Theoretical Calculations of Statistical Heavy-Ion Reactions

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In this work we analyze the following three groups of reactions involving heavy ions: (a) $C^{12}(C^{12},\alpha)Ne^{20}$; (b) $C^{12}(O^{16},\alpha)Mg^{24}$; and (c) $C^{12}(Li^{6},p)O^{17}$. These reactions have been carefully studied: Their excitation functions show strong fluctuations which can be interpreted as due to purely statistical processes. The energy interval over which the excitation functions extend are sufficiently large to include several fluctuations and to permit a meaningful comparison between theory and experiment. We show that the theoretical expression given by the statistical model for the average compound-nucleus (CN) integrated and differential cross sections in terms of the average resonance parameters reproduce the experimental data very well if $\Gamma_J/D^{J\pi}\gg 1$, also when $\langle\Gamma_{\lambda c}{}^{J\pi}\rangle/D^{J\pi}$ and, correspondingly, the transmission coefficients are not much less than unity.

We have used two relations to connect the average resonance parameters to optical-model transmission coefficients. The first is obtained by equalizing the absorption cross section to the CN formation cross section, and leads to the widely used Hauser-Feshbach expression; the second is obtained by equalizing the optical-model transition matrix to the energy average of the reaction-theory transition matrix.

Though it appears a little more appropriate to use the first relation, we cannot draw definite conclusions on this point, because of the uncertainty affecting the experimental values of the cross sections and the average optical-model and level-density parameters.

1. INTRODUCTION

THE Hauser-Feshbach (HF) expression for the average cross section of a statistical reaction proceeding through the formation of a compound nucleus (CN) excited into the region of strongly overlapping levels ($\Gamma_J/D^{J\pi}\gg 1$) is widely used.[1–4] However, some derivations of this formula are based on the assumption that the ratio between the average partial width for the decay of a CN state λ into a given channel c, $\langle\Gamma_{\lambda c}{}^{J\pi}\rangle$, and the average spacing $D^{J\pi}$ of the CN states of spin J and parity π, should be much less than unity.[3,5]

Since in deriving the HF expression one uses the following relation, obtained by equalizing the absorption cross section to the CN-formation cross section, between the average resonance parameters and the optical-model transmission coefficients $T_c{}^J$:

$$T_c{}^J=2\pi\langle\Gamma_{\lambda c}{}^{J\pi}\rangle/D^{J\pi}, \tag{1}$$

the above restriction implies that HF calculations should be valid only if

$$T_c{}^J\ll 1. \tag{2}$$

If the inequality (2) holds, the field of validity of HF calculations is strongly limited: For instance, HF calculations should not work for reactions involving heavy ions and α particles except at very low energy.

Notwithstanding this, Vogt and co-workers[3] have pointed out that good agreement between experimental CN cross-section values and the ones theoretically predicted by HF theory may also be obtained when the condition (2) is not fulfilled. In fact, they have shown that the $C^{12}(C^{12},\alpha_0)Ne^{20}$ reaction, at a mean incident energy of 22.95 MeV in the laboratory system, can be nicely interpreted with HF theory; in this case, the transmission coefficients $T_c{}^J$ for heavy ions and α channels do not satisfy the condition (2), being almost unity. This result seems to us of such great significance as to stimulate more HF calculations on heavy-ion statistical reactions. This will be done in Sec. 2.

In Sec. 3 we use, for calculating integrated and differential cross sections, expressions slightly different from the HF ones, following a suggestion by Feshbach[6] that connects the average resonance parameters to optical-model transmission coefficients with an expression different from (1).

Section 4 is devoted to a brief discussion of related arguments. In Sec. 5, we summarize the results obtained.

2. HF CALCULATIONS

In order to test the capability of HF calculations to fit experimental results when $\Gamma_J/D^{J\pi}\gg 1$ but condition (2) is not satisfied, we used the following procedure: (i) Analyze reactions that, with a great probability, are almost completely of CN type; (ii) use optical-model and level-density parameters not expressly chosen to fit the experimental cross sections but deduced by an independent analysis of experimental data.

