relations and by coincidences of the 1261.3-keV with the 1077.4-keV transitions. Taylor and McPherson[7] and Ramaswamy and Jastram[12] have measured the directional correlation of the 1261.3-1071.4-keV cascade and each assigns a 2-2-0 sequence to the cascade with $\delta = 1.8$ and 2.25, respectively.

There is strong evidence in (p,p')[4] and (d,d')[2] work that the level at approximately 2.37 MeV is a doublet. From (p,t) work,[3] McIntyre reports a level at 2.34 MeV for which the angular distribution has $l = 2$. If this state is indeed a doublet as indicated in the reaction studies (see Table I), then one of these states is undoubtedly the 2+, 2338.7-keV state. Since another level does not appear to be populated in β decay from the 1+ ground state of ^{68}Ga, the second member of the doublet probably has spin 3 or higher or has odd parity.

There is no direct evidence for a level at 2.68 MeV as indicated by Taylor and McPherson[7] though very weak population of such a state cannot be completely excluded. The state at about 2.75 MeV seen in reaction studies (see Table I) is not observed to be populated in the ^{68}Ga decay. This state has been assigned a 3$^-$spin and parity and should not be populated by the 1+ ground state of ^{68}Ga.

A level at 2823.8 keV is now definitely established. The energy of this level is more precisely determined now from the 1746.6-keV energy, which is 47 keV higher than the most recent measurement of Taylor.[8] From the 1746.6-1077.4-keV directional-correlation work, Taylor[8] suggests a 2+ spin assignment for this level.

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Spins of Levels in ^{33}P†

C. E. Moss, R. V. Poore, and N. R. Roberson
Nuclear Structure Laboratory, Duke University, Durham, North Carolina 27706

and

D. R. Tilley
North Carolina State University, Raleigh, North Carolina 27607
and
Nuclear Structure Laboratory, Duke University, Durham, North Carolina 27706

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The method-II angular-correlation technique of Litherland and Ferguson has been applied to the reaction $^{30}\text{Si}(\alpha,p\gamma)^{33}\text{P}$ to determine J^π for the following levels in ^{33}P: $\frac{3}{2}^+$ (1.44 MeV), $\frac{5}{2}^+$ (1.85 MeV), and $\frac{3}{2}^+$ (2.54 MeV). Mixing and branching ratios for the γ decays of these states have also been obtained. The γ-ray spectrum has been examined with a Ge(Li) detector to determine the following excitation energies for these states: 1.435±0.003, 1.850±0.003, and 2.544±0.004 MeV.

I. INTRODUCTION

The shell-model calculations of Glaudemans *et al.*[1] and the intermediate-coupling calculations of Bouten *et al.*,[2] which predicted the properties of many low-lying levels of nuclei in the mass region $28 < A \leq 40$, emphasized the need for experimental studies of all of the low-lying levels in this region. When the former of these calculations was published no excited states of ^{33}P were known. Since then Currie and Evans[3] have located the first three excited states in ^{33}P from a study of the reaction $^{30}\text{Si}(\alpha,p)^{33}\text{P}$ with surface-barrier detectors. Recently Bearse *et al.*[4] have obtained excitation energies for the first excited states from a study of the reaction $^{30}\text{Si}(\alpha,p)^{33}\text{P}$ with a magnetic spectrograph. Bearse *et al.*[4] also obtained l values and spectroscopic factors for the reaction $^{34}\text{S}(d,^3\text{He})^{33}\text{P}$ to the ground state and the second excited state. Davies *et al.*[5] have studied the reaction $^{31}\text{P}(t,p)^{33}\text{P}$ and obtained l values and excitation energies for several levels.

In the present work the method-II angular-correlation technique of Litherland and Ferguson[6] has been applied to the reaction $^{30}\text{Si}(\alpha,p\gamma)^{33}\text{P}$ to determine the spins of the first three excited states of ^{33}P and the

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mixing ratios and branching ratios for the γ decays of these states. In addition, the γ-ray spectrum has been examined with a Ge(Li) detector to determine excitation energies for the first three excited states.

