Gamma Decay of $T=\frac{3}{2}$ States in Al^{25} and P^{29}†

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The yield curve of the residual Al^{25} activity from the $Mg^{24}(p,\gamma)$ reaction was used to locate the lowest two $T=\frac{3}{2}$ states in Al^{25} at $E_p=5.864\pm0.005$ MeV, $E_x=7.916\pm0.006$ MeV and at $E_p=5.936\pm0.005$ MeV, $E_x=7.985\pm0.006$ MeV. Each of the resonances was narrower than the estimated 3-keV spread in the proton beam. γ-ray spectra were measured with a Ge(Li) detector at both resonances. The lower resonance shows strong, approximately equal transitions to the ground state and to the third excited state in Al^{25}, as well as a weaker branch to the second excited state. The decay of the upper resonance is predominantly that to the ground state, but weaker branches go to the first and second states. The decay schemes support a $\frac{5}{2}^+$ assignment for the lower $T=\frac{3}{2}$ state and favor $\frac{3}{2}^+$ for the upper. Rough angular-distribution measurements on the ground-state γ ray further support these assignments and indicate that the radiations are close to pure $M1$. The various γ-ray transition probabilities are well explained by a rotational picture for the low $T=\frac{3}{2}$ states. γ spectra were also taken at the lowest $T=\frac{3}{2}$ resonance in P^{29}, which had previously been located at $E_x=8.374\pm0.005$ MeV, and it was found to feed the first, second, and third excited states.

I. INTRODUCTION

ANALOG–STATE studies have proved to be useful sources of spectroscopic information since these states tend to have simple shell-model configurations but the Coulomb energy difference is such that they can be observed as proton resonances, in which case the reduced widths for the various decay modes can be extracted directly. Because the states are simple, they tend to have large single-particle reduced widths. The actual proton decay rate is inhibited by an isotopic-spin Clebsch-Gordan coefficient that is equal to $(2T+1)^{-1}$. In heavy nuclei this factor is quite formidable and even above the Coulomb barrier is sufficient to make the width of the analog resonances less than their (average) spacing. However, where the neutron excess is small, as in the lighter nuclei, resonances above the Coulomb barrier become so broad as to be unrecognizable. A special situation exists for $T=\frac{3}{2}$ states in nuclei with $T_z=-\frac{1}{2}$, however, as here proton decay is isospin forbidden and thus the analog resonances are again narrow.

The lowest $T=\frac{3}{2}$ states in the $T_z=-\frac{1}{2}$ nuclei of the series $(A=4n+1,\ Z=2n+1)$ have been identified[1] in the β decay of their $T_z=-\frac{3}{2}$ analogs over the range $2\leqslant n\leqslant 10$. The $T_>$ states in this series are proton unstable; indeed, their original identification was in studies of delayed proton emission.[1] Proton decay is energetically possible only to $T=0$ states in the neighboring $(A=4n,\ Z=2n)$ nucleus but, because of the weakness of the electromagnetic interaction, proton emission can successfully compete with γ decay, which is the only energetically available alternative. It has previously been noted[2] that this fact makes it possible to observe the $T_>$ states as radiative-capture resonances. In a capture reaction the yield integrated over a resonance is proportional to $\Gamma_{p_0}\Gamma_\gamma/\Gamma$, where Γ_γ is the total radiation width and Γ_{p_0} is the width for emission of the ground-state proton. The effect of isospin conservation is that Γ_{p_0} and $\Gamma=\Gamma_p+\Gamma_\gamma$ (in which $\Gamma_p=\sum_i\Gamma_{p_i}$ takes account of inelastic scattering) are made smaller than they would be if the proton channel were not isospin-closed. However, unless isospin conservation inhibits the proton decay by the unreasonably large factor of about 10^4 or more, Γ remains nearly equal to Γ_p and the integrated yield remains proportional to Γ_γ—the main effect of isospin conservation being to concentrate the yield in a narrow resonance. Because low-lying $T_>$ states can be expected to have simple shell-model configurations, there may be strong γ decays to low-lying $T_>$ states—in which case the $T_>$ states will appear as strong, sharp capture resonances. Narrow states can be identified by use of proton beams from a tandem Van de Graaff whose energy spread can readily be kept to a few keV. In contrast, the energy spread in the delayed-proton work was about 160 keV. The lowest $T=\frac{3}{2}$ state in P^{29} has, in fact, been observed[2] as a narrow (p,γ) resonance.

Some $T=\frac{3}{2}$ states in $T_z=-\frac{1}{2}$ nuclei have been observed in elastic scattering of protons.[3–5] In this case

† Work performed under the auspices of the U. S. Atomic Energy Commission.
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[1] R. McPherson, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 162, and references cited therein.
[2] D. H. Youngblood, G. C. Morrison, and R. E. Segel, Phys. Letters **22**, 625 (1966).
[3] H. M. Kuan and S. S. Hanna, Phys. Letters **24B**, 566 (1967).
[4] G. M. Temmer and B. Teitelman, Bull. Am. Phys. Soc. **12**, 554 (1967).
[5] B. Teitelman, J. P. F. Sellschop, G. M. Temmer, and G. T. Garvey, Bull. Am. Phys. Soc. **12**, 570 (1967).

the integrated yield is proportional to Γ_{p0} and therefore, contrary to the case in radiative capture, is reduced by the operation of the isospin selection rules. The only discernible characteristic indicating that these states observed in elastic scattering are analog states is their narrowness, and this does not always signify an analog state. Where the $T=\frac{3}{2}$ states can be positively identified, however, proton scattering studies can determine the partial proton widths.[5]

From the spectrum of protons[6] that are emitted following the β decay of Si^{25}, it has been deduced that the lowest $T=\frac{3}{2}$ state in Al^{25} lies at 7.90±0.02 MeV. The state decays mainly to the first excited state in Mg^{24}, with a 17% branch to the ground state. The nucleus Na^{25}, with $T_z=+\frac{3}{2}$ and mass 25, has been studied by means of the $Na^{23}(t,p)$ and $Mg^{26}(t,\alpha)$ reactions.[7] It was found that the first excited state of Na^{25} is at 90 keV and the next higher state is at 1.07 MeV. The β decay[8] of Na^{25} establishes that its ground state is $\frac{5}{2}+$.

