where $U_{\lambda,P}$ is the particular solution, one can calculate the values of A and B which will make $U_\lambda(x)$ satisfy the boundary condition at the core for both values of λ. Reasonably small inaccuracies in the logarithmic derivatives at $x_{\max}$ are of no consequence, since any part of U_λ which is irregular at infinity will damp out rapidly in the process of inward integration.

More recently, the equations have been solved by integrating outward from the core, or from the origin in the case of soft-core potentials, and matching with the functions obtained by inward integration at an intermediate point. This automatically eliminates the appearance of parts of the wave function irregular at the origin.

A measure of the numerical accuracy is provided by the overlap integral (ϕ_{nl}/u_{nl}). When $\rho \leq \rho_{\max}$, it is found that this integral deviates from unity by less than one part in 10^4 with single-precision calculations on the IBM-360 digital computer. If the core-volume contributions are omitted, the deviation grows to about 1%.

PHYSICAL REVIEW VOLUME 174, NUMBER 4 20 OCTOBER 1968

Muon-Capture and Photon-Absorption Calculations in a Configuration-Mixing Model for O^{16}†

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(Received 1 April 1968)

The utility of multiparticle, multihole configuration mixing as a mechanism for resolving certain theoretical-experimental discrepancies for muon capture and photodisintegration on O^{16} is considered. An O^{16} ground state containing a two-particle, two-hole (2p-2h) component is assumed. The usual particle-hole odd-parity states are amended to contain three-particle, three-hole configuration admixtures consistent with the choice of the more complicated ground state. The results of the calculations indicate that the configuration-mixing model can appreciably lower the partial muon-capture rates to the lowest 0^-, 1^-, and 2^- states. Moreover, a shift in the energy distribution of the transition strength, obtained in the model calculations, lowers the capture rate and photodisintegration cross section in and below the giant-resonance region. However, correction terms, arising mainly from $0^+ \rightarrow 1^+$ (2p-2h) allowed Gamow-Teller transitions, seem to preclude a lowering of the total capture rate.

I. INTRODUCTION

INVESTIGATIONS concerned with the capture of muons by complex nuclei have been complicated by the lack of detailed knowledge of the appropriate wave functions for the initial and final nuclear states involved in the capture process. Because of this difficulty, one has been motivated to study muon capture on closed-shell nuclei owing to the more complete information available concerning the nuclear wave functions in such a situation. Generally, however, results for muon capture on such closed-shell nuclei as C^{12}, O^{16}, and Ca^{40} have been less than satisfactory.

We now confine our remarks to O^{16} although some of the comments may be generalized to other "magic" nuclei.

If one assumes the ground state (g.s.) of O^{16} to consist of doubly closed shells, then the final nuclear states reached via muon capture are particle-hole (p-h) states of N^{16}. Historically, isotopic analogs of these p-h states have been extensively studied in connection with electron scattering and photon absorption.[1–3] Similar to the situation for the above processes, it appears that the first forbidden nuclear transitions are of primary importance for calculating the muon-capture rate.[4] Therefore, one might conjecture that the usual procedure for obtaining the p-h states in and below the giant dipole resonance[1] should yield reasonably good final-state wave functions and the theoretically calculated muon capture rate would be in agreement with experiment. Unfortunately, this is not the case. Utilizing coupling constants obtained from general theories concerning weak interactions, the calculated transition rates[5] to particular low-lying states of the daughter nucleus are considerably higher than those obtained from experi-

† Work performed under the auspices of the U. S. Atomic Energy Commission.

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ment.[6,7] Moreover, the total capture rate is overestimated by approximately 30%.

There is at least one other objection to current results for first forbidden transitions in O^{16}. Photon-absorption experiments reveal that approximately 60% of the classical Thomas-Reiche-Kuhn (TRK) dipole sum is exhausted in and below the giant-resonance region.[8] It is also found that a considerable number of photon-absorption-induced nuclear transitions occur to states of higher excitation than the giant resonance.[8,9] Shell-model calculations[2,3] predict the classical TRK sum rule is exceeded in the giant-resonance region and that negligible dipole strength lies above it. This discrepancy becomes especially severe when one considers that exchange and velocity-dependent terms in the nuclear Hamiltonian can sufficiently alter the classical sum-rule result[10] so that in actuality more of the energy enhanced dipole strength may lie above the giant resonance than in it. Although the quasideuteron process[11] may rob some of the strength from the giant resonance, there is no evidence that this mechanism is sufficient to resolve the discrepancy.

The expression for muon capture on a closed-shell nucleus is such that induced transitions resulting in high nuclear excitation are considerably reduced due to energy-dependent factors. The situation is reversed for photon absorption, for in this case, transition strength contained at higher energies is enhanced. Therefore, the difficulties mentioned above with regard to both muon capture and photon absorption on O^{16} could be resolved by a single mechanism that would transfer appreciable transition strength from below 26 MeV to excitation energies 30–60 MeV above the ground state.

Since general agreement exists between the calculated energy levels of the giant-resonance states and the experimentally determined peaks in the photon-absorption reaction, one must remove dipole strength from these states while keeping their excitation energy approximately the same.

A potential mechanism for transferring transition strength to higher energies is suggested by recent developments involving the possibility of appreciable two-particle, two-hole (2p-2h) components in the ground state of O^{16}.[12–14] The presence of such components in the g.s. and the resulting implications concerning the excited-state nuclear wave functions could alter previous results. The motivation for performing the calculations leading to the results reported in this paper has been to ascertain whether the mechanism discussed above is appropriate for eliminating the theoretical-experimental discrepancies discussed earlier.

In Sec. II, we consider the construction of the p-h, (2p-2h), and (3p-3h) wave functions that are employed in the calculations. We also discuss the choice of the ground-state (2p-2h) component utilized in the investigation.

Section III contains the expressions for the muon-capture rate and the photon-absorption cross section. It is noted that a reduction of transition strength in the lowest $J=0^-$, 1^-, and 2^- states and the transferral of strength to higher energy can result from (1) interference between certain transition matrix elements for more complicated configurations and the expression for transitions obtained in familiar p-h calculations and (2) a reduction, arising from exclusion-principle effects, of other transition matrix elements for the complicated configurations. In anticipation of results discussed in later sections, it is pointed out that correction terms, largely due to allowed Gamow-Teller transitions, prevent the over-all capture rate from being reduced even though some strength is robbed from the conventional giant-resonance states.

In Sec. IV, the configuration-mixing model is formulated and utilized in determining $0^+\rightarrow 0^-$ partial muon-capture transition rates. The model is also used in Sec. V to calculate the $0^+\rightarrow 1^-$ and 2^- partial muon-capture rates and to calculate the photon-absorption cross section as a function of nuclear excitation energy. The correction terms mentioned earlier are discussed in greater detail in Sec. VI. Finally, in Sec. VII, conclusions based on the results of the calculations are presented. For muon capture the principal results discussed are the lowering of the partial capture rates to the lowest 0^-, 1^-, and 2^- states, and the apparent inability of the model to lower the prediction for the total capture rate sufficiently despite the removal of approximately 10% of the transition strength from regions in and below the giant resonance. The results of the photodisintegration calculations are also discussed. The interesting result in this case is the 30% reduction of the cross section below 30 MeV and the appearance of appreciable cross section above the giant resonance.

II. WAVE FUNCTIONS

A. General Form

Denoting the closed-shell configuration of O^{16} by $|C\rangle$ and utilizing the occupation number representation, the p-h states assume the form

$$\sum_{j_{zi}t_{zi}} (-1)^{\frac{1}{2}-t_{z2}}(\tfrac{1}{2}t_{z1}\tfrac{1}{2}-t_{z2}|\tfrac{1}{2}\tfrac{1}{2}1T_z)(-1)^{j_2-j_{z2}} \times(j_1j_{z1}j_2-j_{z2}|j_1j_2JJ_z)\eta_1\eta_2^\dagger|C\rangle. \quad (1)$$

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The single-particle-state creation and annihilation operators, η and $\eta^\dagger$, respectively, have the usual fermion anticommutation properties.[15] The subscripts attached to η and $\eta^\dagger$ refer to the set of quantum numbers required to completely specify the single-particle state. In the context of the present discussions, the complete set is

$$1 \to n_1,l_1,j_1,j_{z1},t_1,t_{z1}, \tag{2}$$

where n, l, j, and j_z are the usual labels characterizing eigenfunctions of the harmonic-oscillator Hamiltonian in j-j coupling. (We couple lsj.) The labels t and t_z refer to the isotopic spin ($\frac{1}{2}$ for protons and neutrons) and its z projection ($t_z=+\frac{1}{2}$ for protons).

