Structure-Dependent Axial-Vector Form Factor in Charged-Pion Radiative Decay

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The structure-dependent axial-vector form factor of charged-pion radiative decay, $\pi^+ \to l^+ + \nu + \gamma$, recently investigated by Das, Mathur, and Okubo in the soft-pion limit ($q_\pi \to 0$), has been reexamined in the framework of the "hard pion" calculation due to Schnitzer and Weinberg. The result of our present calculation has been compared with that of the previous authors and also with the presently available data.

RADIATIVE decay of charged pions, e.g., $\pi^+ \to l^+ + \nu + \gamma$, has been of considerable interest,[1] as it can provide us some information about the structure of weak interactions. It proceeds via ordinary inner bremsstrahlung (IB) and the structure-dependent (SD) interactions (both vector and axial-vector can contribute). By means of the conserved-vector-current hypothesis the structure-dependent vector (SDV) contribution can be obtained.[2] The structure-dependent axial-vector (SDA) contribution has recently been calculated by Das, Mathur, and Okubo[3] using current commutation relations, partial conservation of axial-vector current (PCAC), and Weinberg's sum rules. However, they work in the soft-pion limit ($q_\pi \to 0$). In the present calculation we would like to avoid this procedure and follow Schnitzer and Weinberg,[4] to calculate the SDA contribution keeping the pion on the mass shell. However, the SDA form factor obtained in this way has an unknown parameter δ (the same as in Ref. 4). If we take δ equal[5] to $(-\frac{1}{2})$, we find that the SDA form factor is reduced to half the value obtained by DMO. For a further check on our present analysis, we eliminate this unknown parameter δ by using the electromagnetic form factor of the pion in the "hard-pion" model calculation of Ref. 4. This enables us to obtain the SDA form factor $a(\nu)$ in terms of the usual spectral functions and the mean squared radius of the charged pions, where ν is the energy variable. It is interesting to note that in our calculation we obtain the ν dependence of $a(\nu)$, which may be checked by studying the detailed energy spectra of photons and leptons in $\pi^+ \to l^+ + \nu + \gamma$ decay. The ratio of the SDA form factor to the SDV form factor has been obtained and compared with the experimental values obtained by Depommier *et al.*[6] In a similar way the SDA form factor for $K^+ \to l^+ + \nu + \gamma$ can also be obtained following Gupta and Vaishya.[7]

The T-matrix element of $\pi^+(q) \to l^+(p_1) + \nu(p_2) + \gamma(k)$ is given by[8]

$$T = \frac{-ieG\cos\theta}{\sqrt{2}} \left(\frac{m_l m_\nu}{4k_0 q_0 p_{10} p_{20} V^4} \right)^{1/2} \times \left[\epsilon_\mu M_{\mu\nu} l_\nu + F_\pi \bar{u}^{(l)}(p_1) \epsilon_\mu \gamma_\mu \frac{i\gamma\cdot(p_1+k) - m_l}{2p_1\cdot k} \times \gamma\cdot q(1+\gamma_5) v^{(\nu)}(p_2) \right], \quad (1)$$

where

$$M_{\mu\nu} = (2q_0 V)^{1/2} \int d^4x\, e^{-ik\cdot x} \times \langle 0 | T\{ V_\mu^{(\mathrm{em})}(x), [V_{\nu 2}^1(0) + A_{\nu 2}^1(0)] \} | \pi^+, q \rangle \quad (2)$$

and $l_\nu = i\bar{u}^{(l)}(p_1)\gamma_\nu(1+\gamma_5)v^{(\nu)}(p_2)$ is the lepton current. ϵ_μ denotes the polarization vector of the photon, and F_π is the decay constant for $\pi^+ \to l^+ + \nu$ defined by

$$\langle 0 | A_{\mu 2}^1(0) | \pi^+, q \rangle = iF_\pi q_\mu (2qV_0)^{-1/2}. \quad (3)$$

The T-matrix element can be written as a sum of three terms, namely, the IB term, the SDA term, and the SDV term, which are separately gauge-invariant. Thus

$$T = \frac{-ieG\cos\theta}{\sqrt{2}} \left(\frac{m_l m_\nu}{4q_0 k_0 p_{10} p_{20} V^4} \right)^{1/2} \times [M_{\mathrm{IB}} + M_{\mathrm{SDA}} + M_{\mathrm{SDV}}], \quad (4)$$

where

$$M_{\mathrm{IB}} = -F_\pi \epsilon_\mu \left\{ \left(\delta_{\mu\nu} + \frac{q_\mu p_\nu}{\nu} \right) l_\nu - \bar{u}^{(l)}(p_1)\gamma_\mu \frac{i\gamma\cdot(p_1+k) - m_l}{2p_1\cdot k} \gamma\cdot q(1+\gamma_5) v^{(\nu)}(p_2) \right\},$$

$$M_{\mathrm{SDA}} = \nu a(\nu) \epsilon_\mu \left(\delta_{\mu\nu} - \frac{q_\mu k_\nu}{\nu} \right) l_\nu,$$