II. EXPERIMENTAL PROCEDURE FOR CORRELATIONS

The correlation measurements were performed with a 15-cm-diam scattering chamber which has been described previously.[7] A beam of $\approx 0.1\ \mu$A of 8-MeV ^{4}He^{++} particles from a 4-MeV Van de Graaff bombarded a self-supporting foil of $\approx 150\ \mu$g/cm^2 of SiO$_2$ enriched to 95.5% in ^{30}Si. This target was made by evaporating CsI$_2$ and then SiO$_2$ from tantalum boats on to a glass slide and floating off the SiO$_2$. The protons were detected in a 100-μ annular surface-barrier detector collimated to detect particles between 172° and 175°. The observed resolution for protons from the 1.44-MeV level in ^{33}P (see Fig. 2) was 180 keV. The γ rays were detected in a 7.62-cm (diam) $\times$ 7.62-cm (length) NaI(Tl) crystal mounted on an Amperex 58 AVP photomultiplier tube and placed so that the front face of the crystal was 19.8 cm from the target. Measurements were made at $\theta_\gamma = \pm 26°$, $\pm 40°$, $\pm 50°$, $\pm 65°$, and $\pm 90°$ in a random order, and several angles were repeated. Approximately 1000 μC of charge were accumulated at each angle. The data at positive and negative angles were averaged before analysis.

A simplified block diagram of the electronic setup is shown in Fig. 1. Fast rise-time signals were obtained from the surface-barrier detector with a time pickoff circuit which was placed after the preamplifier rather than before it to increase the detection efficiency for low-energy particles. Fast rise-time signals from the γ-ray detector were obtained directly from the anode of the 58 AVP photomultiplier. These signals were fed into a time-to-pulse-height converter (TPHC). The linear signals from the particle detector, the TPHC, and the γ-ray detector were fed to analog-to-digital converters (ADC's). The output of an integral discriminator set on the output of the TPHC, gated these ADC's on when a coincidence occured. The three digitized parameters were then stored on magnetic tape by the Duke University Nuclear Structure Laboratory DDP-224 computer (Honeywell, Computer Control Division). For normalization, a particle monitor spectrum was also obtained by fanning out the particle signal through a fourth ADC which was gated by the fast rise-time singal from the time pickoff circuit. The computer was programmed to display the monitor spectrum and the time and γ-ray spectra in coincidence with as many as four particle windows. All of these spectra were written on the magnetic tape and listed on the line printer at the end of each run.

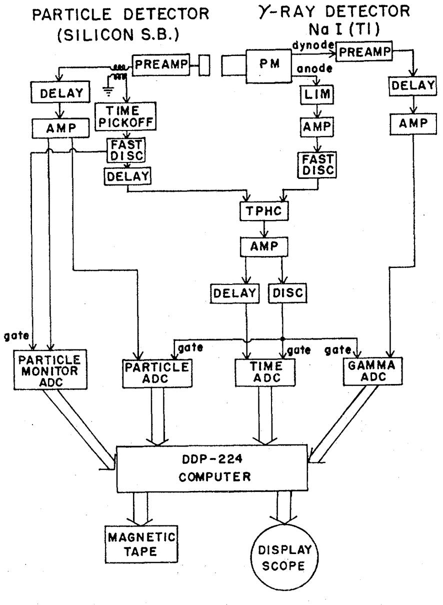

FIG. 1. Block diagram of the electronics.

After completion of the experiment, the data previously stored on magnetic tape were read back into the computer for further processing. In the particle monitor spectrum for each run the peak from the elastic scattering of α particles from ^{30}Si was summed with a light pen and later used to normalize the correlations after being corrected for dead time. In order to analyze the correlation data which was stored event by event, a program was written to total the data in windows set on two of the three parameters and to produce a spectrum in the third parameter. Each of the γ-ray spectra shown in Fig. 2 were obtained by totaling all of the runs with a window set on the corresponding particle group (shown shaded in Fig. 2), with a window ≈ 30 nsec wide set on the peak in the time spectrum to add the true coincidences, and with a window ≈ 30 nsec wide set on a flat part of the time spectrum to subtract off the chance coincidences. After totaling, the spectra were displayed on the scope so that the light pen could be used to sum peaks. The spectra could also be listed and plotted on the line printer.