The HF expressions for the differential and integrated

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cross sections are[2]

$$\sigma_{\alpha\alpha'}(\theta)=\tfrac{1}{4}\lambda_\alpha^2\sum_{ls,l's',JL}\frac{(-1)^{s'-s}}{(2I+1)(2i+1)}\bar{Z}(lJlJ;sL)\times\bar{Z}(l'Jl'J;s'L)\frac{T_{\alpha ls}{}^J T_{\alpha' l's'}{}^J}{\sum_c T_c{}^J}P_L(\cos\theta), \quad (3)$$

$$\sigma_{\alpha\alpha'}=\frac{\pi\lambda_\alpha^2}{(2I+1)(2i+1)}\sum_{lsJ}(2J+1)\times\frac{T_{\alpha ls}{}^J}{\sum_c T_c{}^J}\sum_{l's'}T_{\alpha' l's'}{}^J, \quad (4)$$

where $\sum_c T_c{}^J$ takes into account all the possible decay modes of CN and has the expression

$$\sum_c T_c{}^J=\sum_\nu\sum_{l_\nu,s_\nu,I_\nu}\int_0^{E_\nu\max}T_{l_\nu}(E_\nu)\rho(E_\nu{}^*,I_\nu)dE_\nu, \quad (5)$$

and ν refers to the various possible decays of CN.

The transmission coefficients $T_{\alpha ls}{}^J$, $T_{\alpha' l's'}{}^J$, and $T_c{}^J$ that we used have been calculated with optical-model parameters, obtained by fitting elastic-scattering or total reaction cross sections.

The level densities $\rho(E_\nu{}^*,I_\nu)$ of the various nuclei to which the CN can decay have been calculated by using the Lang and Le Couteur expression[7]

$$\rho(E^*,I)=\frac{\hbar^3}{24\sqrt{8}}(2I+1)\exp\left[-\frac{I(I+1)}{2\sigma^2}\right]a^{1/2}\times g^{-3/2}\frac{\exp[2(aU)^{1/2}]}{(U+t)^2}, \quad (6)$$

with an empirical set of parameters giving good fits of the following quantities: (a) the slow-neutron-resonance spacings; (b) the shape of the energy spectra of particles emitted in statistical reactions; (c) the level widths of nuclei at high excitation energy; and (d) the low energy-level distributions.[8]

The level-density parameters are $R_0\simeq 1.5$F, $g=0.7g_{\mathrm{RIG}}$, and $a=(0.127\,A)$ MeV^{-1}; the effective excitation energy $U=(E^*-\Delta+70/A)$ MeV, where E^* is the usual excitation energy and Δ the pairing energy; in expression (5), $E_\nu{}^{\max}=E_{\mathrm{inc}}+Q^{(\nu)}-\Delta$, $E_\nu{}^*=E_\nu{}^{\max}-E_\nu$.

In the calculation of $\sum_c T_c{}^J$, only neutron-, proton-, and α-decay channels have been taken into account.

The reactions we have studied are (a) $C^{12}(C^{12},\alpha_{0,1})Ne^{20}$,[3] (b) $C^{12}(O^{16},\alpha)Mg^{24}$, leading to the ground state and the first six excited levels of Mg^{24},[9-11] and (c) $C^{12}(Li^6,p)O^{17}$, leading to the ground state and the first three excited levels of O^{17}.[12]

These reactions have been carefully studied: Their excitation functions show strong fluctuations which can be interpreted as due to purely statistical processes. The energy intervals over which the excitation functions extend are sufficiently large to include several fluctuations and to permit a meaningful comparison between theory and experiment.

$C^{12}(C^{12},\alpha_{0,1})Ne^{20}$. The experimental results are taken from Ref. 3. The optical-model parameters used to calculate the transmission coefficients have been taken from Ref. 13 for protons (the derivative imaginary Saxon-Wood form factor has been replaced with an equivalent Gaussian) and from Ref. 14 for α particles. The neutron and $C^{12}+C^{12}$ transmission coefficients are those reported, respectively, in Refs. 15 and 3.

All odd l values in expressions (3) and (4) are forbidden because of the identity of the C^{12} nuclei. The total parity π is positive and both the expressions given for $\sigma_{\alpha\alpha'}(\theta)$ and $\sigma_{\alpha\alpha'}$ must be multiplied by a factor of 2.

