Using the results obtained in the above sections for P, P', ρ, and A_2, taking case (i) for P', we obtain the residue functions of ω as plotted in Fig. 6. We then proceed to present the predictions for the K^+p and K^-p polarizations in Fig. 7. Finally, the predictions for K^-n and K^+n elastic differential cross sections are given in Fig. 8, and their polarizations in Fig. 9. The above calculations are based on the contribution of the Regge poles from the t channel alone.

ACKNOWLEDGMENTS

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Parameters of Low-Energy Λ-K^0 Production

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A pole-resonance model is used to fit the data for $\pi^-p \rightarrow \Lambda K^0$ from threshold to 1200 MeV. Parameters of the three pole contributions are taken from a study of the 1370–2200-MeV region, with form factors employed. The reduced widths of eight resonances are then determined by minimizing χ^2. The most significant contributions to the process are found to be those of the $P_{11}(1470)$ and $S_{11}(1710)$, although other resonances are also important. The results are discussed with consideration given to $SU(3)$ assignments and parity doubling.

1. INTRODUCTION

Despite the theoretical work of many authors on the process $\pi^-p \rightarrow \Lambda K^0$ over a period of several years, one cannot say that the following objectives have been realized: a clear understanding of the contributions of poles and resonances to the low-energy region, and a reliable determination of the pole and resonance parameters. These parameters are important for the assignment of $SU(3)$ multiplets and the determination of D/F ratios of octets, for evaluation of the predictions of symmetry schemes in addition to those of $SU(3)$, and in general for assessing the validity of any dynamical theory of the strong interactions. The present existence of a large collection of experimental data, both differential cross sections and polarization angular distributions, together with the results of recent phase-shift analyses, would seem to eliminate the principal obstacles to achieving the aforementioned objectives. It is our purpose in this paper to present a determination of the low-energy ΛK^0 parameters by fitting the data with pole and resonance contributions. In view of the complexity of the isospin-$\frac{1}{2}$ spectrum of baryon resonances, we can conceive of no reliable way to determine these parameters other than an analysis of the partial-wave amplitudes similar to that given here.

A discussion of most of the earlier models was included in a previous paper[1] and will not be repeated here. However, a relatively recent paper dealing with the low-energy region, which is not mentioned in Ref. 1, is that of Hoffman and Schnitzer.[2] These authors used the Cini-Fubini approximation to the Mandelstam representation to study the region near threshold. The only "well-established" $I=\frac{1}{2}$ resonances at that time were the F_{15} and the D_{13}, and they approximated the latter by a real amplitude. Thus the only imaginary amplitude resulted from the F_{15}, and other important contributions, notably that of the imaginary part of the $S_{11}(1710)$, were not included. This probably accounted for their failure to obtain a good fit to the polarization data.

In this paper, we present a study of the energy region from threshold (768 MeV) to 1200 MeV which was done by varying pole and resonance parameters to fit the data. In order to take account of all three Mandelstam channels, we used poles due to the nucleon, the Σ, and the $K^*(890)$. Resonances which were included are the well-known[3] $S_{11}(1550)$, $S_{11}(1710)$, $P_{11}(1470)$, $D_{13}(1525)$, $D_{15}(1680)$, and $F_{15}(1690)$, as well as two new resonances predicted recently by Donnachie *et al.*,[4] a $P_{11}(1751)$ and a $P_{13}(1863)$. The pole parameters were first studied by using the pole terms alone in a simple model at higher energies,[5] where one-particle exchange should begin to

[1] J. E. Rush and W. G. Holladay, Phys. Rev. **148**, 1444 (1966).

[2] H. Hoffman and H. J. Schnitzer, Nucl. Phys. **76**, 481 (1966).

[3] For references to data on these states, see A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Söding, C. G. Wohl, M. Roos, and W. J. Willis, Rev. Mod. Phys. **40**, 77 (1968).

[4] A. Donnachie, R. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[5] The higher-energy data used are at eight energies 1.4–2.2 GeV, recently published by O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, D. H. Miller, and J. A. Schwartz, Phys. Rev. **163**, 1430 (1967).

dominate. It was found that absorption seemed significant, and form factors were introduced to account for this in part. The calculated value of the $N\Lambda K$ coupling constant[6] was found to make the contribution of the direct-channel nucleon pole too large by two orders of magnitude, and an additional "form factor" was introduced to make this term acceptable. The results of this analysis, with pole parameters fixed, were then extrapolated to lower energy as background for the resonance terms.

Although the pole parameters are probably not reliable for several reasons, we believe that the technique used gives a good representation of the background at low energy, and that the partial widths of resonances which result from the analysis (see Table II) are reasonably accurate. We intend to follow this work with a careful analysis of the higher-energy region, including a comparison of form-factor, absorption, and Regge-pole models, and involving higher-mass resonant states.

In the following sections, we present a brief summary of the kinematics and dynamics involved in this analysis, a description of the numerical calculations, first at high energy, then at low energy, a presentation of the results numerically and graphically, and a summary of the results with discussion and conclusions.

2. KINEMATICS AND DYNAMICS

We designate[1] the four-momenta of the p, Λ, π, and K^0 by p, p', k, and k', their masses by m, m', μ, and μ', and their energies by E, E', ω, and ω', respectively, in the c.m. system. The total c.m. energy is W, and s, t, and u are the usual Mandelstam variables. With

$$x=\cos\theta=\hat{k}'\cdot\hat{k},$$

we may write the differential cross section $d\sigma/d\Omega$ and polarization $P(\theta)$ as

$$d\sigma/d\Omega=|a|^2+|b|^2\sin^2\theta,$$
$$(d\sigma/d\Omega)P(\theta)=2\,\mathrm{Im}ab^*\sin\theta,$$

with

$$a=g+h\cos\theta,\quad b=h,$$

where

$$g=\sum_{l=1}^{\infty}(f_{l-1}{}^{+}-f_{l+1}{}^{-})P_l'(x),\tag{1}$$

$$h=\sum_{l=1}^{\infty}(f_l{}^{-}-f_l{}^{+})P_l'(x),\tag{2}$$

and $f_l{}^{\pm}$ is the partial-wave amplitude for orbital angular momentum l and total angular momentum $j=l\pm\frac{1}{2}$.

In order to represent the pole contributions more easily, we give the amplitudes g and h in terms of invariant amplitudes A and B:

$$g=\frac{1}{8\pi}\left(\frac{|\mathbf{k}'|}{|\mathbf{k}|}\right)^{1/2}\frac{[(E+m)(E'+m')]^{1/2}}{W}\times\{A-[W-\tfrac{1}{2}(m+m')]B\},$$

$$h=\frac{1}{8\pi}\left(\frac{|\mathbf{k}'|}{|\mathbf{k}|}\right)^{1/2}\frac{[(E-m)(E'-m')]^{1/2}}{W}\times\{-A-[W+\tfrac{1}{2}(m+m')]B\}.$$

The only pole on the real axis in the s channel is that due to the nucleon. In the t channel there are contributions from K-π resonances, notably the $K^*(890)$ and the $K^*(1420)$. We included only the $K^*(890)$, neglecting its width. Neglecting Y^* resonances in the u channel, we get only the contribution of the Σ pole. Thus we chose the nearest poles in the t and u channels for simplicity; this means that we should not rely too heavily on the values obtained for the K^* and Σ coupling constants which we obtained.

Using unsubtracted dispersion relations in the appropriate variables, we may calculate the contributions of the pole terms to the amplitudes A and B. For the nucleon and Σ poles we obtain

$$A(N)=\tfrac{1}{2}\sqrt{2}g_{NN\pi}g_{N\Lambda K}(m'-m)/(m^2-s),$$
$$B(N)=-\sqrt{2}g_{NN\pi}g_{N\Lambda K}/(m^2-s),$$

and

$$A(\Sigma)=-\sqrt{2}g_{N\Sigma K}g_{\Sigma\Lambda\pi}\frac{M_\Sigma-\frac{1}{2}(m+m')}{M_\Sigma{}^2-u},$$

$$B(\Sigma)=\sqrt{2}g_{N\Sigma K}g_{\Sigma\Lambda\pi}/(M_\Sigma{}^2-u),$$

where the g_{abc}'s are conventional coupling constants and M_Σ is the Σ mass.