In the present work, we first searched for $T=\frac{3}{2}$ states in Al^{25} by measuring the yield curve of the sum of the captures to the five bound states formed in the $Mg^{24}(p,\gamma)$ reaction. Having identified the two lowest $T=\frac{3}{2}$ states, we next examined their capture-γ-ray spectra and also did some rough angular distributions on the ground-state γ ray. In addition, we measured the γ-ray spectrum from the lowest $T=\frac{3}{2}$ state in P^{29}, which is at 8.37 MeV and has been identified in a previous measurement[2] of the $Si^{28}(p,\gamma)P^{29}$ yield curve.

II. EXPERIMENTAL RESULTS

A. Yield Curves

Yield curves were measured, as before,[2] by counting the residual positron activity in the center crystal of a three-crystal pair spectrometer between bursts of a mechanically chopped beam. Figure 1 shows a portion of the yield curve obtained when protons bombarded an isotopically pure (>99.9%) Mg^{24} target in the form of a self-supporting foil 280 $\mu g/cm^2$ thick. Positrons with energy greater than about 1.3 MeV were counted (the endpoint energy for the β decay of Al^{25} is 3.5 MeV) and their decay curve agreed with the 7.23-sec half-life of Al^{25}. The lower resonance at $E_p=5.864\pm0.005$ MeV represents a state in Al^{25} at 7.916±0.006 MeV, which is in good agreement with the values given by Reeder *et al.*,[6] by Hardy and Skyrme,[9] and by Temmer and Teitelman[4] for the lowest $T=\frac{3}{2}$ state. The spacing of 69±3 keV between the two states is comparable[7] to the 90±10 keV between the ground state and the first excited state in Na^{25}.

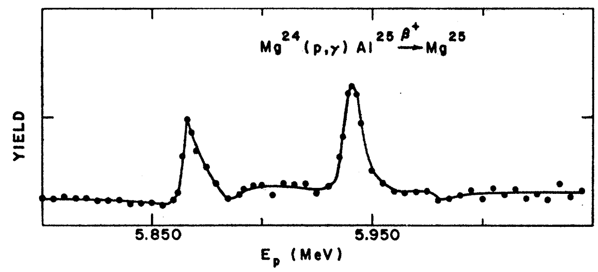

FIG. 1. Yield curve of Al^{25} activity resulting from the capture of protons by Mg^{24}.

The most sensitive direct measure of the width of the resonances is the steepness of the leading edge; in each case the rise occurred in an interval less than 3 keV and is entirely consistent with being due to the energy spread in the proton beam. For the lower resonance, the proton branching ratio is known from the delayed-proton work[6] and thus the limits for the partial widths are $\Gamma_{p0}<0.5$ keV$=\lambda 5\times10^{-5}$ $\Gamma_{s.p.}$ and $\Gamma_{p1}<2.5$ keV $=\lambda 1\times10^{-4}$ $\Gamma_{s.p.}$, where the single-particle width $\Gamma_{s.p.}$ is defined to be $3\hbar^2k_p/2MR$. Similar upper limits are found for the proton widths for the lowest $T=\frac{3}{2}$ state P^{29}. The present results are consistent with those obtained by Teitelman and Temmer,[10] who found the widths of all three resonances to be of the order of 150 eV, in which case the various ground-state and first-excited-state proton branches all have widths $\sim\lambda 10^{-5}\,\Gamma_{s.p.}$.

Particle widths much smaller than the Wigner limit $\Gamma_{s.p.}$ are not, of course, a unique feature of T-forbidden analog resonances; in fact, highly excited nuclei typically have strongly overlapping levels whose widths are very much less than the Wigner limit. These highly excited $T_<$ states are thought to represent complicated configurations wherein a number of nucleons are excited; the narrowness of the states can then be attributed to the long time that elapses before the energy concentrates on a single nucleon (or cluster of nucleons, e.g., an α particle). Only weak γ decay to low-lying levels can be expected from these complicated states, since electromagnetic radiation arises from a one-body operator and low-lying states can be thought of as relatively simple configurations in which only a small number of nucleons are excited. The analog state has but a few nucleons excited and therefore can exhibit strong γ decay, but its particle decays are inhibited by isobaric-spin conservation.

Since the resonances in the present work are too narrow for their shapes to be observed, only the yield integrated over the resonance can be determined. Such integrated yields

$$Y=\frac{1}{\epsilon}\int\sigma dE=\frac{\lambda^2(2J_f+1)}{2\epsilon(2J_p+1)(2J_i+1)}\frac{\Gamma_{p0}\Gamma_\gamma}{\Gamma},$$

[6] P. L. Reeder, A. M. Poskanzer, R. A. Esterlund, and R. McPherson, Phys. Rev. **147**, 781 (1966).

[7] S. Hinds, H. Marchant, and R. Middleton, Nucl. Phys. **31**, 118 (1962); R. Middleton (private communication).

[8] H. E. Gove, G. A. Bartholomew, E. B. Paul, and A. E. Litherland, Nucl. Phys. **2**, 132 (1956).

[9] J. C. Hardy and D. J. Skyrme, Ref. 1, p. 701.

[10] B. Teitelman and G. M. Temmer, Phys. Letters **26B**, 371 (1968).

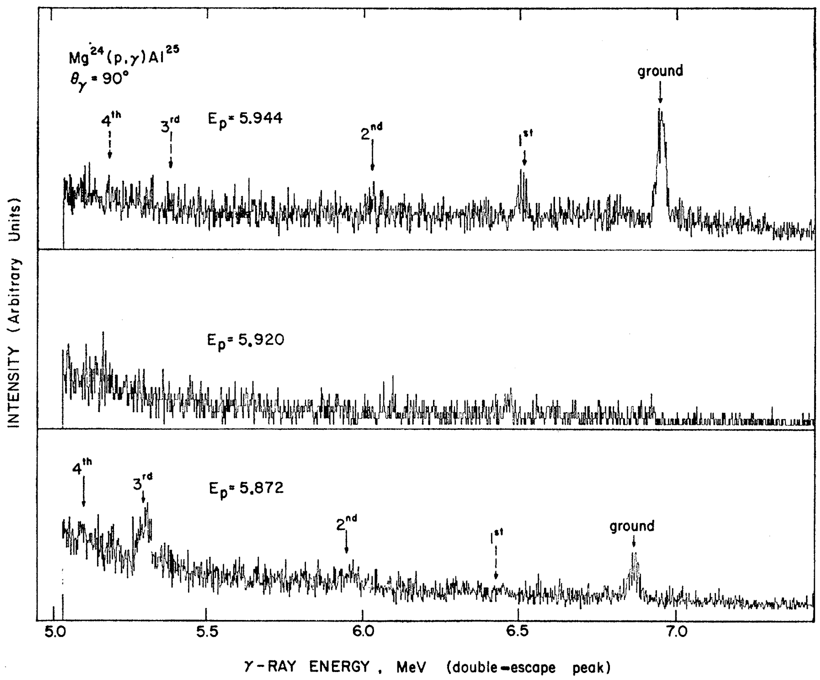

FIG. 2. γ-ray spectra taken on the 5.864- and 5.936-MeV resonances and at an off-resonance energy in $Mg^{24}+p$. Each resonance spectrum was measured at a proton energy at which the yield was at a maximum for a target thick in terms of the energy spread of the system. The Ge(Li) detector was at 90° to the incident beam.