[1] S. G. Brown and S. Bludman, Phys. Rev. **136**, B1160 (1964). This paper contains earlier references to the relevant literature.
[2] V. G. Vaks and B. L. Ioffe, Nuovo Cimento **10**, 342 (1958).
[3] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 859 (1967). Henceforth this paper will be referred to as DMO.
[4] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).
[5] With this value of δ, Schnitzer and Weinberg obtain consistent decay widths for $\rho \to \pi + \pi$ and $A_1 \to \rho + \pi$. However, this value of δ should not be taken very seriously, in view of the large uncertainties in the experimental data.
[6] P. Depommier, J. Heintze, C. Rubbia, and V. Soergel, Phys. Letters **7**, 285 (1963).
[7] K. C. Gupta and J. S. Vaishya, Phys. Rev. **170**, 1530 (1968).
[8] We use notations similar to that in Ref. 3.

and

$$M_{\rm SDV}=F(\nu)\epsilon_{\mu\nu\lambda\sigma}k_\lambda q_\sigma \epsilon_\mu l_\nu.$$

Here $\nu=q\cdot k$; $p=q-k$; and $a(\nu)$ and $F(\nu)$ are the SDA and the SDV form factors, respectively.

Following Ref. 4, the matrix element

$$M_{\mu\nu}{}^A=\frac{i(q^2+m_\pi{}^2)}{F_\pi m_\pi{}^2}\int dx^4dy^4e^{-ik\cdot x+iq\cdot y}\times\langle T(V_\mu{}^{(\rm em)}(x),A_{\nu 2}{}^1(0),\partial_\lambda A_{\lambda 1}{}^2(y))\rangle_0 \quad (5)$$

has the form[9]

$$M_{\mu\nu}{}^A=g_\rho{}^{-1}g_{A_1}{}^{-1}\Delta_{A_1}{}^{\nu\sigma}(p)\Delta_\rho{}^{\mu\eta}(k)\Gamma_{\sigma\eta}(p,q)+(g_\rho{}^{-1}F_\pi p^\nu/p^2+m_\pi{}^2)\Delta_\rho{}^{\mu\eta}(k)\Gamma_\eta(q,p), \quad (6)$$

where

$$\Gamma_{\sigma\eta}(p,q)=(F_\pi M_\rho{}^2M_{A_1}{}^2/g_\rho g_{A_1})\times[-\delta_{\sigma\eta}-M_{A_1}{}^{-2}(\delta_{\sigma\eta}p^2-p_\sigma p_\eta)+F_\pi{}^{-2}g_{A_1}{}^2M_{A_1}{}^{-2}(M_\rho{}^{-2}-M_{A_1}{}^{-2})(\delta_{\sigma\eta}k^2-k_\sigma k_\eta)-\delta g_{A_1}{}^2F_\pi{}^{-2}M_{A_1}{}^{-4}(\delta_{\sigma\eta}q\cdot k-q_\eta k_\sigma)], \quad (7)$$

$$\Gamma_\eta(q,p)=g_\rho{}^{-1}\Big[M_\rho{}^2(p+q)_\eta+\frac{1}{2}\Big\{1-\frac{g_{A_1}{}^2}{F_\pi{}^2M_{A_1}{}^4}(M_{A_1}{}^2-M_\rho{}^2)\Big\}\times\{k^2(p_\eta+q_\eta)+k_\eta(p^2-q^2)\}+\delta g_{A_1}{}^2M_\rho{}^2F_\pi{}^{-2}M_{A_1}{}^{-4}(p_\eta q\cdot k-q_\eta p\cdot k)\Big]. \quad (8)$$

It should be pointed out that in obtaining Eqs. (7) and (8), we have assumed chiral $SU(2)\times SU(2)$, current algebra, PCAC, meson dominance, and smoothness of proper vertices. Weinberg's first sum rule[10]

$$g_{A_1}{}^2M_{A_1}{}^{-2}+F_\pi{}^2=g_\rho{}^2M_\rho{}^{-2} \quad (9)$$

has also been utilized.