The unsubtracted dispersion-relation results for the K^* differ from those obtained by lowest-order Feynman techniques,[7] which give a constant scalar term as well as a tensor term depending on $u-s$. We use the dispersion-relation results for scalar (S), vector (V), and tensor (T) coupling of the $K^*N\Lambda$:

$$A_S(K^*)=B_S(K^*)=0,$$

$$A_V(K^*)=-\sqrt{2}f_{K^*K\pi}f_V\frac{(\mu'^2-\mu^2)(m'-m)}{M_{K^*}{}^2(M_{K^*}{}^2-t)},$$

$$B_V(K^*)=2\sqrt{2}f_{K^*K\pi}f_V/(M_{K^*}{}^2-t),$$

$$A_T(K^*)=\sqrt{2}\frac{f_{K^*K\pi}f_T}{m+m'}\frac{2s+M_{K^*}{}^2-m^2-m'^2-\mu^2-\mu'^2}{M_{K^*}{}^2-t},$$

and

$$B_T(K^*)=2\sqrt{2}f_{K^*K\pi}f_T/(M_{K^*}{}^2-t),$$

[6] J. K. Kim, Phys. Rev. Letters 19, 1079 (1967); C. H. Chan and F. T. Meiere, *ibid.* 20, 568 (1968).

[7] W. G. Wagner and D. H. Sharp, Phys. Rev. 128, 2899 (1962).

where $f_{K^*K\pi}$ is the $K^*K\pi$ coupling constant, f_V and f_T are vector and tensor $K^*N\Lambda$ coupling constants, and M_{K^*} is the mass of the K^*.

To represent resonance terms in the direct channel, we use the Breit-Wigner formula

$$f_l^{\pm}=\frac{1}{2|\mathbf{k}|}\frac{(\Gamma_{l1}\Gamma_{l2})^{1/2}}{W_r-W-\frac{1}{2}i\Gamma}.$$

The partial widths $\Gamma_{l\alpha}$ have the form

$$\Gamma_{l\alpha}=2k_\alpha R v_{l\alpha}\gamma_{l\alpha},$$

where k_α is the c.m. momentum in channel α, R is an interaction radius which was fixed at 1 F, $v_{l\alpha}$ is a barrier-penetration factor,[8] and $\gamma_{l\alpha}$ is a reduced width, assumed to be constant. The momentum-dependent width Γ is given by

$$\Gamma=\Sigma_\alpha\Gamma_{l\alpha},$$

where the sum is over all open channels. To approximate the momentum dependence of Γ, we chose

$$\Gamma=2k_1Rv_{l1}\gamma',\tag{3}$$

where $k_1=|\mathbf{k}|$ is the momentum in the elastic (π-N) channel and γ' is a constant which is chosen so as to give the correct total width at $W=W_r$.

3. HIGHER-ENERGY CALCULATIONS

In a preliminary study[9] of the low-energy region, it was found that the value of the $N\Lambda K$ coupling constant calculated by Kim[6] would not allow a good fit to the data, and that if this coupling constant was allowed to vary in order to give a good fit, the parameter values obtained for the pole terms would not produce a reasonable fit at higher energies. We have thus examined a higher-energy region,[5] using pole terms alone to determine the effects of absorption on these terms.

One may divide the differential cross sections into three parts, which we shall call regions 1 (forward angles), 2 (intermediate angles), and 3 (backward angles). In region 1, there is a strong peak with a maximum at $\theta=0$, indicative of meson exchange, and partially accounted for by the $K^*(890)$ pole. In region 2, the differential cross section is small and slopes gradually downward with increasing θ. In region 3, there is a weaker peak with maximum at $\theta=\pi$.

One may thus consider these three regions to be represented by K^*, N, and Σ poles, respectively. Calculations reveal that the K^*-pole contribution to region 1 is not nearly so sharply peaked as the data require. The absorption model[10] has accounted for such situations, but an extrapolation of the results to low energy would seem unreasonable (for example, S waves are completely absorbed at all energies). The differential cross-section and polarization data from 3 to 4 GeV/c have been fitted by Sarma and Reeder,[11] using $K^*(890)$ and $K^*(1400)$ Regge exchange, but their results should not be valid at low energies.[12] What one might seek as a substitute which would allow extrapolation is a model with less absorption in the lower partial waves. It has been shown[13] that a form factor of the one-pion-exchange (OPE) type[14] represents roughly this kind of model. Thus we multiplied the vector and tensor $K^*(890)$ contributions to the amplitudes g and h by a factor of the form

$$F_{K^*}(t)=(B_{K^*}-M_{K^*}{}^2)/(B_{K^*}-t).$$

Although this form factor did not produce a satisfactory forward peak for any value of B_{K^*}, the results obtained were far superior ot those obtained with the unadorned K^* pole, and they improved as the energy was decreased. Unfortunately, the coupling constants were extremely sensitive to B_{K^*}, whereas the fits to data were not so sensitive. Thus, as predicted by Jackson,[10] the values which we obtain for the K^* coupling constants are not very reliable.

With the backward peak we find problems similar to those of the forward peak, although they are less troublesome because of the smaller (about an order of magnitude smaller) cross-section values in region 3 compared with those in region 1. For consistency, we gave the Σ pole a treatment similar to that accorded the K^* pole, defining

$$F_\Sigma=(B_\Sigma-M_\Sigma{}^2)/(B_\Sigma-u).$$

Because the $NN\pi$ and $N\Lambda K$ coupling constants seem to be known with reasonable accuracy,[6,15] one cannot ignore the N-pole contribution. This creates an additional problem, since it is easily discovered that the N-pole contribution to the total cross section peaks at about 1220 MeV with a value of 67.7 mb,[16] and has a value of 54.6 mb at 900 MeV, compared with the experimental result of 0.73 mb. Thus the nucleon-pole terms would have to be almost exactly cancelled by S- and P-wave contributions with the same energy dependence to allow one to fit the data in region 2. However, one may view the nucleon-pole terms as representing the effect of a point interaction at the πNN and $KN\Lambda$ vertices and modify this by a phenomenological factor to account for absorption in initial and final states (structure at the vertices).

[8] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 390.
[9] J. E. Rush, Bull. Am. Phys. Soc. **13**, 238 (1968).
[10] See J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965).
[11] K. V. L. Sarma and D. D. Reeder, Nuovo Cimento **53A**, 808 (1968).
[12] L. Durand, III, Phys. Rev. **166**, 1680 (1968).
[13] P. Graves-Morris, Nuovo Cimento **50A**, 989 (1967).
[14] E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962).
[15] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).
[16] We have used $g_{NN\pi}g_{N\Lambda K}/4\pi=15.3$.

TABLE I. χ^2 values for various fits.

Fit	χ^2	No. of data	No. of parameters	χ^2/N[a]
A	916	306	8	3.1
B	655	286	8	2.4
BR	488	259	8	1.9
BRN	336	259	8	1.3
C	581	286	11	2.1
S	123	286	187	1.2

[a] N = No. of data − No. of parameters.

