Magnetoresistance of Undoped n-Type Gallium Arsenide at Low Temperatures*

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The magnetoresistance of undoped n-type GaAs has been measured at liquid-helium temperatures employing magnetic field strengths up to 140 kOe. The samples had electron concentrations between 1.7×10^{15} and 4.9×10^{15} cm^{-3} at 77°K. At low magnetic fields, negative magnetoresistance is observed. It is analyzed into a positive and a negative component. The latter is a function of $H/(T+\theta)$, where H is the magnetic field strength, T is the temperature, and θ has a value close to 2°K for each sample. In high magnetic fields, above 30 kOe, the resistivity increases very strongly with magnetic field strength, in a manner similar to that when conduction is due to quantum-mechanical resonance jumping of electrons between donor impurities. To account for both the low-field and high-field magnetoresistance, we suggest that conduction takes place in a set of excited impurity states which are delocalized in zero or low magnetic field but become localized because of shrinkage of the wave functions when a high magnetic field is present.

I. INTRODUCTION

UNDERSTANDING of the low-temperature electrical conductivity of semiconductors having low impurity concentrations is far more advanced for the group-IV elemental semiconductors Ge and Si than for III-V compound semiconductors. This is not only because more experimental and theoretical work has been done on Ge and Si, but also because it is possible to control the properties of these materials better, since they can be purified and doped in a controlled manner down to much lower concentrations than can the compounds.

For this reason, when studying compound semiconductors, one often has to resort to models which have been made and tested experimentally for Ge and Si even though it is not clear how appropriate the models are in view of the differences in band structure, purity, type of binding, electron-lattice interaction, etc., between the material in question and Ge and Si.

In n-type Ge at low temperatures three ranges of impurity conduction are recognized.[1,2] For high-impurity concentrations, conduction takes place in an impurity band. The resistivity ρ_0 and the Hall coefficient R_H are almost independent of temperature, and the low-field magnetoresistance $\Delta\rho/\rho_0$ is negative.[3,4]

For impurity concentrations so low that the distance between donors is large compared to the size of electronic states localized on donors, conduction is caused by phonon-assisted tunnelling, and ρ_0 is characterized by an activation energy ϵ_3.[1,2,5] Phonon assisting is necessary because ionized minority impurities make adjacent majority sites nonequivalent in energy. At higher temperatures, the dc Hall coefficient is found to go through a maximum, but in the liquid helium range, where the hop mechanism causes all conduction, no dc Hall effect has been found. For low fields there is a positive magnetoresistance[6] which is proportional to H^2, while for high fields the magnetoresistance is large and has a more complicated field dependence.[7–11]

In a region of impurity concentration between these two there is a transition, and the conduction has properties not found in either of the other regions. ρ_0 is characterized by an activation energy ϵ_2 which is magnetic-field-dependent.[12,13] A Hall effect has been measured down to the lowest temperatures. It goes through one or two maxima and minima and is an activated function of temperature for the lowest temperatures.

Impurity conduction has also been observed in n-GaAs.[14] One distinct difference from n-Ge is that negative magnetoresistance is found down to much lower impurity concentrations[3,15] and at which the resistivity is an activated function of temperature.

In the present work the magnetoresistance of ingot-grown n-GaAs with electron concentrations between 1.7×10^{15} and 4.9×10^{15} cm^{-3} at 77°K has been measured both in low and high magnetic fields.[16] After discussing our experimental procedure we draw some conclusions about the purity of our samples from data at higher temperatures and from ρ_0 and R_H at liquid-helium

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temperatures. Then we present the results of our magnetoresistance measurements and compare them with the results of previous work, with models for simple semiconductors, and with n-Ge to the extent such a comparison seems relevant.

II. EXPERIMENTAL DETAILS

Single crystal slices of n-GaAs were obtained from the Bell & Howell Research Center. They were not intentionally doped. All the samples except one were cut to a rectangular shape with a diamond wheel; the nonrectangular sample was cut to a bridge shape by means of an ultrasonic cutter. Agreement was found between the results for this sample and for one of rectangular shape from the same slice. The etch employed was $3H_2SO_4:H_2O:H_2O_2$, heated and used for 20 sec. For most measurements, satisfactory contacts were made of 97% Sn 3% Sb, soldered on. However, for measurements of resistivity in magnetic fields up to 140 kOe, and of Hall effect at liquid-helium temperatures, contacts were made this way: 64%Sn-2%Sb-34%In was dissolved in Hg and smeared on to the crystal. The Hg was distilled off and the contact material alloyed to the crystal by heating to 450°C in an atmosphere of H_2.[17]

The samples were checked for spurious effects by means of measurements at 77°K. These revealed that there was no anisotropy when the magnetic field direction was changed in the plane perpendicular to the sample current, and that for all samples the ratio of transverse to longitudinal magnetoresistance was ≥ 10 at low fields. Measurements were always made for two sets of contacts. The results from both sets agreed within 5–10%.

For temperatures below 5°K the samples were immersed in liquid helium, and the desired temperatures were obtained by measuring and controlling the vapor pressure above the bath.

The electrical measurements were made with a potentiometer system using an L & N potentiometer type K2. For conditions where the impedance was below $10^4\,\Omega$, a Keithley 150 AR was used as a null detector. For higher impedances the potentiometer was connected in the feedback loop of an Applied Physics Corporation vibrating-reed electrometer model 30. The electrometer was used as a null detector, showing when the feedback signal from the potentiometer was equal to the input signal from the sample. For the measurements of resistivity in magnetic fields up to 140 kOe at 1.9°K, long time constants and a high noise level made it necessary to use a numerical integrator. For measurements of the Hall effect at liquid-helium temperatures the vibrating-reed electrometer was used as an amplifier of the signal between the Hall probes, the amplified signal being recorded versus magnetic field for both field directions on an X-Y recorder AEI type 1100.