III. ANALYSIS OF THE CORRELATIONS

The method of analysis is similar to that described by Poletti and Warburton[8] for use with method II of Litherland and Ferguson.[6] The angular distribution of the de-excitation γ rays from a state of spin a to a state of spin b is given by

$$W(\theta)=\sum_{\alpha} I(\alpha) \sum_{k} \rho_k(a,\alpha)F_k(a,b,x)Q_kP_k(\cos\theta)\,,$$

where the notation is similar to that of Poletti and Warburton.[8] The phase convention of Rose and Brink,[9]

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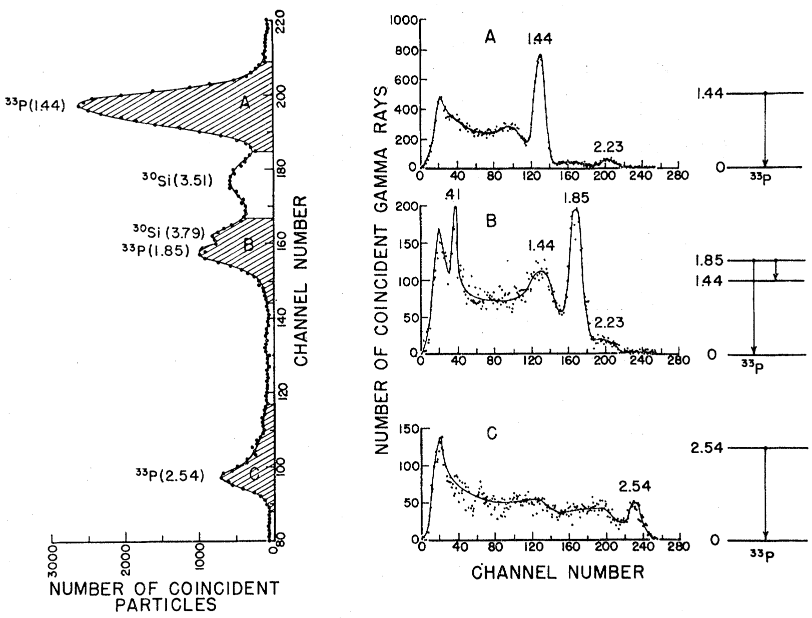

FIG. 2. A coincidence particle spectrum and coincidence γ-ray spectra for the reaction ^{30}Si$(\alpha,p\gamma)^{33}$P at $E_\alpha=8.0$ MeV are shown. To obtain these spectra, the data at all angles were summed after subtracting off chance coincidences. The groups in the particle spectrum are labeled with the residual nucleus and the excitation energy. The γ-ray spectra A, B, and C were obtained in coincidence with the shaded regions A, B, and C in the particle spectrum. The γ-rays are labeled by transition energies given in MeV.

which is equivalent to setting $\sigma=0$ in formula (11) of Poletti and Warburton,[8] was used. A least-squares fit of this formula to the data was performed. Because x occurs nonlinearly, the data were first fitted by varying $I(\alpha)$ for discrete values of x to obtain

$$Q^2(x)=\frac{1}{n}\sum_i\frac{[Y(\theta_i)-W(\theta_i)]^2}{E^2(\theta_i)},$$

which is plotted in Sec. IV. The minima of $Q^2(x)$ with respect to x, which are defined as χ^2, and the corresponding statistical errors in x at the minima were then found by an adaptation of a procedure given by Smith[10] for γ-γ correlations. For the reaction ^{30}Si$(\alpha,p\gamma)^{33}$P ideally only $\alpha=\frac{1}{2}$ is allowed but because of the finite size of the particle detector there may be some contribution from $\alpha=\frac{3}{2}$. The effect of this was seen by repeating the fit with $I(\frac{3}{2})=0.1I(\frac{1}{2})$, and some of these fits are shown in Sec. IV. The programs for these calculations have been given elsewhere.[11] The Q_k were calculated by numerically integrating the formula given by Rose.[12] The coefficients ρ_k, F_k, and P_k were calculated in subroutines.

