Analysis of the $C^{12}(p,\alpha)B^9$ Reaction at 44.5 MeV

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A distorted-wave Born-approximation theory is described which uses a combination of square-well and harmonic-oscillator potentials in the local energy approximation to take account of the finite-range effects in (p,α) direct pickup reactions. A fit was obtained to the forward part of the $C^{12}(p,\alpha)B^9$ angular distribution which gives agreement with the calculated spectroscopic factor. In contrast, no reasonable set of parameters was found which could fit the experimentally observed peak at backward angles, providing some supporting evidence that it is probably caused by an exchange process.

I. INTRODUCTION

HERE we report the analysis of the ground-state angular distribution of the reaction $C^{12}(p,\alpha)B^9$ at 44.5 MeV. This forms part of a systematic attempt at quantitative absolute fits to experimental (p,α) angular distributions in an energy region which is high enough for the process to be considered as entirely nonresonant. Among the (p,α) reactions which have been measured under these conditions the $C^{12}(p,\alpha)B^9$ reaction seems to be unusual in that it shows a rise in the angular distribution at backward angles.[1] It has been suggested that the α-particle structure of the C^{12} target enhances the exchange interactions for this particular reaction. If this is so, then a direct pickup theory may be unable to fit the backward-angle data, even though a fit should be expected in the forward direction where the exchange contributions are thought to be less important.

II. THEORETICAL METHOD

It has been found, in the analysis of (p,α) experimental spectroscopic factors, that there is a large discrepancy in their magnitude, as predicted by the distorted-wave Born-approximation (DWBA) theory, compared to calculations from shell-model calculations, so that the analysis of many (p,α) reactions has been confined to relative values between different final states.[2] It is possible that one of the more serious discrepancies in the DWBA theory lies in the zero-range approximation. In (p,α) pickup reactions the momentum transfer is often much greater than in stripping, and the finite-range corrections, instead of making a small correction, may change the magnitude by an order of magnitude. At the same time they can be expected to considerably alter the shape of the predicted angular distributions.

Exact finite-range calculations[3] involve computing times that are so greatly extended, even with a simple analytical range function, that little or no searching of parameters is possible. The local energy approximation,[4] which corresponds to the lowest-order correction to the zero-range theory, is without this limitation and was adopted for this analysis.

The local energy approximation has previously been formulated for deuteron stripping, and for two-nucleon stripping[5] where the range function was given a Yukawa radial dependence corresponding to the Hulthén neutron-proton potential. The range function, which is expressible approximately in separable form for multinucleon transfers, here involves a proton-"triton" potential for which a wider, shallower potential is more appropriate. It should approximate the shell-model potential of a proton in an α particle. Two convenient analytic forms for the range were investigated, a square well and a harmonic-oscillator well.

In this investigation the zero-range DWBA formula, as used for deuteron stripping, was modified in two ways, apart from the trivial changes due to the mass, spin, and time reversal.

One change involves the local energy approximation for the finite range. If a square well is assumed for the range function of the interaction between the proton and the center of mass of the three nucleons that are picked up, then the DWBA form factor becomes

$$D(K^2)=-\left[\frac{1+(\beta/k)^2}{1+\beta R}\right]^{1/2} \times\frac{\cos(KR)+\beta[\sin(KR)]/K}{1-(K/k)^2}\,\frac{\hbar^2(8\pi\beta)^{1/2}}{2\mu_{pt}},$$

