths, a value of $\Gamma_\gamma=75\pm7$ meV is assumed in the calculations. See the text.

Isotope	Energy range (eV)	$I_\gamma(+)$ (b)	$I_\gamma(-)$ (b)	I_γ(total) (b)
Yb^{168}	0.597–22.6	30 950	···	30 950
Yb^{170}	8.13–1317	326	···	326
Yb^{171}	7.93–354	294	19.2	313
Yb^{172}	139.8–818	23.5	···	23.5
Yb^{173}	4.51–494	390	4.5	394
Yb^{174}	345.0–910	5.0	28.8	33.8
Yb^{176}	148.8–5787	5.3	2.3	7.6

where σ_a, σ_s, and σ_{pm} are the absorption, nuclear scattering, and paramagnetic scattering cross sections, respectively. Now, using the scattering radii R' and the resonance parameters of the present investigations, we generated the nuclear scattering and absorption cross section (due to the contribution from positive energy resonances) for the different isotopes with the aid of the INTTER code.[35] As an illustration, assuming that Yb^{170} and Yb^{172} do not possess bound levels that contribute significantly to the thermal cross sections, we get from the above relation $\sigma_{pm}=5.0\pm1.5$ b and 5.5 ± 1.7 b for Yb^{170} and Yb^{172}, respectively. Furthermore, adopting[36] $\sigma_a=5.5\pm1.0$ for Yb^{176}, we also derive $\sigma_{pm}=5.2\pm1.6$ b for Yb^{176}. Averaging these values, one obtains $\sigma_{pm}=5.2\pm0.9$ b for Yb^{+++} at 0.0253 eV. This is to be compared with a value of 5.0 b obtained by extrapolating $\sigma_{pm}=2.7$ b[37] at 0.0735 to 0.0253 eV, and is to be contrasted[3] with a value of 26.1 ± 2.5 b for Er^{+++}. Furthermore, fitting the difference between the total cross section and the paramagnetic scattering cross section with the relation

$$(\sigma_T-\sigma_{pm})=\sigma_s+\mathrm{b}/E^{1/2},$$

nuclear scattering and absorption cross sections of 72 ± 5 b and 65 ± 5.0 b are obtained for Yb^{174}. The energy dependence of σ_{pm} for Yb is adopted from the calculations of Zimmerman *et al.*[38] Similar approaches applied to Yb^{171} and Yb^{173} indicate that the nuclear scattering cross sections near thermal, due to negative energy resonances, are insignificant for these two isotopes. In view of this, we have chosen the calculated values for the nuclear scattering cross sections for Yb^{171} and Yb^{173}.

This then would lead to absorption cross sections of 48.4 ± 3.3 and 19 ± 2 b for Yb^{171} and Yb^{173}, respectively. We note that these absorption cross sections thus derived are in good agreement with those of Walker,[39] but not with Dobrozemsky *et al.*[40] The results of the latter authors are consistently high, possibly because of the presence of a large component of epithermal neutrons in their spectrum. Finally, weighting all the isotopic cross sections by their relative abundances, one obtains for natural Yb: $\sigma_T=67.6\pm2.0$ b, $\sigma_s=25.6\pm3.5$ b, and $\sigma_s=36.8\pm4.1$ b. These are in very good agreement with previously published values of $\sigma_T=64\pm2$ b,[38] $\sigma_s=30\pm3$ b,[37] and $\sigma_a=37.5\pm4.0$ b.[41] The latter values have been renormalized to the present standard value[14] of the absorption cross section of Au.

Finally, in Table VI, we have listed the calculated values of the absorption resonance integrals for all the isotopes due to the resolved and partially resolved positive energy resonances and from bound levels. Unfortunately, no measurements on the separated isotopes are reported. Only one value for natural Yb is published.[42] In view of this, we have compounded the various isotopic values to get the absorption resonance integral for the element. The value thus obtained is 177 ± 24 b, which is in good agreement with measurements[42] that give 197 ± 20 b. The error on our calculated value is derived from the uncertainties on Γ_n and Γ_γ.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the assistance of M. A. Lone, M. Beer, I. W. Cole, and J. Domish in performing calculations connected with this investigation. Also, we wish to express our thanks to T. E. Stephenson and S. Pearlstein for discussions on the use of INTTER program.

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