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Realistic N-N One-Boson-Exchange Potentials*

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We develop three models which reproduce all of the most recent nucleon-nucleon data almost quantitatively. These models use the regularization technique in a fashion which is interpreted in terms of form factors for the nucleon-meson vertices. Two of the models incorporate recently established heavy scalar mesons: the π_v (1016 MeV) and the η_v (1070 MeV). The third model incorporates, as in prior work, mesons with the same mass as the vector mesons. The fits to all the nucleon-nucleon data are examined, and the results are compared with other models. It appears that one can incorporate very heavy scalar mesons and describe the nucleon-nucleon force quite precisely. Certain ambiguities in the experimental data and phenomenological phase shifts are discussed.

1. INTRODUCTION

SINCE the development of one-boson-exchange models (OBEM)[1–6] many pseudoscalar-, vector-, and scalar-meson (P,V,S) models have been found which explain the major features of nucleon-nucleon scattering data associated with P, D, and higher partial waves. More recently the innermost region of the nuclear force represented in S waves has been treated successfully[7,8] using a regularization technique based upon old P, V, S meson theories with generalized field Lagrangians.[9,10] Here this technique is used in a way which may be interpreted explicitly in terms of nucleon-meson form factors.[11] Furthermore, three P, V, S models are presented which reproduce all of nucleon-nucleon data almost quantitatively.

One of the models considered here embodies the recently established scalar mesons $\pi_v[J^P=0^+, I=1, m(\pi_v)=1016$ MeV$]$ and $\eta_v[J^P=0^+, I=0, m(\eta_v)=1070$ MeV$]$. In addition a weak hypothetical scalar meson, the σ_c ($m_c=416$ MeV, $I=0$) is taken as a substitute for the isoscalar part of 2π continuum exchange effects.

The second model uses isoscalar and isovector scalar mesons degenerate in mass with the corresponding vector mesons, the ω and the ρ. This "five-vector" N-N interaction, put forth some time ago by Green,[10] has its origins in old speculations of five-dimensional unified field theory in which the canonical conjugate of the intrinsic mass is used as a fifth coordinate. Vector and scalar fields combine naturally in such theories into a five vector field. This "π-5ω" model[7,8,12] gains some support from the recent mass formula of Schwinger.[13]

$$m^2=m_v^2[-J+\tfrac{1}{2}(P+1)], \qquad (1)$$

which gives identical masses to 0^+ and 1^- mesons.

Thus far, only a few studies with one-boson-exchange potentials have successfully characterized the S-wave phase shifts. The purely relativistic π-5ω model among the models of Green and Sawada (see Fig. 2 of Ref. 8) achieve good S-wave fits at very high values of the $g^2(5\omega)$. However, then the P-wave fits become very poor. The one-parameter model (see Fig. 3 of Ref. 8) achieves good fits to S waves, the ${}^3P$ phases, and all higher partial waves with the exception of the ${}^1P_1$ and ${}^1D_2$. This highly velocity-dependent model is of considerable interest since it has been successfully used by Kohler and McCarty[14] in a reaction matrix-type calculation and when corrected for the ${}^1P_1$ phases give a good binding energy for O^{16}. It is, however, noteworthy that four purely phenomenological nucleon-nucleon potentials used by them give rather poor O^{16} binding energies. The deficiency of the ${}^1P_1$ and ${}^1D_2$ was largely corrected in two "broken models" using seven parameters (see Figs. 4 and 5 of Ref. 8). Later a two-parameter model was found[15] which preserved the essential features of these seven-parameter models. However, ${}^1P_1$ and ${}^1D_2$ appeared somewhat removed from the phase shifts available in 1966[16] although the error bars for the former were rather large.

The present work was in part motivated by these disparities between theoretical OBEP and earlier reported experimental phase shifts. In part it was initially intended as an attempt to compare theoretical models directly with experimental observables since experimental phase shifts were ambiguous. During the

† On leave from Osaka University.
* Supported in part by the Air Force Office of Scientific Research.
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[15] A. E. S. Green and T. Sawada, International Conference on Nuclear Structure, Tokyo, Japan, 1967, The Institute for Nuclear Study, University of Tokyo, Tanashi-shi, Tokyo, Japan, p. 5.
[16] R. A. Arndt and M. H. MacGregor, Phys. Rev. **141**, 873 (1966).

course of this effort new phase shifts became available which, as it turns out, are less inconsistent although still not fully reconciled. Accordingly, the original motivation lost its major objective. However, the most recent phase-shift analyses of Breit *et al.*[17] and MacGregor *et al.*[18] still have substantial differences especially in the ρ_1 parameter and ^3G parameters. In such a situation direct comparison with experimental observables is particularly useful.

Section 2 of this work is devoted to statements of our regularization–form-factor formalism; Secs. 3 and 4 give the details of the three models and direct comparison with experimental observables; Sec. 5 is devoted to discussions of our results; Sec. 6 is devoted to comparisons with other models.

2. OBEP WITH NUCLEON-MESON FORM FACTOR

Our basic aim here is to give a successful explanation of all nucleon-nucleon parameters, employing the regularization method in a manner so as to serve as an appropriate form factor for the phenomenology of the innermost region. Obviously the form factor should have the most effect on S waves.

Using the formalism of Fock[19] the one-boson-exchange potential may be expressed as[8,9]

$$V=-\int dk\, G^*(\mathbf{k})G(\mathbf{k})/\omega\,, \tag{2}$$

where

$$G(\mathbf{k})=\sum_{i=1}^{2} g(2\pi)^{-1}\Theta_i(1/\sqrt{\omega})\exp(-i\mathbf{k}\cdot\mathbf{x}_i) \tag{3}$$

is the interaction based upon a δ-function form factor and the subscript i specifies two nucleons. Here we take $\hbar=c=1$, $\mathbf{k}$ is the momentum transfer, ω is an energy associated with the exchanged meson ($\omega^2=\mathbf{k}^2+\mu_i^2$), and θ_i represents the Dirac matrices[8]: θ_i represents β_i for a scalar field, $(\boldsymbol{\alpha}_i, iI_i)$ for a vector field, and $i\beta_i\gamma_{5i}$ for a pseudoscalar field.