Using $k^2=0$ in Eqs. (6)–(8), it is straightforward to obtain

$$\epsilon_\mu M_{\mu\nu}{}^A l_\nu=-F_\pi\epsilon_\mu l_\nu\{(\delta_{\mu\nu}+q_\mu p_\nu/\nu)+[\nu b(\nu)\delta g_{A_1}{}^2/F_\pi{}^2M_{A_1}{}^4](\delta_{\mu\nu}-q_\mu k_\nu/\nu)\}, \quad (10)$$

where

$$b(\nu)=M_{A_1}{}^2/(-m_\pi{}^2-2\nu+M_{A_1}{}^2). \quad (11)$$

Substituting this result [Eq. (10)] in Eq. (1) and then comparing it with Eq. (4), we obtain.

$$a(\nu)=-\delta b(\nu)g_{A_1}{}^2/F_\pi M_{A_1}{}^4. \quad (12)$$

[9] Here $\Delta_{1\nu}{}^\sigma(p)$ and $\Delta_\rho{}^{\mu\eta}(k)$ are the covariant spin-1 parts of the unrenoralized axial-vector and vector propagators, respectively. Under the assumption of meson dominance, we have

$$\Delta_{A_1}{}^{\nu\sigma}(p)=g_{A_1}{}^2(\delta_{\nu\sigma}+p_\nu p_\sigma/M_{A_1}{}^2)(p^2+M_{A_1}{}^2)^{-1}$$

and a similar expression for $\Delta_\rho{}^{\mu\eta}(k)$.

[10] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).

The usual charged-pion form factor $F_+(k^2)$ is defined as

$$\langle\pi^+,p|V_\mu{}^{(\rm em)}(0)|\pi^+,q\rangle=F_+(k^2)(p+q)_\mu=g_\rho{}^{-1}\Delta_\rho{}^{\mu\eta}(k)\Gamma_\eta(q,p). \quad (13)$$

From this we see that

$$F_+(k^2)=1+\frac{k^2}{k^2+M_\rho{}^2}\Big[\frac{(1+\delta)g_{A_1}{}^2M_\rho{}^2}{2F_\pi{}^2M_{A_1}{}^4}-\frac{g_\rho{}^2}{2F_\pi{}^2M_\rho{}^2}\Big]. \quad (14)$$

Differentiating with respect to k^2 and rearranging, we get

$$\frac{g_{A_1}{}^2}{M_{A_1}{}^4}\delta=\frac{g_\rho{}^2}{M_\rho{}^4}-\frac{g_{A_1}{}^2}{M_{A_1}{}^4}+2F_\pi{}^2\frac{dF_+(k^2)}{dk^2}\Big|_{k^2=0}. \quad (15)$$

Using it in Eq. (12), we obtain

$$a(\nu)=b(\nu)\Big[\frac{1}{F_\pi}\Big(\frac{g_{A_1}{}^2}{M_{A_1}{}^4}-\frac{g_\rho{}^2}{M_\rho{}^4}\Big)-2F_\pi\frac{dF_+(k^2)}{dk^2}\Big|_{k^2=0}\Big]. \quad (16)$$

If we neglect terms of the order of $m_\pi{}^2/M_{A_1}{}^2$, we get $b(\nu=0)=1$ and Eq. (16) now compares well with the low-energy-theorem result [Eq. (22) of Ref. 3] of DMO.

Using Weinberg's sum rule (Eq. 9) and the current-algebra result[11]

$$g_\rho{}^2M_\rho{}^{-2}\simeq 2F_\pi{}^2, \quad (17)$$

we have from Eq. (12)

$$a(\nu=0)\simeq-\delta F_\pi/M_{A_1}{}^2. \quad (18)$$

This agrees with DMO's result when[12] $\delta=-1$.

Next, with the help of the conserved vector-current hypothesis, the SDV form factor can be written[2] as [see Eq. (8) of Ref. 3]

$$|F(-\tfrac{1}{2}m_\pi{}^2)|=4(2\pi W_{\pi^0})^{1/2}/e^2m_\pi{}^{3/2}, \quad (19)$$

where W_{π^0} is the decay rate of $\pi^0\to 2\gamma$. We may ignore[13] the ν dependence of $F(\nu)$. From Eqs. (18) and (19), the ratio of the SDA to the SDV form factor, γ, is given by

$$|\gamma|\equiv|a/F|\simeq 0.6|\delta|, \quad (20)$$

where we have taken the lifetime[14] of the π^0 as 0.89×10^{-16} sec, and F_π from $\pi\to\mu\nu$ decay. If we use[4] $\delta=-0.5$, we get

$$|\gamma|\simeq 0.3, \quad (21)$$

which is half the value obtained by DMO. This is in

[11] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966); see also D. A. Geffen, Phys. Rev. Letters 19, 770 (1967).

[12] It may be pointed out that pion form factor $F_+(k^2)$ [Eq. (14)] also reduces to the ρ-dominant standard form factor when we put $\delta=-1$.

[13] This is in the same spirit as in Ref. 3.

[14] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 39, 1 (1967).

reasonable agreement with one of the two solutions[15] ($\gamma=0.4$) obtained by Depommier *et al.*, and disfavors strongly the other ($\gamma=-2.1$). Further experiments with better statistics will be of considerable interest in understanding the structure-dependent effects in the weak interactions.

