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K_{l3} Form Factors in Current Algebra with Hard Pion and Kaon

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The K_{l3} form factors $f_\pm(q^2)$ are calculated using the current commutators, partially conserved axial-vector current, the Weinberg-type sum rules, and the Schnitzer-Weinberg technique with hard pions and kaons, which involves the domination of certain vertex functions by vector and axial-vector mesons. The results are very similar to those obtained recently by B. W. Lee using chiral dynamics and field-crurent identity.

I. INTRODUCTION

In the present paper we present calculations of the K_{l3} form factors $f_\pm(q^2)$, using the algebra of currents and the hard-pion and hard-kaon technique of Schnitzer and Weinberg.[1] This technique involves partial conservation of axial-vector current (PCAC) and vector and axial-vector meson dominance of various vertex and spectral functions besides the usual current commutators. Previously we have studied this problem in the soft-pion and soft-kaon approximation and have obtained the K_{l3} form factors at zero momentum transfer.[2] In particular, we obtained

$$f_+(0)=(f_K^2+f_\pi^2-f_\kappa^2)/2f_\pi f_K\,, \tag{1.1}$$

$$\xi(0)\equiv\frac{f_-(0)}{f_+(0)}\simeq\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right), \tag{1.2}$$

where f_K, f_π, and f_κ are, respectively, the decay constants for K_{l2}, π_{l2}, and κ_{l2} decays.

Recently, Lee[3] has obtained the predictions of chiral dynamics and field-current identity on the K_{l3} form factors. Comparison of Eqs. (1.1) and (1.2) with his predictions shows that while Eq. (1.1), which has also been obtained by Glashow and Weinberg[4] with broken chiral symmetry, holds even when the kaon and pion are not soft, Eq. (1.2) is modified with terms proportional to $m_K^2-m_\pi^2$. Thus the soft kaon and soft pion do not appear to be a good approximation for $\xi(0)$. To remedy this, we do not use this approximation in this paper but instead follow the hard-pion (and hard-kaon) technique of SW, still, of course, using current commutators. Our results are very similar to those obtained by Lee.[3] There are, of course, differences and these may be due to his specific form for the effective Hamiltonian for the $K^*K\pi$ vertex. Numerically, the results are almost the same for $f_+(q^2)$, while for $f_-(q^2)$ the predictions are different and may be significant, as discussed below.

Since a number of theoretical papers on the K_{l3} form factors have appeared recently, it is perhaps pertinent to comment on them in the light of the present work and that of Lee. First, we mention the work of various people using the current-algebra and soft-pion technique,[5] which gives the relation

$$f_+(-m_K^2)+f_-(-m_K^2)=f_K/f_\pi\,. \tag{1.3}$$

In order for this relation to be useful, one needs to know the q^2 dependence of the form factors[6] $f_\pm(q^2)$. One way of getting this dependence is through the use of dispersion relations, and it therefore depends on whether the relevant dispersion relations do or do not have subtractions and on the values of the coupling constants which enter when dispersion integrals are dominated by poles. It is well known that the simple K^* pole dominance gives $\xi=f_-(0)/f_+(0)=-0.29$, $\lambda_+=\lambda_-=0.024$, where λ_+ and λ_- determine the q^2 dependence of $f_\pm(q^2)$ in their linear approximations, without recourse to any current-algebra technique. If one now includes the κ meson, it is found that it gives a contribution opposite to that of K^* in $f_-(q^2)$ [no contribution to $f_+(q^2)$] and, for $\Gamma_\kappa\approx 20$ MeV, the contribution from K^* is completely cancelled depending upon the various assumptions used for estimating f_κ and other κ parameters. Since experimentally the κ width is quite uncertain (<30 MeV), people can safely differ. In a recent paper, Matsuda and Oneda[8] apply the dispersion technique to a direct calculation of $f_\pm(q^2)$ and dominate the unsubtracted dispersion-relation integrals by K^* and κ poles. They then make use of current algebra involving charge commutators to fix certain coupling constants which appear in the dispersion relation expressions of $f_\pm(q^2)$ and find the κ contribution to $f_-(0)$ and $\xi(0)$ to be smaller than the K^* contribution, with the result of a small negative value of ξ ($=-0.16$), whereas Lee[3] and we find a small positive value. In our case, the κ contribution does not appear. This is because f_κ turns out to be very small in our approach, and therefore we can use the approximation of conserved strangeness-

