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^{13}N Levels from ^{12}C$(p,p)^{12}$C†

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The polarization and differential cross-section data for ^{12}C$(p,p)^{12}$C on and near the anomalies at $E_p=5.38$, 5.88, and 9.13 MeV, reported in the preceding paper, have been analyzed to obtain level parameters for the corresponding states in ^{13}N. Assignments of $\frac{3}{2}^+$ and $\frac{5}{2}^-$, respectively, for the 5.38- and 5.88-MeV anomalies have been verified. The doublet at 9.13 MeV is assigned spins and parities of $\frac{5}{2}^-$ and $\frac{7}{2}^-$. Quantitative fits to excitation functions in the region of the doublet have been obtained with the following values in the lab system for the resonant energy, total width, and elastic width for these states: $\frac{5}{2}^-$, 9.132 MeV, 33 keV, 8.6 keV; $\frac{7}{2}^-$, 9.132 MeV, 82 keV, 67 keV. It is found that although the two resonant energies may not be exactly equal, they probably do not differ by more than 2 keV.

I. INTRODUCTION

LEVEL parameters for states in ^{13}N observed in ^{12}C$+p$ for bombarding energies between 5 and 10 MeV have been obtained by several groups.[1–5] The assignments for most of the narrow resonances were based on elastic and inelastic scattering cross-section measurements only. It was felt worthwhile to reinvestigate some of these levels by including polarization data along with differential cross-section measurements. Of particular interest is the scattering anomaly at $E_p=9.13$ MeV, which was reported[6] to be a doublet. Although the level assignment of $\frac{7}{2}^-$ to the stronger of the two states is rather well established,[3,5] the assignment for the weaker level was unknown. Following a suggestion resulting from the early stages of the present[7] analysis, a qualitative fit to the cross-section data was obtained by the Rice group,[8] assuming an assignment of $\frac{5}{2}^-$ for the weaker level.

In the present work, the polarization and differential cross-section data for ^{12}C$(p,p)^{12}$C reported by Terrell *et al.*[9] (preceding paper, henceforth referred to as I) for energies on and near the 5.38-, 5.88-, and 9.13- MeV anomalies have been analyzed by means of phase-shift analyses of angular distributions and single-level fits to excitation functions. Spin and parity assignments and other parameters were determined for the ^{13}N levels corresponding to these resonances. These results are discussed in Sec. III.

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1 C. W. Reich, G. C. Phillips, and J. L. Russel, Jr., Phys. Rev. **104**, 143 (1956).

2 L. J. Lidofsky, J. Weil, R. D. Bent, and K. W. Jones, Bull. Am. Phys. Soc. **2**, 29 (1957).

3 G. C. Shute, D. Robson, V. R. McKenna, and A. T. Berztiss, Nucl. Phys. **37**, 535 (1962).

4 N. Nikolic. L, J. Lidofsky, and T. H. Kruse, Phys. Rev. **132**, 2212 (1963).

5 A. C. L. Barnard, J. B. Swint, and T. B. Clegg, Nucl. Phys. **86**, 130 (1966).

6 J. B. Swint, A. C. L. Barnard, T. B. Clegg, and J. L. Weil, Nucl. Phys. **86**, 119 (1966).

7 G. E. Terrell, M. F. Jahns, M. R. Kostoff, and E. M. Bernstein, in *Proceedings of the International Symposium on Polarization Phenomena of Nucleons. 2nd Karlsruhe, 1965*, edited by P. Huber *et al.* (Birkhauser Verlag, Stuttgart, Germany, 1966), p. 489.

8 J. B. Swint, J. S. Duval, Jr., and A. C. L. Barnard, Nucl. Phys. **A93**, 177 (1967).

9 G. E. Terrell, M. F. Jahns, M. R. Kostoff, and E. M. Bernstein, preceding paper, Phys. Rev. **172**, 931 (1968).

II. METHOD OF ANALYSIS

A. General Procedure

Complex phase shifts for ^{12}C$(p,p)^{12}$C have been obtained by Moss and Haeberli[10] from analyses of polarization and differential cross-section data at a number of energies between 4.66 and 8.66 MeV. The more recent results obtained by Barnard *et al.*[5] are essentially in agreement with the Wisconsin phase shifts. These analyses were made at energies which were away from the narrow resonances with the purpose of obtaining the phase shifts which describe the gross structure underlying the resonances. Some of the polarization and cross-section angular distributions reported in I were also made "off" resonance in order to supplement the above-mentioned information concerning the gross-structure phase shifts.

Measurements which are reported in I were also made at energies near the center of the anomalies of interest. Information concerning the spins and parities of the resonances was obtained by comparing the "on"-resonance phase shifts determined from analyses of these data with values interpolated from the slowly varying off-resonance phase shifts. The theoretical considerations involved in obtaining level information from such a comparison are presented in Sec. II B.