For compactness of writing, the p-h state Eq. (1) will be denoted in later sections by

$$\psi(\text{p-h},n_1l_1j_1(n_2l_2j_2)^{-1},J,J_z,T,T_z). \tag{3}$$

The 2p-2h states are given by

$$\frac{1}{N_1}\sum_{j_{zi},t_{zi}}\sum_{J_z'',J_z'}\sum_{T_z'',T_z'}(-1)^{j_3+j_4+J_z'+1+T_z'} \times(j_3-j_{z3}j_4-j_{4z}|j_3j_4J'J_z')(j_1j_{z1}j_2j_{z2}|j_1j_2J''J_z'')(J''J_z''J'J_z'|J''J'JJ_z)(\tfrac{1}{2}-t_{z3}\tfrac{1}{2}-t_{z4}|\tfrac{1}{2}\tfrac{1}{2}T'T_z')$$
$$\times(\tfrac{1}{2}t_{z1}\tfrac{1}{2}t_{z2}|\tfrac{1}{2}\tfrac{1}{2}T''T_z'')(T''T_z''T'T_z'|T''T'TT_z)\eta_1\eta_2\eta_3^\dagger\eta_4^\dagger|C\rangle. \tag{4}$$

The symbol N_1 appearing in Eq. (4) is a normalization factor. In general,

$$N_1=(\sqrt{2})^{[\delta(n_1l_1j_1,n_2l_2j_2)+\delta(n_3l_3j_3,n_4l_4j_4)]}. \tag{5}$$

Because of the frequent reference to 2p-2h states characterized by $(n_1l_1j_1)=(n_2l_2j_2)$, $(n_3l_3j_3)=(n_4l_4j_4)$, $J'=J''=J=T=0$, and $T'=T''=1$, it is convenient to write this special case of Eq. (4) as

$$\psi(\text{2p-2h},(n_1l_1j_1)^2(n_3l_3j_3)^{-2}). \tag{6}$$

For the situation where the 2p-2h state under consideration is a linear combination of the states, Eq. (6), it is convenient to generalize that equation to

$$\psi(\text{2p-2h},\sum_\alpha A_\alpha(n_1l_1j_1)_\alpha{}^2(n_3l_3j_3)_\alpha{}^{-2}). \tag{7}$$

3p-3h states of the following form will be employed in the calculations:

$$\frac{1}{N_1N_2}\sum_{j_{zi},t_{zi}}\sum_{J_z,T_z}\sum_{J_z',T_z'}\sum_{J_z'',T_z''}\sum_{J_z''',T_z'''}(-1)^{j_6-j_{z6}+\frac{1}{2}-t_{z6}}(j_5j_{z5}j_6-j_{z6}|j_5j_6J'''J_z''')(\tfrac{1}{2}t_{z5}\tfrac{1}{2}-t_{z6}|\tfrac{1}{2}\tfrac{1}{2}T'''T_z''')$$
$$\times(-1)^{j_3+j_4+J_z'+1+T_z'}(j_3-j_{z3}j_4-j_{z4}|j_3j_4J'J_z')(\tfrac{1}{2}-t_{z3}\tfrac{1}{2}-t_{z4}|\tfrac{1}{2}\tfrac{1}{2}T'T_z')(j_1j_{z1}j_2j_{z2}|j_1j_2J''J_z'')$$
$$\times(\tfrac{1}{2}t_{z1}\tfrac{1}{2}t_{z2}|\tfrac{1}{2}\tfrac{1}{2}T''T_z'')(J''J_z''J'J_z'|J''J'JJ_z)(T''T_z''T'T_z'|T''T'TT_z)(J'''J_z'''JJ_z|J'''J\bar{J}\bar{J}_z)$$
$$\times(T'''T_z'''TT_z|T'''T\bar{T}\bar{T}_z)\eta_5\eta_6^\dagger\eta_1\eta_2\eta_3^\dagger\eta_4^\dagger|C\rangle. \tag{8}$$

The 3p-3h states above require an additional normalization factor, denoted by N_2, due to exclusion-principle effects. Of particular interest in this paper are 3p-3h states characterized by the conditions for which Eq. (6) is defined. For this case, N_2 assumes the relatively simple form

$$N_2=\left(1-\frac{\delta(n_1l_1j_1,n_5l_5j_5)}{(2j_1+1)}-\frac{\delta(n_3l_3j_3,n_6l_6j_6)}{(2j_3+1)}+\frac{1}{3}\frac{\delta(n_1l_1j_1,n_5l_5j_5)\delta(n_3l_3j_3,n_6l_6j_6)}{(2j_3+1)(2j_1+1)}\right)^{1/2}. \tag{9}$$

The 3p-3h states that are constructed by applying a p-h operator to the 2p-2h state, Eq. (7), will be denoted by

$$\psi(\text{3p-3h},\bar{J},\bar{J}_z,\bar{T},\bar{T}_z,n_5l_5j_5(n_6l_6j_6)^{-1},\sum_\alpha A_\alpha(n_1l_1j_1)_\alpha{}^2(n_3l_3j_3)_\alpha{}^{-2}). \tag{10}$$

B. Ground State of O^{16}

The intent, in this investigation, has been to select a form for the ground state that contains, to a rough approximation, the non-closed-shell part of the wave function. We have, of course, attempted to keep the subesquent calculational complications to a minimum.

Several papers[12,13,16] have demonstrated that the g.s. of O^{16} may contain a substantial 2p-2h deformed com-

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[16] G. E. Brown and A. M. Green, Nucl. Phys. 85, 87 (1966).

ponent. Noting that many of the low-lying states of O^{16} may be fitted into rotational bands and have large $E2$ transitions, Brown and Green[12] (BG) have considered a model in which there exist low-lying deformed states in addition to spherical shell-model states. Taking the unperturbed energies of the deformed states as free parameters adjusted to give the observed levels, they have calculated the mixing of several of the low-lying states and the g.s. It was found that the g.s. contained 76% of the closed-shell configuration with the rest of the wave function being formed from 2p-2h (22%) and 4p-4h (2%) components. The 0^+ state at 6.07 MeV was found to be comprised of 4p-4h (88%), 2p-2h (5%), and closed-shell (7%) components, while a 0^+ state at 11.26 MeV was determined to consist of 2p-2h (73%), 4p-4h (10%), and closed-shell (17%) components. Since the unperturbed energies of the complicated configurations were treated as parameters, one might not feel obligated to take the results seriously. However, additional credence is given the calculation because the calculated branching ratio resulting from $E2$ transitions from the 2^+ state (6.92 MeV) to the 0^+ state (6.07 MeV) and the 0^+ g.s. is considerably closer to the experimental value than was possible with more conventional shell-model calculations.[17]

Later work by BG[16] has discussed in some detail the motivation for the choice of the 2p-2h component to be admixed in the g.s. The form selected is

$$\psi^{J=0,T=0}=\tfrac{1}{2}\sum_{j_{zi},t_{zi}}\sum_{T_z',T_z''}(\tfrac{1}{2}-t_{z3}\tfrac{1}{2}-t_{z4}|\tfrac{1}{2}\tfrac{1}{2}1T_z')$$
$$\times(\tfrac{1}{2}-j_{z3}\tfrac{1}{2}-j_{z4}|\tfrac{1}{2}\tfrac{1}{2}00)(-1)^{T_z}\times(1T_z''1T_z'|1100)$$
$$\times(\tfrac{1}{2}t_{z1}\tfrac{1}{2}t_{z2}|\tfrac{1}{2}\tfrac{1}{2}1T_z'')\{0.808(\tfrac{5}{2}j_{z1}\tfrac{5}{2}j_{z2}|\tfrac{5}{2}\tfrac{5}{2}00)\eta_1(1d_{5/2})^2\eta_2$$
$$-0.038(\tfrac{3}{2}j_{z1}\tfrac{3}{2}j_{z2}|\tfrac{3}{2}\tfrac{3}{2}00)\eta_1(1d_{3/2})^2\eta_2$$
$$+0.588(\tfrac{1}{2}j_{z1}\tfrac{1}{2}j_{z2}|\tfrac{1}{2}\tfrac{1}{2}00)\eta_1(2s_{1/2})^2\eta_2\}$$
$$\times\eta_3^\dagger(1p_{1/2})^{-2}\eta_4^\dagger|C\rangle, \quad (11)$$

where the core is assumed to consist of particles in a deformed basis.

Most likely the 2p-2h component in the g.s. is more complicated than the expression above. But we shall assume that the bulk of the deformity is of this nature. One might guess that the $p_{1/2}{}^{-2}s_{1/2}{}^2$ and $p_{1/2}{}^{-2}d_{5/2}{}^2$ configurations would be admixed more with the g.s. than would other configurations simply because their unperturbed energies are lower.

The calculations of BG reveal the 4p-4h component in the g.s. to be quite small, therefore, we have ignored it completely and for convenience have assumed the g.s. to consist of closed-shell (75%) and 2p-2h (25%) components. The negligible $(d_{3/2})^2(1p_{1/2})^{-2}$ term has been eliminated from the g.s. 2p-2h component. The resulting g.s. wave function which has been used in the calculations has the form

$$0.866|C\rangle+0.5\{\tfrac{1}{2}\sum_{j_{zi}t_{zi}}\sum_{T_z'T_z''}(\tfrac{1}{2}-t_{z3}\tfrac{1}{2}-t_{z4}|\tfrac{1}{2}\tfrac{1}{2}1T_z')$$
$$\times(\tfrac{1}{2}-j_{z3}\tfrac{1}{2}-j_{z4}|\tfrac{1}{2}\tfrac{1}{2}00)(-1)^{T_z'}(1T_z''1T_z'|1100)$$
$$\times(\tfrac{1}{2}t_{z1}\tfrac{1}{2}t_{z2}|\tfrac{1}{2}\tfrac{1}{2}1T_z'')[0.808(\tfrac{5}{2}j_{z1}\tfrac{5}{2}j_{z2}|\tfrac{5}{2}\tfrac{5}{2}00)\eta_1(1d_{5/2})^2\eta_2$$
$$+0.589(\tfrac{1}{2}j_{z1}\tfrac{1}{2}j_{z2}|\tfrac{1}{2}\tfrac{1}{2}00)\eta_1(2s_{1/2})^2\eta_2]\}$$
$$\times\eta_3^\dagger(1p_{1/2})^{-2}\eta_4^\dagger|C\rangle. \quad (12)$$

It has been assumed that the 2p-2h component is constructed on a spherical core.