where ϵ is the stopping power, were measured by determining the height of the step in a thick-target yield curve and comparing this yield with the known yield from the 4.025-MeV resonance[11] in $Ca^{40}(p,\gamma)$ and the 5.833-MeV resonance[2] in $Si^{28}(p,\gamma)$. As shown below (Sec. II B 1), the γ decay of the 5.864-MeV resonance in $Mg^{24}(p,\gamma)$ supports the $\frac{5}{2}^+$ assignment expected for this state by analogy[8] with the ground state of Na^{25}. Taking $\Gamma_{p_0}/\Gamma = 0.17$ for this resonance, as indicated by the delayed-proton spectrum,[6] we find $\Gamma_\gamma = (2.0 \pm 1.0)$ eV for the 7.916-MeV state in Al^{25}, where Γ_γ represents the sum of the radiation widths to the five bound states. The energy resolution in the delayed-proton work[6] was not sufficient to resolve the protons from the two lowest $T=\frac{3}{2}$ states. However, a β-decay branch of comparable strength to the upper state is unlikely since the transition to this state is merely an allowed one while the transition to the lower state is superallowed and expected[12] to have $\log ft \approx 3.3$.

As is discussed below (Sec. II B 1), the γ-ray spectrum favors $J=\frac{3}{2}$ for the 7.985-MeV state. This agrees with the $\frac{3}{2}^+$ assignment for the first excited state in Na^{25} that is favored by the $Na^{23}(t,p)$ angular-distribution data.[7] Taking for Γ_{p_0}/Γ the value 0.5 estimated by Teitelman and Temmer,[10] we find $\Gamma_\gamma = 1.5 \pm 0.8$ eV, where the error does not include the uncertainty of Γ_{p_0}/Γ.

A substantially greater radiation width is inferred from electron scattering measurements that have recently been made[13] on the mirror nucleus Mg^{25}. Here the scattering to the two lowest $T=\frac{3}{2}$ states has been observed. Since the two states are not resolved, only the sum of the radiation widths for the two ground-state transitions is determined; it is found that $\sum(2J+1)\Gamma_{\gamma_0} = 28.0 \pm 4.0$ eV. To the extent that T is a good quantum number, the transition rates are the same in Al^{25} as they are in Mg^{25}. In Al^{25} the radiation widths for the lower $\frac{5}{2}^+$ state can be taken as being well determined, since here the proton branching ratio can be obtained directly from the delayed-proton work.[6] If, then, we take the radiation width given above for the $\frac{5}{2}^+$ state and use the γ-ray branching ratios determined in the present work (Sec. II B 1) and the electron scattering data,[13] we find $\Gamma_\gamma = 8.1 \pm 2.0$ eV for the upper resonance.

B. γ-Ray Spectra

1. Al^{25}

γ-ray spectra on and off each resonance were taken with a Ge(Li) detector. Some of the results are shown in Fig. 2. Good-quality spectra were obtained in about 180 min, the rate at which data could be acquired being limited by the total counting rate in the Ge(Li) detector. γ rays from inelastic scattering to the first excited state of Mg^{24} were the prime contribution to the Ge(Li) counting rate. Individual γ rays were clearly discerned on each resonance but none were apparent in the off-resonance spectra. Since only five transitions from the capturing state in Al^{25} lead to bound states, the failure to observe any of these transitions off resonance indicates that most of the off-resonance yield shown in Fig. 1

[11] D. H. Youngblood, B. H. Wildenthal, and C. M. Class, Phys. Rev. **169**, 859 (1968).

[12] J. C. Hardy and B. Margolis, Phys. Letters **15**, 276 (1965).

[13] L. W. Fagg (private communication).

is background and not from Al^{25}. The chief decays of the 5.864-MeV resonance are the transitions to the $\frac{5}{2}^+$ ground state and the $\frac{7}{2}^+$ third excited state at 1.610 MeV. There is a weaker but definitely established transition to the $\frac{3}{2}^+$ second excited state. The radiation widths for the strong transitions are 10–30 times the single-particle estimate[14] for $E2$ transitions which, since collective enhancement would not be expected to accompany a change in isobaric spin,[15] assures that the transitions are predominantly dipole and therefore the spin of the state must be either $\frac{5}{2}$ or $\frac{7}{2}$. The transition to the second excited state would be some four times the single-particle estimate if the 7.916-MeV state were $\frac{7}{2}$—which in itself renders this assignment unlikely. Positive parity is to be expected, since this is a region of the periodic table in which the valence nucleons are in either the $2s$ or the $1d$ orbit; and, indeed, all low-lying states in this region do have positive parity. The ground state of Na^{25}, which should be the analog of the lowest $T=\frac{3}{2}$ state in Al^{25}, is[8] $\frac{5}{2}^+$. However, the two lowest $T=\frac{3}{2}$ states in the $A=25$ system are so close together that it is possible that their order differs for different values of T_z. As noted above (Sec. II A), the $Na^{23}(t,p)$ data[7] indicate that the first excited state in Na^{25} is $\frac{3}{2}^+$; and since the γ-ray data do not permit this assignment for the lowest $T=\frac{3}{2}$ state in Al^{25}, the ordering must remain the same and therefore the 5.864-MeV resonance is $\frac{5}{2}^+$.

In addition to the 90° data shown in Fig. 2, spectra were taken at 45° and at 135° to the incident beam. In order to position the detector at these angles, it was necessary to remove some of the shielding. This adversely affected the signal-to-background ratio and, in fact, the deterioration of the spectrum was such that only the ground-state transition could be measured accurately. Under these conditions, a number of spectra were taken at the two angles with runs at 90° interspersed. As expected, the intensities at 45° and at 135° were similar and were therefore averaged. The ratio, corrected for finite solid angle, was found to be $W(45)/W(90)=1.14\pm0.15$. This value is close to that expected for a pure $M1$ transition, and an $E2$ admixture of about 6% is indicated. The lack of $E2$ enhancement, $|M|^2\approx0.4$, attests to the isobaric-spin purity of the 7.916-MeV $T_>$ state.