Magnetic fields up to 27 kOe were provided by a Pacific Electric Motor Co. magnet, model 12 C–AT–LI–E, with stabilized power supply QRC–75–5 and the field was monitored with a Rawson Lush rotating coil gaussmeter. Fields up to 75 kOe were provided by a Westinghouse 80-kG Superconducting Solenoid with Magnet Controller model 503. The measurements in fields up to 140 kOe were performed in a Bitter type solenoid at The National Magnet Laboratory. The solenoid currents were monitored and the field strength was found from an independent calibration.

TABLE I. Resistivity, Hall coefficient, and mobility data for n-GaAs samples. The samples are designated according to their carrier concentration at 77°K, such that 3.4-15 means 3.4×10^{15} cm^{-3}. The Hall coefficient was measured in a field of 6 kOe.

Sample Manufacturer's code	1.7-15 404T	2.1-15 404T	3.4-15 226	4.9-15 321
	$T=300$°K			
ρ_0 (Ω cm)	0.24	0.25	0.17	0.14
$1/(R_He)$ (cm^{-3})	3.8×10^{15}	3.9×10^{15}	5.9×10^{15}	7.5×10^{15}
μ_H (cm^2/V sec)	6800	6450	6220	6040
	$T=77$°K			
ρ_0 (Ω cm)	0.28	0.25	0.15	0.097
$1/(R_He)$ (cm^{-3})	1.7×10^{15}	2.1×10^{15}	3.4×10^{15}	4.9×10^{15}
μ_H (cm^2/V sec)	1.3×10^4	1.2×10^4	1.2×10^4	1.3×10^4
	$T=4.2$°K			
ρ_0 (Ω cm)	177	135	12	3.5

III. RESULTS AND ANALYSIS

Table I gives the basic data for the samples. For all specimens the contacts were in a (111) plane; the current direction was in the [112] direction for sample 4.9-15, and the [110] direction for the other samples.

Figure 1 shows how the electrical resistivity varies with temperature in the liquid helium range: ρ_0 increases for decreasing T, being an almost exponential function of $1/T$. For sample 4.9–15 the conductivity is characterized by a single activation energy. For the other samples the slope of the resistivity curves decreases slightly for increasing $1/T$, and the conductivity is somewhat better described by an expression of the form

$$\sigma=\sigma_2\exp(-\epsilon_2/kT)+\sigma_3\exp(-\epsilon_3/kT). \quad (1)$$

However, since our curves would give quite similar values for ϵ_2 and ϵ_3, and we do not have enough data for an accurate examination, we shall not attempt to fit Eq. (1) to our data.

Figure 2 shows the Hall coefficient for sample 2.1-15. It increases as the temperature decreases in the temperature range 5–1.6°K.

In Table II we have given for each of our samples the value of the activation energy ϵ, which was deduced

[17] D. L. Spears (private communication).

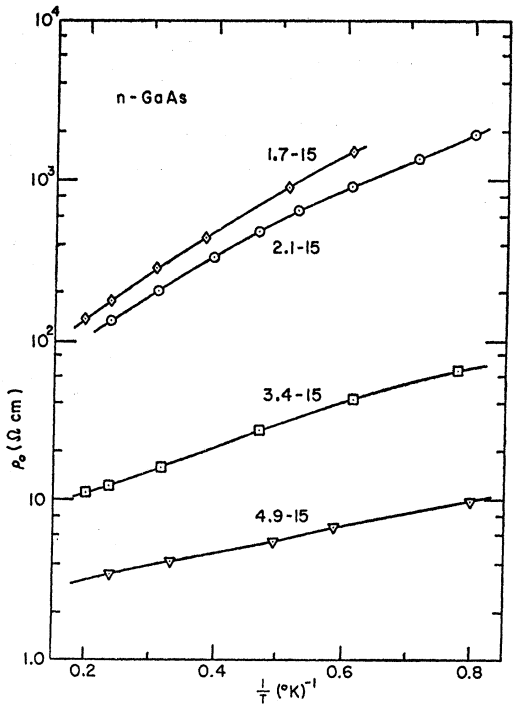

FIG. 1. Resistivity in zero magnetic field versus inverse temperature.

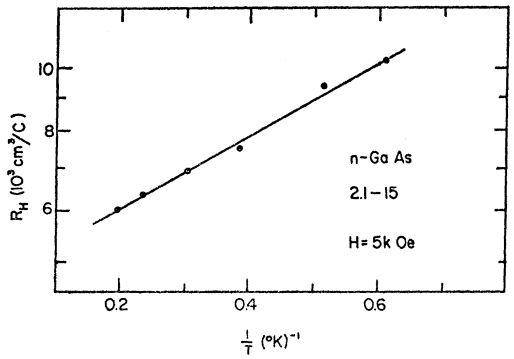

FIG. 2. Hall coefficient at 5 kOe versus inverse temperature for sample 2.1-15.

from the slope of the logρ versus $1/T$ curves at 4.2°K, and the total donor concentration N_D, which was calculated as follows. Ionized impurity scattering was assumed to be the dominant scattering mechanism at 77°K, and we computed the total concentration of ionized impurities, $N_I=N_D+N_A$, by using the Brooks-Herring (BH) formula.[18] The measured values of Hall coefficient and Hall mobility were used, and we set $N_D-N_A=n=1/R_He$. The values of N_D deduced by using the BH formula to obtain N_I and $1/R_He$ to obtain N_D-N_A are somewhat too high since lattice scattering is not negligible in n-GaAs at this temperature,[19] and thus causes the mobility to be lower than it would be if only ionized impurity scattering were present. It is clear, however, that the samples are heavily compensated. The compensation $K=N_A/N_D$, calculated from the concentrations deduced as mentioned above, is given in Table II. The average distance between carriers, r_D, and between donors, R_D, is also given in Table II in units of the effective Bohr radius: $a_0=\hbar^2\kappa/e^2m^*$. For n-GaAs, $a_0=92$ Å if one uses $m^*/m=0.072$ and $\kappa=12.5$.[19] We have set $r_D=(\frac{4}{3}\pi n)^{-1/3}$ and $R_D=(\frac{4}{3}\pi N_D)^{-1/3}$.