III. FORMULAS

A. Muon Capture

The muon-capture process in O^{16},

$$\mu^-+O^{16}\to N^{16}+\nu, \quad (13)$$

proceeds at the rate[5]

$$\Lambda_{\mu c}=\alpha[\Lambda_0+\Lambda_1+\Lambda_2+\Lambda_{\text{corr}}], \quad (14)$$

where Λ_0 is the contribution to the capture rate associated with $J=0^+\to 0^-$ nuclear transitions,

$$\Lambda_0=\sum_a{}'\sum_b\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2\bigg[(G_p-G_A)^2M_{\sigma0}{}^2$$
$$+2(G_p-G_A)\frac{g_A}{m_pC}M_\sigma\cdot M_{\sigma.\text{p}}+\frac{g_A{}^2}{m_p{}^2C^2}M_{\sigma.\text{p}}{}^2\bigg], \quad (15)$$

Λ_1 is associated with $0^+\to 1^-$ transitions,

$$\Lambda_1=\sum_a{}'\sum_b\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2\bigg[G_v{}^2M_1{}^2+G_A{}^2\{M_{\sigma+}{}^2+M_{\sigma-}{}^2\}$$
$$-2\frac{g_v}{m_pC}G_v(Re)M_1\cdot M_{p0}+2\frac{g_v}{m_pC}G_A(Re)$$
$$\times\{M_{\sigma+}\cdot M_{p+}-M_{\sigma-}\cdot M_{p-}\}+\frac{g_v{}^2}{m_p{}^2C^2}M_{\text{p}}{}^2\bigg], \quad (16)$$

and Λ_2 is associated with $0^+\to 2^-$ transitions

$$\Lambda_2=\sum_a{}'\sum_b\left(\frac{\nu_{ab}}{\nu_\mu}\right)^2$$
$$\times[G_A{}^2\{M_{\sigma+}{}^2+M_{\sigma-}{}^2\}+(G_p-G_A)^2M_{\sigma0}{}^2]. \quad (17)$$

$\alpha=\nu_\mu{}^2[(|\varphi_\mu|^2)_{\text{av}}/2\pi\hbar^2C]$, where $h\nu_\mu C\equiv m_\mu C^2$ and $(|\varphi_\mu|^2)_{\text{av}}$ is the square of the muon wave function averaged over the nucleus. m_p is the rest mass of the proton. G_v, G_p, G_A, g_v, and g_A are effective weak interaction coupling constants discussed, for example, in

[17] J. M. Eisenberg, B. M. Spicer, and M. E. Rose, Nucl. Phys. **71**, 273 (1965).

Refs. 4 and 18. The initial nuclear state (g.s. of O^{16}) is denoted by $|a\rangle$ while the final nuclear states (N^{16}) are denoted by $|b\rangle$. Λ_{corr} is a correction term which will be discussed at the end of this subsection.

In general, the nuclear matrix elements (ME) are of the form

$$M_{(\text{op})}=\langle b|\sum_{i=1}^{16}\text{op}(i)e^{-i\nu_{ab}z(i)}\tau^{(-)}(i)|a\rangle, \tag{18}$$

where $h\nu_{ab}$ is the neutrino momentum for the nuclear transition $a\rightarrow b$. For convenience the z axis has been denoted as the direction of neutrino emission. If the usual expansion for the neutrino wave function appearing in Eq. (18) is employed,

$$e^{-i\nu_{ab}z}=\sum_L(-i)^L\{4\pi(2L+1)\}^{1/2}j_L(\nu_{ab}r)Y_{L0}(\theta), \tag{19}$$

then one can classify the various nuclear transitions as $L=0$ (allowed), $L=1$ (first forbidden). $L=2$ (second forbidden), etc. We have retained terms (through third forbidden) which can result in single-particle transitions from the $1p$ shell to the $2s$ or $1d$ shell within the framework of the g.s. and excited states (0^-, 1^-, and 2^-) considered. This requires that in addition to the first forbidden term we retain the third forbidden ($L=3$) term for spin-dependent ($\boldsymbol{\sigma}$) transitions to the 2^- states. We keep the allowed and second forbidden terms in those ME containing the nucleon momentum operator p. Transitions proceeding via other multipole terms than these are included in Λ_{corr}.

Vector coupling the momentum operator (when present) to the spherical harmonic (from the plane-wave expansion) and coupling the resulting sum of operators to the spin operator (when present), one may reduce Eq. (18) to the form

$$\sum_J a_J\langle b|T_J|a\rangle, \tag{20}$$

i.e., a sum of ME, each containing a transition operator that transforms under rotations like an irreducible tensor.

For the configuration-mixing model, the g.s. consists of the closed-shell configuration plus 2p-2h states of the form of Eq. (6) and the final nuclear states of importance are assumed to consist of combinations of the p-h states, Eq. (3), and the 3p-3h states Eq. (10). We list below the general form for the ME of the possible transitions between such states, assuming the single-particle transition operator transforms like an irreducible tensor, T_J:

(a) $|C\rangle\rightarrow|\text{p-h}\rangle$ yields

$$\frac{1}{(2J+1)^{1/2}}(j_1|T_J|j_2), \tag{21}$$

(b) $|\text{2p-2h}\rangle\rightarrow|\text{p-h}\rangle$ yields

$$\frac{(-1)^{j_1+j_2+J}(j_2|T_J|j_1)\delta(p_{12},p_1)\delta(h_{12},h_1)}{[(2J+1)(2j_1+1)(2j_2+1)3]^{1/2}}, \tag{22}$$

(c) $|\text{2p-2h}\rangle\rightarrow|\text{3p-3h}\rangle$ yields

$$N_2\frac{1}{(2J+1)^{1/2}}(j_5|T_J|j_6)\delta(\beta), \tag{23}$$

where the $(j|T_J|j')$ are reduced matrix elements of the single-particle tensor operator T_J between the single-particle states j and j'. In Eqs. (21) and (22), j_1 and j_2 refer to the particle and hole states, respectively, appearing in the p-h wave function. The Kronecker δ function $\delta(p_{12},p_1)(\delta(h_{12},h_1))$ is zero unless the two particles (holes) of the 2p-2h state have the same nlj quantum numbers as the particle (hole) of the p-h state. The symbols N_2, j_5, and j_6 appearing in Eq. (23) have the same meaning as in Eq. (8). The δ function $\delta(\beta)$ is zero unless the first two holes (j_3,j_4) of the 3p-3h state have the same nlj quantum numbers as the two holes in the 2p-2h state with a similar requirement being imposed by the δ function on the first two particle states (j_1,j_2) of the initial and final wave functions. Of course for all transitions the total angular momentum of the final many-particle nuclear state must be the same as the rank of the irreducible tensor T_J since the g.s. has zero angular momentum. In addition, all final states must have unit isotopic spin with the number of neutrons exceeding the number of protons by two owing to the presence of the isotopic spin lowering operator in Eq. (18).

Obviously any change from previous results will be due to the altered behavior of the transition ME between the more complicated states to be introduced in our model. There are two straightforward changes that may be noted at this point. The factor N_2 may cause a considerable reduction in substituting from the $|C\rangle\rightarrow|\text{p-h}\rangle$ to the comparable $|\text{2p-2h}\rangle\rightarrow|\text{3p-3h}\rangle$ transition. Moreover the presence of $|\text{2p-2h}\rangle\rightarrow|\text{p-h}\rangle$ terms can result in interference between this transition and $|C\rangle\rightarrow|\text{p-h}\rangle$ transitions.

The general formulas resulting from the evaluation of the single-particle matrix elements for the various transition operators will not be listed. These are easy to obtain by repeated use of the Wigner-Eckart theorem.[19] For the particular ME required in the present work the results are listed in Appendix A.

The term Λ_{corr}, appearing in Eq. (14), symbolizes the sum of diverse corrections to the total capture rate. We estimate the correction term arising from neglected higher-order velocity-dependent matrix elements is less than 1% of the total rate. [Note that more conventional p-h calculations generally require a 10% correction

[18] L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964).

[19] The results for many of the single-particle matrix elements of interest are listed in Ref. 5.

factor of this type since in such calculations velocity-dependent matrix elements are not included in Λ_0, Λ_1, and Λ_2.] We employ the usual 10% factor with respect to the correction term for higher multipoles (of velocity-independent terms) ignored in the present calculation.[18]

Finally, there exist corrections of a different nature than have been met in previous muon-capture calculations because of the more complicated nuclear configurations involved here. These fall into two general categories. First, there exist 3p-3h states that will not be considered in the calculations although they contain particles and holes in the same orbitals as the 3p-3h states considered in detail in the model. The only difference between these neglected states and the states considered is in the intermediate isospin and angular momentum couplings. Because of the nature of many-particle–many-hole configurations, a multitude of such states exists. One can demonstrate that the appropriate single-particle transition ME linking these neglected states to the assumed g.s. will vanish in most cases. There do exist nonvanishing transitions to neglected states composed of configurations of three identical particles and three identical holes. Except in a few cases these states are not in reality linearly independent of states already included in the model. Calculations involving the few pathological cases indicate that transitions to neglected 3p-3h states belonging to the same configurations as states included in the model, but linearly independent of the model states, account for less than 2% of the total transition rate.

The second, and quantitatively larger, category of configuration-mixing-induced correction terms arises from $|2p\text{-}2h\rangle g.s. \to |2p\text{-}2h\rangle$ excited-state transitions. It is as a result of these transitions that the over-all strength of the capture ME (robbed from low-lying states and the giant-resonance region) is *more* than recovered. The quantitative details of this correction term will be discussed in Sec. VI. We note that more than 75% of the correction arises from allowed Gamow-Teller transitions and that the total correction term amounts to approximately 15% of the capture rate.

B. Photon Absorption

The nuclear photon-absorption cross section for photons of energy E may be evaluated from the expression[18]

$$\sigma_\gamma(E)=\sum_a{}' \sum_b \frac{\pi^2}{411}(E_b-E_a)\,|\langle b|\sum_{i=1}^{A}\tau^{(0)}(i)\mathbf{r}(i)|a\rangle|^2 \times\delta(E-E_b+E_a)\,. \quad (24)$$

The notation utilized above is the same as that used in the expressions for the muon-capture rate.