The main decay from the 5.936-MeV resonance is to the ground state (Fig. 2); substantially weaker branches go to the 0.455-MeV $\frac{1}{2}^+$ first excited state and the 0.949-MeV $\frac{3}{2}^+$ second excited state. Positive parity is again indicated both on theoretical grounds and in analogy to Na^{25}. The strength of the ground-state transition requires it to be dipole and thus restricts the resonance to $\frac{3}{2}^+$, $\frac{5}{2}^+$, or $\frac{7}{2}^+$; and the presence of the transition to the first excited state eliminates $\frac{7}{2}^+$. In analogy to Na^{25}, $\frac{3}{2}^+$ is the expected assignment since the Na^{25} β decay[8] and the $Na^{23}(t,p)$ angular-distribution[7] data require the ground-state doublet to have spins $\frac{5}{2}^+$ and $\frac{3}{2}^+$, and $\frac{5}{2}^+$ has been assigned to the 5.864-MeV resonance. Our measured ratio for the ground-state transition $W(45)/W(90)=0.78\pm0.15$ is close to the value expected for pure $M1$ radiation only if the resonance is $\frac{3}{2}^+$ or $\frac{7}{2}^+$. A $\frac{5}{2}^+$ assignment would require the $E2$ intensity to be 70% of the $M1$. All of the evidence then combines to establish the $\frac{3}{2}^+$ assignment to the 7.985-MeV state. Figure 3 shows the branching ratios for the various transitions that were observed; Table I lists the radiative widths. Corrections have been made for the γ-ray angular distributions that have been taken to be pure $M1$ for the transitions to the Al^{25} excited states.

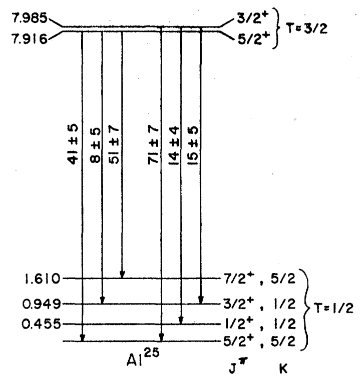

FIG. 3. Energy-level diagram showing the γ-ray decay of the two lowest $T=\frac{3}{2}$ states in Al^{25}. Possible K assignments for the $T=\frac{3}{2}$ states are discussed in Sec. III B.

The region of proton bombarding energy from 7.075 to 7.325 MeV ($9.077\leqslant E_x\leqslant9.317$ MeV) was searched for the resonance corresponding to the analog of the 1.07-MeV second excited state in Na^{25}, which has been observed[16] in the $Mg^{26}(d,He^3)Na^{25}$ reaction to have spin $\frac{1}{2}^+$. This $T=\frac{3}{2}$ state has been observed[9] in the $Al^{27}(p,t)$ reaction to be at 9.17 ± 0.05 MeV in Al^{25}. The state was not observed and an upper limit $\Gamma_{p0}\Gamma_\gamma/\Gamma\leqslant0.33$ eV was determined on the assumption that a resonance $\frac{1}{3}$ as strong as the lowest $T=\frac{3}{2}$ state would have been detected.

2. P^{29}

Figure 4 shows γ spectra taken on and off resonance for the lowest $T=\frac{3}{2}$ state in P^{29}, which is at $E_x=8.374$ MeV. On resonance, transitions to all four bound states are observed; off resonance, decay to the ground state but not to higher states is evident. Thus, in P^{29}, unlike the situation in Al^{25}, there is a measurable amount of off-resonance capture. The intensity of the ground-state γ ray does not materially increase on resonance. It is therefore only transitions to the second, third, and

[14] S. A. Moszkowski, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by Kai Siegbahn (North-Holland Publishing Co., Amsterdam, 1965), p. 881.

[15] E. K. Warburton, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic Press Inc., New York, 1966), p. 99.

[16] D. Dehnhard (private communication).

TABLE I. Partial widths for the various γ-ray transitions from $T=\frac{3}{2}$ to $T=\frac{1}{2}$ states that were observed in the present work. The ratio of the observed width to the single-particle estimate[a] is given under the heading $|M|^2$. Where the $E2$ admixture is not determined, there are three dots in place of the appropriate entry and the transition was assumed to be pure $M1$.

Nucleus	$E_i(T=\frac{3}{2}) \rightarrow E_f(T=\frac{1}{2})$ (MeV)	$J_i^\pi \rightarrow J_f^\pi$	Γ_γ (eV)	$\lvert M_{M1}\rvert^2$	δ^2 ($E2/M1$)	$\lvert M_{E2}\rvert^2$
Al^{25}	$7.916 \rightarrow 0$	$\frac{5}{2}+ \rightarrow \frac{5}{2}+$	0.82	0.087	0.06	0.4
Al^{25}	$7.916 \rightarrow 0.949$	$\frac{5}{2}+ \rightarrow \frac{3}{2}+$	0.16	0.025	...	...
Al^{25}	$7.916 \rightarrow 1.610$	$\frac{5}{2}+ \rightarrow \frac{7}{2}+$	1.02	0.21	...	...
Al^{25}	$7.985 \rightarrow 0$	$\frac{3}{2}+ \rightarrow \frac{5}{2}+$	1.06[b] 5.76[c]	0.11[b] 0.60[c]	0.045	0.4[b] 2.2[c]
Al^{25}	$7.985 \rightarrow 0.455$	$\frac{3}{2}+ \rightarrow \frac{1}{2}+$	0.21[b] 1.14[c]	0.025[b] 0.14[c]	...	...
Al^{25}	$7.985 \rightarrow 0.949$	$\frac{3}{2}+ \rightarrow \frac{3}{2}+$	0.23[b] 1.25[c]	0.030[b] 0.16[c]	...	...
P^{29}	$8.374 \rightarrow 1.277$	$\frac{5}{2}+ \rightarrow \frac{3}{2}+$	0.16	0.024	...	...
P^{29}	$8.374 \rightarrow 2.027$	$\frac{5}{2}+ \rightarrow \frac{5}{2}+$	0.42	0.088	...	...
P^{29}	$8.374 \rightarrow 2.425$	$\frac{5}{2}+ \rightarrow \frac{3}{2}+$	0.22	0.055	...	...