where R is the square-well radius. The other parameters

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[1] R. M. Craig, B. Hird, C. J. Kost, and T. Y. Li, Phys. Letters **21**, 177 (1966).
[2] J. B. Ball and C. B. Fulmer, Phys. Rev. **140**, B330 (1965); R. O. Ginaven, A. M. Bernstein, R. M. Drisko, and J. B. McGrory, Phys. Letters **25B**, 206 (1967); J. A. Nolen, Jr., C. M. Glashausser, and M. E. Rickey, *ibid.* **21**, 705 (1966); L. L. Lee, Jr., A. Marinov, C. Meyer-Boricke, J. P. Schiffer, R. H. Bassel, R. M. Drisko, and G. R. Satchler, Phys. Rev. Letters **14**, 261 (1965); B. F. Bayman and E. Rost (unpublished); R. Sherr, in *Proceedings of the Conference on Direct Interactions and Nuclear Reaction Mechanisms, Padua, 1962* (Gordon and Breach Science Publishers, Inc., New York, 1963).
[3] N. Austern, R. M. Drisko, E. C. Halbert, and G. R. Satchler Phys. Rev. **133**, B3 (1964); R. M. Drisko and G. R. Satchler, Phys. Letters **9**, 342 (1964).
[4] P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) **83**, 701 (1964); Gy. Bencze and J. Zimanyi, Phys. Letters **9**, 246 (1964); E. Ivash, *ibid.* **12**, 327 (1964); J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, *ibid.* **15**, 337 (1965); W. R. Smith (private communication).
[5] Gy Bencze and J. Zimanyi, Nucl. Phys. **81**, 76 (1966).

are defined from the separation energy $\hbar^2\beta^2/2\mu_{pt}$ and the square-well potential depth $\hbar^2(k^2+\beta^2)/2\mu_{pt}$. The operator K is interpreted, in this approximation, as the local momentum transfer and it is possible to express K^2 as a linear function of the three local kinetic energies of relative motion:

$$K^2(r)=(\gamma^2-\gamma/\delta)k_{\alpha A}^2(r) + (1-\gamma\delta)k_{pB}^2(\delta r)+(\gamma/\delta)k_{tA}^2(r),$$

where the local kinetic energy of α with respect to A is $\hbar^2K_{\alpha A}^2/2\mu_{\alpha A}=E_{\alpha A}-V_{\alpha A}(r)$ and $V_{\alpha A}(r)$ is the optical-model potential. In a similar way the proton and triton local kinetic energies may be written down in terms of the corresponding optical potentials. The coordinate transformations involve the constants $\gamma=m_p/m_\alpha$ and $\delta=m_A/m_B$. The DWBA form factor in this approximation is no longer a normalization constant as with the zero-range theory, but is now a function of the radius through the optical potentials. It therefore has to be placed inside the radial integral. However, the expression for $D(K^2)$ is easy to evaluate numerically at each radius and it adds little to the computing time. This square-well form factor is well behaved for all reasonable values of the optical-model potentials. It also tends smoothly to the zero-range form factor $D_0=-(8\pi\beta)^{1/2}\hbar^2/2\mu_{pt}$ in the limit as the square well becomes a δ function with $k^2\to\infty$ and $R\to 0$.

An alternative choice is to assume a harmonic-oscillator range function. The local energy approximation form factor then becomes

$$D(K^2)=-[\pi/\beta^2(6\alpha)^3]^{1/4}(\beta^2+K^2)[\exp(-K^2/12\alpha)] \times\hbar^2(8\pi\beta)^{1/2}/2\mu_{pt}.$$

Unfortunately, when large negative values of K^2 occur, as they do when a deep α optical model is used, then this form factor becomes unphysically large. K^2 represents the change in internal energy in the proton-triton motion in going from the α-A initial configuration to the t-B final configuration. A large negative value therefore implies that the high-momentum components in the internal motion of the α particle of the initial configuration are important. Under these circumstances a harmonic-oscillator potential that has infinite walls and therefore generates the wrong asymptotic form for the wave functions is a poor approximation for the α potential. There seems to be no way to avoid such unrealistic behavior in the finite-range factor when it is derived from a harmonic-oscillator range function. The square-well shape for this range function, on the other hand, is free from this disadvantage, and for the numerical work which is described later in the paper, the square-well expression of $D(K^2)$ was used.

