Effect of Temperature and of Substitutional Mo on the Magnetic Behavior of Superconducting Nb

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The temperature and mean-free-path dependence of the magnetic properties of highly reversible samples of Nb and some of its alloys with Mo are compared with recent generalizations of the Ginzburg-Landau-Abrikosov-Gor'kov theory. The temperature dependence of κ_1 and κ_2 in pure Nb is much more rapid than the theoretical predictions, as in the measurements of Finnemore et al. The disparity between theory and experiment decreases with increasing Mo content. The temperature dependence of the lower critical field of pure Nb is in violent disagreement with theory, while at least qualitative agreement is found in the case of the alloy samples.

INTRODUCTION

THE Abrikosov theory¹ of the magnetic properties of an ideal type-II superconductor, applicable in the temperature range close to T_c , makes three definite predictions relating the shape of the magnetization curve to the Ginzburg-Landau² parameter κ . In the absence of surface effects, the field at which the magnetization vanishes is

$$H_{c2} = \kappa \sqrt{2} H_c, \tag{1}$$

while, for fields close to this, the magnetization follows the law

$$4\pi M = (H - H_{c2}) / \beta (2\kappa^2 - 1), \qquad (2)$$

with $\beta = 1.16$ for a triangular fluxoid lattice.³ The third prediction concerns the lower critical field H_{a1} ; Abrikosov gives a formula valid only for a large κ , but the extension to other values has been made by Harden and Arp.⁴

Attempts⁵⁻¹⁰ to generalize the theory to the whole range of temperatures have started from Gor'kov's11 microscopic formulation. It is found that a single parameter κ does not suffice to describe the three features of the magnetization curve enumerated above. and accordingly it is customary to define parameters $\kappa_1(T)$ and $\kappa_2(T)$ which replace κ in Eqs. (1) and (2), respectively, while the temperature variation of H_{c1} is usually described in terms of a third parameter $\kappa_3(T)$.

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The magnitudes of $\kappa_1(T)$ and $\kappa_2(T)$ thus defined are functions of temperature and electronic mean free path. The temperature dependence of κ_1 for infinite mean free path (clean limit) has been studied by Gor'kov,⁵ and for zero mean free path (dirty limit) by Maki.⁶ The corresponding calculations for $\kappa_2(T)$ have been carried out by Maki and Tsuzuki⁷ for the clean limit, and for the dirty limit by Maki⁶ as well as by Caroli et al.⁸

Helfand and Werthamer⁹ computed the temperature dependence of κ_1 for the intermediate values of the mean free path. Eilenberger's investigation¹⁰ includes the effect of temperature and mean free path on κ_2 as well as on κ_1 , and takes into account the possibility of p as well as s scattering.

The temperature dependence of the lower critical field has been¹² studied completely only in the limit of large κ , but the mean-free-path effects have been elucidated by Neumann and Tewordt13 in the temperature region adjacent to T_c .

In this paper, measurements on pure Nb and on Nb alloyed with molybdenum are compared with the theoretical predictions of Eilenberger¹⁰ (whose work includes the results of earlier investigations) and (for the temperature dependence of H_{c1}) those of Neumann and Tewordt.¹³

The magnetization curves of the samples studied are almost entirely reversible, so that uncertainties stemming from magnetic hysteresis are much less important than in most other work.

EXPERIMENTAL

The niobium sample was prepared from a moderately pure rod by zone melting (several passes) followed by swaging and anneal at temperatures in excess of 2000°K for about 24 h in a vacuum better than 10⁻⁸ Torr. The resulting sample, composed of a few large crystals, exhibited a resistivity ratio of about 2000 at the time magnetization measurements were made. After aging several months at room temperature this was found to have fallen by a factor ~ 2 .

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¹³ L. Neumann and L. Tewordt, Z. Physik 189, 55 (1966).