The experimental integrated cross sections at the mean energy $E_{\mathrm{lab}}=22.95$ MeV are

$$\sigma_{\mathrm{g.s.}}=(19.2\pm 4)\ \mathrm{mb}$$

for the transition to the ground state and

$$\sigma_{1\,\mathrm{exc.}}=(63\pm 14)\ \mathrm{mb}$$

for the transition to the first excited level. The calculated values are, respectively,

$$\sigma_{\mathrm{g.s.}}{}^{\mathrm{calc.}}=18.8\ \mathrm{mb}\quad\text{and}\quad\sigma_{1\,\mathrm{exc.}}{}^{\mathrm{calc.}}=52\ \mathrm{mb}.$$

In Fig. 1, the experimental and calculated differential cross sections (full lines) are reported.

$C^{12}(O^{16},\alpha)Mg^{24}$. The optical-model parameters used to calculate the transmission coefficients of protons and α particles have been taken again from Refs. 13 and 14. The neutron-decay channel transmission coefficients are those of Ref. 15. The transmission coefficients for $O^{16}+C^{12}$ have been calculated with the following optical-model potential:

$$U(r)=V\{\exp[(r-R)/a]+1\}^{-1}+iW\{\exp[(r-R)/b]+1\}^{-1}+V_c(r),$$

where $V=-50$ MeV, $W=-2$ MeV, $R=6.174$ F, and $a=b=0.5$ F.

The Coulomb potential $V_c(r)$ is that due to a uniformly charged sphere of radius $R_c=6.174$ F. The above $O^{16}+C^{12}$ optical-model parameters have been obtained from an analysis of the experimental reaction cross-section data taken in the range 7–16-MeV c.m.

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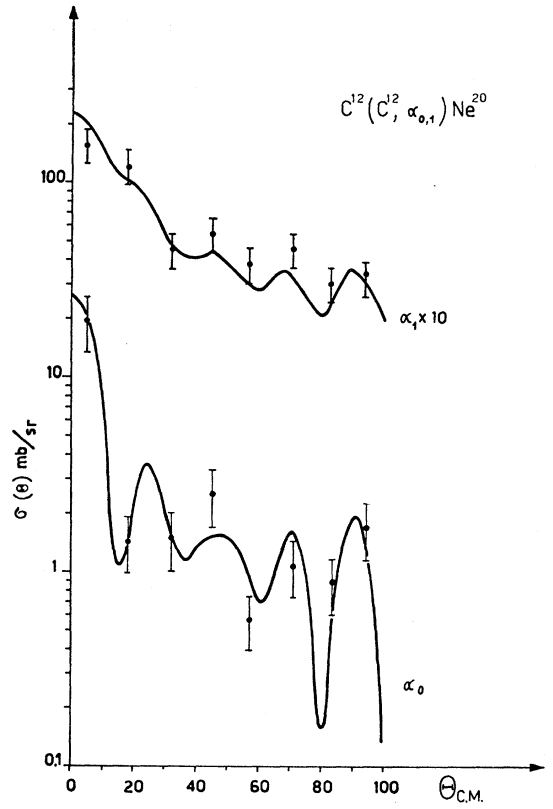

FIG. 1. Comparison between experimental and calculated (full lines) differential cross sections for the reactions $C^{12}(C^{12},\alpha_{0,1})Ne^{20}$ at 22.95-MeV lab-system incident energy. The theoretical calculations have been performed by using the expression (3) (see text).

incident energy[16] (see Fig. 2) and an analysis[17] of the elastic scattering cross-section data at 15-MeV c.m. incident energy. In Fig. 3, the calculated integrated cross sections (full lines) are compared with the experimental results of Drysdale *et al.*[9] The calculated integrated cross sections are also in satisfactory agreement with the experimental results of Halbert *et al.*[10] at about 19-MeV O^{16} lab-system incident energy and seem to confirm that the results of Groce and Lawrence,[11] at the same energy, are an order of magnitude too high. However, we note that the calculated integrated cross section is about 50% lower than the experimental value obtained by Halbert *et al.*[10] at 31.1-MeV O^{16} lab-system incident energy (see Fig. 4).

$C^{12}(Li^6,p)O^{17}$. The experimental results are taken from Ref. 12. The optical-model parameters that we used for neutrons, α's, and Li^6+C^{12} are those reported by Dzubay[12] (the derivative imaginary Saxon-Wood form factors for neutrons, α's, and Li^6+C^{12} have been replaced with an equivalent Gaussian). The optical-model potential for the calculation of transmission coefficients of proton channels is the following:

$$U(r)=(V+cE)\{\exp[(r-R)/a]+1\}^{-1}+iW\exp\{-[(r-R)/b]^2\}+\lambda_\pi^2 V_{s.o.}\frac{1}{r}\frac{df(r)}{dr}\mathbf{l}\cdot\mathbf{s}+V_c(r),$$

where $V=-50$ MeV, $W=-4.6$ MeV, $V_{s.o.}=-10$ MeV, $R=3.22$ F, $a=0.52$ F, $b=1.00$ F, and $c=0.5$.