If we take a form factor $F(\mathbf{k}^2)$ for the nucleon-meson vertex, the new G is given by

$$\bar{G}(\mathbf{k})=F(\mathbf{k}^2)\sum_{i=1}^{2} g(2\pi)^{-1}\Theta_i(1/\sqrt{\omega})\exp(-i\mathbf{k}\cdot\mathbf{x}_i)\,. \tag{4}$$

The Fourier-transform expression for the potential when a form factor is used is

$$\bar{V}=-\theta_1\theta_2 g^2\int dk\frac{(F(\mathbf{k}^2))^2}{\mathbf{k}^2+\mu^2}\exp[i\mathbf{k}(\mathbf{x}_1-\mathbf{x}_2)]\,. \tag{5}$$

In the present study we assume the form factor is

$$F(\mathbf{k}^2)=\Lambda^2/(\mathbf{k}^2+\Lambda^2)\,, \tag{6}$$

which leads to the potential

$$\bar{V}=-\theta_1\theta_2 g^2\left(\frac{\Lambda^2}{\Lambda^2-\mu^2}\right)^2\left[\frac{e^{-\mu r}}{r}-\frac{e^{-\Lambda r}}{r}\left(1+\frac{\Lambda^2-\mu^2}{2\Lambda}r\right)\right]. \tag{7}$$

It is interesting that this potential is obtained in the limit $\Lambda\to U$ of the "well-regulated" superposition of three Yukawa functions used by Green and Sawada[8]:

$$J(r)=\frac{g^2}{r}\left(e^{-\mu r}-\frac{U^2-\mu^2}{U^2-\Lambda^2}e^{-\Lambda r}+\frac{\Lambda^2-\mu^2}{U^2-\Lambda^2}e^{-Ur}\right). \tag{8}$$

Thus the potential given by Eq. (7) may be obtained in the limit

$$\bar{V}=-\theta_1\theta_2\left(\frac{\Lambda^2}{\Lambda^2-\mu^2}\right)^2\lim_{U\to\Lambda}J(r)\,. \tag{9}$$

Green and Sawada primarily used $U=20m_n$ (m_n = nucleon mass) and adjusted Λ phenomenologically, generally to about a few times m_n. Thus apart from the region $r\lesssim 0.01$ F, their function was close to[11]

$$\tilde{J}(r)=g^2\left(\frac{e^{-\mu r}}{r}-\frac{e^{-\Lambda r}}{r}\right). \tag{10}$$

This is the same function used in the most recent model of Bryan and Scott,[20] which also fits the S-wave phases.

In the present work we started with Eq. (8) but while searching on Λ and U we found that the case $U=\Lambda$ gave a better fit to experimental data than $U=20m_n$. After that the entire work was carried out assuming Eq. (7).

We define coupling constants with the following Hamiltonian densities:

$$H_P=g_P F(\mathbf{k}^2)\bar{\psi}i\gamma_5\psi\phi(\mathbf{k})\,, \tag{11}$$

$$H_V=g_V F(\mathbf{k}^2)\bar{\psi}i\gamma_\mu\psi\phi_\mu(\mathbf{k})+(\mu_V/4m_n^2)f_V F(\mathbf{k}^2)\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu}(\mathbf{k})\,, \tag{12}$$

$$H_S=g_S F(\mathbf{k}^2)\bar{\psi}\psi\phi(\mathbf{k})\,, \tag{13}$$

where m_n and μ_ν are the masses of the nucleon and mesons and

$$\sigma_{\mu\nu}=(1/2i)(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\,, \tag{14}$$

$$F_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu\,. \tag{15}$$

These Hamiltonian densities are for isoscalar mesons. They should be multiplied by $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$ for isovector mesons. Expressions for the OBEP's have already been obtained for these Hamiltonians.[8] The results may be placed in the form

$$\bar{V}=\bar{V}_C+\boldsymbol{\sigma}^1\cdot\boldsymbol{\sigma}^2\bar{V}_{\sigma\sigma}+S_{12}\bar{V}_T+\mathbf{L}\cdot\mathbf{S}\bar{V}_{LS}+\bar{V}_\nabla\nabla^2+\bar{V}_\nabla(\mathbf{r}\cdot\nabla)\,. \tag{16}$$

[17] R. E. Seamon, K. A. Friedman, G. Breit, R. D. Haracz, J. M. Holt, and A. Prakash, Phys. Rev. **165**, 1579 (1968); and G. Breit (private communication).