Alloys with molybdenum were prepared by argon arc melting of the constituent metals at AERE Harwell, and subsequent zone melting at zone speeds high enough to avoid segregation. This procedure yielded almost entirely reversible material,¹⁴ and was adopted for the preparation of all the samples used in this work.

Attempts were made to obtain even more reversible samples by annealing in the same way as for Nb, but this proved unsuccessful. Magnetization curves of samples thus treated exhibited much greater hysteresis, together with a long tail above H_{c2} . These effects are believed to result from the preferential evaporation of Mo during annealing, since by etching a layer of metal from the surface the original (reversible) magnetic behavior could be almost (though not quite) restored.

Magnetization measurements were made by a slightly modified version of the vibrating-sample technique reported elsewhere.¹⁵ The sample was situated in liquid helium for measurements in the temperature range below 4.2°K, and in helium exchange gas at the higher temperatures. Temperatures greater than 4.2°K were measured by gas thermometry. The accuracy, having been checked against the susceptibility of

$$Mn(NH_4)_2(SO_4)_2 \cdot 6H_2O_4$$

is estimated to be considerably better than 1%. Further details are given elsewhere.¹⁶

Typical magnetization curves are shown in Fig. 1. In all cases the value of H_c has been computed from the mean area of the curves for increasing and decreasing fields. The magnitude of H_c calculated from the area under the field increasing curve differed from this value by $\sim 2\%$ in the niobium sample, by $\sim 3\%$ in Nb₉₅Mo₅ and by <1% in all other samples.

RESULTS

Since both theory and the present investigation suggest that the magnetic behavior depends on the size of the electronic mean free path relative to the coher-



FIG. 1. Magnetization (arbitrary units) versus field for the Nb sample and for two alloy samples. Both forward and return curves are shown.

ence length, it is necessary to make some estimate of these quantities.

For a pure material of Ginzburg-Landau parameter κ_0 the expression¹⁷

$$\kappa_0 = (1.61 \times 10^{24} T_{c0} \gamma^{3/2} / N^{4/3}) (S_f / S)^2$$
(3)

enables the effective area of the Fermi surface to be computed if the electronic specific heat γ , transition temperature T_{c0} , and the value S_f for a free-electron gas of density N are known. The formula^{17,18}

$$\xi_0 = 0.18kS/12\pi T_{c0}\gamma \tag{4}$$

gives the coherence length of the pure material. The present results for κ_0 , and a γ value¹⁹ of 7.3×10^3 erg/cc °K yield $\xi_0 = 390$ Å. The mean free path may be calculated from¹⁸

$$\langle \rho_n l \rangle = 6\pi^2 h/e^2 S, \tag{5}$$

where ρ_n is the residual resistivity. For the Nb studied in this work these expressions give $\xi_0/l \sim 0.025$, so that the pure limit is approached quite closely.

Accurate estimates of the mean free paths and coherence lengths for the alloys are not easily reached in the absence of detailed knowledge of the Fermi surface. Order-of-magnitude assessment of the mean free path may be made from (5) by assuming S to be the same as in Nb. The coherence length may then be estimated from²⁰

$$\xi = (\xi_0 l)^{1/2},\tag{6}$$

which is applicable in the dirty limit, and may thus be considerably in error for the cleaner alloys.

Values of ξ/l and ξ_0/l calculated according to this prescription are set down in Table I. Also quoted is the value of ξ/l estimated from the relation given by Eilenberger between this quantity and the Ginzburg-Landau parameter κ . The agreement is reasonable in view of the uncertainties inherent in the computation. We estimate κ for the purpose of the above comparison from the extrapolation of $\kappa_1(T)$ and $\kappa_2(T)$ to T_c ; the two values thus obtained are identical within the error of measurement, as expected from (1) and (2).

We consider first the temperature dependence of the thermodynamic critical field H_c , which we obtain from the mean area of the magnetization curves for increasing and decreasing fields as stated above. It is by no means certain that this procedure is better than the use of the increasing field magnetization curve, but the difference between the values of H_c calculated by these two methods do not differ by more than 2%, and the difference is not markedly temperature-dependent.