In addition to angular distribution measurements, polarization and differential cross-section excitation functions in the region of the 9.13-MeV doublet are reported in I. These data were analyzed by means of the one-level approximation to verify the spin and parity assignments obtained from the phase-shift analysis and to obtain the total and elastic widths of these levels.

B. Theory

The elastic scattering of spin-$\frac{1}{2}$ particles from spin-zero nuclei can be described in terms of a set of nuclear phase shifts $\delta_{l,j}$, where $j=l\pm\frac{1}{2}$. Expressions for the

10 S. J. Moss and W. Haeberli, Nucl. Phys. **72**, 417 (1956).

TABLE I. Phase shifts for $^{12}C(p,p)^{12}C$, giving the best fit to the data. The real phase shifts are denoted μ and the nonelastic parameters are denoted τ. The last column, labeled EPP, is the average value of χ^2 per datum. Values in parentheses were not varied in the search.

E(MeV)		$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{5}{2}^-$	$\frac{7}{2}^-$	EPP
5.41	μ	83.4	−19.6	−12.4	8.5	−10.0	(0)	(0)	0.85
	τ	(1.0)	(1.0)	(1.0)	0.64	(1.0)	(1.0)	(1.0)	
5.78	μ	84.2	−25.8	−15.5	38.0	−13.5	0.1	(0)	1.49
	τ	(1.0)	(1.0)	(1.0)	0.94	(1.0)	0.9	(1.0)	
5.89	μ	85.5	−20.8	−11.5	46.0	−11.9	0.5	(0)	1.25
	τ	(1.0)	(1.0)	(1.0)	0.93	(1.0)	0.84	(1.0)	
8.90	μ	75.1	−35.6	−13.3	−30.5	−16.4	2.8	12.0	0.71
	τ	0.82	0.86	0.99	0.70	0.75	1.0	1.0	
9.15	μ	67.8	−38.6	−13.6	−26.8	−18.0	−4.0	120.0	0.19
	τ	0.82	0.82	0.87	0.66	0.73	0.89	0.72	
9.60	μ	71.5	−38.3	−12.5	−25.5	−15.5	1.9	1.0	0.42
	τ	0.88	0.71	1.0	0.63	0.81	0.98	0.98	

differential cross section and polarization in terms of the phase shifts are well known. For the case of pure elastic scattering the $\delta_{l,j}$ are all real; however, at energies where reaction channels are open the phase shifts are, in general, complex and one can write

$$\delta_{l,j}=\mu_{l,j}+i\beta_{l,j}. \tag{1}$$

Usually, the complex phase shifts are expressed in terms of the real part $\mu_{l,j}$ and the nonelastic parameter

$$\tau_{l,j}=e^{-2\beta_{l,j}}. \tag{2}$$

In energy regions away from narrow resonances the μ and τ for all l and j are slowly varying functions of energy. Near a narrow isolated resonance of a particular l and j the diagonal term in the collision matrix for the particular l and j may be approximated[11] by the sum of a slowly varying background part $U_{l,j}{}^0$ plus a resonance part $U_{l,j}{}^R$:

$$U_{l,j}=U_{l,j}{}^0+U_{l,j}{}^R. \tag{3}$$

When the background phase shift is complex the resonance term contains a phase angle φ which shifts the phase of the resonance with respect to the background.[12] In the present case, φ has been taken to be zero since the imaginary parts of the background phase shifts for the resonating partial waves are zero or very small. This assumption is further justified by the fact that satisfactory fits to the data are obtained using $\varphi=0$ (see Sec. III C).

With $\varphi=0$ the nuclear phase shift can be written as

$$\delta_{l,j}=\delta_{l,j}{}^0+\delta_{l,j}{}^R, \tag{4}$$

where the background part $\delta_{l,j}{}^0$ represents the contribution due to nuclear potential scattering and distant levels of the same spin and parity. $\delta_{l,j}{}^R$ is the resonance phase shift which, in general, is complex and may be written in terms of $\mu_{l,j}{}^R$ and $\tau_{l,j}{}^R$ as in Eqs. (1) and (2).

[11] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

[12] D. Robson and A. M. Lane, Phys. Rev. **161**, 982 (1967).