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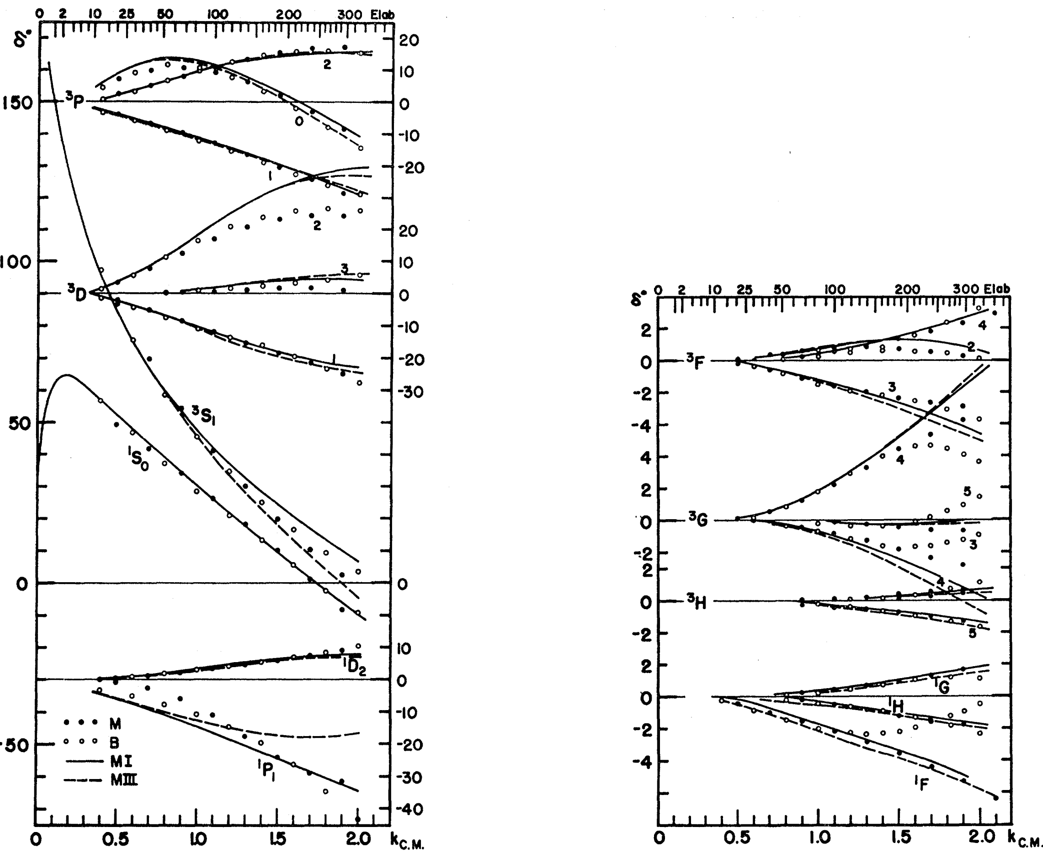

(a) (b)

FIG. 1. (a) Phase shifts of $^3P_{0,1,2}$, $^3D_{1,2,3}$, $^3S_1$, $^1S_0$, $^1D_2$, and $^1P_1$ (deg) versus incident energy (MeV) in laboratory system (upper scale) and incident momentum F^{-1} in center-of-mass system. Solid and broken curves represent calculations of the model I and model III, respectively. Open and solid circles represent Breit *et al.* (Ref. 17) and MacGregor's phase shifts (Ref. 18). (b) Phase shifts of $^3F_{2,3,4}$, $^3G_{3,4,5}$, $^3H_{4,5}$, $^1F_3$, $^1G_4$, $^1H_5$ (deg). (See the figure on meanings of symbols.)

Explicit expressions for the $\bar{V}$'s are listed in Table I. We now express Schrödinger's equation as

$$(\nabla^2/m_n+\bar{V})\psi=(k_{\text{c.m.}}^2/m_n)\psi\,, \tag{17}$$

where $k_{\text{c.m.}}$ is the center-of-mass momentum in the two-nucleon system. After the transformation to eliminate the first-order derivative, we have

$$\psi''-[l(l+1)/r]\psi+(k_{\text{c.m.}}^2-\bar{V}_{\text{eff}})\psi=0\,, \tag{18}$$

TABLE I. Contributions to $\bar{V}(a=1/m_n)$, $J_1=r^{-1}(d/dr)J$, $J_2=r^{-1}(d/dr)J_1$.

	P	S	V	V_{f^5}
V_C		$-J-\frac{1}{4}a^2\langle\nabla^2J\rangle$	J	$(f/g)\times\frac{1}{2}a^2\langle\nabla^2J\rangle$
V_σ	$\frac{1}{12}a^2\langle\nabla^2J\rangle$		$\frac{1}{6}a^2\langle\nabla^2J\rangle$	$(2f/g+f^2/g^2)\times\frac{1}{6}a^2\langle\nabla^2J\rangle$
V_{LS}		$\frac{1}{2}a^2J_1$	$\frac{3}{2}a^2J_1$	$2(f/g)a^2J_2$
V_T	$\frac{1}{12}a^2r^2J_2$		$-\frac{1}{12}a^2r^2J_2$	$-(2f/g+f^2/g^2)\times\frac{1}{12}a^2r^2J_2$
V_Δ		$-a^2J$	$-a^2J$	
V_∇		$-a^2J_1$	$-a^2J_1$	

where

$$\bar{V}_{\text{eff}}=\frac{m_n\bar{V}}{1+\phi}-\frac{1}{4}\left(\frac{\phi'}{1+\phi}\right)^2+k_{\text{c.m.}}^2\left(\frac{\phi}{1+\phi}\right)+\frac{1}{2}\left(\frac{\nabla^2\phi}{1+\phi}\right), \tag{19}$$

$$\phi=\frac{1}{m_n}\sum_{SV}\left[\frac{e^{-\mu r}}{r}-\frac{e^{-\Lambda r}}{r}\left(1+\frac{\Lambda^2-\mu^2}{2\Lambda}r\right)\right]. \tag{20}$$

3. MODELS I AND II

In this section we consider models using the π, ρ, and ω mesons along with scalar mesons[21] $\pi_v(I=1,\ m_{\pi_v}=1016$ MeV) and $\eta_v(I=0,\ m_{\eta_v}=1070$ MeV) which are listed in most recent tables. We consider cases with and without the η. Furthermore, we approximately incorporate 2π exchange effects using an isoscalar scalar meson with a small mass. The isovector part of 2π exchange effects is absorbed in the ρ contribution.[22] Thus we arrive at

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TABLE II. Parameters of the models used.

Meson	g^2	f/g	μ(MeV)	Λ(MeV)
(a) Model I ($\chi^2=58/36$ Breit's phase shifts with $J\leqq 2$)				
π	14.01		138.7	2532.4
η	2.73		548.7	1184.3
ρ	0.78	4.76	763.0	1184.3
ω	8.02	0.0	782.8	1184.3
π_v	4.11		1016.0	1184.3
η_v	4.44		1070.0	1184.3
σ_c	1.96		416.1	1184.3
(b) Model II ($\chi^2=64$)				
π	14.37		138.7	2293.0
ρ	0.81	4.70	763.0	1200.0
ω	7.83	0.0	782.8	1200.0
π_v	4.47		1016.0	1200.0
η_v	3.98		1070.0	1200.0
σ_c	1.97		416.1	1200.0
(c) Model III ($\chi^2=67$)				
π	14.61		138.7	1299.0
ρ	0.65	5.06	763.0	1299.0
ω	9.68	0.0	782.8	1299.0
σ_1	1.01		763.0	1299.0
σ_0	7.32		782.8	1299.0
σ_c	1.52		416.1	1299.0

model I, the $\pi+\eta+\rho+\omega+\pi_v+\eta_v+\sigma_c$ model and model II, without the η. These are the most plausible models in terms of experimentally established mesons.