[1] H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967), referred to hereafter as SW.
[2] Riazuddin, A. Q. Sarker, and Fayyazuddin, Nucl. Phys. (to be published), referred to hereafter as RSF.
[3] B. W. Lee, Phys. Rev. Letters **20**, 617 (1968).
[4] S. L. Glashow and S. Weinberg, Phys. Rev. Letters **20**, 224 (1968).
[5] C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966); M. Suzuki, *ibid.* **16**, 212 (1966); V. S. Mathur, S. Okubo, and L. K. Pandit, *ibid.* **16**, 371 (1966).
[6] See, e.g., R. Oehme, Phys. Rev. Letters **16**, 215 (1966).
[7] V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters **16**, 947 (1966); **16**, 1135(E) (1966). Earlier references for the simple pole-model calculation of $f_\pm(q^2)$ may be traced from here.
[8] S. Matsuda and S. Oneda, **169**, 1172 (1968). There seems to be another work in which a similar approach may have been used: N. Fuchs, Phys. Rev. **170**, 1310 (1968).

changing vector current. Of course, we do not put $m_K=m_\pi$, the approximate conservation of V_λ^{4+i5} being solely due to the negligible value of f_κ. The comparison of our results as well as Lee's with those of Matsuda and Oneda[8] also shows that our predictions for $f_\pm(q^2)$ contain, besides K^*-pole terms, other terms which one can attribute to high-energy contributions from multiparticle states, to a subtraction constant, or to both, in dispersion-theoretic language. Lee regards these contributions as model-dependent, while in our case they are consequences of the PCAC, vector and axial-vector meson dominance, current commutators, and the Weinberg-type sum rules. Our results differ in detail from those given by Lee, as discussed below. This may be due to the specific Lagrangian for $K^*K\pi$ taken by Lee or due to some other approximations which are not obvious or easy to check from Ref. 3.

We also mention the method of Fubini and Furlan,[9] used by d'Espagnat and Gaillard[9] to calculate ξ and λ_-. If λ_+ is given (from experiment), then they are able to predict upper or lower limits for ξ and λ_-, neglecting the so-called leakage term [to outside the $SU(3)$ representation]. Mann and Primakoff[10] have taken into account the leakage term, the parametrization of which does not seem to be unambiguous, as discussed in Ref. 3. Fitelson and Kazes's[11] approach is to find a formal solution of the algebra of currents and saturate it with a vector-meson resonance. That their results for $f_+(q^2)$ are very close to those of K^*-pole dominance is therefore expected.

II. FORMALISM

We begin with a sequence of formulas which in turn define the K_{l3} form factors $f_\pm(q^2)$, the $\langle K\pi|V_\lambda'|0\rangle$ vertex Γ_λ, the $\langle K_A\pi|V_\lambda'|0\rangle$ vertex $\Gamma_{\mu\lambda}$, the $\langle KA_1|V_\lambda'|0\rangle$ vertex $\Gamma_{\nu\lambda}$, and the $\langle K_AA_1|V_\lambda'|0\rangle$ vertex $\Gamma_{\mu\nu\lambda}$, where $V_\lambda'=V_\lambda^4+iV_\lambda^5$ is the strangeness-changing vector current:

$$R_\lambda=\tfrac{1}{2}\sqrt{2}[f_+(q^2)(p-k)_\lambda-f_-(q^2)(p+k)_\lambda]$$
$$=(2\pi)^3(4k_0p_0)^{1/2}\times\langle\pi^0(p),K^+(k)|V_\lambda^{4+i5}(0)|0\rangle,\quad(2.1)$$

