reasonable agreement with one of the two solutions[15] ($\gamma=0.4$) obtained by Depommier *et al.*, and disfavors strongly the other ($\gamma=-2.1$). Further experiments with better statistics will be of considerable interest in understanding the structure-dependent effects in the weak interactions.

The author is grateful to Professor S. N. Biswas for suggesting this investigation and giving constant advice. He is thankful to Dr. K. C. Gupta for helpful discussions. He would also like to thank Professor S. Okubo for clarifying many points and for valuable suggestions.

[15] Experimentally (see Ref. 6) one measures the partial decay rate $d^2W_{\rm SD}$ (due to structure-dependent effects) integrated over certain energy or angular intervals which depends on $(1+\gamma)^2$ and $(1-\gamma)^2$ (see Ref. 1). Thus a single measurement of SD radiation leads to an equation quadratic in γ, which has two solutions, namely, $\gamma=0.4$ and -2.1. Here $\tau_{\pi^0\to2\gamma}=1.05\times10^{-16}$ sec, and the central value of the branching ratio $\Gamma(\pi\to e\nu\gamma;E_e,E_\gamma>48\text{ MeV})/\Gamma(\pi\to\mu\nu)=(3.0\pm0.5)\times10^{-8}$ have been used.

PHYSICAL REVIEW VOLUME 173, NUMBER 5 25 SEPTEMBER 1968

Sum Rules for Meson-Baryon Scattering from Asymptotic Symmetry Requirements

V. P. SETH AND B. K. AGARWAL
Physics Department, Allahabad University, Allahabad, India

AND

Y. M. GUPTA
Tata Institute of Fundamental Research, Colaba, Bombay-5, India

(Received 12 March 1968; revised manuscript received 29 April 1968)

We consider the processes $KN\to KN$ and $K\Sigma\to K\Sigma$. The superconvergent sum rules involving linear combinations of forward elastic scattering amplitudes of these processes are set up and tested.

WHEN a scattering amplitude $M(\nu,\Delta^2)$ for a process $B(p_1)+P(q_1)=B'(p_2)+P'(q_2)$ is odd under crossing and is bounded asymptotically at least to the extent of $\nu|M(\nu)|<$const, then it satisfies a superconvergence relation[1] of the type

$$\int_{-\infty}^{\infty}\text{Im}M(\nu,\Delta^2)d\nu=0\,,\tag{1}$$

where

$$\nu=PQ=\tfrac{1}{4}(s-u),\ \Delta^2=t,\ P=\tfrac{1}{2}(p_1+p_2),\ Q=\tfrac{1}{2}(q_1+q_2).$$

We shall consider elastic meson-baryon scattering and assume that Regge theory accounts for the high-energy behavior of the scattering amplitudes. The forward scattering amplitude is given by const$\times\nu^{\alpha(t=0)}$ as $\nu\to\infty$, where $\alpha(0)$ is the intercept of the leading Regge trajectory.

Costa and Zimerman[2] have pointed out that it is possible to extend the domain of physical application of (1) by making use of symmetry requirements arising from $SU(3)$ invariance on the high-energy Regge behavior of the amplitudes. One now considers a linear combination of the amplitudes, while applying superconvergence, and thereby include amplitudes which would have been otherwise excluded by the requirements of superconvergence. In this paper we apply a similar method to the processes $KN\to KN$ and $K\Sigma\to K\Sigma$.

[1] V. de Alfaro, S. Fubini, A. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966).
[2] G. Costa and A. H. Zimerman, Nuovo Cimento 46, 198 (1966).