The parameters in these models are the coupling constants, the regularization masses and the mass of σ_c. The value of g_π^2 is fixed in the range between 13 and 15, which is obtained from "modified" phase-shift analyses. The quantity $(f/g)_\rho$ is fixed in the range between 3.75 and 5.1, which is consistent with values obtained from electromagnetic form factor. The mass of σ_c is also fixed as $3\mu_\pi=416$ MeV; its variation of mass will be effectively replaced by an appropriate renormalization of g_c^2.

For the mass used in our meson-nucleon form factor

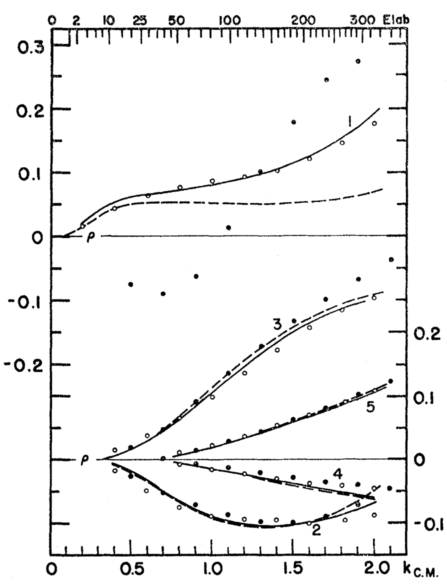

FIG. 2. Mixing parameters ρ_1, ρ_2, ρ_3, ρ_4, and ρ_5.

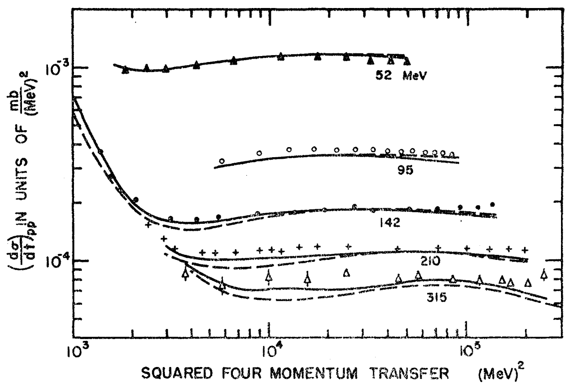

FIG. 3. The p-p differential cross sections at 52, 95, 142, 210, and 315 MeV. Solid and broken curves represent calculations of the models I and III, respectively. Note relations: $t=2k_{\mathrm{c.m.}}^2(1-\cos\theta)$, $d\sigma/dt=(\pi/k_{\mathrm{c.m.}}^2)d\sigma/d\Omega$.

we have as yet no reliable experimental information. In these models we adjust one Λ for the π meson and a common one for all the others. The coupling constants of the ω, ρ, π_v, and σ_c are also adjusted giving seven free parameters without the η and eight with the η.

Automatic search was carried out on these parameters in the aforementioned restricted range of g_π^2 and $(f/g)_\rho$ using the phenomenological phases of Breit *et al.*[17] for total angular momentum $J\leqq 2$. Our χ^2 test is made for the 36 phase shifts at 25, 95, and 330 MeV.[23]

The best parameters are found as shown in Table II(a) and II(b). Each gives $\chi^2=58$ and 64, respectively. These parameters are shown in relation to the phase shifts in Figs. 1 and 2 and low-energy parameters (Table III). The experimental observables are shown in Figs. 3–13. In the calculation of observables higher

TABLE III. Low-energy parameters.[a]

	Experiment	Model I	Model III
Deuteron binding energy MeV	2.22452(1±0.00009)	2.1	2.6
Deuteron quadrupole moment (10^{-27} cm)	2.82±0.01	2.8	2.7
Deuteron magnetic moment (μ_N)	0.85741±0.00008	0.856	0.855
${}^3S_1$ scattering length (10^{-13} cm)	5.399±0.011	5.6	5.2
Effective range $\rho(-\epsilon,-\epsilon)$ (10^{-13} cm)	1.82±0.05	1.8	1.8
D-state probability (%)		5.5	5.0
${}^1S_0$ scattering length (10^{-13} cm)	−23.675±0.095	−24	−23
${}^1S_0$ effective range (10^{-13} cm)	2.69±0.18	2.7	2.7
χ^2 of the model for 36 scattering data	(Breit's phase shifts)	58	67

[a] Gersten and Green (to be published).

[23] For the χ^2 test most of error bars $\Delta\delta$ are taken from energy-independent analysis of MacGregor *et al.* (Ref. 16). As its exceptions, at 25 MeV $\Delta\delta({}^3D_2)$ and $\Delta\delta({}^3D_1)$ are from Breit *et al.* (Ref. 17) and $\Delta\delta({}^3S_1)$ is a quarter of MacGregor's. At 330 MeV, $\Delta\rho_1$ and $\Delta\rho_2$ are taken as twice MacGregor's, because of large differences between Breit *et al.* and MacGregor.