TABLE II. Data and parameters for n-GaAs samples. For the calculation of the donor concentration N_D, see text. K is the compensation ratio N_A/N_D. $r_D=(\frac{4}{3}\pi n)^{-1/3}$, $R_D=(\frac{4}{3}\pi N_D)^{-1/3}$, a_0 is the Bohr radius, and ϵ is the activation energy of the resistivity near 4.2°K.

Sample	1.7-15	2.1-15	3.4-15	4.9-15
N_D (10^{16} cm^{-3})	1.2	1.3	1.6	1.6
K	0.85	0.84	0.78	0.72
r_D/a_0	5.6	5.2	4.5	4.0
R_D/a_0	3.0	2.9	2.7	2.7
ϵ (meV)	0.58	0.51	0.33	0.16

The calculated donor separation in units of a_0 is the same as that of Ge in the ϵ_2 region.[2,12,20] Our samples have other properties similar to those found for Ge in this concentration range at temperatures low enough that all conduction is impurity conduction. The conductivity is characterized by one or two activation energies [see Eq. (1)]. The activation energy decreases rapidly as the impurity concentration increases. (The calculated donor concentration in samples 3.4-15 and 4.9-15 is the same. Analogous to n-Ge the former has both a higher activation energy and larger compensation than the latter.[20]) The observed Hall effect, which has an activation energy considerably lower than that of the resistivity, is also analogous to n-Ge for temperatures so low that the residual electrons in the conduction band do not contribute to the Hall effect.

To investigate the possibility that phonon-assisted hop conduction might be important, we have calculated the resistivity, according to the theory by Miller and Abrahams (MA),[21] for n-GaAs with carrier and donor concentrations as deduced for our samples. For sample 2.1-15 we find at 4.2°K: $\rho_{MA}=320\,\Omega$ cm, activation energy $\epsilon_3=0.9$ meV, compared to the measured values $\rho=135\,\Omega$ cm and $\epsilon=0.5$ meV. Thus, phonon-assisted hopping would be expected to play a minor role in this sample. In the samples of higher impurity concentration, the overlap of the wave functions is stronger and this mechanism is even less likely to be of importance.

Several models have been proposed to explain the conduction in the ϵ_2 region, of which none has proved completely satisfactory so far. Fritzsche[2] suggested that

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[21] A formula for the resistivity due to phonon-assisted tunnelling according to Miller and Abrahams's theory, adapted to semiconductors with a simple conduction band, is found in Ref. 14.

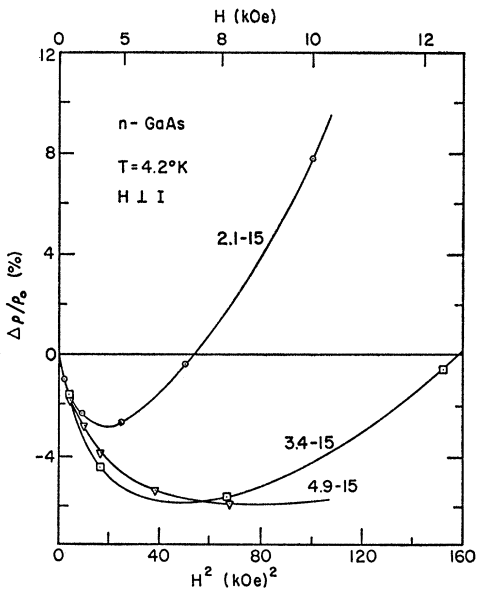

FIG. 3. Magnetoresistance of three samples at 4.2°K for low transverse magnetic fields.

ϵ_2 is related to the energy between the ground state and excited states. He proposed that the excited states were due to so-called D^- ions, i.e., donor atoms with an additional electron. A theory based on this model was formulated by Mikoshiba,[22] who assumed the D^- wave functions could be approximated by screened $1s$ hydrogen wave functions, and the interaction by a screened Coulomb potential. He set the energy difference between the D^- states and the ground state equal to the ionization energy minus the exchange integral between two neighboring D^- states. Stress measurements on p-type Ge [23] and resistivity measurements on n-type Ge over a wide range of compensations[20] were in qualitative agreement with Mikoshiba's theory.

Nishimura[24] made use of the tight-binding approximation for the D^- states, using an exponentially decaying potential and wave function. He calculated expressions for σ_2 and ϵ_2. Quantitative comparison with experiment was not possible for σ_2 since the expression contained a relaxation time not deduced from measurable quantities. The calculated ϵ_2 fit experimental values fairly well for low donor concentrations in the ϵ_2 range, but were larger than the measured values for higher concentrations. The discrepancies were tentatively explained by electrostatic screening effects and interaction between the donor ground states causing a "band" of ground-state levels. However, more recent theoretical work[25] has purported to show that the energy of the D^- states is much higher than was believed previously, and rather than merging with the donor ground states as the donor separation decreased, they would remain close to the bottom of the conduction band.