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[15] Experimentally (see Ref. 6) one measures the partial decay rate d^2W_{SD} (due to structure-dependent effects) integrated over certain energy or angular intervals which depends on $(1+\gamma)^2$ and $(1-\gamma)^2$ (see Ref. 1). Thus a single measurement of SD radiation leads to an equation quadratic in γ, which has two solutions, namely, $\gamma=0.4$ and -2.1. Here $\tau_{\pi^0\to 2\gamma}=1.05\times10^{-16}$ sec, and the central value of the branching ratio $\Gamma(\pi\to e\nu\gamma;E_e,E_\gamma>48\text{ MeV})/\Gamma(\pi\to\mu\nu)=(3.0\pm0.5)\times10^{-8}$ have been used.

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Sum Rules for Meson-Baryon Scattering from Asymptotic Symmetry Requirements

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We consider the processes $KN\to KN$ and $K\Sigma\to K\Sigma$. The superconvergent sum rules involving linear combinations of forward elastic scattering amplitudes of these processes are set up and tested.

WHEN a scattering amplitude $M(\nu,\Delta^2)$ for a process $B(p_1)+P(q_1)=B'(p_2)+P'(q_2)$ is odd under crossing and is bounded asymptotically at least to the extent of $\nu|M(\nu)|<$const, then it satisfies a superconvergence relation[1] of the type

$$\int_{-\infty}^{\infty}\operatorname{Im}M(\nu,\Delta^2)d\nu=0\,,\tag{1}$$

where

$$\nu=PQ=\tfrac{1}{4}(s-u),\ \Delta^2=t,\ P=\tfrac{1}{2}(p_1+p_2),\ Q=\tfrac{1}{2}(q_1+q_2).$$

We shall consider elastic meson-baryon scattering and assume that Regge theory accounts for the high-energy behavior of the scattering amplitudes. The forward scattering amplitude is given by const$\times\nu^{\alpha(t=0)}$ as $\nu\to\infty$, where $\alpha(0)$ is the intercept of the leading Regge trajectory.

Costa and Zimerman[2] have pointed out that it is possible to extend the domain of physical application of (1) by making use of symmetry requirements arising from $SU(3)$ invariance on the high-energy Regge behavior of the amplitudes. One now considers a linear combination of the amplitudes, while applying superconvergence, and thereby include amplitudes which would have been otherwise excluded by the requirements of superconvergence. In this paper we apply a similar method to the processes $KN\to KN$ and $K\Sigma\to K\Sigma$.

[1] V. de Alfaro, S. Fubini, A. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966).

[2] G. Costa and A. H. Zimerman, Nuovo Cimento 46, 198 (1966).

For the combinations of processes

$$(KN\to KN)+(\bar{K}N\to\bar{K}N)\tag{2}$$

and

$$(K\Sigma\to K\Sigma)+(\bar{K}\Sigma\to\bar{K}\Sigma)\,,\tag{3}$$

we get contributions only from the ρ trajectory. The invariant amplitude for elastic meson-baryon scattering is given by

$$T_{fi}=\bar{u}(p_2)(-A+i\gamma\cdot QB)u(p_1)\,,\tag{4}$$

where u is the Dirac spinor, and A and B are scalar functions depending only on the Mandelstam variables. For each process considered separately, the amplitudes behave as

$$-A_{KN}{}^{I=1}=C_1(pq)^\alpha[x^\alpha-O_1x^{\alpha-2}+\cdots]\,,$$
$$-B_{KN}{}^{I=1}=C_2(pq)^{\alpha-1}[x^{\alpha-1}-O_2x^{\alpha-3}+\cdots]\,,$$

where

$$x=-\nu/pq\,,$$
$$C_1=(\pi\epsilon_R/m)Rg_{\rho KK}[2f_{\rho NN}{}^V-\alpha(2f_{\rho NN}{}^V-4mf_{\rho NN}{}^T)]\,,$$
$$C_2=\pi\epsilon_R Rg_{\rho KK}\alpha[2f_{\rho NN}{}^V-4mf_{\rho NN}{}^T]\,.$$

Here, $\nu=\frac{1}{4}(s-u)$, p, and q are the center-of-mass momenta in the t channel, α is the trajectory parameter, $\epsilon_R=\operatorname{Re}\alpha'$,

$$R=\frac{2^{\alpha+1}\Gamma(\alpha+\frac{1}{2})}{3\sqrt{\pi}\,\Gamma(\alpha+1)}(\alpha+\tfrac{1}{2})\,,$$

the f's and g's are the coupling constants, and m is the mass of the baryon. For ρ the $\alpha(0)$ is 0.57. Therefore, the amplitudes A and B are clearly not superconvergent