IV. RESULTS FROM THE CORRELATIONS

A. 1.44-MeV Level

Figure 2 shows the γ-ray spectrum obtained in coincidence with the protons corresponding to the state at 1.44 MeV in ^{33}P after the chance coincidences have been substracted. It was assumed that the 2.23-MeV γ rays came from the high-energy tail of the ^{30}Si$(\alpha,\alpha')^{30}$Si (3.51 MeV) group. The 3.51-MeV level of ^{30}Si is known to decay through the first excited state at 2.23 MeV.[13] The γ-ray spectrum obtained in coincidence with the ^{30}Si (3.51) group was used to subtract this contribution from the sum of the counts in the 1.44-MeV photopeak. The correlation is shown in Fig. 3(a) and $Q^2(x)$ versus arctan$x$ is shown in Fig. 3(b) for various values of the spin of the 1.44-MeV level. The dashed curve shows the effect of setting $I(\frac{3}{2})=0.1I(\frac{1}{2})$. Only $J=\frac{3}{2}$ gives a fit and $x=0.63\pm0.17$. The curve in Fig. 3(a) is for this value of x. Since Davies *et al.*[5] reported $l=2$ for the reaction ^{31}P$(t,p)^{33}$P to this level, the parity is positive and the decay is an $E2$-$M1$ mixture.

B. 1.85-MeV Level

Figure 2 shows the γ-ray spectrum obtained in coincidence with the protons corresponding to the state at 1.85 MeV in ^{33}P after the chance coincidences have been subtracted. It was assumed that the 2.23-MeV γ rays came from the low-energy tail of the ^{30}Si (3.51) group and from the ^{30}Si (3.79) group, and again the contribution from these γ rays has been subtracted. The correlation for the 1.85-MeV photopeak is shown in Fig. 4(a) and $Q^2(x)$ versus arctan$x$ is shown in Fig. 4(b) for various values of the spin of the 1.85-MeV level. The dashed curve shows the effect of setting $I(\frac{3}{2})=0.1\times I(\frac{1}{2})$. Only $J=\frac{5}{2}$ gives a fit, and $x=0.06\pm0.04$ or 2.4 ± 0.3. The curve in Fig. 4(a) is for these values of x. Since Bearse *et al.*[4] reported $l=2$ for the reaction ^{34}S$(d,^3$He$)^{33}$P to this level and Davies *et al.*[5] reported $l=2$ for the reaction ^{31}P$(t,p)^{33}$P to this level, the parity is positive and the decay is an $M3$-$E2$ mixture. The

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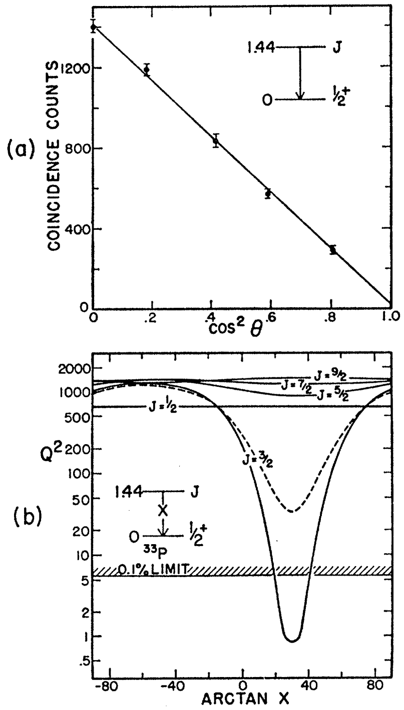

Fig. 3. (a) The angular correlation obtained from the photopeak of the 1.44-MeV γ ray from the 1.44-MeV level is shown. The curve was calculated with the parameters corresponding to the minimum of Q^2 shown in (b) with $J=\frac{3}{2}$ and $I(\frac{3}{2})=0$. (b) Plots of Q^2 versus arctan$x$ for $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$, and $\frac{9}{2}$ are shown. The dashed curve shows the effect of setting $I(\frac{3}{2})=0.1\ I(\frac{1}{2})$ for $J=\frac{3}{2}$. Only $J=\frac{3}{2}$ gives a fit below the 0.1% confidence level.