For the nucleon pole alone, the partial-wave decomposition represented by Eqs. (1) and (2) is

$$g_N = f_0^+,$$
$$h_N = f_1^-.$$

The effect of a point interaction in the Breit-Wigner formula may be represented by $R \to 0$ or, for P waves,

$$v_1(k_\alpha R) = (k_\alpha R)^2/[1+(k_\alpha R)^2] \to (k_\alpha R)^2$$

in the partial widths. We were therefore led to consider replacing h_N with $h_N F_N(s)$, with

$$F_N(s) = A/[(B_N+\mathbf{k}^2)(B_N+\mathbf{k}'^2)]^{1/2},$$

where B_N is arbitrary and A is chosen so that $F_N(m^2)=1$. For the amplitude g_N the argument based on barrier factors produces no change. One may claim, however, that S-wave absorption is at least as important as P-wave absorption, and that some factor is also needed for g_N. This is certainly true empirically. In fact, between 800 and 2500 MeV, g_N^2/h_N^2 ranges from 625 to 9, and the total cross section with h_N^2 alone reaches 5.0 mb at 1900 MeV, at least an order of magnitude too large. Thus both g_N and h_N must be suppressed, and we chose to do this by also replacing g_N with $g_N F_N(s)$.

In the subsequent study of the higher-energy region, as in all low-energy work, we performed a computer fit to the data by defining the quantity χ^2 to be a sum over all data of the squares of the differences between experimental and calculated values weighted by the squares of reciprocals of experimental errors. In those instances where a total cross section had not been measured, we obtained the values by extrapolation of measured results. (This means that some adjustment of those values is tolerable.) We then used the computer to minimize χ^2, employing a rather simple procedure. For the higher-energy data, we obtained several "fits," none of which was particularly good, as explained earlier, but each of which improved as the energy decreased. Only the coupling constants were varied by computer, the numbers B_N, B_Σ, and B_{K^*} being varied "by hand." We then attempted preliminary fits to the low-energy data, fixing all pole parameters and varying resonance parameters. In this way we selected the only set of pole parameters which gave any promise of fitting the low-energy data. The values obtained were as

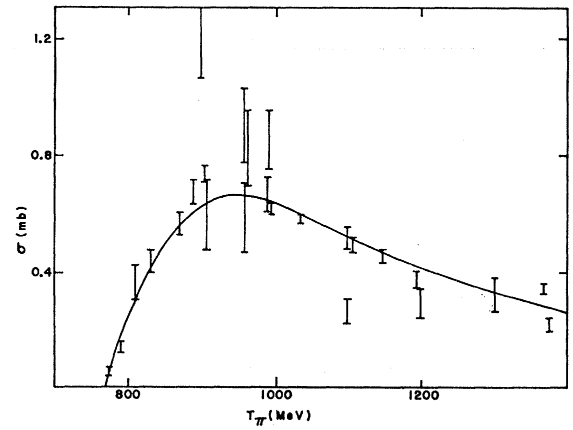

FIG. 1. Total cross section for fit B.

follows:

$$g_{NN\pi}g_{N\Lambda K}/4\pi = 15.3, \quad \text{(fixed)}$$
$$g_{N\Sigma K}g_{\Sigma\Lambda\pi}/4\pi = -2.3,$$
$$f_{K^*K\pi}f_V/4\pi = -1.4,$$
$$f_{K^*K\pi}f_T/4\pi = 0.8,$$
$$B_N = 0.28m^2, \quad B_\Sigma = 1.2M_\Sigma^2,$$

and

$$B_{K^*} = 1.2M_{K^*}^2.$$

After this discussion, we must make the somewhat anticlimactic comment that the background (pole terms) is not particularly significant [see Fig. 9(a)]. The contributions of the pole terms at low energy are primarily to S and P waves, and are much less than those which we obtain for the $S_{11}(1710)$ and the $P_{11}(1470)$, expecially below 1000 MeV. Thus "double counting" because of the inclusion of both direct- and crossed-channel terms should not be a serious problem, except perhaps in regard to the $P_{11}(1751)$ and the $P_{13}(1863)$. We shall comment on this again at the end of Sec. 6.

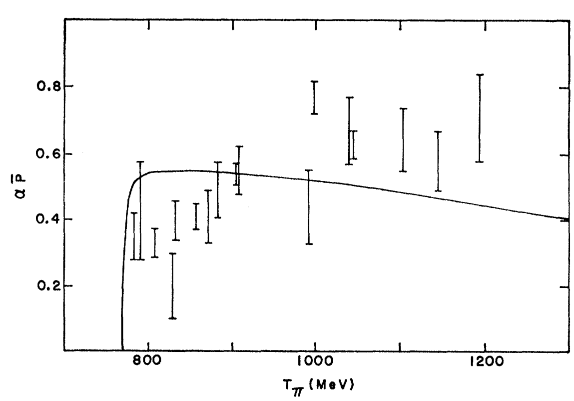

FIG. 2. Average polarization $\bar{P}$ times decay asymmetry parameter $\alpha = -0.67$ for fit B.

TABLE II. Resonance parameters for fit B.

Resonant states	$(\gamma_{\pi p}\gamma_{\Lambda K})^{1/2}$ (MeV)	$\gamma_{\pi p}$ (MeV)	$\gamma_{\Lambda K}$ (MeV)	$\Gamma_{\Lambda K}$ (MeV)	$\Gamma_{\Lambda K}/\Gamma$	(χ^2)[a]	$(\gamma_{\Lambda K}/\gamma_{\pi N})^{1/2}$[b]	α[c]
$S_{11}(1550)$	0.87	5.4	0.14			660	0.13	1.30
$S_{11}(1710)$	9.23	27	3.2	8.6	0.029	1816	0.28	1.08
$P_{11}(1470)$	31.6	26	38			1542	0.98	0.03
$P_{11}(1751)$	−2.41	12	0.48	1.12	0.0034	679	0.16	1.26
$P_{13}(1863)$	2.66	6.4	1.11	4.1	0.014	728	0.34	0.99
$D_{13}(1525)$	9.37	17	5.2			844	0.45	0.82
$D_{15}(1680)$	2.27	12	0.43	0.10	0.00058	727	0.15	1.27
$F_{15}(1690)$	4.87	25	0.91	0.020	0.00015	684	0.15	1.27

[a] Values of χ^2 with the corresponding resonance removed and all other resonance parameters varied. Compare with $\chi^2=655$ for fit B.
[b] Note that $\gamma_{\pi p}=\frac{2}{3}\gamma_{\pi N}$.
[c] D-F mixing parameter.