Utilizing the same techniques employed in evaluating the muon-capture transition ME, we may obtain the result given in Appendix A for the photodisintegration ME. [See Eq. (A7).]

IV. $0^+ \to 0^-$ TRANSITIONS

If we assume an O^{16} g.s. containing 2p-2h admixtures, then the usual p-h wave functions should probably be altered in the interest of consistency. To illustrate this point, let us first consider a series of calculations performed at an early stage of the investigation.

Suppose one utilizes the g.s., Eq. (12), and considers transitions to the usual p-h odd-parity states. (For example, the wave functions of Elliott and Flowers,[2] Gillet and Vinh Mau,[20] or Lewis[1,21] would be appropriate.) A substantial reduction in the transition rates to the p-h states is obtained, compared with conventional calculations that employ a pure closed-shell g.s. This results because a portion of the g.s., the 2p-2h component, has reduced or negligible transition rates to the p-h states. Moreover, the non-negligible transitions are frequently opposite in sign to the usual $|C\rangle \to |p\text{-}h\rangle$ transitions so that destructive interference occurs.

Since there may exist important transitions to odd-parity 3p-3h final states from the 2p-2h component in the g.s., one should proceed to calculate these matrix elements. The question of the excitation energies of the 3p-3h states is solved by simply taking the unperturbed single-particle and -hole energies of the individual particles and holes comprising the 3p-3h state and adding them. Carrying out the calculation indicated above one finds that the 3p-3h states do not contain much transition strength because of energy-dependent factors which depress higher-energy transitions.

Of course, there is the possibility that some of these highly excited 3p-3h states may be admixed with the usual p-h states and that this might affect the transition strength. In fact, however, accomplishing a diagonalization of the matrix resulting from allowing a Serber-force residual interaction[22] to mix the 3p-3h and p-h configurations, one determines that there is little mixing of these states. Therefore, the reduction in transition strength is still obtained.

The fact that the p-h and 3p-3h configurations remain substantially unmixed is partly due to an energy gap between the two groups of states. This energy gap is caused by the high unperturbed configuration energy characteristic of the 3p-3h states. However the gap may, in actuality, be smaller, since if the g.s. of O^{16} is deformed, then the conventional interpretation of the levels of neighboring nuclei (in regard to their yielding single-particle and single-hole energies) should be reevaluated.

In most cases, all one has done to form the p-h and 3p-3h states is to apply a p-h operator to the closed-shell and 2p-2h configurations. Therefore, if one postulates a certain amount of mixing in the ground state,

[20] V. Gillet and N. Vinh Mau, Nucl. Phys. **54**, 321 (1964). We use wave functions obtained in their approximation I.

[21] Only Lewis's 1^- wave functions may be found in Ref. 1. The 0^- and 2^- wave functions are given in Ref. 18.

[22] The Serber-force residual interaction of Ref. 1 has been used.

TABLE I. Energies and wave functions for the $J^{\pi}=0^-$ states of O^{16}. The simple particle-hole basis states have been amended to include three-particle, three-hole admixtures consistent with the assumed form for the ground state.[a,b]

	Amplitudes of amended basis states					
Energy (MeV)	$0.866\,\psi$(p-h,1) $0.5\,\psi$(3p-3h,1)	$0.866\,\psi$(p-h,2) $+0.5\,\psi$(3p-3h,2)	$-0.5\,\psi$(p-h,1) $+0.866\,\psi$(3p-3h,1)	$-0.5\,\psi$(p-h,2) $+0.866\,\psi$(3p-3h,2)	ψ(3p-3h,3)	ψ(3p-3h,4)
50.0	−0.008		−0.005		−0.054	0.999
40.1	0.020	−0.008	0.092	−0.114	0.988	0.054
38.3	0.009	−0.002	−0.042	0.992	0.117	0.005
27.3	−0.067	0.991	0.118	0.008	−0.001	
26.0	−0.027	−0.120	0.987	0.052	−0.086	
14.3	0.997	0.064	0.033	−0.004	−0.024	0.007

[a] We have employed the Serber-force residual interaction of Ref. 1.
[b] A shortened notation has been adopted for the basis states. Reading from left to right in the table the basis states are those denoted by Eqs. (29), (31), and (33)–(36), respectively.

based on something other than a conventional residual interaction mixing calculation, then approximately the same degree of mixing for the p-h and 3p-3h basis states should be assumed, otherwise the calculation seems inconsistent.

A more consistent model is formulated below for the 0^- states of N^{16}.

If the usual closed-shell configuration is replaced by a g.s. of the form

$$\alpha|C\rangle+\beta|2\text{p-}2\text{h}\rangle\,, \tag{25}$$

then the following replacement is made in the simple p-h basis states:

$$\alpha|\text{p-h}\rangle+\beta|3\text{p-}3\text{h}\rangle\,, \tag{26}$$

where the $|C\rangle$, $|\text{p-h}\rangle$, $|2\text{p-}2\text{h}\rangle$, and $|3\text{p-}3\text{h}\rangle$ states are each normalized so that

$$\alpha^2+\beta^2=1\,. \tag{27}$$

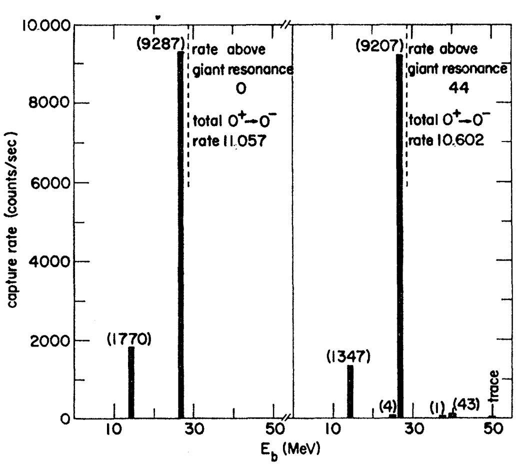

FIG. 1. The O^{16} partial rates for muon capture leading to the 0^- states of N^{16}. The left bar graph is obtained from a calculation employing a closed-shell ground state and particle-hole configurations as excited states. The bar graph on the right is obtained by including two-particle, two-hole configurations in the ground state and amending the particle-hole states to include three-particle, three-hole configurations. The Serber-force residual interaction and the oscillator parameter of Ref. 1 have been adopted. The particle-hole wave functions used are those of Lewis (Ref. 21), while the weak-interaction coupling constants are those assumed in Ref. 18.

For example, assuming a g.s. of the form Eq. (12) one would replace the simple p-h basis state

$$\psi(\text{p-h},2s_{1/2}(1p_{1/2})^{-1},0,0,1,-1) \tag{28}$$

by

$$0.866\psi(\text{p-h},\,2s_{1/2}(1p_{1/2})^{-1},\,0,\,0,\,1,\,-1) +0.5\psi(3\text{p-}3\text{h},0,0,1,-1,2s_{1/2}(1p_{1/2})^{-1} \times[0.808(1d_{5/2})^2(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{1/2})^{-2}])\,, \tag{29}$$

while the p-h basis state

$$\psi(\text{p-h},1d_{3/2}(1p_{3/2})^{-1},0,0,1,-1) \tag{30}$$

would become

$$0.866\psi(\text{p-h},1d_{3/2}(1p_{3/2})^{-1},0,0,1,-1) +0.5\psi(3\text{p-}3\text{h},0,0,1,-1,1d_{3/2}(1p_{3/2})^{-1}, \times[0.808(1d_{5/2})^2(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{1/2})^{-2}])\,. \tag{31}$$

The unperturbed configuration energies of these amended basis states are, in general, slightly different than those given for the usual p-h states in conventional calculations. The small variations in the configuration energies, induced by configuration mixing, may be determined by reexamining the usual procedure for obtaining the p-h energies. In earlier research, the experimentally determined single-particle and single-hole levels in O^{17} and O^{15}, respectively, have been used to yield the unperturbed p-h energies. However, the interpretation of these levels in neighboring nuclei as single-particle and single-hole states is now suspect. For, if the g.s. of O^{16} contains a 2p-2h component, then the low-lying states O^{17}, for example, would be expected to consist of a 3p-2h state as well as the usual single-particle orbit outside the closed shell. Similarly, one would expect the states of O^{15} to contain a 2p-3h component as well as the usual single-hole state.

We have assumed that the relevant states of O^{17} and O^{15} contain 3p-2h and 2p-3h normalized wave-function amplitudes, respectively, with the same coefficients as the 2p-2h components in the ground state. Utilizing these more complicated states we have proceeded to determine the unperturbed energies of the states Eqs. (29) and (31). The alteration in the configuration en-

TABLE II. Energies and wave functions for the $J^{\pi}=0^{-}$ states of O^{16} as determined by a multiparticle, multipole configuration-mixing model.[a,b]

Energy (MeV)	Amplitudes of amended basis states					
	$0.866\,\psi$(p-h,1) $0.5\,\psi$(3p-3h,1)	$0.866\,\psi$(p-h,2) $+0.5\,\psi$(3p-3h,2)	$-0.5\,\psi$(p-h,1) $+0.866\,\psi$(3p-3h,1)	$-0.5\,\psi$(p-h,2) $+0.866\,\psi$(3p-3h,2)	ψ(3p-3h,3)	ψ(3p-3h,4)
50.0					−0.070	0.998
40.0					0.998	0.070
38.3			−0.070	0.998		
27.3	−0.070	0.998				
25.4			0.998	0.070		
14.4	0.998	0.070				

[a] We have used the particle-hole amplitudes of Lewis (Refs. 1 and 21). [b] The notation for the amended basis states the same as in Table I.

ergies arises because exclusion-principle effects force different normalization factors N_2 for the 3p-3h, 3p-2h, and 2p-3h states. These dissimilar factors in turn complicate the usual procedure and one cannot obtain the energies of the basis states by use of the levels of neighboring nuclei alone. The result is that certain contributions to the unperturbed energy must be evaluated by using a residual interaction.[22] The net contribution of these terms is small and, therefore, the inaccuracy obtained owing to the uncertainty in the residual interaction is minimal.