The energy dependences[11] of $\mu_{l,j}{}^R$ and $\tau_{l,j}{}^R$ are

$$\mu_{l,j}{}^R=\tfrac{1}{2}\tan^{-1}\left[\frac{(E_R-E)\Gamma_e}{(E_R-E)^2+\frac{1}{4}\Gamma^2-\frac{1}{2}\Gamma_e\Gamma}\right],$$

$$\tau_{l,j}{}^R=\left\{\frac{[(E_R-E)^2+\frac{1}{4}\Gamma^2-\frac{1}{2}\Gamma_e\Gamma]^2+[(E_R-E)\Gamma_e]^2}{[(E_R-E)^2+\frac{1}{4}\Gamma^2]^2}\right\}^{1/2}. \tag{5}$$

Here E is the incident energy, E_R is the resonant energy, and Γ_e and Γ are the elastic and total widths, respectively. In writing Eqs. (5) the level shift has been set equal to zero, since for the present case if has been assumed that the total width of the resonance is small so that the energy variation of the level shift can be neglected. It will also be assumed that the energy variation of all of the partial widths is small so that they can be considered constant.

Discussions of the energy dependence of the resonant phase shift in the one-level approximation have recently been given elsewhere.[13–15] A qualitative description of the behavior of the resonant phase shift is as follows: Three different cases arise for the energy variation of $\mu_{l,j}{}^R$, which depends on whether Γ_e/Γ is greater than, equal to, or less than $\frac{1}{2}$. For Γ_e/Γ greater than $\frac{1}{2}$, $\mu_{l,j}{}^R$ always increases monotonically with energy starting at small values, passing through $\frac{1}{2}\pi$ at E_R, and continuing toward π. The rate of increase of $\mu_{l,j}{}^R$ with respect to energy depends on the particular value of Γ_e/Γ. For Γ_e/Γ exactly equal to $\frac{1}{2}$, $\mu_{l,j}{}^R$ is double-valued at $E=E_R$, having the values $\pm\frac{1}{4}\pi$. The phase shift increases from small values to $\frac{1}{4}\pi$ as E is increased to E_R. As E increases above E_R the phase shift increases from $-\frac{1}{4}\pi$ to zero. For Γ_e/Γ less than $\frac{1}{2}$, $\mu_{l,j}{}^R$ is antisymmetric with respect to E_R, with the value zero at $E=E_R$. It increases from small values as E increases from below E_R and has maximum and minimum values at

$$E_R\pm(\tfrac{1}{4}-\tfrac{1}{2}\Gamma_e/\Gamma)^{1/2}\Gamma. \tag{6}$$

[13] W. G. Weitkamp and W. Haeberli, Nucl. Phys. **83**, 46 (1966).

[14] B. Hoop, Jr., and H. H. Barschall, Nucl. Phys. **83**, 65 (1966).

[15] J. S. Duval, Jr., A. C. L. Barnard, and J. B. Swint, Nucl. Phys. **A93**, 164 (1967).

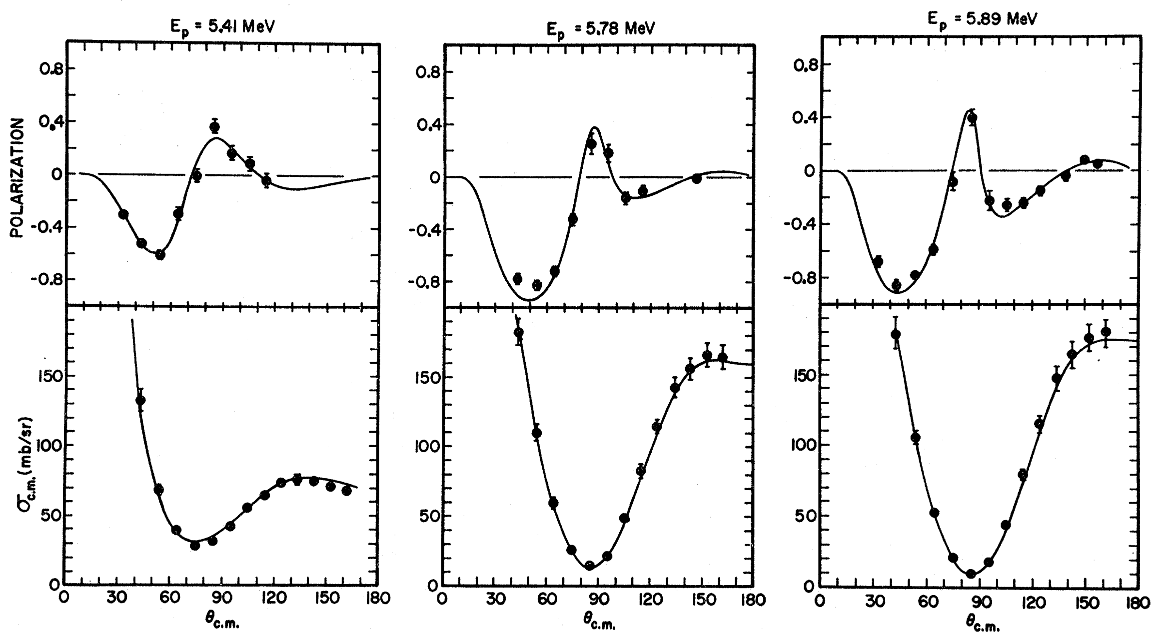

FIG. 1. Polarization and differential cross-section angular distributions for ^{12}C$(p,p)^{12}$C at laboratory energies of 5.41, 5.78, and 5.89 MeV. The curves were calculated from the phase shifts determined in the analyses.