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 $\rho_4(\mu\Omega~{
m cm})$ Sample $T_{c}(^{\circ}\mathrm{K})$ $\kappa(T_c)$ ξ/l ξ/l ρ_{300} / ρ_4 ξ_0/l Nb80M020 4.243.9 2.69 7.10 7.3 2.73.7 $Nb_{85}Mo_{15}$ 5.30 3.6 3.15 6.05 6.3 2.5 3.3 Nb90M010 6.38 3.3 4.104.724.9 2.2 3.0 Nb95Mo5 7.84 2.3 6.69 3.07 3.21.8 1.7Nb 9 20 0.83 ~ 2000 $\sim 2.5 \times 10^{-2}$ ~ 0.025

TABLE I. Details of samples. Compositions quoted are at. % (nominal). The last column gives ξ/l obtained from κ with the aid of Ref. 10. Details of the calculation of this same quantity as given in the penultimate column are to be found in the text.

Figure 2 shows the ratio of H_c to its value H_{c0} at T=0, plotted against the square of the reduced temperature. The solid line represents a "parabolic" variation of H_c , while the broken line is the BCS prediction. The temperature variation of the critical field of pure Nb is closer to the parabolic law than to the BCS law. Finnemore *et al.*²¹ have noticed this behavior and attribute it to a moderately strong coupling.

In spite of the rather large scatter of the experimental points a distinct tendency for the temperature variation to change, with increasing Mo content, from parabolic to BCS is discernable in Fig. 2. This suggests that the coupling becomes weaker as the Mo content is increased, as is reasonable in view of the strong fall in T_c with increasing Mo content (Table I).



FIG. 2. Thermodynamic critical field as a function of temperature. Solid line represents a parabolic variation and the dotted line the BCS prediction. For clarity the experimental points for some samples have been displaced upwards and the theoretical curves redrawn.

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It is customary to analyze the temperature dependence of H_{c2} in terms of the parameter κ_1 defined by $\kappa_1 = H_{c2}/H_c\sqrt{2}$. The relevant theories, however, proceed by calculating H_{c2} and dividing by the BCS value of H_c to give

$$\kappa_1' = H_{c2}/\sqrt{2}H_c(BCS)$$

Since we wish to compare our observed values of H_{c2} to those predicted, it is convenient to work in terms of κ_1' rather than κ_1 . For the dirtiest alloys the temperature variation of H_c is essentially that predicted by BCS so that $\kappa_1' \approx \kappa_1$, but for pure Nb and the cleaner alloys the temperature variation of H_c differs from the BCS prediction so that $\kappa_1' < \kappa_1$.

Figure 3 shows the temperature variation of κ_1' in the five samples studied. The parameter κ' is defined as the limiting value of κ_1' for $T \rightarrow T_c$. For each sample, the theoretical¹⁰ temperature dependence of κ_1' is shown for the clean limit (upper curve) and for the dirty limit (lower curve). In the samples of shortest electronic mean free path (Nb₈₅Mo₁₅ and Nb₈₀Mo₂₀) the temperature dependence of κ_1' is in accordance with theory within the limit of experimental error. As the electronic mean free path is reduced the temperature dependence of κ_1' becomes stronger. This increase is qualitatively in accord with theory,¹⁰ but its magnitude is considerably greater than expected. Thus, the temperature variation in Nb₉₀Mo₁₀ is already in excess of the prediction for the clean limit, and the ratio $\kappa_1'(0)/\kappa_1(T_c)$ for pure Nb is well in excess of the pure-limit prediction, as noted by Finnemore et al.21

A discrepancy between theory and experiment in the pure limit also occurs in the case of the parameter κ_2 (Fig. 4). Again, the observed temperature variation is considerably stronger than the theoretical estimate.