[a] Reference 14.
[b] Derived by use of the value Γ_{p0}/Γ from Ref. 10.
[c] Derived by use of the electron scattering data of Ref. 13.

fourth excited states that contribute to resonant capture. The ground state of Al^{29} is known[17] to be $\frac{5}{2}+$ and, since the first excited state of Al^{29} does not come until 1.40 MeV, a $\frac{5}{2}+$ assignment can be safely assumed for the lowest $T=\frac{3}{2}$ state in P^{29}. This assignment, together with the proton branching ratio determined by the delayed-proton work, has been utilized[2] in extracting a total radiation width (to all bound states) $\Gamma_\gamma=0.8\pm0.3$ eV for the state. The branching ratios are shown in Fig. 5; the various partial widths are listed in Table I.

The region of proton bombarding energy from 7.20 to 7.38 MeV was covered in the $Si^{28}(p,\gamma)$ reaction in a search for the analog of the 1.40-MeV $\frac{1}{2}+$ first excited state[18] in Al^{29}. A resonance $\frac{1}{5}$ as strong as the lowest $T=\frac{3}{2}$ state would have been detected, yet none was found with the upper limit $\Gamma_{p0}\Gamma_\gamma/\Gamma \leqslant 0.4$ eV.

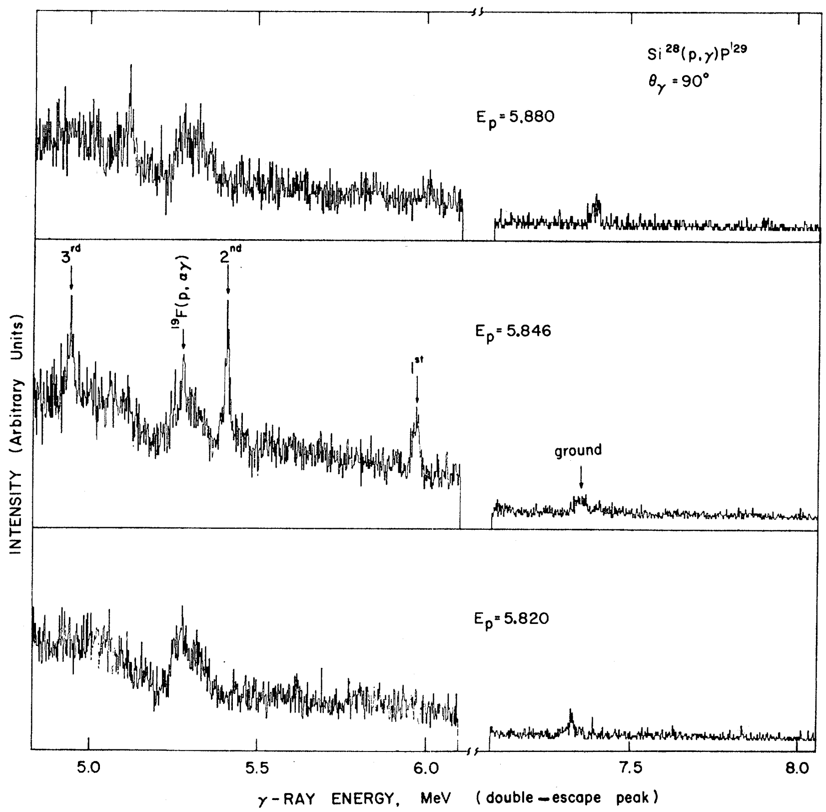

FIG. 4. γ-ray spectra taken on and off the 5.833-MeV resonance in $Si^{28}+p$. The resonance spectrum was measured at a proton energy at which the yield was at a maximum for a target thick in terms of the energy spread of the system. The Ge(Li) detector was at 90° to the incident beam.

[17] A. A. Jaffe, F. de S. Barros, P. D. Forsyth, J. Muto, I. J. Taylor, and S. Ramavataram, Proc. Phys. Soc. (London) **76**, 914 (1960).
[18] R. C. Bearse, J. L. Yntema, and D. H. Youngblood, Phys. Rev. **167**, 1043 (1968).

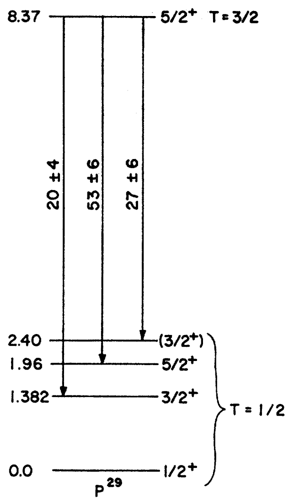

FIG. 5. Energy-level diagram showing the γ-ray decay of the lowest $T=\frac{3}{2}$ state in P^{29}.

III. DISCUSSION

A. Particle Widths

In the case of isospin-allowed analog resonances in heavy nuclei, "external" mixing of the analog state induces a "micro-giant resonance" among the densely packed background of $T_<$ states.[19] In fact, a considerable fraction of the width of the analog state may be the spreading width which characterizes this mixing and leads to the decay "down" into the compound nucleus rather than to direct emission of a nucleon. In light nuclei, however, the density of $T_<$ states in the region of the lowest $T_>$ states is considerably reduced. Therefore, the mixing becomes less and particle widths are the main contributors to the total width. In the region in which the density of $T_<$ levels is too low to be compatible with the existence of a "micro-giant resonance" but nevertheless is high enough to give a good probability for the occurrence of a nearby $T_<$ state, the analog state is sometimes found to split into two states. A graphic example of the effect of the level density on the mixing of the analog states is seen in the $T=\frac{5}{2}$ states in K^{41}, as observed in the high-resolution studies[20] of $A^{40}+p$. In these spectra, the analog states at bombarding energies of 1.87 and 2.45 MeV appear as micro-giant resonances, while the state at about $E_p=1.095$ MeV appears split into two sharp resonances separated by 15 keV.

In the case of an isospin-forbidden analog resonance the analog state has no initial proton width. Rather, such width as it is observed to have is acquired because the Coulomb interaction induces isospin mixing with nearby states of the same spin and parity. Spreading of the T-forbidden state into the nearby states by "external" mixing is expected to be small if not zero. Of course, the spacing of $T=\frac{1}{2}$ states of the same spin and parity at the nuclear excitation of the $T=\frac{3}{2}$ states in the present work is greater than the expected spreading width—even if the decay of the states were isospin allowed. The same is true for the other $T=\frac{3}{2}$ states in $T_z=-\frac{1}{2}$ nuclei that have been studied[2,4,5] as well as for the $T=2$ states in $T_z=0$ nuclei,[21] for which it is the density of $T=1$ levels that is relevant. To date, therefore, it has been possible to study only isolated isobaric-spin-forbidden resonances. Further (p,γ) studies such as those reported here could be useful in examining the question of spreading of isospin-forbidden resonances at excitation energies where there is a higher density of $T_<$ states.