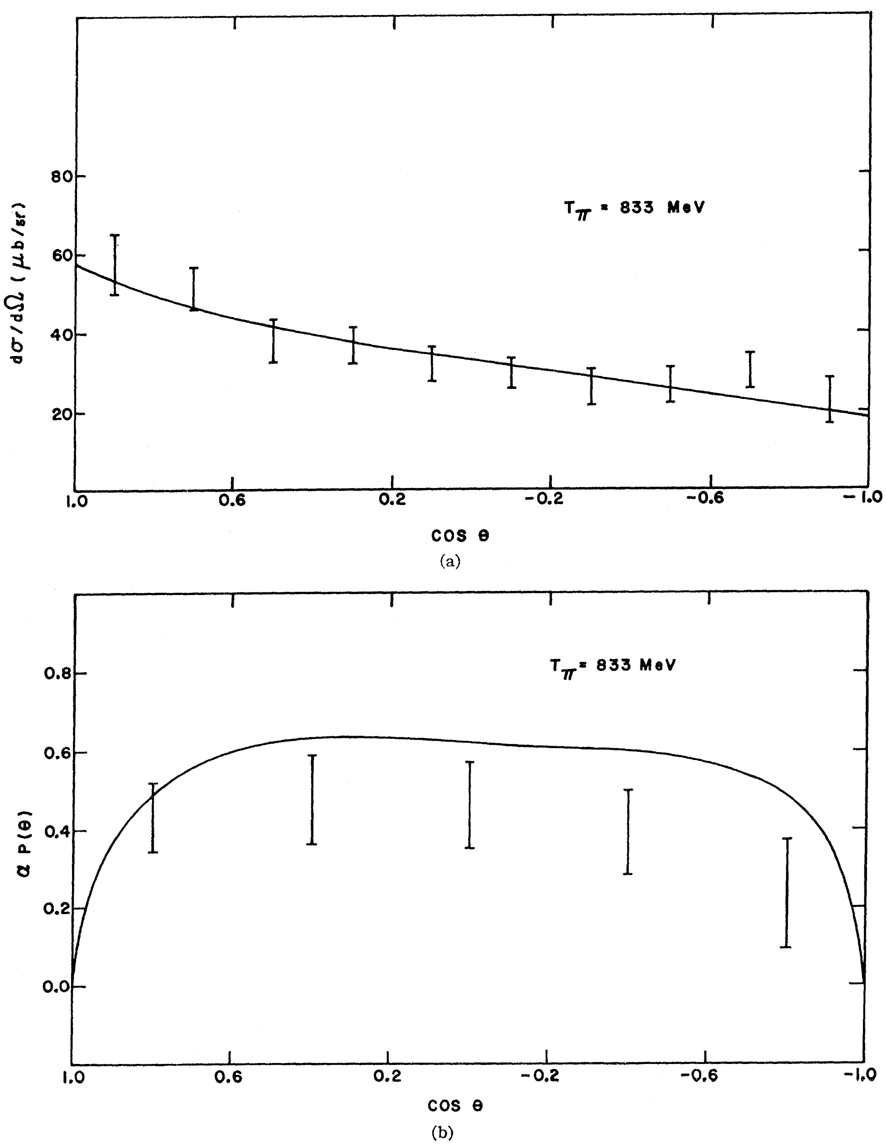

FIG. 3. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)$ for fit BRN at $T_\pi=833$ MeV. $\chi^2{}_{BRN}=16.5$; $\chi_S{}^2=1.6$; normalization factor $N=1.00$.

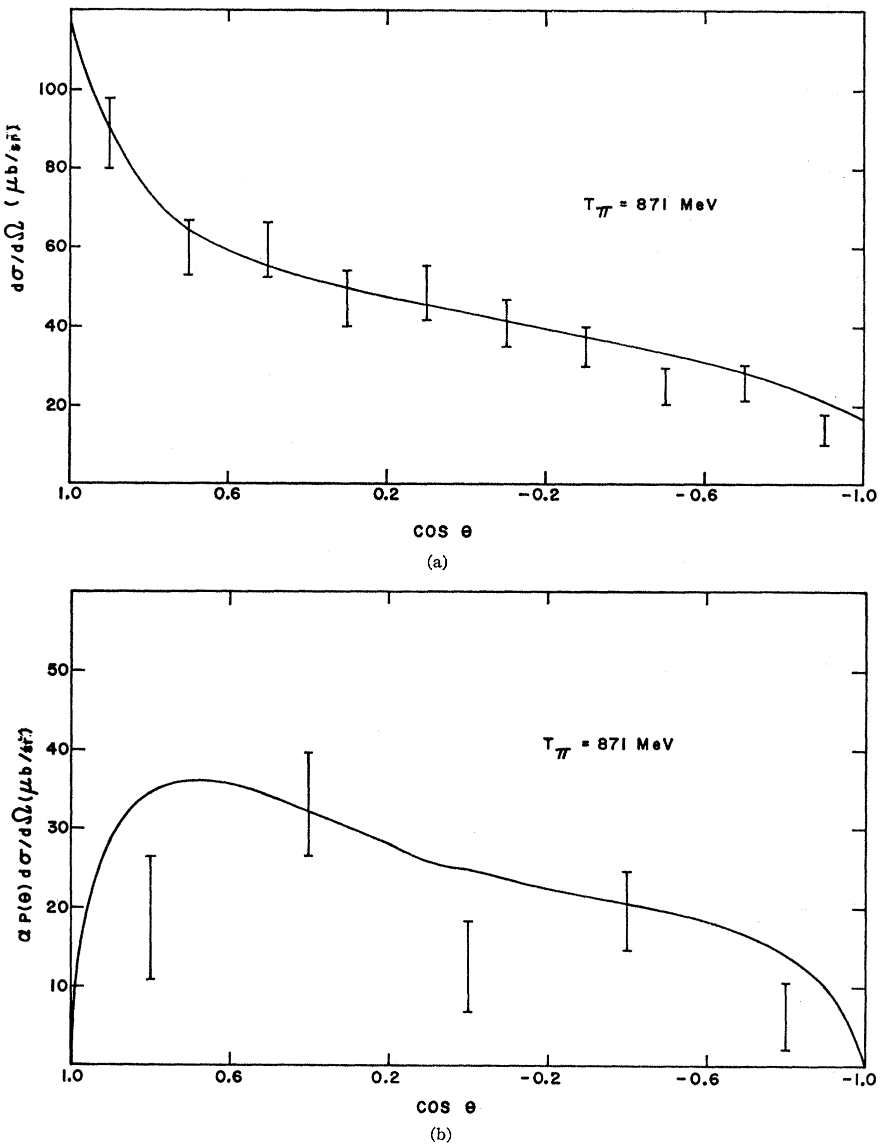

FIG. 4. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)d\sigma/d\Omega$ for fit *BRN* at $T_\pi=871$ MeV. $\chi^2{}_{BRN}=19.1$; $\chi_S{}^2=6.1$; $N=0.98$.

4. LOW-ENERGY CALCULATIONS

In addition to the data listed in Ref. 5, there are now available angular distributions and polarizations at 784, 808, 833, 858, 882, and 907 MeV,[17] polarizations at 1038 and 1170 MeV,[18] differential cross section and polarization at 1038 MeV,[19] and differential cross sections and polarizations at 997, 1103, 1145, and 1194 MeV.[20] These have been combined with data at 791, 829, and 871,[21] at 890,[22] and at 905 MeV,[23] to give a total of 306 data points. Much of the preliminary low-energy analysis was done with the data at 833, 858, 871, 905, 997, 1038 (Ref. 19), 1103, and 1145 MeV, totaling 165 data points.

The masses and widths of the $S_{11}(1550)$, $S_{11}(1710)$, $P_{11}(1470)$, $D_{13}(1525)$, $D_{15}(1680)$, and $F_{15}(1690)$ were

[17] L. B. Auerbach, D. Bowen, J. Dobbs, K. Lande, A. K. Mann, F. J. Sciulli, H. Huto, D. H. White, and K. K. Young, Nuovo Cimento **47A**, 19 (1967); the energy values are averages.

[18] Y. S. Kim, G. R. Burleson, P. I. P. Kalmus, A. Roberts, C. L. Sandler, and T. A. Romanowski, Phys. Rev. **151**, 1090 (1966).

[19] J. A. Anderson, University of California Lawrence Radiation Laboratory Internal Report No. UCRL-10838, 1963 (unpublished). These data were not used in the final analysis.

[20] T. O. Binford, Ph.D. thesis, University of Wisconsin, 1965 (unpublished). We thank Dr. R. Hartung for providing these data.

[21] L. Bertanza, P. L. Connolly, B. B. Culwick, F. R. Eisler, T. Morris, R. Palmer, A. Prodell, and N. P. Samios, Phys. Rev. Letters **8**, 332 (1962).

[22] J. Keren, Phys. Rev. **133**, B457 (1964).

[23] J. A. Anderson, F. S. Crawford, B. Crawford, R. L. Golden, L. J. Lloyd, G. Meisner, and L. R. Price, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 271.