The energy variation of $\tau_{l,j}{}^R$ is qualitatively the same for all values of Γ_e/Γ. The values are symmetric with respect to E_R with a minimum at $E=E_R$. As $|E-E_R|$ is increased $\tau_{l,j}{}^R$ approaches unity. The minimum value of $\tau_{l,j}{}^R$ is simply related to Γ_e/Γ by

$$\tau_{l,j}{}^R(E_R)=\pm[1-2\Gamma_e/\Gamma]. \tag{7}$$

In this expression the minus sign applies for $\Gamma_e/\Gamma>\frac{1}{2}$, and the plus sign applies for $\Gamma_e/\Gamma<\frac{1}{2}$.

A comparison of on-resonance phase shifts with interpolated background phase shifts, at least in principle, will immediately lead to a spin and parity assignment for the anomaly. The phase shifts for the nonresonant partial waves will agree with the interpolated values, whereas the resonant phase shift will be different. In practice the sensitivity of this method will depend on the accuracy in the determination of the on-resonance phase shifts, and the accuracy in the interpolation of the background phase shifts. For very small values of Γ_e/Γ, high accuracy is required in the determination of the phase shifts since the values of $\mu_{l,j}{}^R$ and $\tau_{l,j}{}^R$ will not be very different from zero and unity, respectively.

This method of assigning spins and parities to resonances has been applied successfully in the present case to the three anomalies studied.

C. Phase-Shift Analysis

The polarization and differential cross-section angular distributions reported in I were phase-shift analyzed using the Wisconsin phase-shift program SCRAM. A description of this program is given in Ref. 10. The procedure was to start with a trial set of phase shifts which were modified by a gradient search routine to minimize an error function χ^2, given by

$$\chi^2=\sum\left[\frac{\sigma_c(\theta)-\sigma_e(\theta)}{\Delta\sigma_e(\theta)}\right]^2+\sum\left[\frac{P_c(\theta)-P_e(\theta)}{\Delta P_e(\theta)}\right]^2, \tag{8}$$

where $\sigma_c(\theta)$ and $\sigma_e(\theta)$ are calculated and experimental cross sections, respectively, $\Delta\sigma_e$ is the uncertainty in $\sigma_e(\theta)$, $P_c(\theta)$ is the calculated polarization, etc.

In general, complex phase shifts for partial waves through $l=3$ were considered. However, for energies between 5 and 6 MeV the previous work[5,10] has shown that the gross structure can be described with all nonelastic parameters equal to unity except for $\tau_{d_{3/2}}$. Therefore, in the analysis presented here for this energy range most of the nonelastic parameters were set equal to unity and not varied in the search. Only $\tau_{d_{3/2}}$ and the nonelastic parameter for a partial wave which was being investigated for resonance behavior were allowed to vary. For the same reason both the real and imaginary parts of the f-wave phase shifts were set equal to zero and not varied in this energy range unless resonance behavior was being investigated for one of the f waves. In the analysis of the angular distributions between 8.90 and 9.60 MeV all 14 parameters were allowed to vary.

The results of the phase-shift analyses of the six angular distributions are given in Table I. The measurements at 5.78, 8.90, and 9.60 MeV are off-resonance but near the anomalies of interest. The measurements at 5.41, 5.89, and 9.15 MeV are the on-resonance data. The calculated polarizations and differential cross sec-