For the alloys, comparison of κ_2 with theory requires knowledge of the ratio of *s* to *p* scattering. The theoretical curves in Fig. 5 are drawn for the case of pure *s* scattering, with values of ξ/l equal to 2 (upper curve) and 6 (lower curve), marking the approximate range appropriate to these alloys (Table I). For the dirtier alloys, the temperature dependence of κ_2 is in reasonable agreement with theory. As the mean free path is reduced the temperature variation of κ_2 becomes stronger than the theoretical prediction. This disparity would be even greater were a more reasonable ratio of p to s scattering assumed.

Thus the behavior of the parameter κ_2 is rather similar to that of κ_1' in that there is fairly good agreement with theory for the dirtier samples, and a progressive departure from theory with increasing elec-



FIG. 3. The ratio κ_1'/κ as a function of reduced temperature. The broken curves represent the theoretical prediction (Ref. 10) for the clean limit (upper curve) and the dirty limit (lower curve).

tronic mean free path until, in the pure limit, the variation of both κ_2 and κ_1' is much stronger than expected.

Measured values of H_{c1} will be compared with the calculations of Neumann and Tewordt,¹³ which are valid for all values of H_{c1}/H_c , although only in the region $T \approx T_c$ (the calculation by Maki¹² is valid for all T, but its validity is confined to the limit of large κ and it is inapplicable to our samples).







FIG. 5. The ratio κ_2/κ for the alloy samples. Theoretical (Ref. 10) curves are drawn for $\xi/l=2$ (upper curve) and 6 (lower curve) for the case of pure s scattering.

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FIG. 6. The ratio H_{e1}/H_e for the Nb samples. Open circles are for the observed H_e and closed circles for the BCS H_e . The line represents the theoretical prediction of Ref. 13.

Open circles in Fig. 6 show the observed ratio H_{c1}/H_c as a function of T/T_c . H_c is obtained from the magnetization curve as described above. For comparison we show also values of H_{c1}/H_c (BCS) (closed circles). The difference between these two sets of points emphasizes the disparity between the experimental value of H_c and the BCS value (Fig. 2). The short line is the theoretical estimate¹³ of H_{c1}/H_c , valid for $T \approx T_c$. Agreement as to the magnitude of H_{c1}/H_c is reasonably good, but the predicted temperature dependence differs even in sign from that observed.

Values of H_{c1}/H_c for the alloy samples are plotted as a function of T/T_c in Fig. 7. The short lines are the theoretical curves $(T \approx T_c)$ for 0.88 ξ_0/l equal to 2 (upper line) and 10 (lower line), which covers the approximate range of ξ_0/l in these alloys (Table I). Agreement of the observed and predicted magnitudes of H_{cl} is good in all cases. The scatter of the experimental data and its scarcity in the relevant temperature region precludes any detailed comparison with the theoretical temperature dependence of H_{c1}/H_c , but we can at least conclude that the theory is not qualitatively in error, as is the case for the pure sample. There seems to be a tendency for H_{c1}/H_c to decrease rather more rapidly with temperature than the calculations suggest, but firm conclusions must await more accurate measurements in the region near to T_c .

DISCUSSION

The temperature variation of κ_1' and κ_2 exhibits a systematic dependence on impurity content. Samples of short electronic mean free path behave in a manner more or less consistent with theory, but the experimental observations show an increasing departure from theory as the mean free path increases, and the disparity becomes quite pronounced in the limit of long mean free path.

The addition of Mo to Nb introduces factors other than the straightforward change in mean free path considered in the theory.¹⁰ Figure 2 suggests that one

such factor is a change in the coupling strength. This is interesting in view of the suggestion²² that the temperature dependence of κ_1' , at least, may be influenced by coupling strength. On the other hand, recent theoretical considerations²³ indicate that this factor is not likely to be of sufficient importance to account for anomalies of the magnitude observed here.

It is more likely that the observed deviations are related to anisotropy associated with the nonspherical Fermi surface.²⁴ The tendency for the temperature dependence of κ_1' and κ_2 to become rapidly weaker with increasing Mo content is compatible with the expected fall in anisotropy effects with decreasing electronic mean free path.