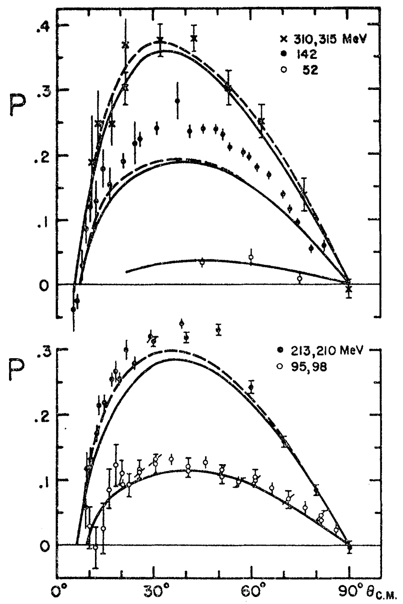

FIG. 4. The p-p polarization P at 310 (315), 142, and 52 MeV; 213 (210) and 98 (95) MeV. See the caption of Fig. 3.

partial waves than $J=5$ are approximated by Born terms of one-pion-exchange contributions which are dominant in the outer part of the nuclear force.[24]

Most observables are almost quantitatively reproduced for p-p and n-p scattering data and deuteron

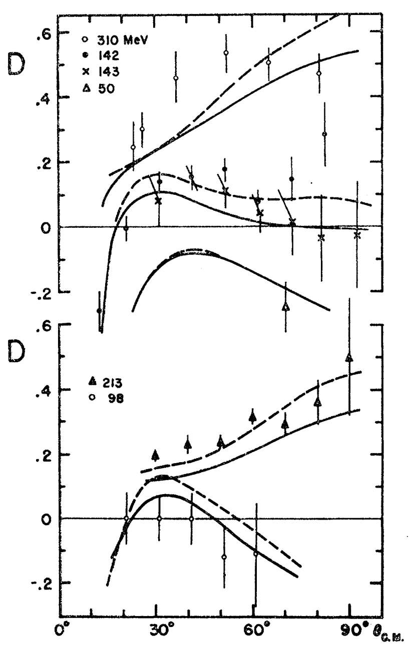

FIG. 5. The p-p depolarization D at 310, 142 (143), and 50 MeV; 213 and 98 MeV. See the caption of Fig. 3.

[24] M. Taketani, S. Nakamura, and M. Sasaki, Progr. Theoret. Phys. (Kyoto) 6, 581 (1951); Suppl. No. 3 (1956).

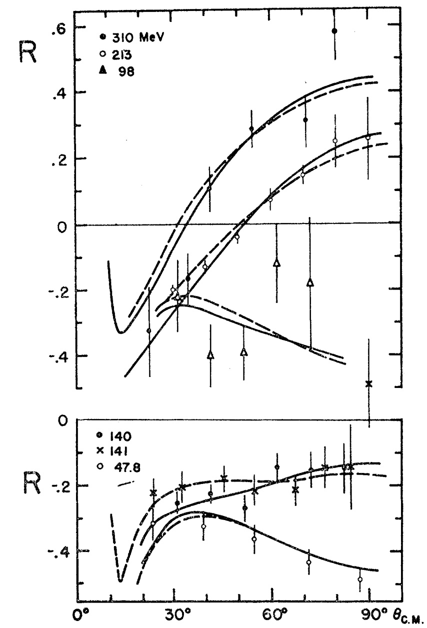

FIG. 6. The p-p rotation of polarization R at 310, 213, and 98 MeV; 140 (141) and 47.8 MeV. See the caption of Fig. 3.

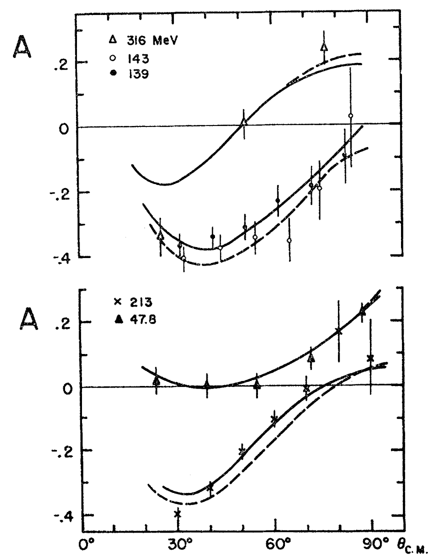

FIG. 7. The p-p triple scattering parameter A at 316, 143, and 139 MeV; 213 and 47.8 MeV. See the caption of Fig. 3.

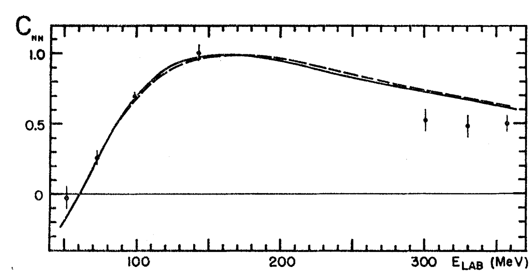

FIG. 8. The p-p spin correlation parameter C_{NN} at 90° versus incident laboratory energy (MeV). See the caption of Fig. 3.

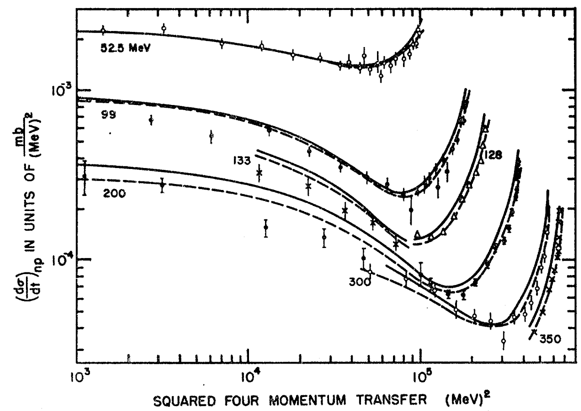

FIG. 9. The n-p differential cross section at 52.5, 99, 128, 133, 200, 300, and 350 MeV. See the caption of Fig. 3.

parameters. Both models give about equally good fits. So in the figures plots from model II are omitted.

Note that the p-p polarizations at 142 MeV are a little lower than the experimental values shown. However, recent experiments at 138, 140.7, and 147 MeV as well as the phenomenological analyses of Breit *et al.*[17] also have lower polarizations in this energy neighborhood.