Within the spirit of the model and based on the results of BG,[12] there exists an even-parity state 11.26 MeV above the g.s. which we shall approximate by

$$0.866\psi(\text{2p-2h},0.808(1d_{5/2})^2(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{3/2})^{-2})-0.5|C\rangle. \quad (32)$$

This requires we construct excited odd-parity states of the form

$$0.866\psi(\text{3p-3h},0,0,1,-1,2s_{1/2}(1p_{1/2})^{-1},0.808(1d_{5/2})^2\times(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{1/2})^{-2})-0.5\psi(\text{p-h},2s_{1/2}(1p_{1/2})^{-1},0,0,1,-1) \quad (33)$$

and

$$0.866\psi(\text{3p-3h},0,0,1,-1,1d_{3/2}(1p_{3/2})^{-1},[0.808(1d_{5/2})^2(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{1/2})^{-2}])-0.5\psi(\text{p-h},1d_{3/2}(1p_{3/2})^{-1},0,0,1,-1). \quad (34)$$

This second group of states have unperturbed configuration energies approximately 11 MeV above the corresponding states, Eqs. (29) and (31).

Ideally there should exist a third set of states, orthogonal to the states Eqs. (29), (31), (33), and (34). These states, included for completeness, have the form

$$\psi(\text{3p-3h},0,0,1,-1,2s_{1/2}(1p_{1/2})^{-1},[0.589(1d_{5/2})^2(1p_{1/2})^{-2}-0.808(2s_{1/2})^2(1p_{1/2})^{-2}]) \quad (35)$$

and

$$\psi(\text{3p-3h},0,0,1,-1,1d_{3/2}(1p_{3/2})^{-1},[0.589(1d_{5/2})^2(1p_{1/2})^{-2}-0.808(2s_{1/2})^2(1p_{1/2})^{-2}]). \quad (36)$$

Interpreting the O^{15} and O^{17} levels in the conventional manner and utilizing a residual interaction[22] to calculate terms unobtainable from experiment, the configuration energies of the states Eqs. (35) and (36) have been determined. We would conjecture that these states are strongly mixed with other high-lying states and therefore their transition strength (which will be quite small) should be considered as strength distributed in states over an energy band of 5–10 MeV.

Conventional calculations consider two $J=0^-$ particle-hole basis states which are admixed by a residual interaction. The eigenvectors of the diagonalized two-by-two residual interaction matrix are employed as final nuclear states $|b\rangle$ for $0^+\rightarrow 0^-$ muon-capture-induced nuclear transitions. In the present scheme, we have six 0^- basis states. These six states are allowed to mix via a residual interaction as in conventional calculations.[22] The resulting six eigenstates of the diagonalized interaction matrix are shown in Table I.

Utilizing the formulas of Appendix A we have calculated the muon-capture rate to the 0^- states. The results

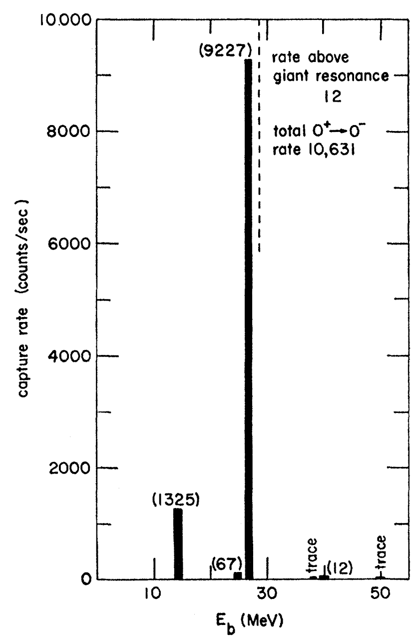

FIG. 2. The $0^+\rightarrow 0^-$ partial muon-capture rates for O^{16} as determined by including two-particle, two-hole configurations in the ground state and using a multiparticle, multihole configuration-mixing model to obtain the 0^- nuclear states. The 0^- wave functions are taken from Table II. The oscillator parameter and weak interaction coupling constants are the same as in Fig. 1.

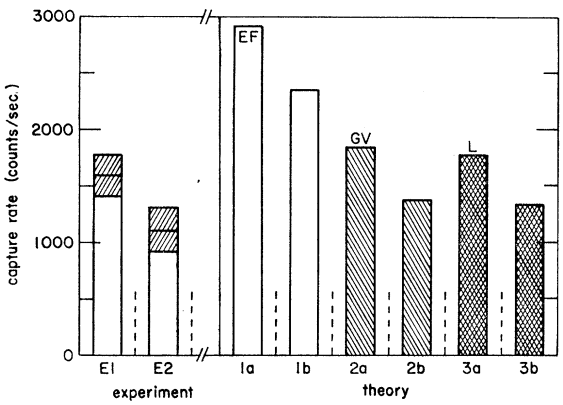

FIG. 3. Comparison of theoretical and experimental results for the O^{16} partial muon-capture rate leading to the lowest 0^- state of N^{16}. The experimental results shown in graphs E1 and E2 are taken from Refs. 7 and 6, respectively. Graphs 1a, 2a, and 3a are results obtained utilizing nuclear states given by conventional particle-hole calculations. For graphs 1a, 1b, and 2a, 2b we have adopted the values assumed for the coupling constants in Ref. 4 and an oscillator parameter $b=1.8$ F. For graphs 3a, 3b the same parameters are used as in Fig. 1. The 0^- wave functions used in obtaining graphs 1a, 2a, and 3a are those of Elliott and Flowers (Ref. 2) (EF), Gillet and Vinh Mau (Ref. 20) (GV), and Lewis (Ref. 21) (L), respectively. Graphs 1b, and 2b, 3b are obtained from applications of a multiparticle, multipole configuration-mixing model and the assumption of a ground state containing two-particle, two-hole admixtures. Graphs 1b, 2b, and 3b employ the amplitudes resulting from the particle-hole calculations used in the corresponding graphs 1a, 2a, and 3a. However the admixture amplitudes are now assumed to apply to the amended basis states.

are illustrated in Fig. 1 along with results obtained in a conventional p-h calculation (note the same residual interaction is used in both calculations). Comparison of the results reveals that the only appreciable difference occurs for the lowest 0^- state, where the transition-matrix-element strength is reduced in the case of the multiparticle, multihole configuration-mixing calculation. Unfortunately the current theoretical-experimental situation with regard to muon-capture-induced nuclear transitions to the lowest 0^- state is clouded by the variation in the two experimental results available (cf. Fig. 3).

Since the calculation of the configuration energies and the evaluation of the ME of the residual interaction between the six complicated 0^- states is at best a tedious task, it is useful to adopt the following simplifying procedure.

We amend the simple particle-hole basis states as in Eqs. (29), (31), and (33)–(36). However, the energies and admixture amplitudes of the final 0^- wave functions are obtained from conventional p-h calculations. For example, Lewis[21] has found that the 0^- state at 14.41 MeV has the form

$$0.998\psi(\text{p-h},2s_{1/2},(1p_{1/2})^{-1},0,0,1,-1) +0.070\psi(\text{p-h},1d_{3/2}(1p_{3/2})^{-1},0,0,1,-1), \quad (37)$$

therefore (with our choice of a g.s.), we assume there exists a state at 14.41 MeV of the form

$$\begin{aligned}0.998[&0.866\psi(\text{p-h},2s_{1/2}(1p_{1/2})^{-1},0,0,1,-1)\\&+0.5\psi(\text{3p-3h},0,0,1,-1,2s_{1/2}(1p_{1/2})^{-1},\\&\{0.808(1d_{5/2})^2(1p_{1/2})^{-2}+0.589(2s_{1/2})^2(1p_{1/2})^{-2}\})]\\+0.070[&0.866\psi(\text{p-h},1d_{3/2}(1p_{3/2})^{-1},0,0,1,-1)\\&+0.5\psi(\text{3p-3h},0,0,1,-1,1d_{3/2}(1p_{3/2})^{-1},\\&\{0.808(1d_{5/2})^2(1p_{1/2})^{-2}\\&+0.589(2s_{1/2})^2(1p_{1/2})^{-2}\})]. \quad (38)\end{aligned}$$

The results of generalizing this technique to the six 0^- states are shown in Table II. The state at 25.4 MeV (38.3 MeV) is taken to be 11 MeV above the state

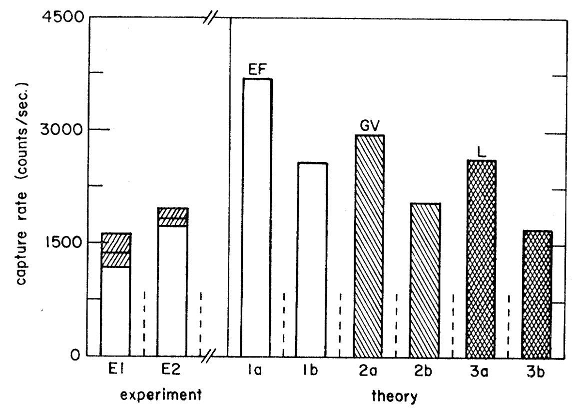

FIG. 4. Comparison of theoretical and experimental results for the O^{16} partial muon-capture rate leading to the lowest 1^- state of N^{16}. The experimental results shown in graphs E1 and E2 are taken from Refs. 7 and 6, respectively. The relevant information and procedures used to obtain the six theoretical predictions 1a, 1b, 2a, 2b, 3a, and 3b are identical, for corresponding graphs, to those of Fig. 3. Graphs 1a, 2a, and 3a illustrate the results of conventional calculations while graphs 1b, 2b, and 3b are obtained by using nuclear wave functions determined from a multiparticle multihole configuration-mixing model.

denoted by Eq. (29) [Eq. (31)] because the even-parity state Eq. (32) is separated from the g.s. by that amount. Adopting the conventional values for single-particle and single-hole configuration energies one determines that the two states at highest energy should be approximately 25 MeV above the states represented by Eqs. (29) and (31), respectively. (This probably results in an exaggeration of the excitation energy of the two highest-lying levels.) The above procedure for finding the 0^- wave functions will also be utilized in the case of the 1^- and 2^- states and is the kernel of the configuration-mixing model.