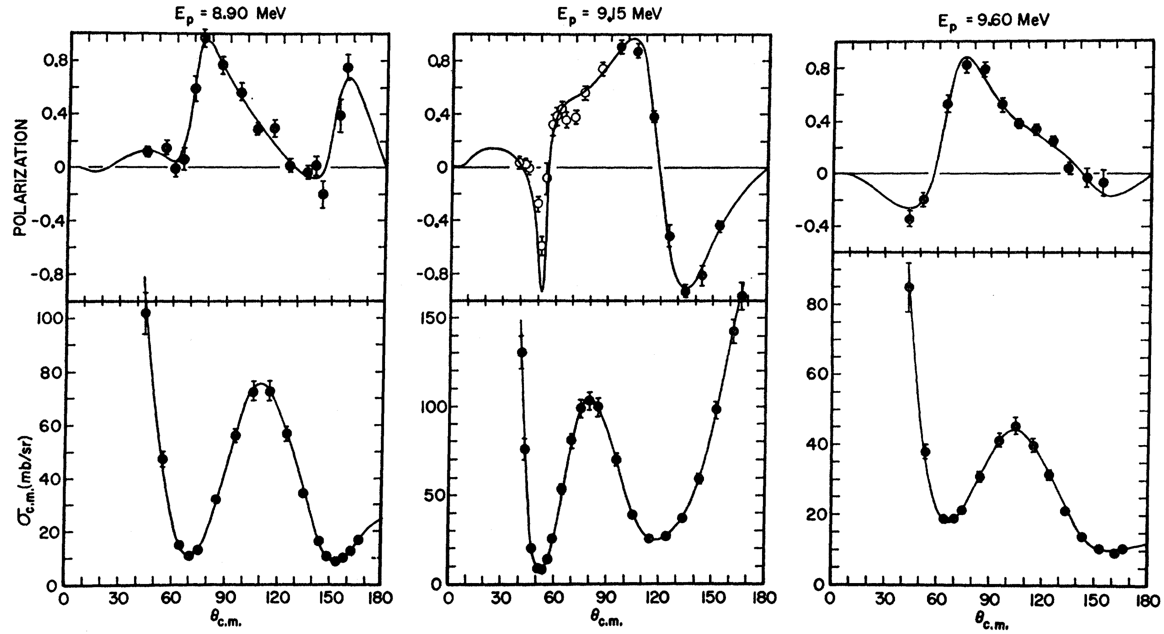

FIG. 2. Polarization and differential cross-section angular distributions for $^{12}C(p,p)^{12}C$ at laboratory energies of 8.90, 9.15, and 9.60 MeV. The curves were calculated from the phase shifts determined in the analysis. At 9.15 MeV the forward-angle polarizations (open circles) were not included in the data to be fit (see the text).

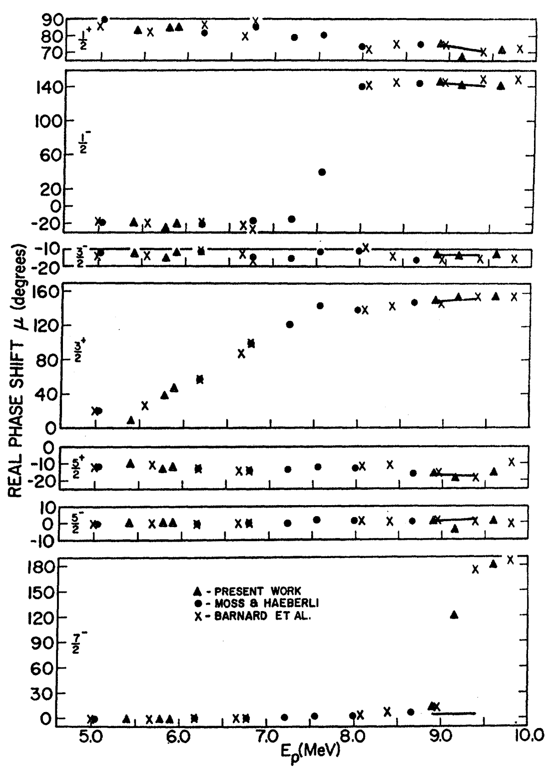

FIG. 3. The real parts of the phase shifts, μ, determined in the present analyses compared with previous results at nearby energies obtained by Moss and Haeberli (Ref. 10) and by Barnard *et al.* (Ref. 5). The present results indicate resonance behavior at 5.41 MeV ($\frac{3}{2}^+$) and 9.15 MeV ($\frac{5}{2}^-$ and $\frac{7}{2}^-$). See also Fig. 4.

tions are in good agreement with the experimental values. The calculations are shown along with the experimental data in Figs. 1 and 2. It should be noted that essentially equivalent fits to the angular distributions were often obtained with slightly different parameters. These variations of a degree or two in the phase shifts resulted from using different starting values. Such trivially different phase shifts were considered to be the same set, the variations merely reflecting the uncertainty in the determination of the actual values. The results given in Table I correspond to the lowest values of χ^2 obtained.

The values of μ and τ derived in the present analyses are compared with the previous results[5,10] at nearby energies in Figs. 3 and 4. Except for the resonating phase shifts the present results fit well with the previous values. Of course, the resonating phase shifts deviate from the slowly varying background values. A more detailed description of the phase-shift analyses of the on-resonance angular distributions is given in Sec. III, where the individual anomalies are discussed.