The n-p total cross sections are a little higher than the experimental at high energies. The deviations come mainly from $^3D_2$ partial waves.

4. MODEL III

In this section we consider two five-dimensional vector mesons, the (ω,σ_0) and the (ρ,σ_1). We consider the isoscalar scalar meson σ_0 and isovector scalar meson

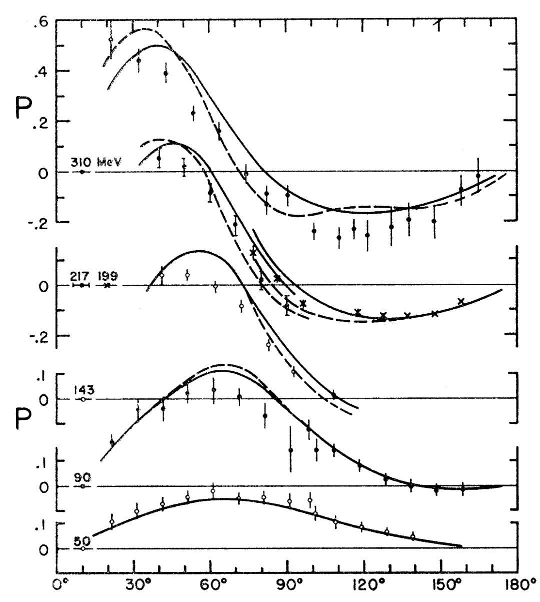

FIG. 10. The n-p polarization at 310, 217 (199), 143, 90, and 50 MeV. See the caption of Fig. 3.

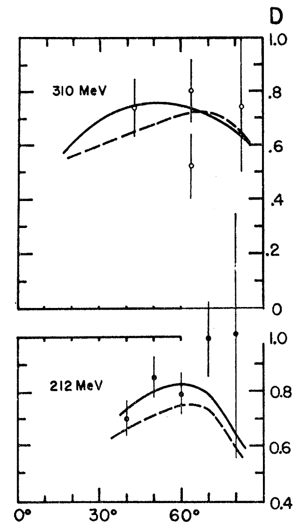

FIG. 11. The n-p rotation of polarization at 310 and 137 MeV. See the caption of Fig. 3.

σ_1 as the fifth components of five-vectors whose first four components are the ω and ρ, respectively. We denote this model as model III. These σ_0 and σ_1 have the same mass as ω and ρ, respectively. Schwinger[13] has recently found an empirical mass formula for mesons which predicts scalar mesons with the same masses as

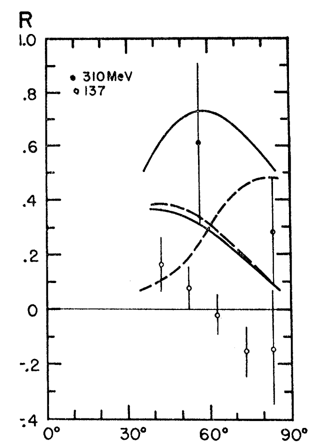

FIG. 12. The n-p depolarization at 310 and 212 MeV. See the caption of Fig. 3.

vector meson. Rigorously speaking, the scalar meson should also have the same coupling constants as ω and ρ. However, we break this symmetry by about 50% in this model. Regularization parameters are taken in common for all mesons. Thus the completely free parameters are the direct coupling constants of ρ, ω

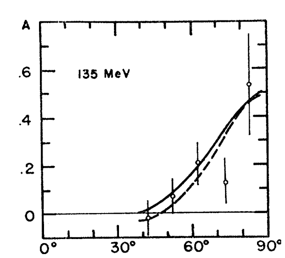

FIG. 13. The n-p triple scattering parameter at 135 MeV. See the caption of Fig. 3.

and σ_c and Λ, and restricted free parameters are the coupling constants of σ_1 and σ_0. The parameters g_π^2 and $(f/g)_\rho$ are fixed as in the previous models. The best parameters are found in the same way as in models I and II [Table II(c)]. This model reproduces the phase shifts and observables as shown in Figs. 1 and 2, and Figs. 3–13 and low-energy parameters in Table III.

This model gives $\chi^2=67$ and fits to experimental observables are about as good as those of model I or II. In spite of restricted parameters, most of p-p and n-p scattering data and the deuteron data are reproduced almost quantitatively.

Fits to the n-p differential cross section are somewhat better in this model than in the model I or II; however, the p-p total cross sections at high energies are somewhat better in I than III.

5. DISCUSSION

A. Coupling Constants

Table II shows the best parameters for the three models. It will be noted that the coupling constants are quite reasonable.

The g_π^2's are 14.01, 14.37, and 14.61 for the models I, II, and III, which are quite consistent with results of MacGregor's phase-shifts analysis[16] ($g_\pi^2=14.72\pm0.83$) or those of Breit[17] ($g_\pi^2=14.7$).

The parameter $(f/g)_\rho$ is also in a range consistent with the value obtained from electromagnetic form factors. If we take only the ρ-meson model for the interpretation of isovector part of electromagnetic form factor we get $(f/g)_\rho=5.0$. This model gives some deviation at high momentum transfer ($\mathbf{k}^2\gtrsim 10\ \mathrm{F}^{-2}$). A ρ-meson form-factor model corrected for 2π dissociation gives $(f/g)_\rho=3.8$. So our values $(f/g)_\rho=4.76$, 4.70, and 5.06 for our models are just a little larger than that given by electromagnetic form-factor analyses, but still in an acceptable range.

It is interesting that g_ω^2 are not very different for our three models; 8.02, 7.83, and 9.68 for models I, II, and III.

The direct ρ coupling constants obtained are 0.78, 0.81, and 0.65 for models I, II, and III, respectively. These are quite comparable with the value 0.65, which Sakurai proposed for g_ρ^2 from the universality of strong interactions.[25] In our models, however, the ρ-exchange contribution might be included both as an intrinsic ρ-exchange contribution and isovector part of the 2π exchange contribution.[22] Accordingly, it is not imperative that our g_ρ^2agree with 0.65, even if the hypothesis of universality is valid.