The $0^+\rightarrow 0^-$ partial muon-capture rate has been calculated utilizing as final nuclear states the wave functions listed in Table II. The results do not differ appreciably, for those states containing non-negligible strength, from those obtained earlier using the wave functions of Table I as final nuclear states (compare Figs. 1 and 2).

It is instructive to consider whether the particular p-h admixture amplitude chosen is critical to obtain the reduction in the rate to the lowest 0^- state. Therefore, we have used the amplitudes resulting from three different p-h calculations. The various rates determined, illustrated in Fig. 3, demonstrate that a reduction occurs independent of which amplitudes are adopted. From a mathematical viewpoint, the reduction is obtained due to the difference in sign of the terms E_1 and E_2 (see Appendix A). For a conventional p-h calculation E_2 would vanish and E_1 would be larger. One expects that the reduction effect would be largest for the lowest-lying 0^- state since it contains the largest component of the $2s_{1/2}(1p_{1/2})^{-1}$ configuration, whose transition matrix element with the $|2\text{p-}2\text{h}\rangle$ g.s. component is the source of the destructively interfering term E_2.

V. $0^+ \rightarrow 1^-$ AND 2^- TRANSITIONS

The simplifying technique (mp-mh configuration-mixing model) discussed in Sec. IV has been used to obtain the appropriate 1^- and 2^- states of O^{16} and N^{16}.

A. Muon Capture

Using the wave functions given by the configuration-mixing model we have calculated the O^{16} partial muon-capture rates resulting in nuclear transitions to 1^- and 2^- states of N^{16}. The relevant experimental and theo-

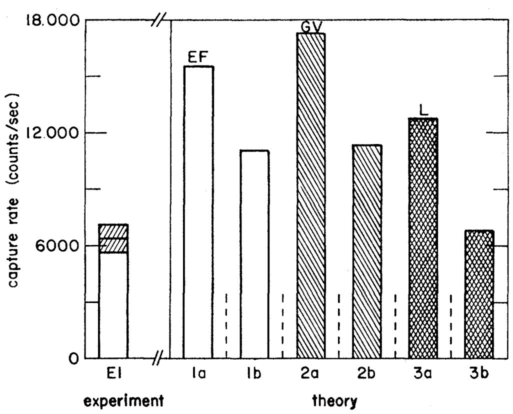

FIG. 5. Comparison of theoretical and experimental results for the O^{16} partial muon-capture rate leading to the lowest 2^- state of N^{16}. The experimental result shown in graph E1 is taken from Ref. 6. The information regarding the six graphs resulting from the theoretical calculations may be determined from the caption of Fig. 3. Graphs 1a, 2a, and 3a result from conventional calculations while graphs 1b, 2b, and 3b are obtained by employing the configuration-mixing model.

retical results for the partial capture rates to the lowest 1^- and 2^- states are shown in Figs. 4 and 5. We have again chosen the particle-hole admixture amplitudes given by Elliott and Flowers, Gillet and Vinh Mau, and Lewis as those to be reinterpreted in the manner prescribed by the configuration-mixing model. The results indicate that the configuration-mixing procedure lowers the theoretical prediction in each case and that thereby the theoretical-experimental situation is substantially improved.

The reduction in the rate to the lowest 1^- state occurs largely because of the destructive interference between the terms E_6 and E_2 for the spin-dependent matrix element multiplying $G_A{}^2$ (see Appendix A). One obtains positive reinforcement between the terms E_4 (which like E_6 is lessened by configuration mixing) and E_2 appearing in the vector spin-independent matrix element multiplying $G_V{}^2$. Therefore, configuration mixing results in a larger percentage reduction in the spin-dependent part of the matrix element for the transition rate to the lowest 1^- state than for the spin-independent part.

The decrease in the term E_8 results in a large reduction in the rate to the lowest 2^- state. Whereas for the lowest 0^- and 1^- states the rate reduction resulted partly from interference between $|(1p_{1/2})^{-2}(2s_{1/2})^2\rangle_{\text{g.s.}} \rightarrow |1p_{1/2}{}^{-1}2s_{1/2}\rangle$ transitions and $|C\rangle \rightarrow |1p_{1/2}{}^{-1}2s_{1/2}\rangle$ terms, for the 2^- states the interfering transition is of the type $|(1p_{1/2})^{-2}(1d_{5/2})^2\rangle_{\text{g.s.}} \rightarrow |1p_{1/2}{}^{-1}d_{5/2}\rangle$ and is not important to the rate reduction.

The partial capture rates leading to the full spectrum of 1^- and 2^- states considered are shown in Figs. 6 and 7. In each figure, the results obtained by including configuration mixing in the ground and excited nuclear states are compared with those obtained in a more conventional calculation employing Lewis's wave functions. In general, using the configuration-mixing model results in a lowering of the partial rate in and below the giant resonance with some transition strength now appearing in the region 30–50 MeV above the ground state. Figures 1, 2, 6, and 7 demonstrate that the muon-capture rate (associated with $0^+ \rightarrow 0^-$, 1^-, and 2^- nuclear transitions) above the giant resonance is quite small. This, as mentioned in the Introduction, is because energy-dependent factors suppress transitions resulting in high nuclear excitation.

B. Photon Absorption

The predictions of the configuration-mixing model with respect to the O^{16} photodisintegration cross section

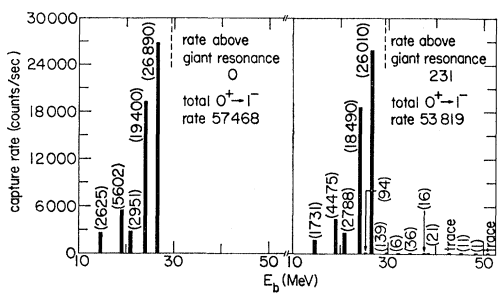

FIG. 6. The partial rates for muon capture on O^{16} leading to the 1^- states of N^{16}. The left bar graph is obtained from a calculation employing a closed-shell ground state and the particle-hole wavefunctions of Lewis (Ref. 1) as the 1^- states. The bar graph on the right is obtained by using 1^- wave functions given by a multiparticle, multipole configuration-mixing model and including two-particle, two-hole configurations in the ground state. The oscillator parameter and weak-interaction coupling constants are the same as in Fig. 1.

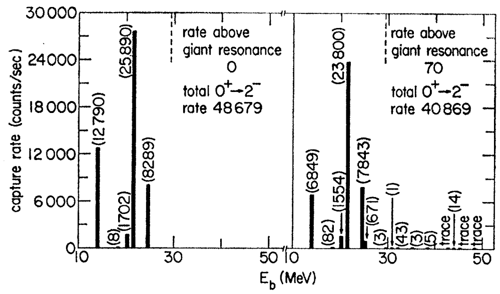

FIG. 7. The partial rates for muon capture O^{16} leading to the 2^- states of N^{16}. (See caption of Fig. 6.)

are shown in Fig. 8. Once again, for the purposes of comparison, the results are also illustrated of a calculation employing a closed-shell ground state and the p-h wave functions of Lewis. The configuration-mixing model predicts a substantially lower cross section below 27 MeV and indicates that 25% of the integrated cross section from this process should lie above the giant-resonance region.

Finally, we note that an obvious additional effect of the model discussed here is an increased number of states to which transitions may occur both in the cases of muon capture and photon absorption.

VI. $|2p\text{-}2h\rangle_{g.s.} \to |2p\text{-}2h\rangle_{f.s.}$ CORRECTIONS

The configuration-mixing model under consideration does not yield information regarding $|2p\text{-}2h\rangle_{\text{ground}} \to |2p\text{-}2h\rangle_{\text{final state}}$ transitions. It might be conjectured that such transitions are negligible and that therefore one is justified in ignoring them. Unfortunately, in the case of muon capture, such does not seem to be the situation.

For the purpose of convenience we have assumed the excitation energy of the 2p-2h final states to be 30 MeV. Utilization of unperturbed configuration energies obtained by simply adding the single-particle and single-hole energies from the levels of neighboring nuclei indicate that the $|2p\text{-}2h\rangle$ states may in actuality lie in a broad region from approximately 20 to 60 MeV above the ground state, although a group of states contributing approximately one-half of the 2p-2h correction are within 1 MeV of 30-MeV excitation energy. Of course these states may also be mixed with the p-h and 3p-3h states used as basis states in the configuration-mixing model. However, this complication is beyond the present introductory investigation into the effects of more complicated configuration mixing.

We have considered allowed and first forbidden matrix elements of velocity-independent single-particle operators that result in transitions to the five 2p-2h configurations listed in Table III. The calculations reported in Table III indicate that the allowed Gamow-Teller transitions dominate the correction.

Since allowed transitions are unimportant in conventional calculations of the O^{16} muon-capture rate, one might wonder about their prominence in the present work. The trivial difference lies in the fact that, in the present situation, the ground state contains particles in unfilled shells so that it is possible to have transitions to particles in the same orbit as the initial state with an accompanying spin and isospin flip.