The Coulomb potential is again that due to a uniformly charged sphere of radius $R_c=3.22$ F. The parameters, different from the ones reported by Dzubay, have been chosen in order to obtain a better fit to the elastic scattering data for protons on O^{16}.[18] These proton optical-model parameters, that we used for $p+O^{17}$, give a slightly better fit in the case of elastic scattering $p+O^{16}$ than those recently suggested by Stevens *et al.* for $p+O^{18}$.[19]

The comparison between calculated (full lines) and experimental cross sections is given in Fig. 5.

The agreement between the experimental and the above-calculated cross sections is very satisfactory, both for the reaction $C^{12}(C^{12},\alpha)Ne^{20}$ and the reaction $C^{12}(O^{16},\alpha)Mg^{24}$. (The best agreement for the latter reaction is obtained with the experimental results of Drysdale and co-workers.[9])

In the case of the reaction $C^{12}(Li^6,p)O^{17}$, the calculated HF cross sections fit with more accuracy the transitions to the second and third level of the residual nucleus O^{17} than those to the ground state and the first excited level. This reflects the great sensitivity of the calculated cross-section values as to the optical-potential parameters for the initial and final channels. In fact, the transmission coefficients both for Li^6+C^{12} and $p+O^{17}$ show a resonance structure for different l values which strongly depends on the shape and depth of the optical potential used.

The ground state and first excited state of O^{17} have even parity; the second and third levels have odd parity, and therefore different combinations of ingoing and outgoing angular momenta are present in the products $T_{\alpha l s}{}^J \cdot T_{\alpha' l' s'}{}^J$ appearing in expressions (3) and (4); even-even and odd-odd for p_0 and p_1 transitions, even-odd and odd-even for p_2 and p_3 transitions. The resonance structure of the transmission coefficients strongly affects the calculated cross sections; it increases the calculated values for p_0 and p_1 transitions and lowers the ones corresponding to p_2 and p_3 transitions.

Taking into account the preceding discussion, we consider the agreement between theory and experiment quite satisfactory also in the case of the reaction

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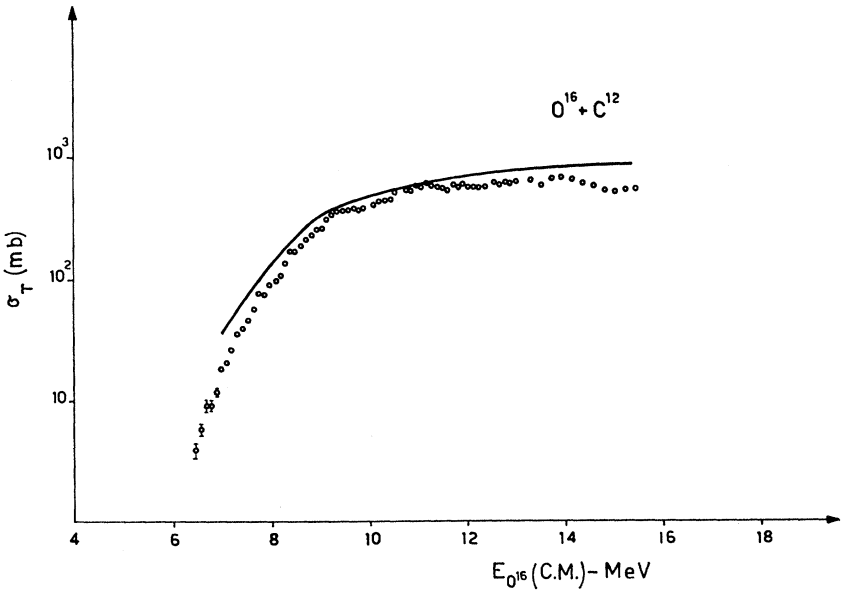

FIG. 2. Comparison between experimental and calculated (full line) total reaction cross section for the reaction $O^{16}+C^{12}$.