In the present work the widths for the forbidden proton decays provide in principle a measure of the isobaric-spin purity of the resonances. Proton widths ranging up to a good fraction of a single-particle width are found for low-lying analog states whose decay is allowed. The proton decays from the $T=\frac{3}{2}$ states observed here are at least 10^4 times slower. This implies that the amplitude for isospin-mixed impurities is no greater than about 1%.

B. γ Widths

It has been shown[22] that Al^{25} is deformed and that its low-lying levels can be described in terms of rotational bands of a nucleus that has a positive deformation.[23] In this picture [Fig. 6(a)] the $\frac{5}{2}^+$ ground state and the

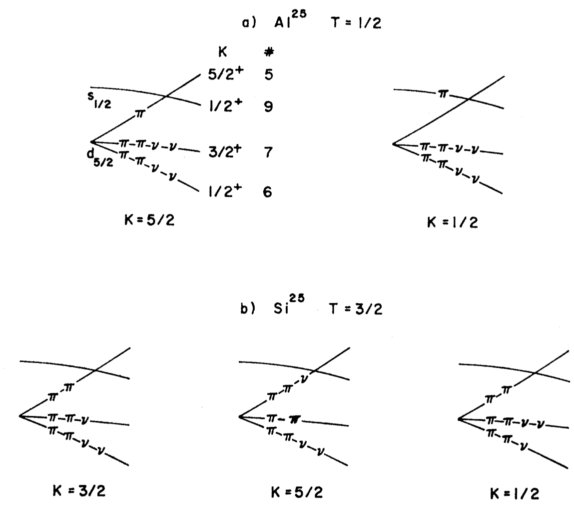

FIG. 6. Schematic diagram showing the filled Nilsson orbitals (a) for intrinsic states of the $K=\frac{5}{2}$ and $K=\frac{1}{2}$ bands that comprise the low-lying $T=\frac{1}{2}$ states in Al^{25} and (b) for intrinsic states of the $K=\frac{3}{2}$, $K=\frac{5}{2}$, and $K=\frac{1}{2}$ bands that comprise the low-lying $T=\frac{3}{2}$ states in Si^{25}; the analogous $T=\frac{3}{2}$ states in Al^{25} are obtained by T^+ operation.

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TABLE II. Transition rates predicted on the assumption that the $T=\frac{3}{2}$ states are members of a $K=\frac{3}{2}$ band described by the Nilsson model.

$(T_iJ_iK_i \to T_fJ_fK_f)$	Γ_γ (calc) $\eta=+6$	$\eta=+\infty$	Γ_γ (obs)	$\log ft$ (calc) $\eta=+6$	$\log ft$ (obs)[a]
$(\frac{3}{2}, \frac{5}{2}^+, \frac{3}{2}^+ \to \frac{1}{2}, \frac{5}{2}^+, \frac{5}{2}^+)$	1.0	0.26	0.82	5.25	5.3
$(\frac{3}{2}, \frac{5}{2}^+, \frac{3}{2}^+ \to \frac{1}{2}, \frac{7}{2}^+, \frac{5}{2}^+)$	1.26	0.33	1.02	4.9	5.4
$(\frac{3}{2}, \frac{3}{2}^+, \frac{3}{2}^+ \to \frac{1}{2}, \frac{5}{2}^+, \frac{5}{2}^+)$	3.59	0.94	1.06[b] 5.76[c]		

[a] P. M. Endt and C. Van der Leun, Nucl. Phys. **A105**, 1 (1967).
[b] Derived by use of the value of Γ_{p0}/Γ from Ref. 10.
[c] Derived by use of the electron scattering data of Ref. 13.

$\frac{7}{2}^+$ third excited state are the two lowest members of a $K=\frac{5}{2}^+$ band Nilsson orbit 5; and the $\frac{1}{2}^+$ first excited state, $\frac{3}{2}^+$ second excited state, and $\frac{5}{2}^+$ fourth excited state are the first three states of a $K=\frac{1}{2}^+$ band built upon Nilsson orbit 9. It is of interest to see whether the $T=\frac{3}{2}$ states can also be described in terms of rotational bands, and in this light we have examined our γ-ray decay data.

For the same deformation, the lowest state in the $A=25$, $T=\frac{3}{2}$ system would be $\frac{3}{2}^+$ with the odd neutron in Si^{25} (or the odd proton in Na^{25}) in Nilsson orbit 7 [Fig. 6(b)] and there would then be a rotational band built upon this level. The next lowest bands are expected [Fig. 6(b)] to be $K=\frac{5}{2}^+$ (the odd nucleon in orbit 5) and $\frac{1}{2}^+$ (the odd nucleon in orbit 6). Band mixing could be expected to take place and of particular importance would be a repulsion between the $\frac{5}{2}^+$ levels in the $K=\frac{3}{2}^+$ and in the $K=\frac{5}{2}^+$ (and perhaps also in the $K=\frac{1}{2}^+$) bands (Fig. 7). Thus the fact that the lowest state in the system is $\frac{5}{2}^+$ can be explained. The $\frac{5}{2}^+$ ground state (i.e., the lowest $T=\frac{3}{2}$) then has components of the $K=\frac{3}{2}^+$ and $K=\frac{5}{2}^+$ and perhaps also $K=\frac{1}{2}^+$ bands, with the $K=\frac{3}{2}^+$ providing the major portion of the wave function; the $\frac{3}{2}^+$ first excited state is largely $K=\frac{3}{2}$. Upon examining the orbits that are occupied in the various bands (Fig. 6), it becomes apparent that the only allowed $M1$ $T=\frac{3}{2} \to T=\frac{1}{2}$ transitions are from the $T=\frac{3}{2}$, $K=\frac{3}{2}^+$ to the $T=\frac{1}{2}$, $K=\frac{5}{2}^+$ bands. This follows both from the $\Delta K=0, \pm 1$ selection rule for dipole transitions and from the fact that the electromagnetic operator is a single-particle operator so that a transition must change the orbit of one and only one nucleon. The experimental results show that, indeed, the strong transitions are to the members of the $\frac{5}{2}^+$ band.