An alternative type of excited states responsible for the ϵ_2 conduction was proposed by Frood,[26] who suggested that the conductivity takes place in delocalized, excited atomic states which are merged into the conduction band. Mycielski[27] suggested that the mechanism involved thermally activated electrons jumping over the Coulomb barrier between the donor states. Piezoresistance data did not support this idea.[23,28]

All the models mentioned do not take into account the different spacings between various impurities, and none seems totally satisfactory. Matsubara and Toyozawa[29] approached impurity banding from the random lattice point of view, using a Green's-function method. If the ϵ_2 mechanism is conduction in a band due to excited states, a modification of their theory, with wave functions more suitable than those of the hydrogen $1s$ state which they used, might perhaps give a better understanding of ϵ_2 conduction.

A. Magnetoresistance at Low Magnetic Fields

Some of the characteristic features of the low-field magnetoresistance are shown in Figs. 3 and 4. Figure 3

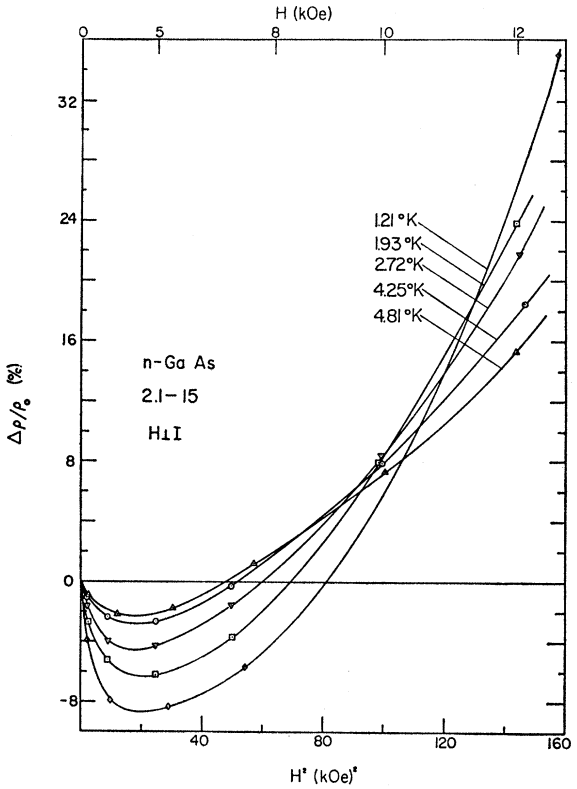

FIG. 4. Magnetoresistance of sample 2.1-15 for low transverse magnetic fields at various temperatures.

[22] Mikoshiba's theory has been outlined in Ref. 23.
[23] F. H. Pollak, Phys. Rev. 138, A618 (1965).
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[26] D. G. H. Frood, Proc. Phys. Soc. (London) 75, 185 (1960).
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[28] H. Fritzsche, Phys. Rev. 125, 1552 (1962).
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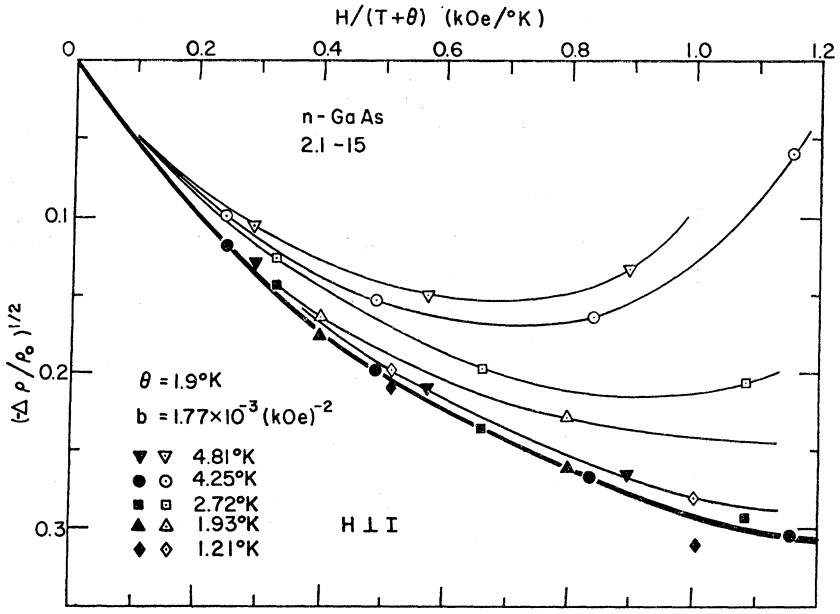

FIG. 5. Measured and negative component of the low field transverse magnetoresistance for sample 2.1-15. The open symbols indicate measured values, the full symbols indicate the deduced negative component. The parameters θ and b were chosen to give a best fit of Eq. (2) to the measured data.

shows the magnetoresistance at 4.2°K in transverse magnetic fields up to about 12 kOe for three samples, and Fig. 4 shows magnetoresistance for sample 2.1-15 in the same field range at various temperatures.

The magnetoresistance $\Delta\rho/\rho_0$ is found to be negative for the weakest fields, but for a value of $H=H_{\text{cross}}$ it becomes positive. For a given temperature H_{cross} is higher the larger the carrier concentration, and for a given sample it is higher for lower temperatures. For the minimum value of $\Delta\rho/\rho_0$, denoted $(\Delta\rho/\rho_0)_{\min}$, it is found that $|(\Delta\rho/\rho_0)_{\min}|$ is smaller for the lower-concentration samples, and for a given sample the magnitude increases for decreasing temperature. For the weakest fields, $|\Delta\rho/\rho_0|$ approaches proportionality with H^2, with a proportionality constant that increases with decreasing temperature.