$C^{12}(Li^6,p)O^{17}$. The good agreement between calculated and experimental values of the cross sections in all the cases considered seems to confirm the validity of HF calculations beyond the limit established by condition (2).

In fact, it must be emphasized, such agreement is not due to a choice *ad hoc* of the parameters used in the calculations: All of them were *a priori* fixed in all the cases considered, and therefore the agreement reflects a quality of the theory and is not simply the result of the choice of the parameters.

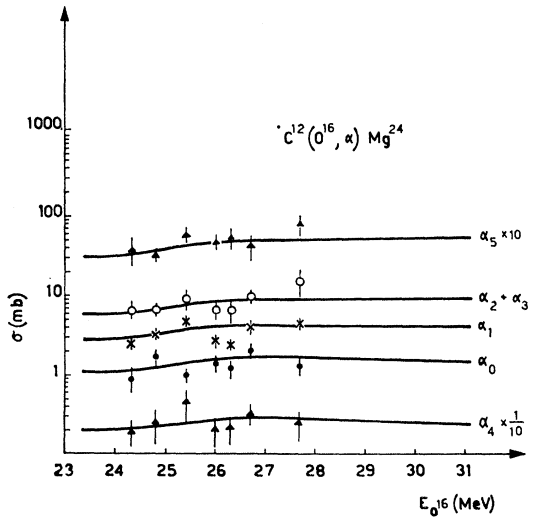

FIG. 3. Comparison between experimental (Ref. 9) and calculated (full lines) integrated excitation functions for the reactions $C^{12}(O^{16},\alpha)Mg^{24}$. The energy of O^{16} is given in lab system. The theoretical calculations have been performed by using the expression (4) (see text). α_0, ●; α_1, ×; $(\alpha_2+\alpha_3)$, ○; α_4, ▲; α_5, ▲.

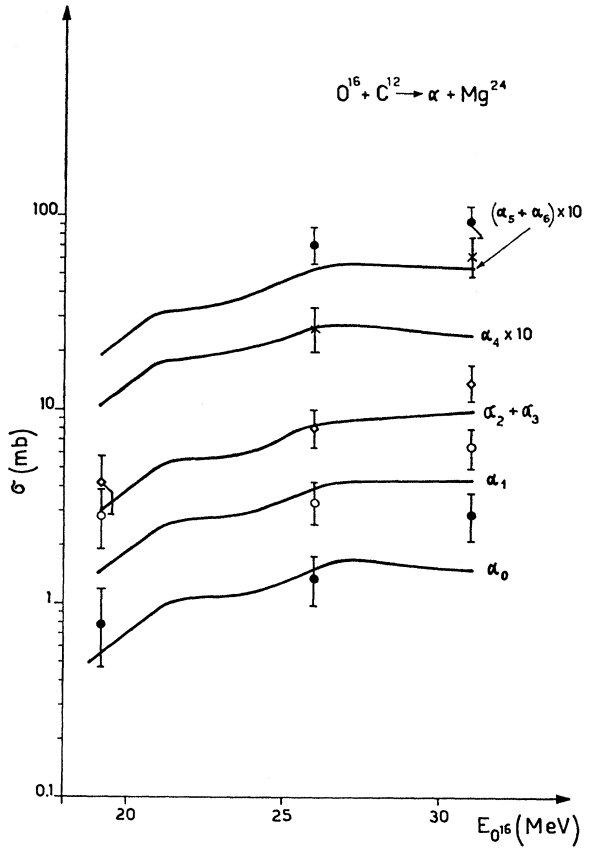

FIG. 4. Comparison between experimental and calculated (full lines) integrated excitation functions for the reactions $C^{12}(O^{16},\alpha)Mg^{24}$ leading to the ground state and first six excited states of Mg^{24}. The energy of O^{16} is given in the lab system. The values at 19.2 and 31.1 MeV are taken from Ref. 10; the value at 26 MeV is from Ref. 9. The theoretical calculations have been performed by using the expression (4) (see text). α_0, ●; α_1, ○; $(\alpha_2+\alpha_3)$, ◇; α_4, ×; $(\alpha_5+\alpha_6)$, ●.