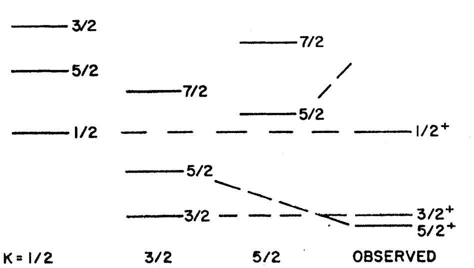

FIG. 7. Approximate unperturbed positions of the levels in the various $T=\frac{3}{2}$ bands (solid lines) with the postulated shifting of the $\frac{5}{2}^+$ levels (dotted lines) as a result of their mutual repulsion.

In the Nilsson model, the $M1$ transition strength of an interband transition $I_1K_1 \to I_2K_2$ is given by[24]

$$B_{M1}(I_1 \to I_2) \propto (I_1 1 K_1 K_2 - K_1 | I_2 K_2)^2 \times \langle \psi_{K_2} | Q_{K_2-K_1} | \psi_{K_1} \rangle^2, \quad (1)$$

where $\langle \psi_{K_2} | Q_{K_2-K_1} | \psi_{K_1} \rangle$ is the reduced $M1$ matrix element between the two intrinsic states. Thus a check on the above picture is to see whether the absolute transition probabilities for $M1$ transitions from $K=\frac{3}{2}$ to $K=\frac{5}{2}$ can account for the observed transition strengths. Such calculations have been made for several assumed deformations and the results are shown in Table II.

The speeds of the various β transitions from the ground state of Na^{25} can also be calculated by use of the same model; the values so obtained are likewise given in Table II. A qualitative agreement is apparent in that the model predicts the transitions to be severely inhibited, with values of $\log ft$ in the 5–5.5 range, and, indeed, the experimental values reported are 5.3 and 5.4, respectively. However, in contradiction to experiment, the model predicts the reduced transition speed to the $\frac{7}{2}^+$ state to be the greater, the discrepancy in the ratio being about a factor of 3. We note that because of its lower energy the transition to the 1.61-MeV state is only about a 5% branch and therefore its reported intensity might contain considerable experimental error. Neither of the two leading impurities in the Na^{25} ground-state wave functions—i.e., neither the $K=\frac{5}{2}$ admixture nor the promotion of two nucleons to Nilsson orbit 9 discussed later in this section—can contribute to the β decay to the $K=\frac{5}{2}$ band for the same reason that neither of these components can contribute to the $M1$ transition rate. The discrepancy must be caused by still smaller parts of the wave function but, since the main component produces such a small β decay rate, it is possible that very small components in the wave function can have a major effect.

Independent of assumptions about the deformation of the nucleus, a meaningful check may be obtained from the ratio of the intensities of various transitions to members of a band from the same initial state. Apart from the usual E_γ^3 factor, the speed of an interband

[24] A. K. Kerman, in *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Co., Amsterdam, 1959), Vol. 1, p. 427.

transition $I_1K_1 \rightarrow I_2K_2$ is simply proportional to the square of the Clebsch-Gordan coefficient of Eq. (1). On the basis of this relationship, the prediction for transitions for the lower resonance to the $\frac{5}{2}^+$ and $\frac{7}{2}^+$ members of the $K=\frac{5}{2}^+$ band is that

$$\frac{|M|^2(\frac{5}{2},\frac{3}{2}\rightarrow\frac{5}{2},\frac{5}{2})}{|M|^2(\frac{5}{2},\frac{3}{2}\rightarrow\frac{7}{2},\frac{5}{2})}=0.40.$$

This is in good agreement with the experimental value 0.41 ± 0.07. The relationship also predicts that the ratio for transitions to the ground state from the two resonances is

$$\frac{|M|^2(\frac{5}{2},\frac{3}{2}\rightarrow\frac{5}{2},\frac{5}{2})}{|M|^2(\frac{3}{2},\frac{3}{2}\rightarrow\frac{5}{2},\frac{5}{2})}=0.29.$$

The experimental value is 0.79 ± 0.12 if the value of Γ_{p0}/Γ given by Teitelman and Temmer[10] is used to determine the radiation widths for the $\frac{3}{2}^+$ resonance but is 0.15 ± 0.02 if the electron scattering data[13] are used. The theoretical result assumes the same fraction of $K=\frac{3}{2}$ band in both $T=\frac{3}{2}$ states and is therefore most likely an overestimate, since the mixing is likely to be greatest for the lowest $T=\frac{3}{2}$ state, for which admixtures of $K=\frac{5}{2}^+$ and possibly $K=\frac{1}{2}^+$ are expected. These admixtures cannot contribute to the transitions to the $T=\frac{1}{2}$, $K=\frac{5}{2}^+$ lower band and therefore should lead to dilution of the strength of such transitions. On the other hand, the second $T=\frac{3}{2}$ state is expected to be more nearly pure $K=\frac{3}{2}^+$, since the nearest $\frac{3}{2}^+$ state will be the third state of the $\frac{1}{2}^+$ band, and should be quite far away in energy. Thus, band mixing is expected to weaken the ground-state transition from the 7.916-MeV state relative to that from the 7.985-MeV state. The theoretical picture is in agreement with experiment only if the widths derived from the electron scattering are the more correct. If these larger widths are accepted as the correct ones, the experimental value of the ratio $\Gamma_{\gamma0}(\frac{5}{2},\frac{3}{2}\rightarrow\frac{5}{2},\frac{5}{2})/\Gamma_{\gamma0}(\frac{3}{2},\frac{3}{2}\rightarrow\frac{5}{2},\frac{5}{2})$ is less than the theoretical ratio by somewhat more than would be expected from the band mixing; but in view of the uncertainties in the various measurements, the discrepancy does not appear to be a serious one.

The transitions to the members of the $T=\frac{1}{2}$, $K=\frac{1}{2}^+$ band still remain to be explained. The intrinsic low-lying $T=\frac{1}{2}$ states can be expected to be well represented by a single Nilsson orbit, orbit 5 for the $K=\frac{5}{2}$ band and orbit 9 for the $K=\frac{1}{2}$ band. Therefore the impurities that are responsible for these transitions must be sought in the $T=\frac{3}{2}$ states. Indeed if the nuclear deformation is about the same for the $T=\frac{3}{2}$ states as it is for the low-lying $T=\frac{1}{2}$ bands, the orbits 5 and 9 will lie close in energy. It is therefore quite likely that the $T=\frac{3}{2}$ states contain admixtures in which a pair of protons is raised from orbit 5 to orbit 9 (Fig. 6). Magnetic-dipole transitions to members of the first $T=\frac{1}{2}$, $K=\frac{1}{2}^+$ are then possible from the $T=\frac{3}{2}$ states with $K=\frac{3}{2}^+$ (in which case the nucleon makes a transition from orbit 9 to orbit 7) or with $K=\frac{1}{2}^+$ (with the nucleon making a transition from orbit 9 to orbit 6). On the assumption that the upper resonance is more nearly pure $K=\frac{3}{2}$, the ratio

$$\frac{|M|^2(\frac{3}{2},\frac{3}{2}\rightarrow\frac{1}{2},\frac{1}{2})}{|M|^2(\frac{3}{2},\frac{3}{2}\rightarrow\frac{3}{2},\frac{1}{2})}=1.2,$$

which agrees reasonably well with the experimental value 0.8 ± 0.3. The large uncertainty arises from the weakness of the observed transitions. In the next paragraph we show that the $K=\frac{3}{2}^+$ admixture is also preferred in the lower resonance.