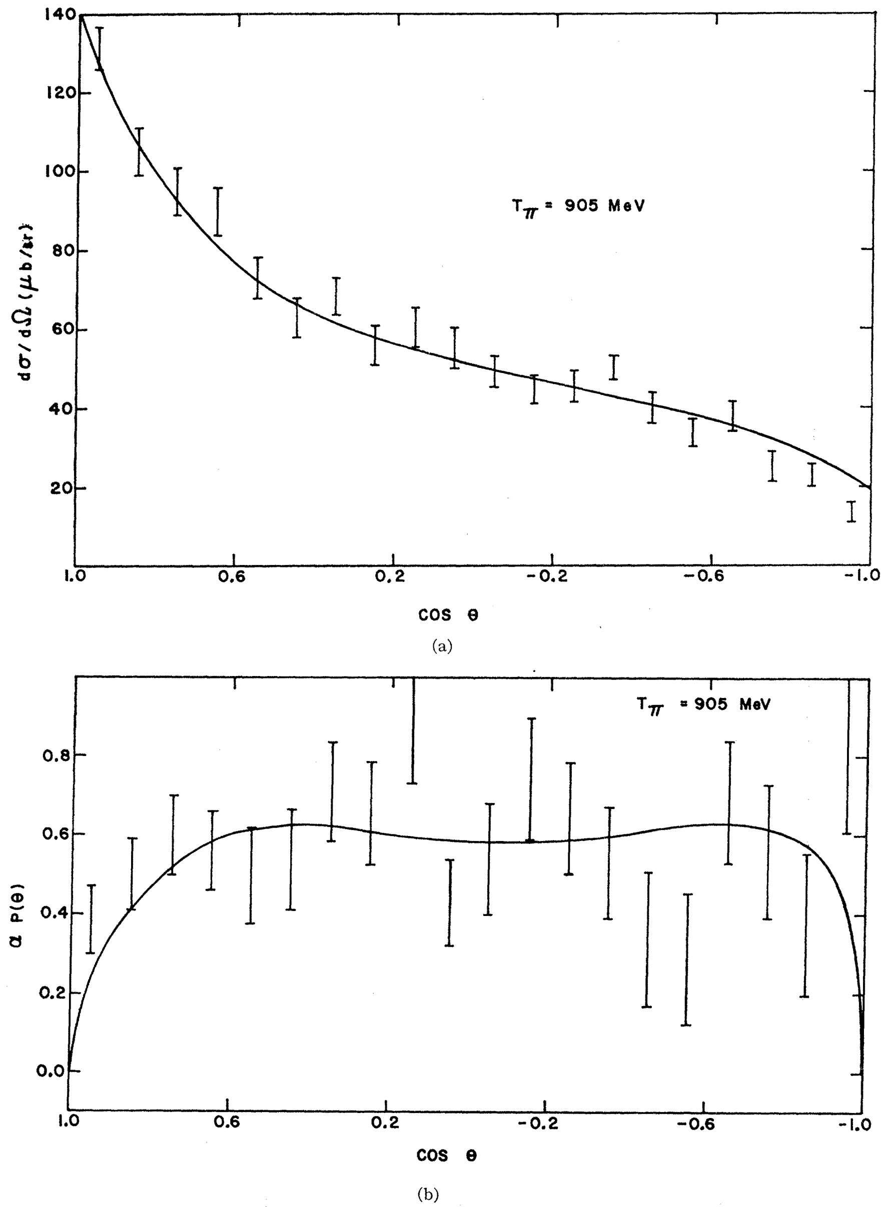

FIG. 5. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)$ for fit *BRN* at $T_\pi=905$ MeV. $\chi^2_{BRN}=54$ [with two $P(\theta)$ points omitted]; $\chi_S{}^2=33$; $N=1.14$.

taken from the compilation of Rosenfeld *et al.*,[3,24] with the γ' constants [Eq. (3)] chosen accordingly. The masses and widths of the $P_{11}(1751)$ and the $P_{13}(1863)$ were taken from Donnachie *et al.*[4] Thus, with the pole parameters fixed as described in Sec. 3, there remained only eight variable parameters at low energy, namely, the eight products $(\gamma_{l1}\gamma_{l2})^{1/2}$ for the various resonances. These parameters were determined for the 306 data points mentioned previously, and are labeled "fit *A*."

During the course of the calculations, certain modifications suggested themselves. First, the measured total cross section of Ref. 19 was too small (by half) to agree with other values at nearby energies, and for this reason the differential cross-section data could not be fitted reasonably. Removal of Ref. 19 (20 points) from the program reduced the value of X^2 by 261. We label this "fit *B*."

Second, some experimental values are clearly incorrect. We have used the value $\alpha=-0.67$ for the decay asymmetry parameter of the Λ,[25] and so we cannot obtain values of $\alpha P(\theta)$ larger than 0.67. We may eliminate experimental points with values of $\alpha P(\theta)$ significantly larger than 0.67. This was done without any additional variation of parameters, since the effect of overly large

[24] Much of the preliminary work was done with the masses and widths taken from an earlier compilation of Rosenfeld *et al.*, dated September 1967 (unpublished). The change to the latest values reduced χ^2 by about 40.

[25] The most recent measured value is -0.647 ± 0.016; see Ref. 3.

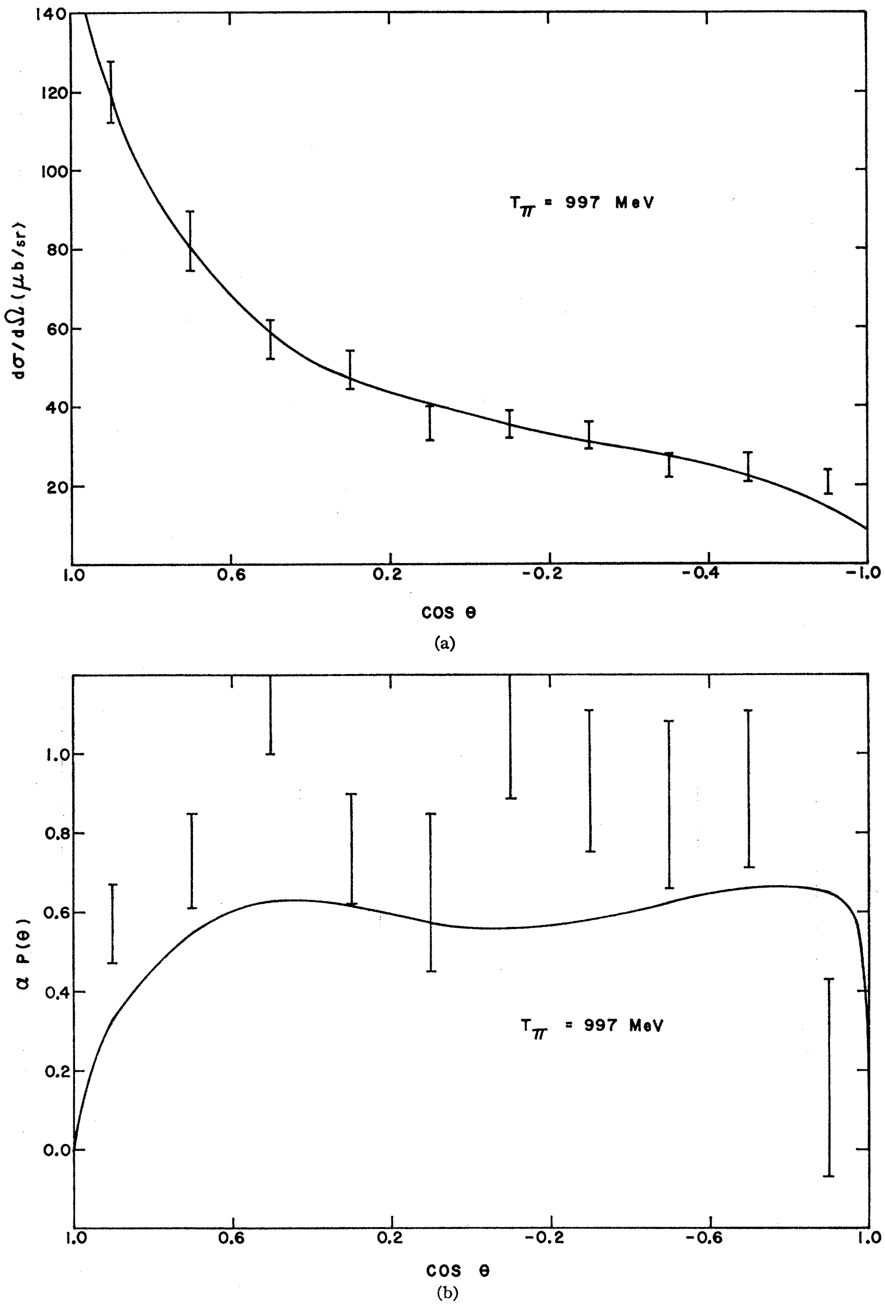

FIG. 6. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)$ for fit BRN at $T_\pi=997$ MeV. $\chi^2_{BRN}=19.0$ [with five $P(\theta)$ points omitted]; $\chi_S{}^2=10.1$; $N=0.94$.