For the second modification, which was made to the deuteron-stripping DWBA theory, the harmonic-oscillator potential is probably the better choice. This is the factor that represents the transformation of the internal coordinates of the α particle to those of a proton and a triton cluster. Only bound-state properties are of interest in this transformation so that only low internal momenta are important. Assuming harmonic-oscillator wave functions, the transformation provides an extra normalization constant in the scattering amplitude of magnitude

$$64(3\alpha\tau)^{3/2}/(3\tau+4\alpha)^3,$$

where $\alpha=0.05429$ fm^{-2} is an oscillator size parameter for the α particle, and $\tau=0.05956$ fm^{-2} is the corresponding triton parameter. These numerical values were obtained by matching the rms radius of the harmonic wave functions to the experimental electron scattering radius in each case.[6] The square well for the range function was also adjusted so that the rms value of the eigenfunctions matched the electron scattering radius of the α particle with the appropriate center-of-mass corrections to give $R=2.7835$ fm, and then the experimental proton-triton separation energy was used to determine the square-well depth. A value of 38.7 MeV was obtained in this way.

The above assumptions are, physically, the rough equivalent of treating both the α particle and the triton as having harmonic-oscillator internal wave functions, but assuming that the relative motion of one nucleon with respect to the other three is that for an eigenstate of a square well. No new free variables were introduced by either of the extra terms that were added to the normal DWBA theory.

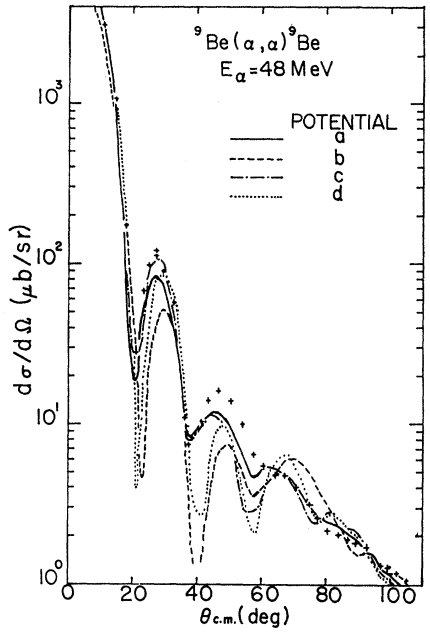

FIG. 1. Optical-model fits to the elastic scattering of α particles from Be^9. The experimental points are those obtained by Summers-Gill (see Ref. 10). The parameters corresponding to each curve are given in Table I.

[6] N. K. Glendenning, Phys. Rev. 137, B102 (1965).

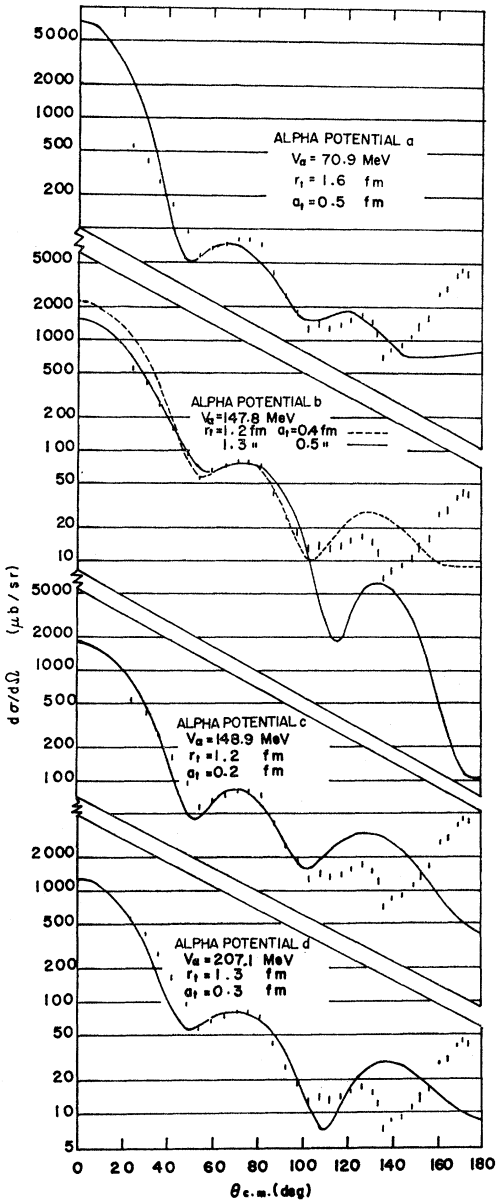

FIG. 2. DWBA fits to the reaction $C^{12}(p,\alpha)B^9$ at 44.5 MeV. The triton potentials that best fit the forward part of the angular distribution are shown for each of the α optical potentials of Table I. The LS spectroscopic factor of 1.498 has been included in the theoretical curves.