For the samples reported here we have found that in transverse fields below 6–8 kOe the magnetoresistance may be decomposed into two components, hereafter referred to as the negative and the positive component:

$$\frac{\Delta\rho}{\rho_0}=-f\left(\frac{H}{T+\theta}\right)+bH^2, \qquad (2)$$

where θ and b are constants, which have different values for each sample. The function f approaches proportionality with $[H/(T+\theta)]^2$ for the lowest fields and tends to saturate for higher fields. Figure 5 shows the measured transverse magnetoresistance and the negative component f, deduced for sample 2.1-15, f being found by adjusting b and θ to give the smoothest fit to the observations. Values of the parameters deduced in this way are given in Table III.

The behavior of the square root of the negative component resembles a Curie-Weiss-type dependence on magnetic field and temperature for these samples. Such behavior of the resistivity has been observed both for dilute magnetic alloys[30] and for some highly doped semiconductors.[3,31,32] For the latter it is believed that the magnetic moments causing this behavior are the spins of electrons semilocalized around donor impurities.[4] Some donors are pictured as being close enough to each other so that the electronic states form an impurity band. Conduction is via electrons in this band and is limited by scattering of the electrons. However, some donor sites are far enough apart so that electrons are localized around them most of the time, causing these donors to have a localized magnetic moment, which still has some probability of interacting with the delocalized electrons. With application of a magnetic field, the magnetic moments become aligned, the amount of inelastic scattering is reduced, and the resistivity decreases by an amount proportional to the square of the magnetization. Between the magnetic moments there may be a weak ferromagnetic or antiferromagnetic interaction. This leads to a magnetoresistance due to

TABLE III. Parameters in Eq. (2) for n-GaAs samples. The values are deduced by making a best fit to measurements.

Sample	2.1-15	3.4-15	4.9-15
θ (°K)	1.9	2.1	2.2
b (10^{-3} kOe^{-2})	1.77	1.56	1.50

[30] A. N. Gerritsen, Physica **25**, 489 (1959).
[31] R. P. Khosla and R. J. Sladek, J. Phys. Soc. Japan Suppl. **21**, 557 (1966).
[32] Y. Katayama and S. Tanaka, Phys. Rev. **153**, 873 (1967).

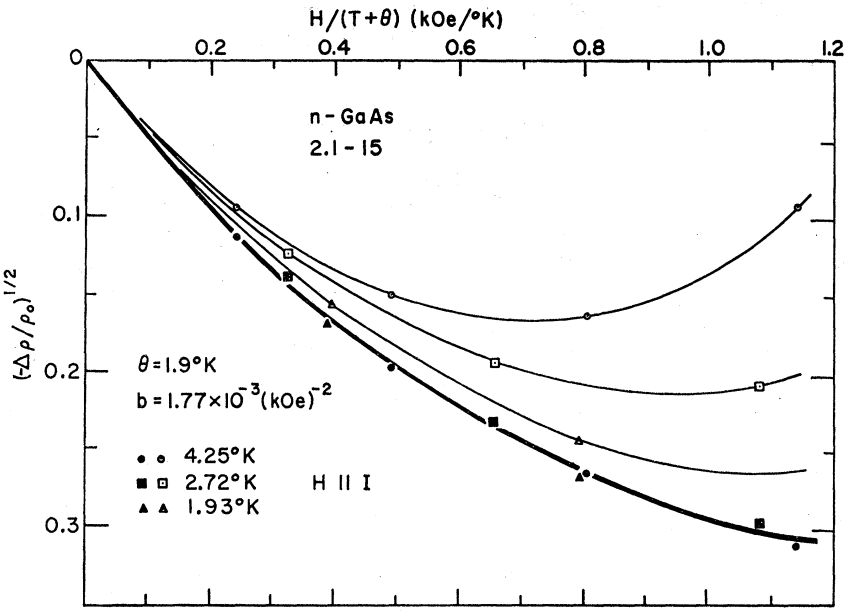

FIG. 6. Measured and negative component of the low field longitudinal magnetoresistance for sample 2.1-15. The open symbols indicate measured values, the full symbols indicate the deduced negative component.

semilocalized spins,

$$\left(\frac{\Delta\rho}{\rho_0}\right)_s = -f\left(\frac{H}{T+\theta}\right), \tag{3a}$$

and in the limit of weak and strong fields,

$$\left(\frac{\Delta\rho}{\rho_0}\right)_s \sim -\left(\frac{H}{T+\theta}\right)^2, \quad \text{for} \quad H \to 0 \tag{3b}$$

$$\left(\frac{\Delta\rho}{\rho_0}\right)_s \to -\text{const}, \quad \text{for} \quad H \to \infty . \tag{3c}$$

The constant θ appears due to the coupling between the magnetic moments and is positive for antiferromagnetic coupling. A characteristic feature of this mechanism is that there is no dependence on the angle between magnetic and electric field in the crystal.[32]

As mentioned above, Eqs. (3) appear to be satisfied by the negative component of the magnetoresistance which we have deduced for our samples, with a suitable choice of θ. We also made measurements in weak longitudinal fields for one crystal. Within the uncertainty of the measurements, we found the same magnetoresistance as in transverse fields. (See Fig. 6.) The observation that the magnetoresistance at low fields is isotropic agrees with previous work on n-GaAs.[15]

The low-field behavior reported here seems characteristic of a narrow range of impurity concentrations for n-GaAs. We have found that for higher impurity concentrations, when the material is degenerate, there is negative magnetoresistance up to higher fields, but it has a different character and cannot be described in the form given by Eq. (2). For this reason Toyozawa's theory[4] does not seem capable of providing an adequate explanation without some modifications. On the other hand, the negative magnetoresistance becomes less pronounced as the concentration decreases. (Additional evidence for this is provided by data on epitaxially grown material.[33]) Therefore, we do not believe that the negative magnetoresistance can be due to hopping of carriers between antiferromagnetically ordered impurity states, as has been suggested to explain negative magnetoresistance in low concentration p-type Si.[34]