D. Excitation Function Analysis

A computer search program was written to fit the polarization and differential cross-section excitation functions measured in the region of the 9.13-MeV anomaly. Polarizations and cross sections were calculated as a function of energy from phase shifts obtained by adding a resonance contribution to slowly varying background phase shifts. The one-level approximation described in Sec. II B was used for the resonance contribution. The partial level widths were assumed to be constant. The background phase shifts which were obtained by interpolation of the off-resonance values were allowed a linear energy variation in the region to be fit.

The program utilizes a gradient search routine starting with initial input values of Γ and Γ_e/Γ, which are modified in the search to reduce an error function defined as in Eq. (8). In the present case (the 9.13-MeV doublet) the four parameters describing the two resonances of different spin and parity were searched for simultaneously. The program as originally written allowed the resonance energy E_R to be varied in the search in addition to the width parameters. However, the search on E_R proved to be unsuccessful, at least in the present application. Therefore it was necessary to repeat the search for best-fit width parameters for several different fixed values of the resonance energies. The results of the analysis are discussed in Sec. III.

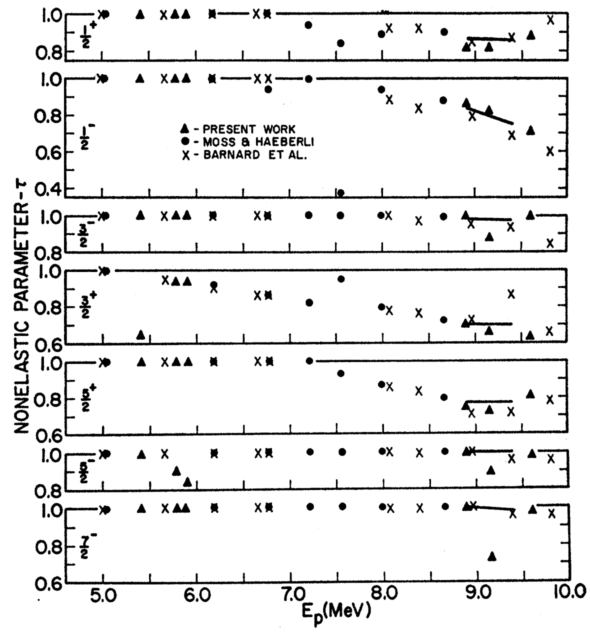

FIG. 4. Nonelastic parameters τ determined in the present analyses compared with the previous results at nearby energies obtained by Moss and Haeberli (Ref. 10) and by Barnard *et al.* (Ref. 5). The present results indicate resonance behavior at 5.41 MeV ($\frac{3}{2}^+$), 5.89 MeV ($\frac{5}{2}^-$), and 9.15 MeV ($\frac{5}{2}^-$ and $\frac{7}{2}^-$). See also Fig. 3.

III. RESULTS

A. 5.38-MeV Anomaly

The spin and parity of the scattering anomaly at 5.38 MeV in ^{12}C(p,p)^{12}C have been rather well established[1,3–5] as $\frac{3}{2}^+$. The present investigation of this resonance may be viewed primarily as a test of the method employed.

The phase shifts derived from the 5.41-MeV angular distribution clearly indicate an assignment of $\frac{3}{2}^+$ to this resonance. As seen in Figs. 3 and 4, both $\mu_{d_{3/2}}$ and $\tau_{d_{3/2}}$ deviate from the slowly varying background values whereas the μ and τ for other partial waves are in good agreement with the background values. An investigation of the possibility of explaining the 5.41-MeV data in terms of resonance behavior for a different partial wave was made in the following manner: Using the background phase shifts for l and j as starting values in SCRAM, searches were made allowing only the two parameters μ and τ, corresponding to one particular l and j, to vary. This was done for each possible spin for all $l\leq 3$. The one reasonable fit to the data was obtained when the $d_{3/2}$ phase shift was varied. It is therefore concluded that the only assignment consistent with the present results is $\frac{3}{2}^+$, which is in agreement with the previous work.

B. 5.88-MeV Anomaly

The first spin and parity assignments[1–3] to the 5.88-MeV resonance were $\frac{5}{2}^+$; however, more recent results[4,5] have changed the assignment to $\frac{5}{2}^-$.

Because of the small value of Γ_e/Γ for this resonance the determination of a definite spin and parity in the present work proved to be more difficult than for the case considered above. A systematic search for resonance behavior for all of the partial waves through $l=3$ was made in the analysis of the 5.89-MeV angular distribution. The scheme of the search was to use a grid of starting phase shifts based on the fact that the background phase shifts were rather well determined. For example, nine searches were made starting $\mu_{s_{1/2}}$ at the background value plus $n\times 20°$, where n is an integer between zero and 8. The starting values for $\tau_{s_{1/2}}$ and the phase shifts for the other partial waves were the background values. This procedure was repeated for all seven l and j values.