Fietz and Webb²⁵ have recently reported an inves-



FIG. 7. Temperature dependence of H_{el}/H_e for the alloy samples. The theoretical prediction (Ref. 13) for $T \approx T_e$ is indicated by lines drawn for $0.88\xi_0/l=2$ (upper curve) and 10 (lower curve).

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tigation similar to ours, with Ti rather than Mo as the solvent. Their conclusions are similar to ours, except that they find an anomalously strong variation of κ_1' and κ_2 for all values of ξ/l , whereas in the present work the temperature dependence of κ_1' and κ_2 is compatible with the theory, within the experimental error, for the samples of shortest mean free path.

The theory of Neumann and Tewordt¹³ accounts very well for the magnitude of H_{c1}/H_c in all samples. Their estimate of the temperature dependence of this quantity is in violent disagreement with our observations on pure Nb. In the case of the alloy samples, their calculation is at least in qualitative agreement with the data. There is some indication that the theory underestimates the rate of change with temperature, but the accuracy of measurement does not warrant definite conclusion.

One would, perhaps, expect to find systematic errors in H_{c1} due to surface effects.²⁶ The fact that theory¹³ gives a good account of the observed magnitude of H_{cl} , and the almost complete absence of hysteresis in the alloy samples, suggests that this is not so. The reason why surface effects are absent is not clear; it is presumably related to some special surface condition in these samples.

ACKNOWLEDGMENTS

We are obliged to P. E. Madsen, C. H. Thomas, and G. Taylor for assistance in preparing the specimens, and to K. Mendelssohn for his critical interest. Financial support was provided by the Ministry of Technology.

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PHYSICAL REVIEW

VOLUME 173, NUMBER 2

10 SEPTEMBER 1968

Ultrasonic Attenuation in Superconducting Mercury and Mercury-Cadmium Single Crystals*†

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(Received 29 March 1968)

This paper presents the results of low-temperature ultrasonic-attenuation studies in mercury single crystals, polycrystals, and mercury single crystals with varying concentrations of cadmium impurity. The frequency ranged from 10 to 130 MHz and the doping concentration from 0.01 to 0.10% by weight. The major emphasis is a systematic study of the deviations of α_t/α_n for longitudinal ultrasound from that predicted by the BCS theory. These deviations are a drop in α_s/α_n with decreasing temperature near T_o which is greater than that predicted by BCS. The rapidity of the drop increased with increasing frequency in all pure samples, was frequency-independent, and decreased with doping concentration in the doped samples. The deviations observed cannot be explained adequately by dislocation attenuation or by multiple anisotropic energy gaps.

I. INTRODUCTION

URING the past several years there has been a) growing interest in the significant deviations from theoretically predicted values displayed by the attenuation of longitudinal ultrasound in superconductors. Bardeen, Cooper, and Schrieffer (BCS),¹ Tsuneto,² and others calculate that the ratio of the electronic attenuation in the superconducting state to that in the normal state should be

$$\alpha_s/\alpha_n = 2\{1 + \exp[\Delta(T)/kT]\}^{-1}, \quad (1)$$

where $\Delta(T)$ is the superconducting energy gap. Measurements of this ratio in pure superconductors seldom follow Eq. (1), the deviation usually appearing near T_c as a too rapid decrease of α_s/α_n with decreasing temperature. The most extreme example of this effect yet reported is by Deaton,³ who worked with a very pure unstrained lead crystal, but other workers have observed the too rapid drop in other materials: polycrystalline mercury,4 single-crystal mercury,5 singlecrystal tin,6-8 single-crystal lead and lead doped with

^{*} Supported in part by the National Science Foundation and

the National Aeronautics and Space Administration. † Based on a thesis submitted by C P. Newcomb in partial fulfillment of the requirements for the Ph.D. degree at Rensselaer

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