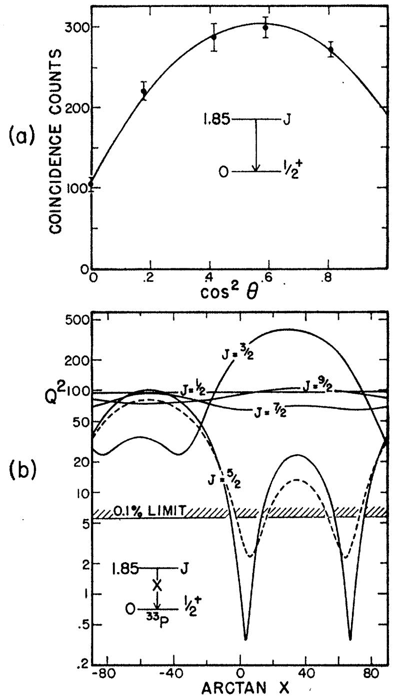

Fig. 4. (a) The angular correlation obtained from the photopeak of the 1.85-MeV γ ray from the 1.85-MeV level is shown. The curve was calculated with the parameters corresponding to one of the minima of Q^2 shown in (b) with $J=\frac{5}{2}$ and $I(\frac{3}{2})=0$. The curve for the other minimum is identical. (b) Plots of Q^2 versus arctan$x$ for $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$, and $\frac{9}{2}$ are shown. The dashed curve shows the effect of setting $I(\frac{3}{2})=0.1\ I(\frac{1}{2})$ for $J=\frac{5}{2}$. Only $J=\frac{5}{2}$ gives a fit below the 0.1% confidence level.

value $x=0.06\pm0.04$ is favored because a large amount of $M3$ is unlikely.

To obtain the branching ratio to the 1.44-MeV level, runs of approximately equal length at the angles previously mentioned in Sec. II were added together and the photopeaks were summed with a light pen. Then the total efficiencies and photofractions[14] were used to calculate the branching ratio of 6±2% to the 1.44-MeV level.

C. 2.54-MeV Level

Figure 2 shows the γ-ray spectrum obtained in coincidence with the protons corresponding to the state at 2.54 MeV in ^{33}P after chance coincidences have been subtracted. The correlation for the 2.54-MeV photopeak is shown in Fig. 5(a) and $Q^2(x)$ versus arctan$x$ is shown in Fig. 5(b) for various values of the spin of the 2.54-MeV level. The dashed curve shows the effect of setting $I(\frac{3}{2})=0.1I(\frac{1}{2})$. Both $J=\frac{3}{2}$ and $J=\frac{1}{2}$ give fits but $J=\frac{3}{2}$ is favored. Since Bearse *et al.*[4] reported $l=2$ for the reaction ^{34}S$(d,{}^3\mathrm{He})^{33}$P to this level and Davies *et al.*[5] reported $l=2$ for the reaction ^{31}P$(t,p)^{33}$P to this level, $J^\pi=\frac{3}{2}^+$, and the decay is an $E2$-$M1$ mixture. The values for the mixing ratio are $x=-0.16\pm0.04$ or 2.6 ± 0.3. The curve in Fig. 5(a) is for these values of x.

The correlation results are summarized in Table I.

Table I. Results of correlation measurements.

Level No.	J^π	Transition	% Branching	x[a]
1	$\frac{3}{2}^+$	$1\rightarrow0$	100	0.63 ± 0.17
2	$\frac{5}{2}^+$	$2\rightarrow0$	94±2	0.06 ± 0.04 or 2.4 ± 0.3
		$2\rightarrow1$	6±2	···
3	$\frac{3}{2}^+$	$3\rightarrow0$	100	-0.16 ± 0.04 or 2.6 ± 0.3

[a] The phase convention is that of Ref. 9.

V. Ge(Li) SPECTRUM

The following measurements were made to determine accurate excitation energies for the first three excited states in ^{33}P. The same target described above was placed in a 9-cm-diam scattering chamber and bombarded with ≈30 nA of 8-MeV ^{4}He^{++} particles. The γ rays were detected without coincidence in a 20-cc coaxial Ge(Li) detector at 90° with the front of the detector 10 cm from the target. The signals were fed to a 2048-channel ADC connected to the computer. The resolution of the system was 7 keV for a 0.84-MeV γ ray.

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[14] J. B. Marion, *1960 Nuclear Data Tables* (U. S. Government Printing Office, National Academy of Science–National Research Council, Washington 25, D. C., 1960), Part 3.