B. Form Factor

The pionic form-factor parameter $\Lambda_\pi=2532$ and 2293 MeV is obtained for the models I and II and $\Lambda_\pi=1299$ MeV is obtained for model III. These values are bigger than that which is obtained from reaction the $p+p\rightarrow p+n+\pi^+$ at the 1-GeV region on the basis of one-pion-exchange model modified with a regulator or form factor. Previously, Ueda[11] reproduced $p+p\rightarrow p+n+\pi^+$ in the sub-GeV region when using the form factor

$$\{F_L(\mathbf{k}^2)\}^2=L^2/(\mathbf{k}^2+L^2)\,, \tag{21}$$

with the regulator value $L=400$–600 MeV. At momentum transfer $\mathbf{k}^2=36\mu_\pi^2$ which corresponds to the maximum momentum transfer at $E_{\mathrm{lab}}=330$ MeV we have

$F(36\mu_\pi^2)=0.90$ for $\Lambda_\pi=2532$ MeV of model I,

$F(36\mu_\pi^2)=0.88$ for $\Lambda_\pi=2293$ MeV of model II,

$F(36\mu_\pi^2)=0.71$ for $\Lambda_\pi=1300$ MeV of model III,

whereas for the form factor of Eq. (21) we have

$F_L(36\mu_\pi^2)=0.43$–0.58 for $L=400$–600 MeV of Eq. (21).

At present, however, the concept of the pionic form factor is not well defined. For example, the one-pion production from proton-proton collision can also be reproduced by the K-matrix method without the form factor.[26] Thus this difference between a sub-GeV result and an elastic energy result need not be taken too seriously.

C. η Contribution

The η meson apparently does not play an important role in our models since almost equally good fits are obtained with or without the η (model I or II). Actually we get $\chi^2=58$ with η and $\chi^2=63$ without η. The η contribution is easily replaced by other mesons with adjustment of parameters and slight change of other parameters. This situation will be also the same for the X_o meson (mass$=960$ MeV, $I=0$, pseudoscalar).

D. $\mathbf{L}^2$ Force

The phase shifts for the ${}^1D_2$ state deviate somewhat at high energy from the experimental phase shifts. We found that when an accurate fit to $\delta({}^1S_0)$ is sought, a fit to $\delta({}^1D_2)$ is sacrificed a little. It seems difficult to reproduce both of these phases accurately since these two states are described only with the central potential in our models. If, however, there were an appropriate $\mathbf{L}^2$ force, simultaneous fits to both states are quite easily attained. Indeed, we find that if we add a weak $\mathbf{L}^2$ potential to model I

$$\bar{V}_{L^2}=G\frac{1}{r}\left[e^{-\mu r}-e^{-\Lambda r}\left(1+\frac{\Lambda^2-\mu^2}{2\Lambda}r\right)\right], \tag{22}$$

[25] J. J. Sakurai, Phys. Rev. Letters **17**, 1021 (1966).

[26] M. Kikugawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) **37**, 88 (1967).

where

$$G=-1.0, \quad \mu=782.8 \text{ MeV}, \quad \Lambda=1184.3 \text{ MeV};$$

both $\delta(^1S_0)$ and $\delta(^1D_2)$ fit the experimental phase shifts (Table IV) rather well.

E. Further Improvements

The models I, II, and III yield $\chi^2=58$, 64, and 67, respectively, when 36 "data" are taken from Breit's phase shifts[17] with $J\leq 2$ at 25, 95, and 330 MeV. Approximately $\frac{2}{3}$ of the χ^2 values come from the 330-MeV region. In the higher-energy region some corrections which should be made to the present model become more important. These corrections may be (1) some contributions from heavy mesons like ϕ_0, A_1, etc., (2) relativistic corrections which may amount to 15% at 300 MeV, and (3) the auxiliary condition for the vector mesons which are not taken into account here. We can expect further improvement in the present models by taking into account these corrections.

6. COMPARISON WITH OTHER MODELS

Let us compare the present results with the two-[15] and seven-parameter models of Green and Sawada (GS),[8] where π, ρ, ω, σ_1, σ_0, and σ_c are taken and the regularized potential of Eq. (8) is used. In the two-parameter model the free parameters are g_c^2 and a common Λ. All the other parameters are fixed by reasonable constraints. In spite of these strong restrictions, fits to experimental observables below 150 MeV are quite good. In higher-energy regions, however, the deviations are appreciable. The seven-parameter models naturally give better fits and actually fit the phase shifts of MacGregor *et al.*[16] up to 210 MeV very well. In the "broken" models studies the η contributions were shown not to be essential. The most important deviations in the two- and seven-parameter GS models occur in $\delta(^3P_2)$, $\delta(^1D_2)$, and ρ_1. The present models give considerable improvements in the fits to these phases primarily because of the form factor used and the more relaxed restriction on g_ρ^2 and $(f/g)_\rho$.

Very recently Bryan and Scott[20] have presented an OBEP model which includes π, ρ, ω, η, and σ_0, and σ_1 with adjustable masses and employs a simple regularization of the form of Eq. (10). Their model resembles the two- and seven-parameter models of Green and Sawada, and gives good qualitative fits to experiment particularly at energies below 142 MeV. However, similar deviations appear in $\delta(^1D_2)$ and $\delta(^3P_2)$ at high energy. These deviations cause poor fits to the *p-p* differential cross sections at small angles at 315, 213, and 142 MeV, and to the polarization and A parameter at 310 MeV. The present models give a considerable improvement in these observables.

It is interesting to compare the present coupling constants with those which are obtained from data

TABLE IV. Effects of $\mathbf{L}^2$ potential.