$\alpha P(\theta)$ experimental values is to constrain the calculated values of $\alpha P(\theta)$ to be large, which they almost certainly should be. The elimination of 23 such points from χ^2, along with four isolated data points which were clearly out of line, reduced χ^2 by 167. We call this result "fit BR."

Third, the fits to differential cross sections depend on measured total cross-section values, which in the past have been somewhat unreliable (note also that some values were obtained by extrapolation). They are also based on a specific energy dependence of resonance terms, which is by no means firmly established theoretically. In an attempt to compensate for this, we multiplied the calculated differential cross section by a factor N and adjusted N independently at each energy to minimize χ^2 at that energy, without changing any other parameters. The resultant fit we label "fit BRN."

Finally, it is interesting to discover to what extent fit B can be improved by varying coupling constants for the Σ and K^* poles. We consequently allowed these parameters and all eight resonance parameters to vary and obtained "fit C," which process reduced χ^2 by 74.

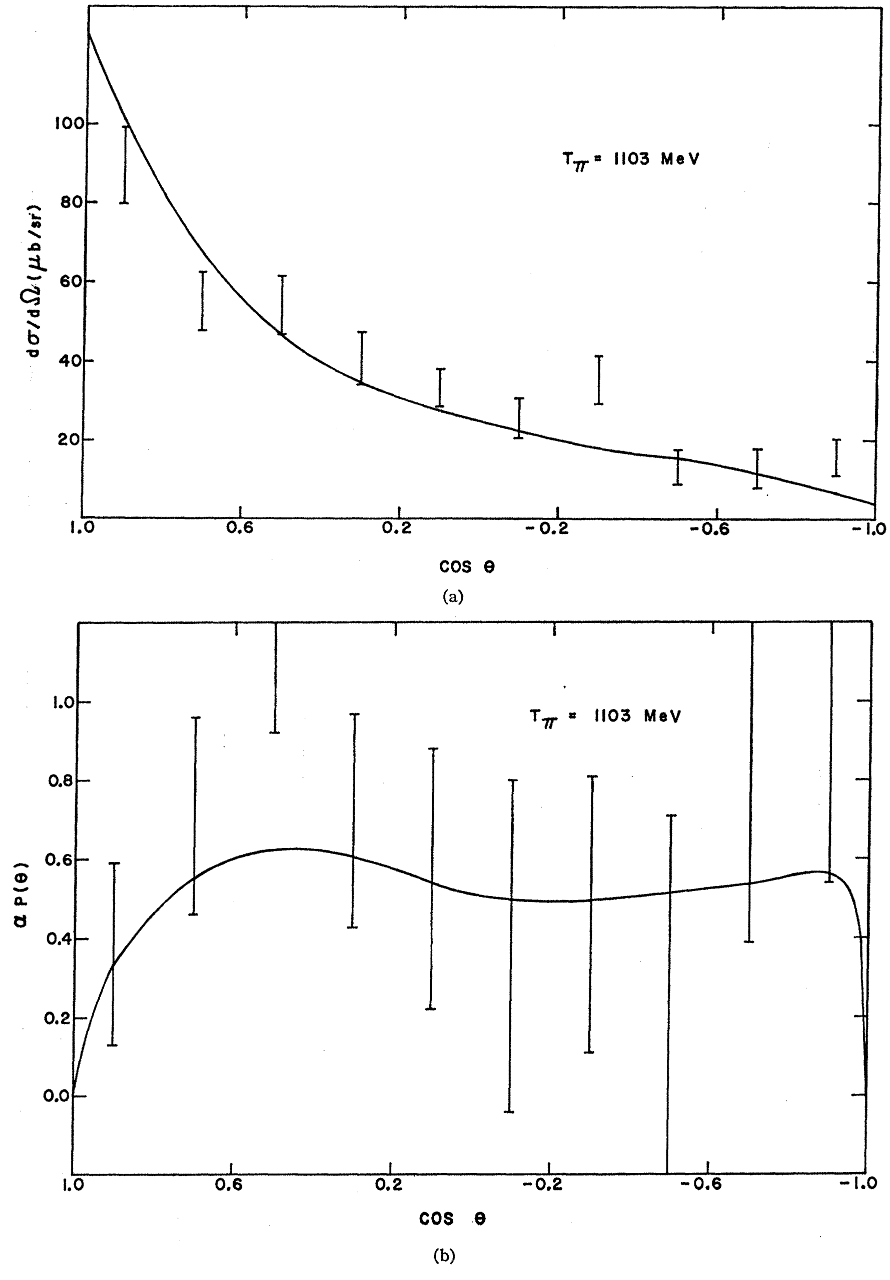

FIG. 7. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)$ for fit *BRN* at $T_\pi=1103$ MeV. $\chi^2_{BRN}=19.5$ [with three $P(\theta)$ points omitted]; $\chi_S^2=9.3$; $N=0.86$.

Of course, the new pole parameters would not be expected to fit the higher-energy data nearly so well (they do not).

For comparison, we have considered the expansions[26]

$$\frac{d\sigma}{d\Omega}=\sum_{n=0}^{5} a_n \cos^n\theta$$

and

$$\alpha P(\theta)\frac{d\sigma}{d\Omega}=\sum_{n=0}^{4} b_n \cos^n\theta$$

and obtained separate fits to the data at each energy with a_n and b_n chosen arbitrarily; this we call "fit *S*." χ^2 values for fits *A*, *B*, *BR*, *BRN*, *C*, and *S* are shown in Table I.

5. RESULTS

The results of fit *BRN* are compared with representative data in Figs. 1–8. For each set of data we also quote the value of χ^2 from fit *BRN* and that from the cosine-series fit, χ_S^2.

The resonance parameters obtained in fit *B* are shown in Table II. The traditional branching ratios of the

[26] The upper limits on the sums were chosen to agree with the set of resonances listed earlier.