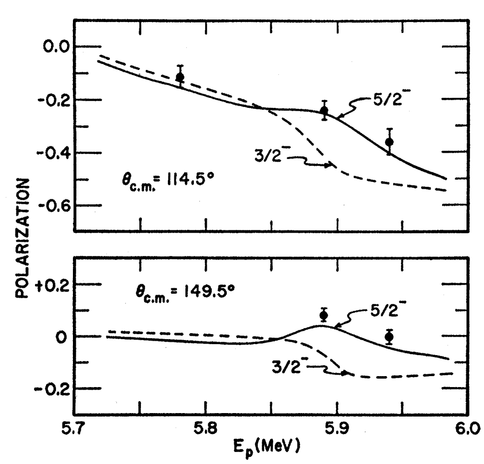

FIG. 5. Polarization excitation functions at center-of-mass angles of 114.5° and 149.5° in the region of the 5.88-MeV anomaly. The solid and dashed curves were calculated assuming $\frac{5}{2}^-$ and $\frac{3}{2}^-$ resonances, respectively.

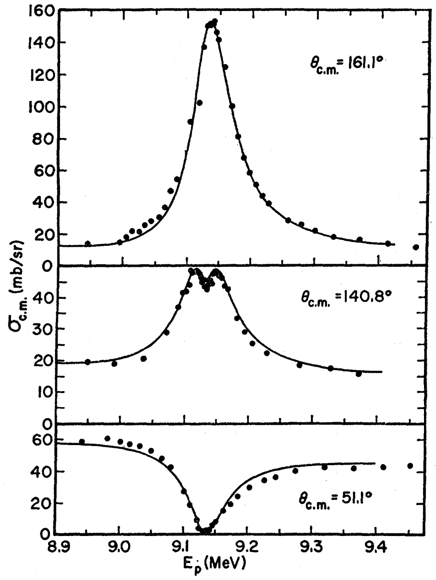

FIG. 6. Differential cross-section excitation functions at center-of-mass angles of 51.1°, 140.8°, and 161.1° in the region of the 9.13-MeV doublet. The smooth curves were calculated for $\frac{5}{2}^-$ and $\frac{7}{2}^-$ resonances using the resonance parameters given in Table I.

Although a number of local minima in the error surface were found in these searches, there were only two different sets of phase shifts which resulted in reasonable fits to the data. One of these sets was consistent with an assignment of $\frac{5}{2}^-$ to the resonance; the other indicated a $\frac{3}{2}^-$ assignment. The results which indicated an assignment of $\frac{5}{2}^-$ were preferred since the values of the nonresonating phase shifts were all within 1° or 2° of the expected values, whereas for the $\frac{3}{2}^-$ possibility the nonresonating phase shifts all differed by 3° to 6° from the expected values. The $\frac{5}{2}^-$ assignment is further supported by the fact that an improvement in the fit to the 5.78-MeV angular distribution was obtained by allowing the value of $\tau_{f_{5/2}}$ to deviate from the background value of unity (see Fig. 4). A more definitive choice between the two possibilities was made by comparing the energy dependence of the polarization at several angles with that expected for $\frac{3}{2}^-$ and $\frac{5}{2}^-$ assignments. This comparison is shown in Fig. 5. The theoretical curves in Fig. 5 were calculated using values of $E_R=5.88$ MeV, $\Gamma=70$ keV, and $\Gamma_e/\Gamma=0.1$ for both cases. The latter value is consistent with the two sets of phase shifts. The qualitative behavior of the theoretical curves is not very sensitive to the values assumed for the resonance parameters. As seen, the experimental values follow the $\frac{5}{2}^-$ curves, thus confirming this assignment.

Since the phase shifts have been determined at an energy very close to E_R, an estimate of Γ_e/Γ may be obtained from the value of $\tau_{f_{5/2}}{}^R(E_R)\approx 0.84$ using Eq. (7). The result is $\Gamma_e/\Gamma\approx 0.08$, which is in reasonable agreement with the value 0.1 previously reported.[4,5]

TABLE II. Resonance parameters for the 9.13-MeV doublet in $^{12}C(p,p)^{12}C$. E_R is the resonant energy, Γ is the total width, and Γ_e/Γ is the ratio of the elastic-to-total width. The energies are given in the laboratory system.