We suggest that conduction in our material is due to delocalized electrons in energy levels above the donor ground state and that the magnetoresistance is due to the field dependence of the mobility of carriers in the band formed by these levels. The band could be formed either by overlapping D^- states as discussed earlier, or by excited atomic states. Since measurements of resistivity and the Hall effect[14,35] show that the carriers have to be activated to get from these impurity states into the conduction band, we do not believe that the band of excited states overlaps or is a tail of the conduction band.[26] Since the donor separation is about 3 Bohr radii, the first excited atomic states centered on neighboring donors overlap considerably in the hydrogenic model, and electrons, once they are excited to these levels, will most likely exhibit bandlike conduction. The random array of impurities could give rise to both an impurity band and the semilocalized spins proposed by Toyozawa, the semilocalized spins having an antiferromagnetic coupling as indicated by the positive sign of θ obtained when Eq. (2) is fitted to our data.

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[34] M. Pollak, Bull. Am. Phys. Soc. 8, 422 (1963).
[35] D. V. Eddolls, Phys. Status Solidi **17**, 67 (1966).

An alternative source of localized magnetic moments would be the spin of localized electrons in the donor ground state.

B. Magnetoresistance at High Magnetic Fields

Figure 7 shows the resistivity of sample 3.4-15 in magnetic fields up to 75 kOe. After the magnetoresistance has become positive, it increases very strongly with magnetic field strength. For high fields the increase is approximately proportional to $\exp(\text{const}\times H)$. The high-field magnetoresistance is larger the lower the temperature. Similar measurements for sample 2.1-15 show that for a given temperature and field, the purer sample (2.1-15) has the larger magnetoresistance.

The very large magnetoresistance suggests that at high magnetic fields the conduction mechanism is controlled by the overlap of wave functions of the charge carriers, with the effect of the magnetic field being to decrease the overlap.

Two different models have been suggested for the effect of a strong magnetic field on jumping conduction. One is for the case when phonon assistance is necessary for the hopping process,[8,9,11] the other is for resonance jumping.[36] Use will be made of the latter since it is appropriate for stronger overlap than is the former.

In their theory for the effect of strong magnetic fields on a hydrogenlike atom, Yafet *et al.*[37] used a wave function of Gaussian shape, i.e., for H in the z direction:

$$\psi=[(2\pi)^{3/2}a_\perp^2 a_\parallel]^{-1/2}\exp\left(-\frac{x^2+y^2}{4a_\perp^2}-\frac{z^2}{4a_\parallel^2}\right). \quad (4)$$

The two parameters $a_\perp$ and $a_\parallel$, the transverse and longitudinal effective Bohr radii, were considered to be functions of the magnetic field and were found by performing a variational calculation to minimize the ground-state eigenvalue of the Hamiltonian. Yafet *et al.* gave a plot of $a_\perp(H)/a_0$ and $a_\parallel(H)/a_0$ versus the quantity $\gamma=\hbar\omega/2\ \mathrm{Ry}_\kappa^*$, where ω is the cyclotron frequency and $1\ \mathrm{Ry}_\kappa^*$ is the ground-state energy of a hydrogen atom in a medium of dielectric constant κ, the electron having mass m^*. For n-GaAs, $\gamma=1$ for $H=69$ kOe. This approach is presumably suitable for high fields, and in the limit $\gamma\gg1$ $a_\perp(H)$ approaches $a_e(H)$, the radius of the orbit of a free electron in a magnetic field. However, the limiting value of $a_\perp(H)$ and $a_\parallel(H)$ when H approaches zero is not the correct value a_0 but it is $0.94a_0$.

Sladek[36] took the electron mobility to be proportional to the diffusion constant D, which is determined by the frequency ν_j at which an electron jumps between two donor ions, and the square average of the jump distance in the direction of the electric field. For large enough donor separation the jump frequency is given by

$$\nu_j=(2/\hbar)K_e,$$

where K_e is the exchange integral. Using the wave function of Yafet *et al.*, Sladek deduced an expression for K_e and for the diffusion constant for transverse and for longitudinal fields. R, the distance between the neutral donor and the ionized donor to which the electron jumps, is considered to be the same for all jumps. The mobility was then found by the simple Einstein relation

$$\mu(H,T)=eD(H)/kT.$$

To explain our high magnetic field resistivity data, we assume that at high fields all the mobile carriers n conduct via this mechanism, and the resistivity becomes

$$\rho(H,T)=[ne\mu(H,T)]^{-1}. \quad (5)$$

Furthermore, we assume that n may vary with temperature, but not with magnetic field strength. Then the magnetic field dependence of ρ in transverse fields is contained in the variation of D_{trans} through the parameters $a_0/a_\perp(H)$ and $a_0/a_\parallel(H)$. D also depends on the impurity concentration through R/a_0. For the range of impurity concentrations and fields in our case, the first term in Eq. (9) of Ref. 36 will dominate, and the exponential term in $[a_0/a_\perp(H)]^2$ will determine the field dependence, since calculations show that it changes much more rapidly with the field than both the factor in front of it and the integral following it. Thus, for our

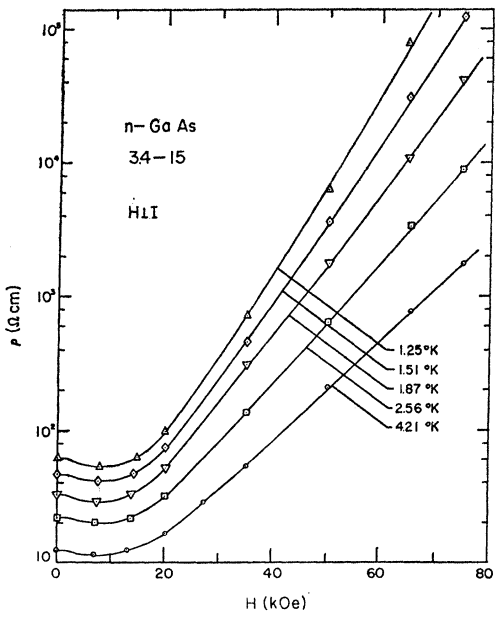

FIG. 7. Resistivity versus magnetic field strength at various temperatures for sample 3.4-15.