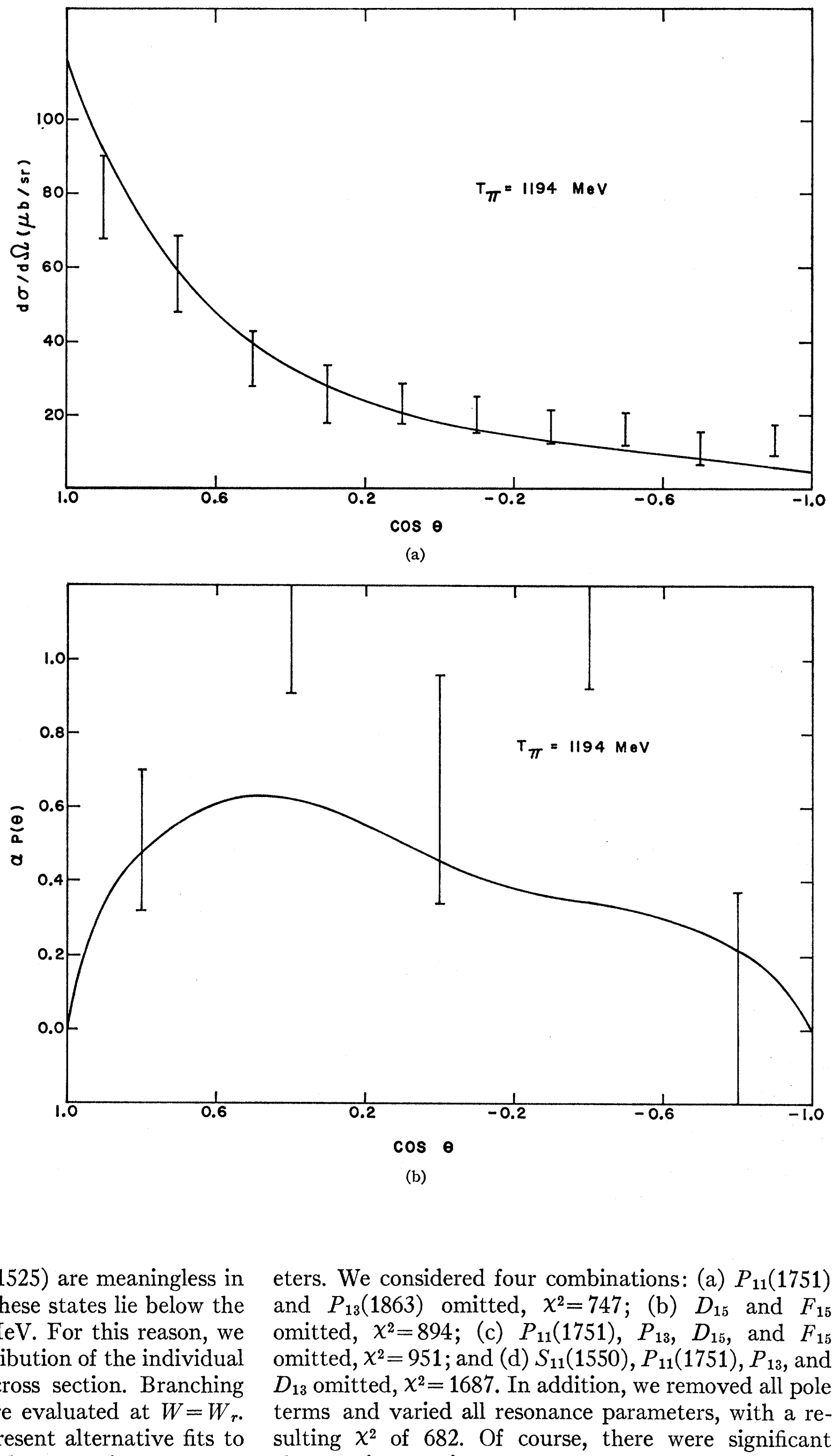

FIG. 8. (a) $d\sigma/d\Omega$ and (b) $\alpha P(\theta)$ for fit BRN at $T_\pi=1194$ MeV. $\chi^2{}_{BRN}=10.2$ [with two $P(\theta)$ points omitted]; $\chi_S{}^2=5.5$; $N=0.88$.

$S_{11}(1550)$, $P_{11}(1470)$, and $D_{13}(1525)$ are meaningless in this case, since the masses of these states lie below the threshold value of $W=1613$ MeV. For this reason, we have plotted in Fig. 9 the contribution of the individual resonant states to the total cross section. Branching ratios for the other states were evaluated at $W=W_r$. The χ^2 values in Table II represent alternative fits to fit B, determined by removing the respective resonance and varying the other resonance parameters.

It is interesting to remove certain combinations of resonances and vary the remaining resonance parameters. We considered four combinations: (a) $P_{11}(1751)$ and $P_{13}(1863)$ omitted, $\chi^2=747$; (b) D_{15} and F_{15} omitted, $\chi^2=894$; (c) $P_{11}(1751)$, P_{13}, D_{15}, and F_{15} omitted, $\chi^2=951$; and (d) $S_{11}(1550)$, $P_{11}(1751)$, P_{13}, and D_{13} omitted, $\chi^2=1687$. In addition, we removed all pole terms and varied all resonance parameters, with a resulting χ^2 of 682. Of course, there were significant changes in certain resonance parameters, notably a considerable increase of $P_{11}(1751)$. The parameters determined with background included are expected to give a better representation of the resonant states.

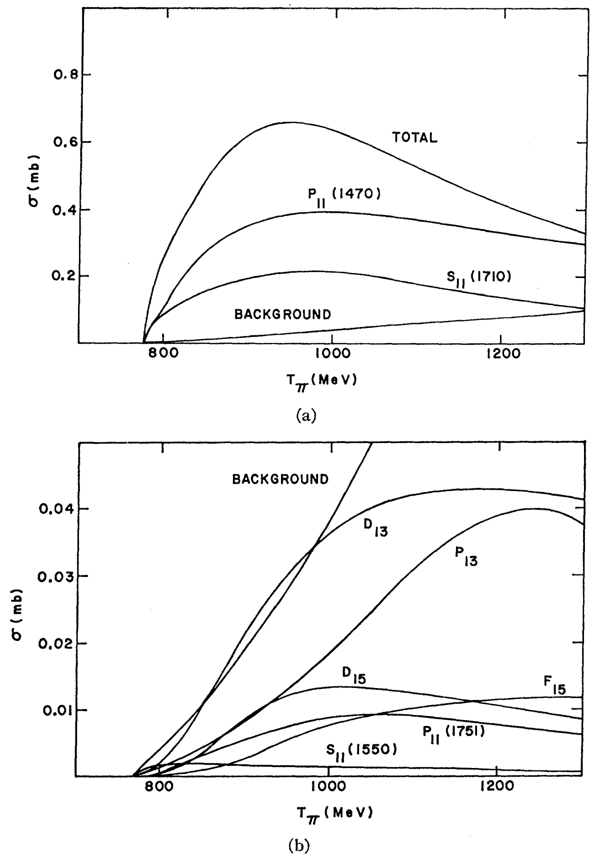

FIG. 9. Individual contributions of each resonance and of the background (pole terms) to the total cross section for fit B compared with the curve in Fig. 1. Note the change of scale in (b). Interference terms are included in the total.

6. DISCUSSION AND CONCLUSIONS

Let us proceed to examine the results of this analysis and to consider some of the possible implications. By studying the χ^2 column in Table II, we see that the $S_{11}(1710)$ and $P_{11}(1470)$ are crucial to an understanding of Λ-K^0 production in the present context, and that the $D_{13}(1525)$ is certainly significant. We also see that the $S_{11}(1550)$ is quite superfluous, and that it makes the smallest contribution in Fig. 9. [Note the change of scale in Fig. 9(b).] We might add that in other fits of this analysis, with various resonance combinations, it was not uncommon to obtain much smaller values for the reduced width of the $S_{11}(1550)$, and that it is quite likely that the $S_{11}(1550)$ does not decay into the Λ-K^0 channel at all.

The roles of the other resonances are more difficult to assess. One is aided by the list of values for $(\gamma_{\Lambda K}/\gamma_{\pi N})^{1/2}$ in Table II. (Note the distinction $\gamma_{\pi p}=\frac{2}{3}\gamma_{\pi N}=2\gamma_{\pi n}$, where $\pi p\equiv\pi^- p$ and $\pi n\equiv\pi^0 n$.) We observe the interesting fact that this ratio (which we shall call G, the ratio of coupling constants) is the same for the D_{15} as for the F_{15}. This fact calls to mind the recent parity-doublet scheme of Barger and Cline[27] in which these two resonances were paired as "MacDowell symmetric states with degenerate masses." It is tempting, therefore, to suppose that the equality of G for the D_{15} and F_{15} is no accident, but further evidence for parity doubling. One is uneasy regarding the small value of χ^2 obtained without the F_{15}, but we must add that the partial width of the F_{15} remained quite stable during the course of this analysis, except when the D_{15} was removed. This fact is in marked contrast to the behavior of the $S_{11}(1550)$.