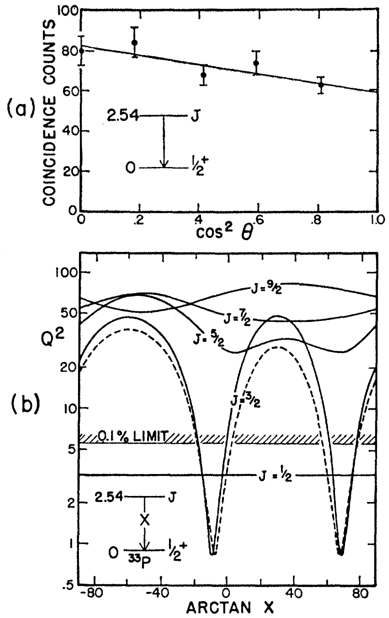

FIG. 5. (a) The angular correlation obtained from the photopeak of the 2.54-MeV γ ray from the 2.54-MeV level is shown. The curve was calculated with the parameter corresponding to one of the minima of Q^2 shown in (b) with $J=\frac{3}{2}$ and $I(\frac{5}{2})=0$. The curve for the other minimum is identical. (b) Plots of Q^2 versus arctan$x$ for $J=\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{9}{2}$ are shown. The dashed curve shows the effect of setting $I(\frac{5}{2})=0$. $I(\frac{1}{2})$ for $J=\frac{3}{2}$. Both $J=\frac{3}{2}$ and $J=\frac{5}{2}$ give a fit below the 0.1% confidence but $J=\frac{5}{2}$ is excluded because $l=2$.

The γ-ray energies used for calibration were 0.66162 MeV from ^{137}Cs, and 2.61425 and 1.59224 MeV from RdTh (^{208}Tl).[14] A quadratic fit to these values gave. the results shown in Table II for the excitation energies in ^{33}P. The values given by Bearse *et al.*[4] and by Currie and Evans[3] are also shown. The energies of other γ rays in the spectrum from the reactions ^{30}Si$(\alpha,n)^{33}$S and ^{30}Si$(\alpha,\alpha')^{30}$Si were in good agreement with Endt and Van der Leun.[13]

TABLE II. Results of Ge(Li)-counter measurements.

	Excitation energies (MeV)		
Level No.	Present work	Bearse *et al.*[a]	Curie and Evans[b]
1	1.435±0.003	1.436±0.008	1.43±0.01
2	1.850±0.003	1.852±0.008	1.81±0.01
3	2.544±0.004	2.540±0.008	2.43±0.03

[a] Reference 4.
[b] Reference 3.

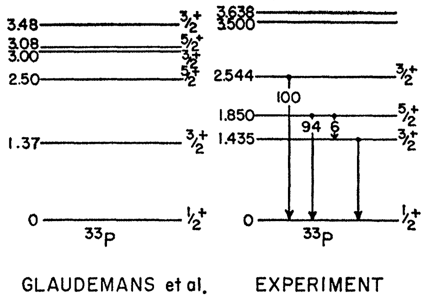

FIG. 6. The levels in ^{33}P predicted by the shell-model calculation of Glaudemans *et al.* (Ref. 1) and the experimental levels found in the present work and the work of Bearse *et al.* (Ref. 4) are shown.

VI. DISCUSSION

Figure 6 shows the levels in ^{33}P predicted by the shell-model calculation of Glaudemans *et al.*[1] and the experimental levels found in the present work and in the work of Bearse *et al.*[4] The predicted excitation energy of the lowest $\frac{3}{2}^+$ level is in good agreement with the observed excitation energy of this level. As pointed out previously[3,4] the observed $\frac{5}{2}^+$ level at 1.850 MeV probably arises from the excitation of the $d_{5/2}$ shell which was assumed to be closed by Glaudemans *et al.*[1] and hence this level should not be associated with the predicted $\frac{5}{2}^+$ level at 2.50 MeV. The argument was based on the fact[1] that a $d_{5/2}$ level was observed in ^{31}P at 2.23 MeV and the fact that the spectroscopic factor for the reaction ^{34}S$(d,{}^3$He$)^{33}$P to the 1.850-MeV level was large as expected for a $d_{5/2}$ level. The second $\frac{3}{2}^+$ level in ^{33}P is observed at 2.544 MeV, and the second predicted $\frac{3}{2}^+$ level is at 3.00 MeV. The $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$ sequence predicted by Bouten *et al.*[2] agrees with experiment.

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