III. THEORETICAL SPECTROSCOPIC FACTOR

The nuclear structure information that enters the reaction calculation can be simplified to a single multiplying constant, the spectroscopic factor, when only one value of the radial quantum number is allowed. All three transferred nucleons were therefore assumed to have originated from the p shell of C^{12}. This seems reasonable in that any s-shell nucleons would have to be removed in pairs so as to conserve parity. The calculation of the spectroscopic factor then involves recoupling the three p-shell nucleons into internal and external cluster motion. In order to obtain an analytic expression for the spectroscopic factor, the single-particle states were assumed to be harmonic-oscillator states for which the Talmi[7] transformations could be used. The three-nucleon cluster, which is transferred in the reaction, was taken to be in the space-symmetric internal state of the triton, with the state of mixed symmetry being neglected. This calculation was performed for pure LS and for pure jj coupling, and the resulting values of the spectroscopic factor were found to be 1.498 and 0.791, respectively.

IV. DWBA SEARCH

The DWBA part of the calculation requires many empirical input parameters. In order to obtain fits that represent a meaningful test of the theory, these parameters were obtained, as far as possible, from independent experimental data. It turned out that such a procedure was largely, but not entirely, possible and that some curve fitting was necessary on a few critical parameters.

The proton-C^{12} elastic scattering is reasonably well documented and a set of optical-model parameters given by Fricke and Satchler[8] for 40-MeV scattering was used. Their values were $r=1.18$ fm, $a=0.7$ fm, $V=37.6$ MeV, $W=5.2$ MeV, $r'=1.4$ fm, and $a'=0.7$ fm, where the dashed parameters refer to the imaginary Saxon-Woods potential. The spin-orbit term of the Fricke-Satchler potential was omitted. By making small changes in each of these parameters in turn it was found that none of them played a critical part in determining the shape or magnitude of the DWBA differential cross section. The proton channel parameters therefore seem to be known with adequate precision, for the present investigation.

The α-B^9 distorted-wave parameters on the other hand were found to be decisive in locating the structure in the angular distribution, and were therefore required with good accuracy. Experimental elastic scattering data are available[9] for the mirror system α-Be^9 at 48 MeV, which is close to the required channel energy. These data were analyzed by an automatic search code.

TABLE I. Best-fit optical-model fits to the α-Be^9 elastic scattering data at 48 MeV of Summers-Gill (see Ref. 9).

	V (MeV)	r (fm)	a (fm)	W_{vol} (MeV)	W_{sur} (MeV)	χ^2
(a)	70.9	1.550	0.660	23.87	0	21.6
(b)	147.8	1.403	0.539	0	17.26	81.0
(c)	148.9	1.491	0.587	29.52	0	30.8
(d)	207.1	1.545	0.475	26.98	0	61.7

[7] I. Talmi, Helv. Phys. Acta **25**, 185 (1052); Yu. F. Smirnov, Nucl. Phys. **27**, 177 (1961); **39**, 346 (1962).
[8] M. P. Fricke and G. R. Satchler, Phys. Rev. **139**, B567 (1965).
[9] R. G. Summers-Gill, Phys. Rev. **109**, 1591 (1958).

In order to avoid the Vr^2 type of optical-potential ambiguity, the search was started from a value of $r_\alpha = 1.5$ fm in each trial. Three distinct regions, each corresponding to a different number of half-waves inside the potential well, were identified by their values of V_α. To reduce the number of parameters, either pure-surface or pure-volume absorption was assumed. Usually, the volume absorption fits were somewhat better than with the corresponding surface absorption potential. Four solutions are shown in Table I, and the corresponding fits to the experimental differential cross section are shown in Fig. 1. It has been suggested that the potential for the elastic scattering of composite particles should be the sum of the nucleon elastic scattering potential or perhaps somewhat less,[10] so that the solutions beyond about 250 MeV were not examined. The calculations of the DWBA angular distributions were quite insensitive to the Coulomb radius parameter, so fixed values in the region of 1.25 fm were assumed for all three optical potentials.