For the combinations of processes

$$(KN\to KN)+(\bar{K}N\to\bar{K}N)\tag{2}$$

and

$$(K\Sigma\to K\Sigma)+(\bar{K}\Sigma\to\bar{K}\Sigma)\,,\tag{3}$$

we get contributions only from the ρ trajectory. The invariant amplitude for elastic meson-baryon scattering is given by

$$T_{fi}=\bar{u}(p_2)(-A+i\gamma\cdot QB)u(p_1)\,,\tag{4}$$

where u is the Dirac spinor, and A and B are scalar functions depending only on the Mandelstam variables. For each process considered separately, the amplitudes behave as

$$-A_{KN}{}^{I=1}=C_1(pq)^\alpha[x^\alpha-O_1x^{\alpha-2}+\cdots]\,,$$
$$-B_{KN}{}^{I=1}=C_2(pq)^{\alpha-1}[x^{\alpha-1}-O_2x^{\alpha-3}+\cdots]\,,$$

where

$$x=-\nu/pq\,,$$
$$C_1=(\pi\epsilon_R/m)Rg_{\rho KK}[2f_{\rho NN}{}^V-\alpha(2f_{\rho NN}{}^V-4mf_{\rho NN}{}^T)]\,,$$
$$C_2=\pi\epsilon_RRg_{\rho KK}\alpha[2f_{\rho NN}{}^V-4mf_{\rho NN}{}^T].$$

Here, $\nu=\frac{1}{4}(s-u)$, p, and q are the center-of-mass momenta in the t channel, α is the trajectory parameter, $\epsilon_R=\text{Re}\alpha'$,

$$R=\frac{2^{\alpha+1}\Gamma(\alpha+\frac{1}{2})}{3\sqrt{\pi}\Gamma(\alpha+1)}(\alpha+\tfrac{1}{2})\,,$$

the f's and g's are the coupling constants, and m is the mass of the baryon. For ρ the $\alpha(0)$ is 0.57. Therefore, the amplitudes A and B are clearly not superconvergent

TABLE I. $(KN \to KN)+(\bar{K}N \to \bar{K}N)$.

Resonant states	$I,(J^P)$	Mass (MeV)	Value of $(2/\pi)\int \mathrm{Im}A_{KN}{}^{I=1}d\nu$	Value of $(4/\pi)\int\nu\, \mathrm{Im}B_{KN}{}^{I=1}d\nu$
Λ	$0,(\frac{1}{2}^+)$	1115	5.6700	−3.7130
Y_0^*	$0,(\frac{1}{2}^-)$	1405	1.0721	0.3865
Y_0^*	$0,(\frac{3}{2}^-)$	1520	4.8103	−8.0703
Y_0^*	$0,(\frac{1}{2}^-)$	1670	−0.1938	−0.1232
Y_0^*	$0,(\frac{3}{2}^-)$	1700	1.5232	−3.2824
Y_0^*	$0,(\frac{5}{2}^+)$	1820	1.9110	−1.8818
Y_0^*	$0,(\frac{7}{2}^-)$	2100	13.7334	−30.1912
Σ	$1,(\frac{1}{2}^+)$	1190	−0.3000	0.3450
Y_1^*	$1,(\frac{3}{2}^+)$	1385	−21.9560	−50.8991
Y_1^*	$1,(\frac{5}{2}^-)$	1770	8.6060	−12.9109
Y_1^*	$1,(\frac{5}{2}^+)$	1910	−1.0808	2.3751
Y_1^*	$1,(\frac{7}{2}^+)$	2035	6.2306	−9.3039

because of the presence of the terms like x^α and $x^{\alpha-1}$. However, the following combinations are superconvergent:

$$C_1A_{K\Sigma}-C_1'A_{KN}\sim\nu^{\alpha(0)-2}, \tag{5}$$

$$CB_{2K\Sigma}-C_2'B_{KN}\sim\nu^{\alpha(0)-3}, \tag{6}$$

where C_1' and C_2' are defined for the $K\Sigma \to K\Sigma$ processes in the same way as C_1 and C_2 are for the $KN \to KN$ processes. The corresponding superconvergent sum rules are

$$\int_{-\infty}^{\infty} \mathrm{Im}(C_1A_{K\Sigma}{}^{I=1}-C_1'A_{KN}{}^{I=1})d\nu=0, \tag{7}$$

$$\int_{-\infty}^{\infty} \nu\,\mathrm{Im}(C_2B_{K\Sigma}{}^{I=1}-C_2'B_{KN}{}^{I=1})d\nu=0, \tag{8}$$

and

$$\int_{-\infty}^{\infty} \mathrm{Im}(C_2B_{K\Sigma}{}^{I=1}-C_2'B_{KN}{}^{I=1})d\nu=0. \tag{9}$$