	$\delta(^1S_0)$	$\delta(^1D_2)$	$\delta(^1G_4)$	χ^2
Without $\mathbf{L}^2$ potential	-9.9	7.8	1.7	58
With $\mathbf{L}^2$ potential	-9.9	9.7	1.9	43
Experiment (Breit)	-9.4	10.5	1.15	
Experiment (MacGregor)	-9.26 ± 1.56	9.22 ± 0.67	1.53 ± 0.33	

which are not affected by the innermost region. Previously Sawada, Watari, Yonezawa and one of the authors (T. U.) presented OBE models (referred as SUWY) which include π, ρ, ω, σ_0 ($m_{\sigma_0}=540$ MeV) with η and without η, and reproduced all phase shifts well except the two S states.[1,27] They obtained the coupling constants $g_\omega^2=10$, $f_\omega/g_\omega=0.6$, $g_\rho^2=0.4$, $f_\rho/g_\rho=4$, and $g_{\sigma_0}^2=5.2$ with fixed $g_\pi^2=14.4$, and, in the case with η, $g_\eta^2\lesssim 10$.[27] These values were determined from data which do not include the effects of the innermost region. Our present values for g_ω^2, g_ρ^2, g_η^2, and f_ρ/g_ρ are quite consistent with those values. Our value for g_c^2 which is to simulate the 2π continuum is dependent upon the choice of the effective scalar mass.[27] The present values go with the choice[8] $m_c=3\mu_\pi$.

We note that our OBEP models have, with a few exceptions, achieved fits to the Livermore and Yale phase shifts within the range of uncertainty or differences between these sets of phase shifts. Their differences in part reflect differing methods of data selection and in part differences of theoretical approach, particularly in the use of auxiliary information in the interpretation of *n-p* data which, by themselves are still quite ambiguous. The Livermore solutions have been restricted somewhat by the choice of their energy-dependent forms which extend smoothly into the inelastic domain. The Yale solutions, which in some instances inflect more strongly at higher energies, are based upon specific charge-dependent assumptions which are used as constraints in treating the experimental data.

At low energies, we tend to agree with the Yale's ρ_1 and $^1P_1$. At higher energies, our models tend to the trends of the Livermore phase shifts, particularly the $^3G_3$, $^3G_4$, $^1F_3$, and $^1P_1$. We differ somewhat from both sets in our $^3D_2$ phases. From the relatively minor nature of these discrepancies, it should be clear that only minor additional potential components would be needed to make our model consistent with the best current interpretations of experimental *p-p* and *n-p* phase shifts as obtained from the two most extensive analyses of the experimental data.

The recent Livermore study uses 23 free parameters in their energy-dependent forms to fit 14 $I=1$ phase shifts and 22 free parameters for 11 $I=0$ phase shifts. The Yale study does not specify their number of free

[27] S. Ogawa, S. Sawada, T. Ueda, W. Watari, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) Suppl. 39, 140 (1967).

parameters but it is probably of the same order. It might also be noted that comparable numbers of free parameters are used in phenomenological potential models. These traditionally use hard cores, phenomenological middle-range potentials, and OPEP outer potentials as illustrated by the Hamada-Johnston[28] and Yale[29] potentials, which achieved precise representations of the experimental data available several years ago, and the recent Tamagaki and Watari[30] potential. From these studies one might conclude that the experimental nucleon-nucleon data intrinsically requires say 40 ± 10 parameter forms for its description. If this is correct, then a theoretical model may be characterized as having meaningful content over and beyond the elastic scattering data used in its adjustment only if it provides a substantial reduction in the number of free parameters. A recent study of Kishi, Sawada, and Watari (KSW)[31] is noteworthy in that by using OBEP's rather than phenomenological potentials for the middle region in conjunction with a quadratic $\mathbf{L}\cdot\mathbf{S}$ force and $\mathbf{L}^2$ force and hard cores, they require only about $\frac{1}{3}$ this number of "intrinsic" parameters to achieve rather precise fits to empirical phase shifts including the two S states. Our present models use only about $\frac{1}{6}$ the number of intrinsic parameters and also give a rather precise description of the experimental data. The present work differs from the KSW work in (a) the use of velocity-dependent (p^2) forces which follow in a straightforward way from vector and scalar OBE interactions and (b) in the use of regularization which we now interpret explicitly in terms of meson-nucleon form factors. The form-factor feature is probably more physically meaningful than the phenomenological hard core because it can be involved in the field-theoretical formulation and can be extended to the relativistic energy region. If we were to augment present interactions by either a weak $\mathbf{L}^2$ term or perhaps OBEP components arising out of axial-vector and tensor mesons which are known to exist at somewhat higher masses than π_v and η_v, it is very likely that we can resolve any remaining discrepancies between theory and experiment.

[28] T. Hamada and I. D. Johnston, Nucl. Phys. **34**, 382 (1962).
[29] K. E. Lassila, M. H. Hull, H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962).
[30] R. Tamagaki and W. Watari, Progr. Theoret. Phys. (Kyoto) Suppl. **39**, 23 (1967).
[31] Y. Kishi, S. Sawada, and W. Watari, Progr. Theoret. Phys. (Kyoto) **38**, 892 (1967).

7. SUMMARY

We have presented three OBEP models which use p^2 meson theoretical potentials, an explicit nucleon-meson form factor (Eq. 6) based upon the regularization techniques, and a weakly coupled light scalar meson σ_c as a substitute for the isoscalar part of two-pion exchange. We have shown that these models reproduce all of the nucleon-nucleon data almost quantitatively using quite reasonable parameters. The first two models also include strongly coupled scalar mesons whose existence is now experimentally establighed; thus they are rather realistic and plausible representations of nuclear forces.

The fact that they achieve fits to N-N data with only eight free parameters which are similar to the fits of phenomenological potential model or energy-dependent phase-shift representations employing 40 ± 10 parameters lends credence to these simple OBEP models. For applications to nuclear physics, one is frequently involved in calculations "off the energy shell." For such purposes a meson theoretic potential with relatively few parameters, in the opinion of the authors, probably furnishes a more reliable basis for extrapolation into an untested domain than does phenomenological representation of the elastic scattering data. Furthermore, the "soft" features of our regularized potentials should alleviate some of the well-known difficulties in the nuclear many-body problem associated with infinite hard cores.

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