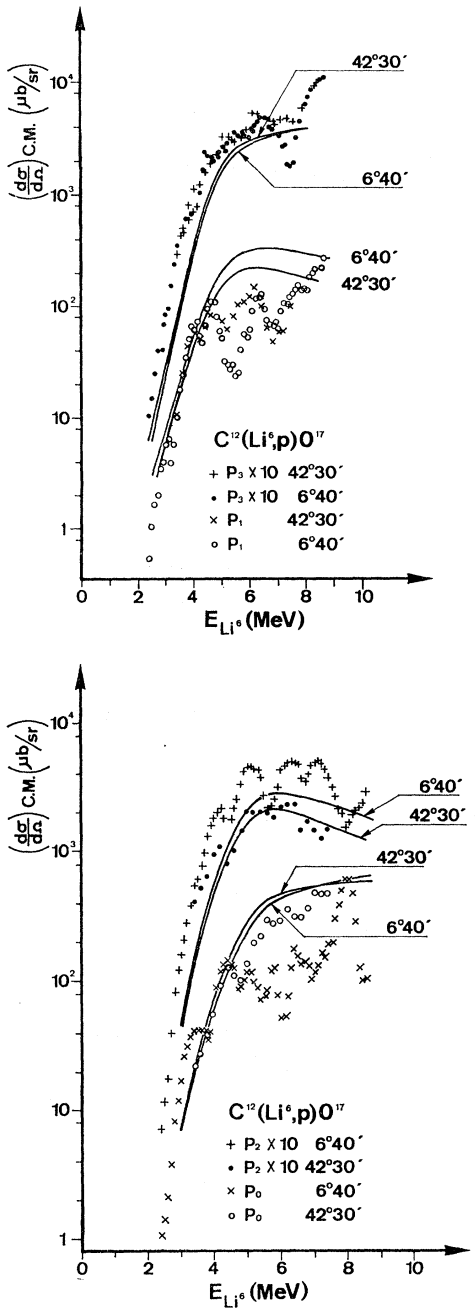

FIG. 5. Comparison between experimental and calculated (full lines) differential excitation functions for the reactions $C^{12}(Li^6,p)O^{17}$ leading to the ground state and first three excited states of O^{17}. The energy of Li^6 is given in lab system. The theoretical calculations have been performed by using the expression (3) (see text).

In HF theory, one assumes that the optical-model absorption cross section equals the CN-formation cross section. The experimental evidence seems to suggest that for neutron, proton, α, and heavy-ion channels, in the light-nuclei region, such approximation is not unrealistic.

3. ALTERNATIVE TO HF EXPRESSION

The connection between the optical-model transition matrix and the energy average of the reaction-theory transition matrix suggests that, when one neglects direct contributions to the absorption cross section, a more correct relation between optical-model transmission coefficients and average resonance parameters could be[6]

$$T_c{}^J = 2\pi\langle\Gamma_{\lambda c}{}^{J\pi}\rangle/D^{J\pi} - \pi^2\langle\Gamma_{\lambda c}{}^{J\pi}\rangle^2/(D^{J\pi})^2. \quad (7)$$

The use of the relation

$$\pi\langle\Gamma_{\lambda c}{}^{J\pi}\rangle/D^{J\pi} = 1-(1-T_c{}^J)^{1/2} \quad (8)$$

in deriving the expression for the theoretical cross section of CN reactions does not modify strongly the results which we obtain with the usual HF expression when, for ingoing and outgoing channels, $T_c{}^J \leq 0.5$, but increases considerably the calculated cross sections when the transmission coefficients $T_c{}^J$ approach unity.