It is then natural to consider the $P_{11}(1751)$ in comparison with the $S_{11}(1710)$, since this is another suggested parity doublet.[27] We find the ratio G to be almost twice as large in magnitude for the $S_{11}(1710)$ as for the $P_{11}(1751)$ and of opposite sign. The dominant role of the $P_{11}(1470)$, however, leads us to examine the situation more carefully. In the energy region under consideration ($W=1613$ to 1850 MeV), the expected values for the width of the $P_{11}(1470)$ are somewhat larger than the values obtained from Eq. (3) because of the opening of many additional channels. One might attempt to take account of this by varying the γ' of the $P_{11}(1470)$ [Eq. (3)] in addition to the other eight resonance parameters. Taking γ_0' as the original value of γ' used in fit B, we found the optimum value to be $\frac{3}{2}\gamma_0'$, with a reduction in χ^2 of 24. The resulting resonance parameters, labeled as "fit D," are shown in Table III.

One observes in Table III that G for the $P_{11}(1751)$ is now comparable in magnitude with that for the $S_{11}(1710)$, but that there has been a change in the relative relationship of the D_{15} and F_{15}. Thus these sorts of correlations, though somewhat suggestive, would have dubious value because of the uncertainties in the parameters, even if their significance were clear. We note the considerable decrease in the ratio for the $S_{11}(1550)$ as further indication that it makes essentially no contribution to Λ-K^0 production. This is in contrast to its marked significance in $\pi^- p\rightarrow n\eta$.[28] In our analysis, there is definitely *no* correlation between the $P_{11}(1470)$ and the $S_{11}(1550)$, raising a question about the pairing of these resonances, as given by Barger and Cline.

The D_{13} also lies below threshold and presents a problem similar to that of the $P_{11}(1470)$, but the problem is not so easily handled because of the less important role of the D_{13}. We might regard the reduced width of this resonance, given in Table II, as an approximate upper limit on the actual value, although such a limit would not apply to the ratio of coupling constants of the D_{13}. The $P_{13}(1863)$, on the other hand, has a quoted mass slightly greater than the upper limit of the energy range of this study. Thus the Γ values representing the P_{13} should be somewhat larger than those used. Furthermore, it is difficult to fix its reduced width without study-

[27] V. Barger and D. Cline, Phys. Letters **26B**, 85 (1967); Phys. Rev. Letters **20**, 298 (1968).
[28] S. R. Deans and W. G. Holladay, Phys. Rev. **165**, 1886 (1968).

TABLE III. Resonance parameters of fit D. See footnotes in Table II.

Resonant states	$(\gamma_{\pi p}\gamma_{\Lambda K})^{1/2}$ (MeV)	$\gamma_{\Lambda K}$ (MeV)	$\Gamma_{\Lambda K}$ (MeV)	$\Gamma_{\Lambda K}/\Gamma$	$(\gamma_{\Lambda K}/\gamma_{\pi N})^{1/2}$	α
$S_{11}(1550)$	0.126	0.0029			0.019	1.47
$S_{11}(1710)$	8.70	2.8	7.5	0.025	0.26	1.11
$P_{11}(1470)$	41.4	45			0.89	0.16
$P_{11}(1751)$	−3.73	1.16	2.7	0.0082	0.25	1.12
$P_{13}(1863)$	2.64	1.09	4.0	0.014	0.34	0.99
$D_{13}(1525)$	7.30	3.1			0.35	0.98
$D_{15}(1680)$	1.60	0.21	0.049	0.00028	0.11	1.33
$F_{15}(1690)$	5.31	1.08	0.024	0.00018	0.17	1.24

TABLE IV. Comparison of the parameters of fit B $[\gamma_{\Lambda K}(B)]$ with those obtained with no background $[\gamma_{\Lambda K}(R)]$.

Resonant states	$\gamma_{\Lambda K}(B)$ (MeV)	$\gamma_{\Lambda K}(R)$ (MeV)
$S_{11}(1550)$	0.14	0.56
$S_{11}(1710)$	3.2	2.9
$P_{11}(1470)$	38	37
$P_{11}(1751)$	0.48	9.2
$P_{13}(1863)$	1.11	12.3
$D_{13}(1525)$	5.2	8.4
$D_{15}(1680)$	0.43	0.17
$F_{15}(1690)$	0.91	0.62

ing data at energies $W>W_r$. If the P_{13} exists as a resonant state, then the value given for $\gamma_{\Lambda K}$ is probably a lower limit.

It is perhaps useful to calculate D/F ratios for each of the resonant states in the limit of exact $SU(3)$ symmetry, assuming each state in turn to be a member of an octet. We have therefore given the mixing parameter α for PBB' coupling,[29] defined so that the D coupling of the nucleon is represented by $\alpha g_{NN\pi}$. The values of α are given by

$$\alpha=\tfrac{1}{2}[3\pm3(\gamma_{\Lambda K}/\gamma_{\pi N})^{1/2}]$$

and we have removed the ambiguity in sign by choosing $\alpha<\frac{3}{2}$. These values are listed in Tables II and III.

Tripp *et al.*[30] have considered $SU(3)$ assignments for baryon resonances. Of the group of resonances which we have studied, the $S_{11}(1550)$, D_{15}, and F_{15} were assigned to octets, and plots were made which represented separate D/F-ratio predictions for other members of the octets and other decay modes. Our results are in relatively good agreement for the D_{15} and $S_{11}(1550)$, but do not agree very well for the F_{15}. One cannot set a lower limit on the reduced widths of the D_{15} and F_{15} by considering only the χ^2 values of Table II. But the results in the last paragraph of Sec. 5 show that at least one of these resonances contributes. Furthermore, if the partial width of the D_{15} (F_{15}) is larger, that of the F_{15} (D_{15}) almost certainly is smaller. By studying the results with each resonance omitted, we may claim that $(\gamma_{\Lambda K}\gamma_{\pi p})^{1/2}$ for one resonance can be no more than twice as large, even if the corresponding quantity for the other resonance vanishes. Thus good agreement with the results of Tripp *et al.* may be hard to obtain, although at present one can probably attribute the discrepancy to symmetry breaking.

A problem which we consider more puzzling is that of the nucleon-pole term: How does one reconcile a predicted cross section of 55 mb (for the nucleon pole) with the measured value of 0.73 mb? The *ad hoc* form factor which we introduced is obviously too drastic to be believable; it simply served to produce a reasonable background. The idea of cancelling this large contribution with other pole terms is interesting but seems to be fruitless. We intend to devote further study to this matter.

As a final consideration of the accuracy of the resonance parameters, we compare the parameters from fit B with those obtained with all pole terms omitted (Sec. 5). The results are shown in Table IV. We see that $\gamma_{\Lambda K}$ for the $P_{11}(1751)$ increased by a factor of almost 20, while the parameters of the $P_{11}(1470)$ and $S_{11}(1710)$ changed very little. A comparison of the values in Table IV should give a reasonable picture of the accuracy with which the partial widths have been determined. In almost all instances, this accuracy is quite significant in comparison with all other values currently available.

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[29] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); see P. Carruthers, *Introduction to Unitary Symmetry* (Interscience Publishers, Inc., New York, 1966).

[30] R. D. Tripp, D. W. G. Leith, A. Minten, R. Armenteros, M. Ferro-Luzzi, R. Levi-Setti, H. Filthuth, V. Hepp, E. Kluge, H. Schneider, R. Barloutaud, P. Granet, J. Meyer, and J. P. Porte, Nucl. Phys. **B3**, 10 (1967).