The t-B^9 system was treated by the DWBA code as a particle in a real optical-potential well whose depth is adjusted to give a bound system at the separation energy. Independent data do not seem to be available on the parameters of this well, and as the angular distributions were quite sensitive to the values of these parameters, a two-dimensional search was made covering the range from 1.0 to 1.9 fm for the well radius r_t, and 0.1 to 0.9 fm for the surface thickness a_t. The search involved two types of fitting. An attempt was first made to simulate the rise at backward angles, but no combination of parameters within this range was found that would produce sufficient backward rise in the differential cross section. A search was then made to fit only the forward regions where the exchange terms are probably small enough to be neglected. The best solutions are shown in Fig. 2 for each of the α-Be^9 optical potentials. These curves were normalized in the region of the peak near 75°, and the corresponding experimental spectroscopic factors as shown in Table II.

TABLE II. Triton spectroscopic factors for the C^{12} ground state.

α optical potential	Bound-system parameters r_t (fm)	a_t (fm)	Experimental spectroscopic factor	Theoretical spectroscopic factor jj coupling	LS coupling
(a)	1.6	0.5	1.62		
(b)	1.2	0.4	0.99		
(b)	1.3	0.5	1.29		
(c)	1.2	0.2	0.48		
(d)	1.3	0.3	0.94		
				0.79	1.498

[10] Brookhaven National Laboratory Report No. BNL-948, 1965, Vol. III, p. 1057 (unpublished).

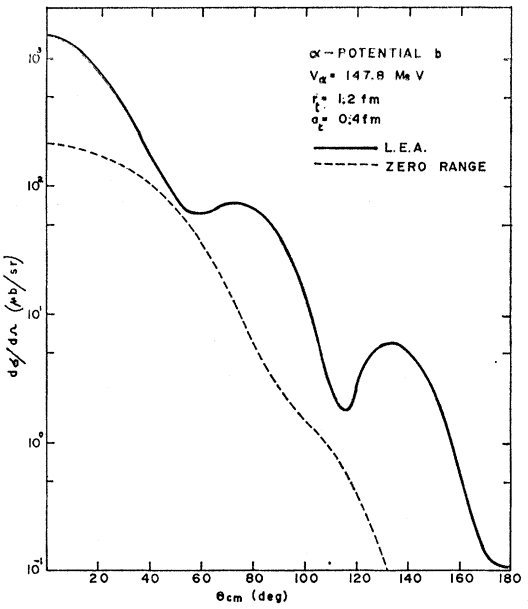

FIG. 3. The effect of the finite-range corrections on the angular distribution. The α potential corresponds to solution (b) of Table I and the triton potential had values of $r_t = 1.2$ fm and $a_t = 0.4$ fm, for both curves.

To illustrate the large effect that the finite range corrections have both on the over-all magnitude and on the shape of the angular distribution, the zero-range and the finite-range predictions were compared directly when all the other parameters had the same values. The result is shown in Fig. 3.

V. DISCUSSION

From this analysis it appears that a reasonable fit to the forward region of the angular distribution can be obtained with reasonable triton parameters for all the four possible α potentials. There is not a clear best choice between the fits, so that it does not seem probable that the α-potential ambiguity can be reduced by DWBA analysis of (p,α) reactions, even when more is known about the triton potential.

The absolute magnitude is predicted extremely well. The finite-range correction term was found to change the zero-range predictions by about a factor of 5 when the shallow α potential wells are used and by a factor of about 30 with the deep wells. When the finite-range correction was used, agreement within a factor of 2 between experiment and theory was obtained for the whole range of α potentials. In reactions where exchange is not favored and where the shape should be fitted over a wider angular range, it should be possible to extract absolute spectroscopic information from (p,α) reactions, particularly when the triton parameters are better known.