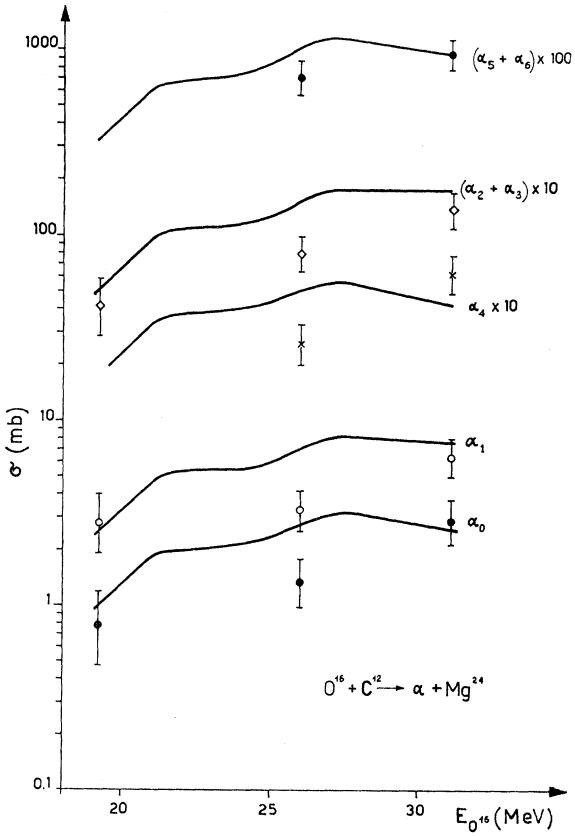

FIG. 6. Comparison between experimental and calculated (full lines) integrated excitation functions for the reactions $C^{12}(O^{16},\alpha)Mg^{24}$ leading to the ground state and first six excited states of Mg^{24}. The energy of O^{16} is given in the lab system. The values at 19.2 and 31.1 MeV are taken from Ref. 10; the value at 26 MeV is from Ref. 9. The theoretical calculations have been performed by using the expression (9) and relating the optical-model transmission coefficients to the average resonance parameters by means of the expression (8) (see text). α_0, ●; α_1, ○; $(\alpha_2+\alpha_3)$, ◇; α_4, ×. $(\alpha_5+\alpha_6)$, ●.

TABLE I. Values of the calculated differential cross sections (for $\theta_{c.m.}=6°\,42'$ and $\theta_{c.m.}=42°\,30'$) in μb/sr for the reactions $C^{12}(Li^6,p)O^{17}$ leading to the ground state and first three excited states of O^{17}. The theoretical calculations have been performed by using the expression (3) (see text).

E (MeV)	$\sigma(\theta)_{p_0}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_1}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_2}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_3}$ (μb/sr) 6° 42′	42° 30′
3.00	6.88	7.64	6.79	5.79	5.39	4.63	2.50	2.83
4.25	102.51	111.83	86.19	69.60	93.48	77.62	54.51	61.62
5.41	352.94	381.75	272.20	192.78	259.53	200.42	227.54	259.01
6.75	512.59	519.73	351.36	219.21	256.74	182.42	345.39	375.28
8.00	611.13	535.14	311.47	178.69	196.95	132.41	405.90	401.59

TABLE II. Values of the calculated differential cross sections (for $\theta_{c.m.}=6°\,42'$ and $\theta_{c.m.}=42°\,30'$) in μb/sr for the reactions $C^{12}(Li^6,)p$ O^{17} leading to the ground state and first three excited states of O^{17}. The theoretical calculations have been performed by using the expression (9) and relating the optical-model transmission coefficients to the average resonance parameters by means of the expression (8) (see text).

E (MeV)	$\sigma(\theta)_{p_0}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_1}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_2}$ (μb/sr) 6° 42′	42° 30′	$\sigma(\theta)_{p_3}$ (μb/sr) 6° 42′	42° 30′
3.00	6.23	6.81	6.22	5.34	4.91	4.24	2.24	2.53
4.25	95.23	103.89	80.30	65.37	91.89	76.98	52.26	58.97
5.41	396.91	429.71	302.55	221.34	331.28	263.16	261.03	296.44
6.75	663.63	694.52	473.68	296.60	367.45	265.39	459.99	510.33
8.00	773.83	700.95	420.36	241.54	260.82	178.27	506.25	510.69

To discuss this point more specifically, we rewrite the theoretical expression (4) for the average CN cross section $\sigma_{\alpha,\alpha'}$ in terms of the average resonance parameters:

$$\sigma_{\alpha,\alpha'}=\frac{\pi\lambda_\alpha^2}{(2i+1)(2I+1)}\sum_{Jls,l's'}(2J+1)\times\frac{2\pi}{D^{J\pi}}\frac{\langle\Gamma_{\alpha ls}{}^{J\pi}\rangle\langle\Gamma_{\alpha' l's'}{}^{J\pi}\rangle}{\Gamma_J}. \quad (9)$$

In the case of the reactions here examined, we can estimate that the use of formula (8) does not increase significantly the value of the total width Γ_J because the most important contribution to it is due to transmission coefficients corresponding to low-energy outgoing particles due to the very fast increase with excitation energy of the level density of the various residual nuclei to which the CN can decay. In the case of the three reactions we examined, the difference between the value of Γ_J calculated by using expressions (8) or (1) depends weakly on J.

In all the energy range, this difference, on the average, is equal to 30% of the Γ_J value calculated by using (1) for $J=0$, going to 20% of this value for $J=10$.