[36] R. J. Sladek, J. Phys. Chem. Solids **5**, 157 (1958).
[37] Y. Yafet, R. W. Keyes, and E. N. Adams, J. Phys. Chem. Solids **1**, 137 (1956).

conditions, a good approximation to the theory is

$$D_{\text{trans}}(H)=\text{const}\times\exp\left[-\frac{R^2}{8a_0^2}\left(\frac{a_0}{a_\perp(H)}\right)^2\right]. \quad (6)$$

The resistivity is given by

$$\rho(H,T)=\text{const}\times\frac{T}{n}\exp\left[\frac{R^2}{8a_0^2}\left(\frac{a_0}{a_\perp(H)}\right)^2\right]. \quad (7)$$

Thus, in this approximation, the resistivity is expected to have an exponential dependence on the quantity $[a_0/a_\perp(H)]^2$.

The logarithm of the resistivity of each sample measured up to 75 kOe was found to be a linear function of $[a_0/a_\perp(H)]^2$ above 30 kOe. Representative data are shown for sample 2.1-15 in Fig. 8. For a given temperature the field dependence of the resistivity which we observe for strong fields can be accounted for by means of Eq. (7). Since the form of the wave functions used in arriving at Eq. (7) would presumably be even more appropriate above the maximum field strength achievable with our superconducting solenoid, we made measurements on one sample, 1.7-15, employing fields up to 140 kOe at the National Magnet Laboratory. Figure 9 shows the results. They indicate that up to the highest fields employed the resistivity continues to be an exponential function of $[a_0/a_\perp(H)]^2$.

Recently, attempts have been made to find better trial functions for the hydrogenlike atom in a magnetic field.[38] Unfortunately, integrations involving them have to be performed numerically, and calculations become quite complicated. For our case, when excited states are believed responsible for the conduction, the wave functions of the first excited state of Ref. 38 might be a better choice than the simple Gaussian shape. However, at distances far from the donor nucleus the important feature of the wave function in a strong magnetic field is that it varies almost like $\exp[-C(x^2+y^2)]$, where x^2+y^2 is the square of the component of the distance from the nucleus which is perpendicular to the magnetic field direction.

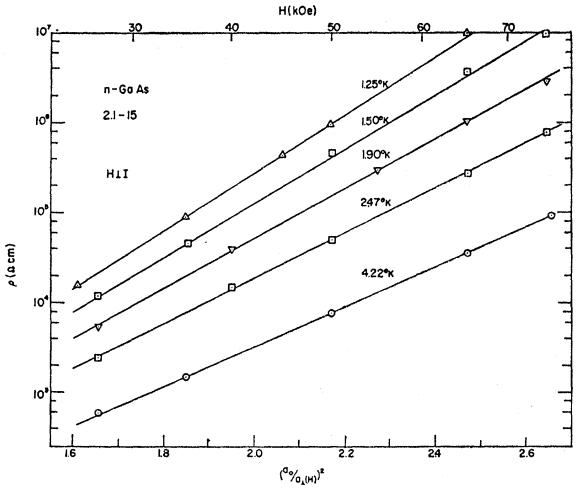

FIG. 8. Resistivity versus $[a_0/a_\perp(H)]^2$ in high transverse magnetic fields at various temperatures for sample 2.1-15.

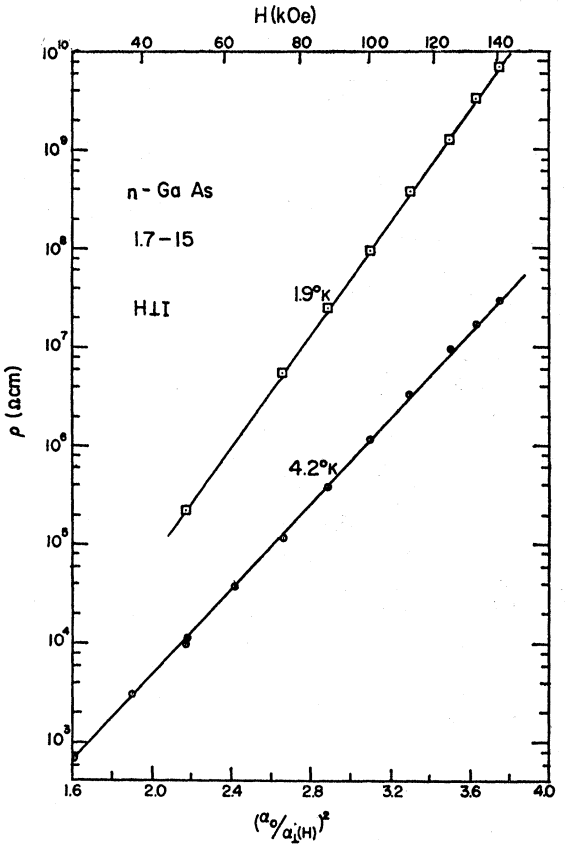

FIG. 9. Resistivity versus $[a_0/a_\perp(H)]^2$ in high transverse magnetic fields at two temperatures for sample 1.7-15.