J^π	E_R(MeV)	Γ(keV)	Γ_e/Γ
$\frac{5}{2}^-$	9.132	33	0.26
$\frac{7}{2}^-$	9.132	82	0.81

C. 9.13-MeV Anomaly

The double structure of the 9.13-MeV anomaly was first reported by Swint *et al.*[6] The evidence for two resonances was the appearance of double peaks in the cross-section excitation functions at center-of-mass angles of 90° and 140.8°. An assignment[5] of $\frac{7}{2}^-$ was made to the stronger resonance in agreement with a previous assignment.[3]

The first evidence for the spin and parity assignment for the weaker resonance was obtained from the present[7] analysis of the 9.15-MeV angular distribution. Only the back-angle polarization data were used in the phase-shift search at this energy. The forward-angle polarization data were not included because of possible energy and angular resolution problems due to the rapidly varying structure observed at forward angles in both the angular distribution and the excitation function (see Figs. 2 and 7). Attempts to fit the 9.15-MeV angular distribution with resonance behavior for a single partial wave were unsuccessful. However, these results provided some evidence for a $\frac{7}{2}^-$ assignment to one of the states. Next, resonance behavior in a second partial wave was sought by allowing the $\frac{7}{2}^-$ parameters to vary

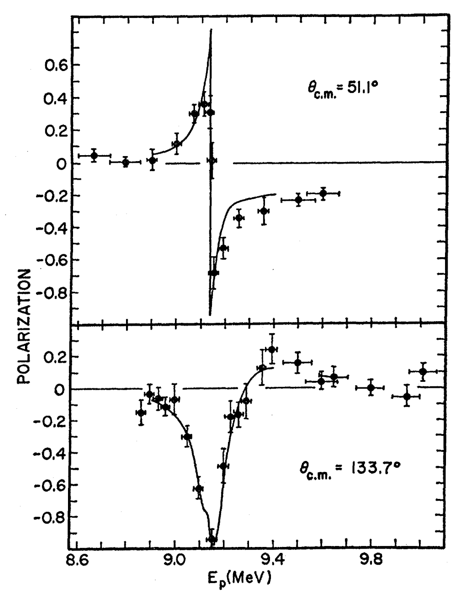

FIG. 7. Polarization excitation functions at center-of-mass angles of 51.1° and 133.7° in the region of the 9.13-MeV doublet. The horizontal bars on the data points indicate the target thickness. The smooth curves were calculated for $\frac{5}{2}^-$ and $\frac{7}{2}^-$ resonances, using the resonance parameters given in Table I.

along with the parameters for each of the other partial waves one at a time. The remaining phase shifts were held fixed at the background values. The data could be fit only when the $\frac{5}{2}^-$ phase shift was allowed to vary along with the $\frac{7}{2}^-$ phase shift. Additional searches were made using a variety of starting phase shifts and allowing all 14 parameters to vary. The results of these searches agreed with the above conclusion that resonance behavior in both the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ phase shifts was required to fit the data.

These spin and parity assignments for the two resonances were verified by fitting the polarization and differential cross-section excitation functions with the computer program discussed in Sec. II D. Because of the resolution problems previously discussed, the polarization measurements near the center of the resonance at 51.1° were not included in the data to be fit. The background values of μ and τ used for the analysis are indicated by the straight lines from 8.9 to 9.4 MeV in Figs. 3 and 4. The level parameters giving the best fit to the data are given in Table II. Polarization and differential cross-section excitation functions calculated with these parameters are compared with the measurements in Figs. 6 and 7. The double peak observed at 90° by the Rice group is also reproduced by these parameters. Searches were made with a number of combinations of values for the two resonant energies. It was found that if the resonant energies differed by more than 2 keV, the near symmetry of the two peaks relative to the minimum at 140.8° was destroyed. It is therefore concluded that although the two resonant energies may not be exactly equal, they probably do not differ by more than 2 keV. The values of the level widths giving the best fit to the data depended to some extent on the exact values used for the background phase shifts; however, the results obtained never differed by more than 10% from those given in Table II.

Some of the level parameters derived in the present analysis differ significantly from the assignments made by Swint *et al.*[8] In particular, the resonant energies for the two levels given by these authors differed by 7 keV; the total width for the $\frac{5}{2}^-$ state was given as 12 keV. While the Rice parameters qualitatively reproduce the features of the excitation functions, the new parameters given here provide a much more quantitative reproduction of the data (see Figs. 6 and 7). Part of the difference between the absolute values for the resonant energies given here and those given by Swint *et al.*[8] is due to a 10-keV difference in the absolute energy scales (see I, Sec. IV).

IV. CONCLUSIONS

The spin and parity assignments of $\frac{3}{2}^+$ and $\frac{5}{2}^-$ to the anomalies in ^{12}C(p,p)^{12}C at 5.38 and 5.88 MeV, respectively, have been verified including polarization data along with differential cross-section measurements. Assignments of $\frac{5}{2}^-$ and $\frac{7}{2}^-$ have been obtained for the doublet near $E_p = 9.13$ MeV. The resonant energies for these two f-wave resonances are found to be the same within a few keV.

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