Because of crossing symmetry, relation (9) is trivial. We saturate relations (7) and (8) by keeping contributions from all the known[3] single-particle states as shown in Tables I and II. In the s channel, we have multiplied the amplitudes by the appropriate isotopic-spin pro-

[3] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

TABLE II. $(K\Sigma \to K\Sigma)+(\bar{K}\Sigma \to \bar{K}\Sigma)$

Resonant state	$I,(J^P)$	Mass (MeV)	Value of $(3/2\pi)\int \mathrm{Im}A_{K\Sigma}{}^{I=1}d\nu$	Value of $(3/\pi)\int\nu\, \mathrm{Im}B_{K\Sigma}{}^{I=1}d\nu$
N	$\frac{1}{2},(\frac{1}{2}^+)$	940	0.3000	0.9330
N^*	$\frac{1}{2},(\frac{1}{2}^+)$	1400	−3.4020	4.8422
N^*	$\frac{1}{2},(\frac{3}{2}^-)$	1525	0.8539	−1.4418
N^*	$\frac{1}{2},(\frac{1}{2}^-)$	1570	−0.5172	−0.1506
N^*	$\frac{3}{2},(\frac{3}{2}^+)$	1236	40.4899	−33.7506
N^*	$\frac{3}{2},(\frac{1}{2}^-)$	1670	0.5129	0.2384
Ξ	$\frac{1}{2},(\frac{1}{2}^+)$	1315	−3.7500	−2.0430
Ξ^*	$\frac{1}{2},(\frac{3}{2}^+)$	1530	−21.9949	−37.0968

jection operators. In evaluating the contributions, we have used the Born amplitudes for states with masses less than the threshold, and the Breit-Wigner formula for those above the threshold. We finally get the following sum rules:

$$10.0128\frac{\pi^2\epsilon_R R g_{\rho KK}}{m'}[2f_{\rho\Sigma\Sigma}{}^V-\alpha(2f_{\rho\Sigma\Sigma}{}^V-4m'f_{\rho\Sigma\Sigma}{}^T)]$$
$$=8.3284\frac{\pi^2\epsilon_R R g_{\rho KK}}{m}$$
$$\times[2f_{\rho NN}{}^V-\alpha(2f_{\rho NN}{}^V-4mf_{\rho NN}{}^T)], \tag{10}$$

$$58.6346\pi^2\epsilon_R R g_{\rho KK}\alpha[f_{\rho\Sigma\Sigma}{}^V-2m'f_{\rho\Sigma\Sigma}{}^T]$$
$$=45.6461\pi^2\epsilon_R R g_{\rho KK}\alpha[f_{\rho NN}{}^V-2mf_{\rho NN}{}^T]. \tag{11}$$

To test these sum rules we put $f_{\rho\Sigma\Sigma}=2f_{\rho NN}$ using the universal vector-coupling hypothesis and introduce the mixing parameter for tensor coupling by the relation $f_{\rho\Sigma\Sigma}{}^T=2\alpha'f_{\rho NN}{}^T$. If we take $f_{\rho NN}{}^T/f_{\rho NN}{}^V=(\mu_p-\mu_n)/2m=1.9628$, we get $\alpha'=0.34$ from (10) and $\alpha'=0.44$ from (11). Obviously, both solutions have the correct sign and the values are also consistent with the usually accepted value[4] $\alpha'=0.3$–0.4.

One of us (V.P.S.) is grateful to C.S.I.R., New Delhi, for financial assistance.

[4] P. Carruthers, *Introduction to Unitary Symmetry* (Interscience Publishers, Inc., New York, 1966), p. 122.