From these considerations it appears that the field dependence of the resistivity in high transverse magnetic fields can be explained in terms of resonance jumping between states whose wave functions shrink in the presence of the field. However, the slope of the $\log\rho$-versus-$[a_0/a_\perp(H)]^2$ curves has a marked temperature dependence. The model does not seem able to account for this. We have assumed n, the number of mobile carriers, to be independent of H. The slope according to the model becomes, from Eq. (7),

$$\frac{\partial(\ln\rho)}{\partial[a_0/a_\perp(H)]^2}=\frac{R^2}{8a_0^2}, \quad (8)$$

or, in other words, this slope depends on donor separation and Bohr radius alone. The observed slope, deduced for samples 2.1-15 and 3.4-15, has been plotted in Fig. 10. In both cases we get a temperature dependence of

[38] D. M. Larsen, J. Phys. Chem. Solids 29, 271 (1968).

the form

$$\frac{\partial(\ln\rho)}{\partial[a_0/a_1(H)]^2}=C_1+\frac{C_2}{T}, \tag{9}$$

where $C_1=4.2$ and $C_2=4.1$°K for sample 2.1-15, and $C_1=2.8$ and $C_2=6.1$°K for sample 3.4-15. If we equate C_1 to $R^2/8a_0^2$, we obtain $R/a_0=5.8$, $R/a_0=4.7$ for samples 2.1-15 and 3.4-15, respectively, which is qualitatively of the expected magnitude. However, Eqs. (8) and (9) suggest that R increases as the temperature decreases. This does not seem possible. R is the average jump distance and depends on donor separation and compensation only.

A temperature dependence of ρ/ρ_0 could enter into this model if n changed with magnetic field as well as with temperature. A change of n with H could occur, in principle, because of an increase in the energy difference between the ground state and the excited states in which conduction is believed to take place. If we believe that the slope of each resistivity curve in Fig. 1 indicates roughly the energy difference, then a large relative change of ϵ, due to the magnetic field, would be necessary to account for the observations. For the hydrogenlike model, and the wave functions used in Ref. 38, it is found that the energy difference between the ground state and the first excited state is the same for $\gamma=1$ as it is for zero magnetic field. Furthermore, if there were a large change of ϵ with magnetic field, we would not expect the linear dependence of $\log\rho$ on $[a_0/a_1(H)]^2$, since the latter requires that n be independent of H.

Some simplifications which may not be justified have been introduced into our discussion of the resonance jump model. The donors have been assumed to be isolated, whereas the influence of neighboring donors should have been considered. Furthermore, the donors have been assumed to be regularly spaced, and the effect of compensating acceptors has not been taken into account.

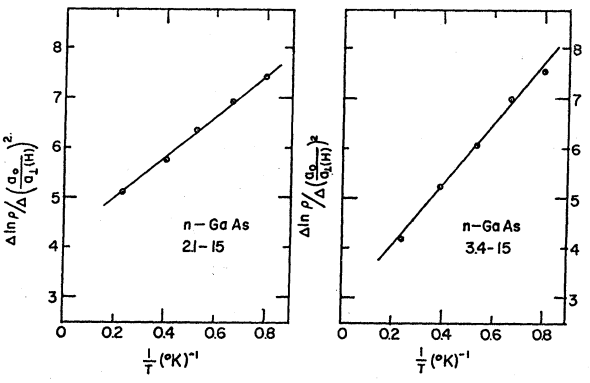

FIG. 10. $\Delta\ln\rho/\Delta[a_0/a_1(H)]^2$ versus inverse temperature for two samples.

For n-Ge in the ϵ_2 region, the effect of a moderate magnetic field is to increase the activation energy by an amount proportional to H^2.[12,13] A simple explanation of this in terms of a change in effective Bohr radius to order H^2 predicted an effect considerably smaller than that which was observed.[12] A different explanation was proposed in terms of a D^- band with antiparallel spins for the two electrons on a D^- ion.[13] This was supported by measurements on As-doped Ge but not on Sb-doped Ge.[13] Recently, part of the temperature dependence of the magnetoresistance found in P-doped Ge was attributed to the split of the ground state into a singlet and a triplet.[10] For n-Ge in which phonon-assisted hopping is responsible for conduction, the temperature dependence of ρ/ρ_0 in very strong magnetic fields has been explained in terms of hopping with reversal of spin.[9] It is found that the magnetic field will cause a mixing of the singlet state and the triplet state. Since for n-GaAs only the band at $\mathbf{k}=0$ contributes to the donor wave function for our material and temperature range, the ground state is not split, and it does not seem that such an effect could be possible.

IV. CONCLUSION

Electrical conduction at low temperatures has been investigated in n-type GaAs having impurity concentrations between those at which phonon-assisted hop conduction and degenerate, impurity conduction occur.

It is suggested that conduction is via electrons in a group of impurity levels which lies below the conduction band but above the energy level associated with the ground state of a shallow donor impurity.

In zero or weak magnetic fields, conduction in these levels is more or less bandlike. Evidence for the bandlike character is provided by the fact that the weak-field magnetoresistance is similar to that exhibited by degenerate n-Ge and n-InSb in which conduction is thought to be due to carriers in a metallic impurity band or in the conduction band which is modified perhaps by impurities.

In strong magnetic fields conduction is still thought to be due to electrons in excited impurity levels. The conduction process seems to involve quantum-mechanical resonance jumping of electrons between donors, since the magnetoresistance is correlated with the estimated shrinkage of the impurity wave functions by the magnetic field.

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