These same impurities are, of course, responsible for the β decays from the Na^{25} ground state to the $K=\frac{1}{2}^+$ states in Mg^{25}. The β-decay matrix element to $K=\frac{1}{2}^+$ states is very large for the configuration with $K=\frac{3}{2}^+$ arising from raising two protons from orbit 5 to orbit 9. This is because the transition from orbit 9 to orbit 7 is nearly a pure spin-flip transition, and is much bigger than the transition connecting orbit 9 to orbit 6, or the transition from orbit 5 to orbit 7 which determines the strength of β decay to the $K=\frac{3}{2}^+$ states from the dominant part of the $T=\frac{3}{2}$ ground state. In fact, this large matrix element is not very sensitive to deformation, and one can estimate what amount of admixture in the $T=\frac{3}{2}$ ground state will account for the $\log ft$ observed in decay to the $\frac{3}{2}^+$, $K=\frac{1}{2}$ state. The intensity of admixture needed lies between 8% for $\eta=+4$ and 5% for $\eta=+\infty$, so it will not seriously perturb the rotational picture for the $T=\frac{3}{2}$ states.[25] Since the same matrix elements contribute to $M1$ γ decay, only small impurities are needed in order to account for the γ transitions observed to the $K=\frac{1}{2}^+$ states. (An alternative explanation of the observed β decay to the $K=\frac{1}{2}^+$ states in terms of an axially asymmetric model of Mg^{25} has recently been proposed,[26] but it appears to be less successful.)

Another experimental observation that the rotational picture explains is the failure to observe the resonance that corresponds to the third $T=\frac{3}{2}$ state that is near 9.17 MeV. This $\frac{1}{2}^+$ state is the lowest member of the $\frac{1}{2}^+$ band and no $M1$ strength would be contributed by the main component in the wave function. Transitions would arise through the impurities in the wave functions that are described above but these, of course, will be weak. The combination of low spin—so that $(2J+1)$ is small—and small transition strength makes it plausible that the resonance corresponding to the third $T=\frac{3}{2}$ state was not detectable in the present work. We also note that transitions to the barely unbound fifth and sixth excited states (which are the first two members of the $K=\frac{1}{2}$ band based on Nilsson orbit 11)

[25] D. Kurath (private communication).
[26] S. Das Gupta, Nucl. Phys. **A97**, 481 (1967).

should be completely excluded on the model. Therefore, transitions to bound states can be relied upon to account for the bulk of the β activity of Al^{25}.

It thus appears that all of the experimental data reported here fit well into a rotational picture with a nuclear deformation similar to that which fits the spectrum of low-lying states. More detailed knowledge of the relative $K=\frac{3}{2}^+$ purity of the $T=\frac{3}{2}$ states is necessary to determine if a single value of the deformation parameter can simultaneously explain all of the measured values.

It is tempting to try to apply this same picture to other cases—in particular to P^{29}. However, for mass 29 a clear-cut rotational description of the low-lying states does not seem to be possible. Therefore, even if such a picture could be developed for the $T=\frac{3}{2}$ states, it would be difficult to test. Other cases may be more favorable but such have yet to be investigated.

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Pion Double-Charge-Exchange Cross-Section Measurements*

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The differential cross section for the pion double-charge-exchange reaction $V^{51}(\pi^+,\pi^-)$, at an incident energy of 31 MeV and a production angle of 29°, has been measured to be $0.52_{-0.19}^{+0.71}$ μb/sr. The apparatus was sensitive to a pion-energy difference of 7 to 19 MeV, which covers the predicted isobaric analog state of $\Delta T_Z=2$. A similar measurement for $Li^7(\pi^+,\pi^-)$ gave a differential cross section of $0.165_{-0.060}^{+0.225}$ μb/sr. Particle trajectories were photographed in thin-foil spark chambers. The charge and momenta of outgoing particles were determined with a magnet, and their range was determined in a thick-plate spark chamber.

I. INTRODUCTION

A PION double-charge-exchange reaction is a pion nuclear reaction of the type $\pi^\pm+(Z,N)\rightarrow\pi^\mp+(Z\pm2,N\mp2)$. A number of calculations and experiments relating to these reactions have been performed since 1963. It has been expected that these reactions might be useful in producing new nuclides with large excesses of neutrons or protons, in exciting isobaric analog states with $\Delta T_3=\pm2$, and in testing for correlations of nucleons within the nucleus.[1–5] These expectations are not yet experimentally well fulfilled.

Several theoretical estimates have been made of the double-charge-exchange cross section for low-energy incident pions. Kohmura predicted a total double-charge-exchange cross section of 15 μb for 10-MeV positive pions on O^{18}.[6] Kerman and Logan give tables which, when interpolated for 30-MeV pions on V^{51}, predict a total cross section of about 1 μb.[7] Parsons, Trefil, and Drell give a forward differential cross section for 30-MeV pions on O^{18} of about 0.06 μb/sr to the Ne^{18} analog state and about an equal cross section to other Ne^{18} excited states.[8]

The theory of Koltun and Reitan[9] predicts a large cross section at low energy. This theory uses a phenomenological single-charge-exchange Hamiltonian in second-order Born approximation. In the zero-energy limit, it gives an isotropic-differential cross section

$$d\sigma/d\Omega=7.6(q'/q)|\langle f|T_+T_+|i\rangle|^2\,\mu\text{b/sr} \qquad (1)$$

for the reaction $\pi^+\rightarrow\pi^-$. The T_+ is the nuclear isospin raising operator and the squared matrix element is equal to $(N-Z)(N-Z-1)/2$. Because of the raising operators, the final state should be an isobaric analog of the initial state with $\Delta T_3=2$. The q and q' are the initial and final pion momenta. They differ because of

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