The number of 2p-2h states to which transitions may occur from the ground state, Eq. (12), is large. For example, consider the first forbidden transitions to the $(1p)^{-2}(1d1f)$ configuration. In this case, there are ten linearly independent states that contain transition strength (ignoring different magnetic substates of the total angular momentum).

The total O^{16} muon-capture rate may be determined by adding the various partial rates and corrections. The results listed in Table IV indicate, as a consequence of the 2p-2h corrections, that the total capture rate calculated in the present model is slightly higher than that predicted using a simpler particle-hole model to obtain nuclear wave functions. Further discussion of this result is postponed to the next section.

The situation with regard to the 2p-2h correction terms for the photon-absorption cross section is considerably more encouraging. We have determined that the correction term in this case is less than 1% of the total cross section and is therefore negligible.

TABLE III. The contributions to the O^{16} muon-capture rate resulting in capture-induced $|2p\text{-}2h\rangle \to |2p\text{-}2h\rangle$ nuclear transitions.[a]

	Allowed		First forbidden		
Configuration	σ indep.	σ dep.	σ indep.	σ dep	Totals
$(1p)^{-2}(2s)^2$	2	4790			4792
$(1p)^{-2}(1d)^2$	9	9044			9053
$(1p)^{-2}(1d1f)$			408	1281	1689
$(1p)^{-2}(1d2p)$			35	113	148
$(1p)^{-2}(2s2p)$			245	1030	1275
Totals	11	13 834	688	2424	16 957

[a] An excitation energy of 30 MeV was assumed for the 2p-2h final nuclear states. The oscillator parameter and weak-interaction coupling constants are the same as in Fig. 1. The wave function Eq. (12) is taken as the ground state.

VII. SUMMARY AND CONCLUSION

Motivated by (a) the large experimental-theoretical discrepancies in the O^{16} partial muon-capture rates associated with nuclear transitions to the lowest 1^- and 2^- states of N^{16}, (b) the slight experimental-theoretical disagreement in the total O^{16} muon-capture rate, (c) the current overestimation of the O^{16} photon-absorption cross section in the giant-resonance region, (d) the lack of dipole strength calculated to be present in states above the giant resonance, and (e) the predictions of Brown *et al.* that the ground state of O^{16} may contain non-negligible amplitudes of more complicated configurations than the closed shell, we have considered the effect of special kinds of configuration mixing on nuclear transition matrix elements associated with muon capture and photon absorption in O^{16}.

After reporting briefly on calculations performed that seem to alleviate the experimental-theoretical discrepancies by treating configuration mixing differently in the ground state and final states, we have formulated a more consistent model.

In the configuration-mixing model, one essentially postulates that there should be similar multiparticle, multihole configuration mixing in the final states (reached via O^{16} muon capture and photon absorption) as in the O^{16} ground state and certain other low-lying even-parity states of the parent nucleus. Utilizing a simplified version of a 2p-2h component which we conjecture contains many of the features of the complicated 2p-2h admixtures to be found in the ground state, we have amended the usual p-h states following a prescription discussed in Sec. IV. The following change has been made in the ground state:

$$|C\rangle \rightarrow \alpha|C\rangle + \sum_i \beta_i^{(2p-2h)}|2p\text{-}2h\rangle_i \tag{39}$$

and, as a result, the simple p-h basis states have been altered:

$$|p\text{-}h\rangle \rightarrow \alpha\,\mathrm{Op(p\text{-}h)}|C\rangle + \sum_i \mathrm{Op}^i(p\text{-}h)\beta_i^{(2p-2h)}|2p\text{-}2h\rangle_i, \tag{40}$$

where Op(p-h) represents a particle-hole creation operator. The operator appears under the summation sign because of the normalization procedure adopted which requires, in addition, that

$$|p\text{-}h\rangle \rightarrow \alpha|p\text{-}h\rangle + \sum_i \beta_i^{(3p-3h)}|3p\text{-}3h\rangle_i \tag{41}$$

with

$$\beta_i^{(3p-3h)} \equiv \beta_i^{(2p-2h)} \tag{42}$$

and where the states $|p\text{-}h\rangle$ and $|3p\text{-}3h\rangle_i$ are each normalized.

Additional final nuclear states have been obtained by carrying out the above procedure on two other linearly independent even-parity states containing $|C\rangle$ and the $|2p\text{-}2h\rangle_i$. This procedure changes the number of final 0^- nuclear states considered from two to six and the number of 1^- and 2^- states each from five to fifteen.

After first completing a lengthy calculation to determine the mixing of the more complicated 0^- basis states, the following shortened procedure was adopted: One uses the admixture amplitudes and energy levels obtained in conventional particle-hole calculations but reinterprets the admixtures as applying to the amended-configuration-mixed basis states of the form of Eq. (40) as opposed to the simple p-h states.

We have then proceeded to calculate the appropriate transition ME between the configuration-mixed ground state and the amended-configuration-mixed final states and obtained the results listed below.

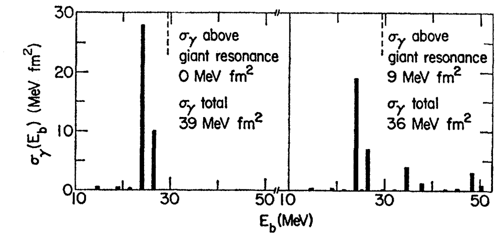

FIG. 8. The calculated photon-absorption cross section for O^{16}. The left bar graph is obtained from a conventional calculation using Lewis's (Ref. 1) wave functions. The results on the right are obtained by using 1^- wave functions given by the configuration-mixing model and including two-particle, two-hole configurations in the ground state. The oscillator parameter is the same as that assumed for Fig. 1.

Utilizing conventional values for the coupling constants and applying the configuration-mixing model one finds that:

(a) The partial muon-capture rates associated with transitions to the lowest 0^-, 1^-, and 2^- states of N^{16} are reduced and the experimental-theoretical discrepancy is eliminated.

(b) The partial capture rates to the totality of 0^-, 1^-, and 2^- states is reduced, the reduction in the case of the 2^- states being approximately 16%. However, due to corrections arising from allowed Gamow-Teller transitions, the total capture rate remains essentially unchanged (2% increase).

(c) The effects of configuration mixing are different for the muon-capture spin-dependent and spin-independent matrix elements so that, for example, a substantially larger percentage reduction occurs for the spin-dependent ME than for the spin-independent term in the case of transitions to the lowest 1^- state.

(d) The photodisintegration cross section in the giant-resonance region is reduced by 30%. Most of the lost matrix-element strength subsequently reappears in nuclear states above 30 MeV excitation.

(e) The general pattern of the results mentioned above is obtained independent of which particle-hole admixture amplitudes are adopted.

The difficulties with respect to overestimation of the total muon-capture rate may stem from an oversimplifi-

TABLE IV. Comparison of theory and experiment for the total muon-capture rate on O^{16}.

(Units 10^3/sec)	$(0^+\rightarrow 0^-)$ Λ_0	$(0^+\rightarrow 1^-)$ Λ_1	$(0^+\rightarrow 2^-)$ Λ_2	Λ_{corr}[e]	Total
Model I[a]	11.1	57.5	48.7	12.9	130.2
Model II[b]	10.6	53.8	40.9	28.5	133.8
Expt[c]					98
Expt[d]					97

[a] A conventional calculation using a closed-shell ground state and particle-hole final states.
[b] A multiparticle, multihole configuration-mixing model.
[c] J. Barow, J. C. Sens, P. J. Duke, and M. A. R. Kemp, Phys. Letters 9, 84 (1964).
[d] M. Eckhouse, thesis, Carnegie Institute of Technology, Department of Physics, Pittsburgh, 1963 (unpublished).
[e] See Secs. III and VI for discussions of the correction term.

cation of the ground-state wave function. For if terms [such as the $(p_{3/2})^{-2}(d_{3/2})^2$ configuration] existed in the g.s. which could cause destructive interference of the transition amplitudes in the important states in the giant resonance, then a substantial reduction is possible. Moreover, if the 2p-2h g.s. component is built on a deformed core then the reduced overlap of the initial and final states might change the results (although a consistent treatment of deformation in both the initial and final states probably precludes much of a reduction).

Since the effects of mp-mh configuration mixing are felt somewhat differently for the cases of the partial muon-capture rates and the photon-absorption cross section, it seems an oversimplification to relate the reduction of the total muon-capture rate directly to a reduction of the photon-absorption cross section as was done in Ref. 18. Nevertheless, it is true that the first forbidden contributions to both processes are reduced in the present model, which is in general agreement with the conclusions of Foldy and Walecka.[18]

Of course there are many approximations attendant to the procedure we have adopted. For example, the question of spurious states has not been considered. However, the simplifications used seem in keeping with an introductory investigation of the effects of complicated configuration mixing on weak and electromagnetic interaction-induced nuclear transitions for closed-shell nuclei. This is especially true if one wishes to keep the problem tractable and retain some of the appeal of the particle-hole model.

APPENDIX: MUON-CAPTURE AND PHOTON-ABSORPTION MATRIX ELEMENTS

Let $\alpha\equiv$ coefficient of closed-shell configuration in the ground-state wave function.

$\beta,\gamma\equiv$ coefficients of normalized $\psi(\text{2p-2h}, (1d_{5/2})^2(1p_{1/2})^{-2})$ and $\psi(\text{2p-2h}, (2s_{1/2})^2(1p_{1/2})^{-2})$ wave functions, respectively, in the ground state.