The use of expression (8) changes much more appreciably the product $\langle\Gamma_{\alpha ls}{}^{J\pi}\rangle\langle\Gamma_{\alpha' l's'}{}^{J\pi}\rangle$ appearing in the numerator of (9), increasing quite strongly the calculated value of the cross sections.

Let us examine more specifically the various cases.

$C^{12}(C^{12},\alpha)Ne^{20}$. The integrated calculated cross section becomes $\sigma_{g.s.}{}^{calc.}=28.55$ mb for the ground-state transition and $\sigma_{1\,exc.}{}^{calc.}=82.08$ mb for the first-excited-level transition. The calculated values are 30–40% higher than the experimental ones reported in Ref. 3.

It must be noted that the experimental values of Vogt and co-workers[3] agree very well with those of Leachman and co-workers.[20]

$C^{12}(O^{16},\alpha)Mg^{24}$. The comparison between the calculated cross sections and the experimental values reported in Refs. 9 and 10 is given in Fig. 6. We now have good agreement between theory and the experimental data of Halbert and co-workers[10]; the theoretical cross sections, however, are definitively too high with respect to the experimental values reported by Drysdale and co-workers.[9]

In this case, the experimental uncertainty does not allow us to assess which of the two relations (1) or (8) is preferable.

$C^{12}(Li^6,p)O^{17}$. Table I reports the values of the cross sections we calculated by means of the usual HF formula (3) as reported in Sec. 2. Table II reports the cross-section values calculated by relating the average resonance parameters to the transmission coefficients by means of expression (8). The comparison of the two tables shows that the use of expression (8) does not affect the cross-section values in the energy range 3–5 MeV but, on the other hand, increases the calculated cross-section values in the energy range 5–8 MeV. This increase leads to a less satisfactory fit for p_0 and p_1 transitions and to better agreement for p_2 and p_3 transitions. On the whole, the use of (3), as in Sec. 2, gives a slightly better fit to the experimental results.

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To summarize, the comparison between experimental and calculated values of the cross sections here considered seems to suggest that the use of relation (1) between average resonance parameters and optical-model transmission coefficients leads to slightly better results than the use of relation (8). However, we must keep in mind that the uncertainty in the experimental values and the average parameters used (the optical-model parameters and level-density parameters) prevents us from reaching a definite conclusion.

4. FURTHER REMARKS

In a recent work, on the basis of some particular models, Moldauer[21] suggests the following relation between the transmission coefficients and the average resonance parameters:

$$T_c^J = 1 - \exp(-2\pi\langle\Gamma_{\lambda c}^{J\pi}\rangle/D^{J\pi}). \quad (10)$$

The use of this formula increases the calculated cross-section values even more than the use of formula (8). In particular, for the cases here examined, the cross-section values calculated using the Moldauer relation (10) in expression (9) are systematically too high.

In deriving expression (9), the fundamental assumption is made that the reduced-width amplitudes $\gamma_{\lambda c}$ (whose square is proportional to $\Gamma_{\lambda c}^{J\pi}$) are uncorrelated with respect to both the channel index c and the resonance index λ.[3,4,22] The possibility of higher-order dynamical correlations with respect to the resonance index λ is claimed by several authors,[21] but we have neglected such correlations, since at present the experimental results do not seem to require that they be taken into account.[22]

5. CONCLUSION

The theoretical expressions given by the statistical model for the average CN integrated and differential cross sections, in terms of the average resonance parameters, reproduce the experimental data very well if $\Gamma_J/D^{J\pi} \gg 1$, also when $\langle\Gamma_{\lambda c}^{J\pi}\rangle/D^{J\pi}$ and, correspondingly, the transmission coefficients are not much less than unity.

We have used two relations to connect the average resonance parameters to the optical-model transmission coefficients. The first is obtained by equalizing the absorption cross section to the CN-formation cross section, the second by equalizing the optical-model transition matrix to the energy average of the reaction-theory transition matrix. Though it appears a little more appropriate to use the first relation, which leads to the widely used HF expression, we can hardly draw any definite conclusion on this point, because of the uncertainty affecting the experimental values of the cross sections and the average optical-model and level-density parameters. However, we must stress the fact that until the experimental cross sections are affected by a percentage error of the order of 20–30%, no definite conclusion on this subject can be drawn in most cases.

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