$A, B, C, D, E, F\equiv$ coefficients of a normalized $\psi(\text{p-h}, n_1l_1j_1(n_2l_2j_2)^{-1}, J, J_z, 1, -1)$ wave function in a particular final nuclear state where the symbols $A, \cdots, F$ refer to the different p-h configurations:

$$A\to(2s_{1/2},1p_{1/2}^{-1}),\ B\to(1d_{5/2},1p_{1/2}^{-1}),\ C\to(1d_{3/2},1p_{1/2}^{-1}),\ D\to(2s_{1/2},1p_{3/2}^{-1}),\ E\to(1d_{5/2},1p_{3/2}^{-1}),\ F\to(1d_{3/2},1p_{3/2}^{-1}).$$

$N, O, P, Q, R, S, T, U, V, W, X, Y\equiv$ coefficients of a normalized $\psi(\text{3p-3h}, \bar{J}, \bar{J}_z, 1, -1, n_5l_5j_5(n_6l_6j_6)^{-1}, (n_1l_1j_1)^2\times(n_3l_3j_3)^{-2})$ wave function in a particular final nuclear state where the symbols $N, \cdots, Y$ refer to the different 3p-3h configurations:

$$\begin{aligned}
&N\to\{(2s_{1/2},1p_{1/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\}, && O\to\{(1d_{5/2},1p_{1/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\},\\
&P\to\{(1d_{3/2},1p_{1/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\}, && Q\to\{(2s_{1/2},1p_{3/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\},\\
&R\to\{(1d_{5/2},1p_{3/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\}, && S\to\{(1d_{3/2},1p_{3/2}^{-1}),1d_{5/2}^21p_{1/2}^{-2}\},\\
&T\to\{(2s_{1/2},1p_{1/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\}, && U\to\{(1d_{5/2},1p_{1/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\},\\
&V\to\{(1d_{3/2},1p_{1/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\}, && W\to\{(2s_{1/2},1p_{3/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\},\\
&X\to\{(1d_{5/2},1p_{3/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\}, && Y\to\{(1d_{3/2},1p_{3/2}^{-1}),2s_{1/2}^21p_{1/2}^{-2}\}.
\end{aligned}$$

The partial muon-capture rates Λ_0, Λ_1, and Λ_2 may now be evaluated within the framework of the initial and final states listed above. We obtain for the partial rates to a particular 0^-, 1^-, or 2^- state the following:

$$\begin{aligned}\Lambda_0(\text{part. state})=(\nu_{ab}/\nu_\mu)^2e^{-z^2/2}[\{(G_p-G_A)^2[(E_1-E_2)R_1+E_3R_2]^2\}+\{2(G_p-G_A)(g_Ah/m_pC)[(E_1-E_2)R_1+E_3R_2]\\
\times[E_1R_3-E_3R_4+E_5R_2]\}+\{(g_A{}^2h^2/m_p{}^2C^2)[E_1R_3-E_3R_4+E_2R_5]^2\}],\quad\text{(A1)}\end{aligned}$$

$$\begin{aligned}\Lambda_1(\text{part. state})=(\nu_{ab}/\nu_\mu)^2e^{-z^2/2}[\{G_v{}^2[(E_4+E_2)R_1+E_5R_2]^2\}+\{2G_A{}^2[(E_6+E_2)R_1+E_7R_2]^2\}\\
+\{(-2g_vG_vh/m_pC)[(E_4+E_2)R_1+E_5R_2]\times\tfrac{1}{3}[E_4(R_3-2R_6)+E_5(-R_4+2R_7)-E_2R_8]\}+\{(4g_vG_Ah/m_pC)\\
\times[(E_6+E_2)R_1+E_7R_2]\times\tfrac{1}{3}[E_4(R_3+R_6)+E_5(-R_4-R_7)-E_2R_9]\}+\{(2g_v{}^2h^2/9m_p{}^2C^2)([E_4(R_3+R_6)\\
+E_5(-R_4-R_7)-E_2R_9]^2+[E_4(R_3-2R_6)+E_5(-R_4+2R_7)-E_2R_8]^2)\}],\quad\text{(A2)}\end{aligned}$$

$$\begin{aligned}\Lambda_2(\text{part. state})=(\nu_{ab}/\nu_\mu)^2e^{-z^2/2}[\{2G_A{}^2[\sqrt{3}E_8R_1+(6\sqrt{2}/5)E_9(R_2+\tfrac{1}{6}R_{10})+\tfrac{1}{5}\sqrt{3}E_{10}(R_2-4R_{10})\\
-\tfrac{3}{5}(\sqrt{7})E_{11}(R_2-(4/21)R_{10})-\tfrac{2}{5}\sqrt{3}E_{12}(R_2+R_{10})]^2\}+\{(G_p-G_A)^2[2E_8R_1+\tfrac{1}{5}(\sqrt{6})E_9(4R_2-R_{10})\\
+\tfrac{2}{5}E_{10}(R_2+6R_{10})-\tfrac{2}{5}(\sqrt{21})E_{11}(R_2+(2/7)R_{10})-\tfrac{4}{5}E_{12}(R_2-\tfrac{3}{2}R_{10})]^2\}],\quad\text{(A3)}\end{aligned}$$

where

$$z=\nu_{ab}b_{\text{osc.}}\ (b_{\text{osc.}}\equiv(\hbar/m_p\omega)^{1/2}),$$

$$E_1=\alpha A+\frac{\gamma T}{\sqrt{12}}+\frac{\beta N}{\sqrt{2}},\quad E_2=+\frac{\gamma A}{2\sqrt{3}},\quad E_3=\alpha F+\beta S+\gamma Y,$$

$$E_4=\alpha A+\sqrt{2}\alpha D+\frac{\gamma T}{(\sqrt{12})}+\gamma W+\frac{\beta N}{\sqrt{2}}+\sqrt{2}\beta Q,$$

$$E_5=-\alpha C-\frac{3}{\sqrt{5}}\alpha E+\frac{\alpha F}{\sqrt{5}}-\frac{\gamma V}{\sqrt{2}}-\frac{3\gamma X}{\sqrt{5}}+\frac{\gamma Y}{\sqrt{5}}-\frac{\beta P}{\sqrt{2}}-\frac{\sqrt{6}}{2}\beta R+\frac{\beta S}{\sqrt{5}},$$

$$E_6=-\alpha A+\frac{\alpha D}{\sqrt{2}}-\frac{\gamma T}{(\sqrt{12})}+\frac{\gamma W}{2}-\frac{\beta N}{\sqrt{2}}+\frac{\beta Q}{\sqrt{2}},$$

$$E_7=-\frac{\alpha C}{2}+\frac{3\alpha E}{2\sqrt{5}}+\frac{2\alpha F}{\sqrt{5}}-\frac{\gamma V}{2\sqrt{2}}+\frac{3\gamma X}{2\sqrt{5}}+\frac{2\gamma Y}{\sqrt{5}}-\frac{\beta P}{2\sqrt{2}}+\frac{\sqrt{6}\beta R}{4}+\frac{2\beta S}{\sqrt{5}},$$

$$E_8=\frac{\alpha D}{\sqrt{2}}+\frac{\gamma W}{2}+\frac{\beta Q}{\sqrt{2}},\quad E_9=\frac{\alpha B}{2}+\frac{\gamma U}{2\sqrt{2}}+\frac{(\sqrt{13})\beta O}{12}-\frac{\beta B}{12},$$

$$E_{10}=\frac{\alpha C}{2}+\frac{\gamma V}{2\sqrt{2}}+\frac{\beta P}{2\sqrt{2}},\quad E_{11}=\frac{\alpha E}{2}+\frac{\gamma X}{2}+\frac{\beta R}{2}\left(\frac{5}{6}\right)^{1/2},$$

$$E_{12}=\tfrac{1}{2}\alpha F+\tfrac{1}{2}\gamma Y+\tfrac{1}{2}\beta S;$$

$$R_1=-\frac{\sqrt{2}z}{3}\left[1-\left(\frac{z}{2}\right)^2\right],\qquad R_2=\frac{(\sqrt{10})z}{3}\left(1-\frac{z^2}{10}\right),$$

$$R_3=\frac{\sqrt{2}}{b_{\text{osc.}}}\left(1-\frac{z^2}{12}+\frac{z^4}{24}\right),\qquad R_4=\frac{\sqrt{10}}{b_{\text{osc.}}}\left(1-\frac{z^2}{3}+\frac{z^4}{60}\right),$$

$$R_5=\frac{\sqrt{2}}{b_{\text{osc.}}}\left(1+\frac{z^2}{4}-\frac{z^4}{24}\right),\qquad R_6=\frac{\sqrt{2}}{3b_{\text{osc.}}}z^2\left(1-\frac{z^2}{8}\right),$$

$$R_7=\frac{2}{b_{\text{osc.}}}\left(\frac{2}{5}\right)^{1/2}\frac{7}{6}z^2\left(1-\frac{z^2}{14}\right),\qquad R_8=\frac{\sqrt{6}}{b_{\text{osc.}}}\left(1+\frac{z^2}{4}-\frac{z^4}{8}\right),$$

$$R_9=\frac{\sqrt{6}}{b_{\text{osc.}}}\left(1+\frac{z^2}{4}\right),\qquad R_{10}=\frac{z^3}{3\sqrt{10}}.\tag{A4}$$

From conservation of energy

$$\hbar\nu_{ab}C=m_\mu C^2-E_{ba}+\kappa\tag{A5}$$

$$=108-E_{ba},\tag{A6}$$

where E_{ba} is the energy difference between the nuclear states $|a\rangle$ and $|b\rangle$. The constant κ contains corrections arising from the binding energy of the muon, the neutron-proton mass difference, and the Coulomb energy difference in $|a\rangle$ and $|b\rangle$.[18]

Utilizing the same notation as above, the photodisintegration cross section associated with the nuclear transition $|a\rangle\to|b\rangle$ may be written

$$\sigma_{ab}=(\pi^2/616)(b_{\text{osc.}})^2(E_b-E_a)\{\sqrt{2}(E_2+E_4)+\sqrt{10}E_5\}^2.\tag{A7}$$