$$M_\lambda^{abc}=\int d^4xd^4y\,e^{-i(kx+py)}\times\langle T\{\partial_\mu A_\mu^a(x)\partial_\nu A_\nu^b(y)V_\lambda^c(0)\}\rangle_0$$
$$\equiv\frac{if_af_bm_a^2m_b^2}{2(k^2+m_a^2)(p^2+m_b^2)}\Gamma_\lambda^{abc}(k,-p),\quad(2.2)$$

$$M_{\nu\lambda}^{abc}=\int d^4xd^4y\,e^{-i(kx+py)}\times\langle T\{\partial_\mu A_\mu^a(x)A_\nu^b(y)V_\lambda^c(0)\}\rangle_0$$
$$\equiv\frac{f_am_a^2g_b^{-1}}{(k^2+m_a^2)}\Delta_{\nu\sigma}^{A(b)}(-p)\Gamma_{\sigma\lambda}^{abc}(k,-p)-\frac{f_am_a^2f_b}{2(k^2+m_a^2)(p^2+m_b^2)}p_\nu\Gamma_\lambda^{abc}(k,-p),\quad(2.3)$$

$$M_{\mu\nu\lambda}^{abc}=\int d^4xd^4y\,e^{-i(kx+py)}\times\langle T\{A_\mu^a(x)A_\nu^b(y)V_\lambda^c(0)\}\rangle_0$$
$$\equiv2ig_a^{-1}g_b^{-1}\Delta_{\mu\tau}^{A(a)}(k)\Delta_{\nu\sigma}^{A(b)}(-p)\Gamma_{\tau\sigma\lambda}^{abc}(k,-p)+\frac{ig_a^{-1}f_bp_\nu}{(p^2+m_b^2)}\Delta_{\mu\tau}^{A(a)}(k)\Gamma_{\tau\lambda}^{abc}(k,-p)$$
$$+\frac{ig_b^{-1}f_ak_\mu}{(k^2+m_a^2)}\Delta_{\nu\sigma}^{A(b)}(-p)\Gamma_{\sigma\lambda}^{abc}(k,-p)-\tfrac{1}{2}i\frac{f_af_bk_\mu p_\nu}{(k^2+m_a^2)(p^2+m_b^2)}\Gamma_\lambda(k,-p),\quad(2.4)$$

where $q=-(p+k)$, and $\Delta_{\mu\nu}^V$ and $\Delta_{\mu\nu}^A$ are the covariant spin-1 part of the unrenormalized vector and axial-vector propagators:

$$\Delta_{\mu\nu}^V(k)\equiv\int d\mu^2\rho^V(\mu^2)[\delta_{\mu\nu}+k_\mu k_\nu/\mu^2]\times[\mu^2+k^2]^{-1},\quad(2.5)$$

$$\langle V_\mu^a(x)V_\nu^b(0)\rangle_0=(2\pi)^{-3}(\tfrac{1}{2}\delta_{ab})\int d^4k\,\theta(k_0)\,e^{ik\cdot x}\times\{\rho^V(-k^2)[\delta_{\mu\nu}-k_\mu k_\nu/k^2]+\rho_\sigma^V(-k^2)k_\mu k_\nu\},\quad(2.6)$$

$$\Delta_{\mu\nu}^A(k)\equiv\int d\mu^2\rho^A(\mu^2)[\delta_{\mu\nu}+k_\mu k_\nu/\mu^2]\times[\mu^2+k^2]^{-1},\quad(2.7)$$

$$\langle A_\mu^a(x)A_\nu^b(0)\rangle_0=(2\pi)^{-3}\tfrac{1}{2}\delta_{ab}\int d^4k\,\theta(k_0)e^{ik\cdot x}\times\{\rho^A(-k^2)[\delta_{\mu\nu}-k_\mu k_\nu/k^2]+\rho_p^A(-k^2)k_\mu k_\nu\}.\quad(2.8)$$

The decay constants g_V^2, g_A^2, f_p^2, and f_σ are defined as

[9] B. d'Espagnat and M. K. Gaillard, Phys. Letters **25B**, 346 (1967); S. Fubini and G. Furlan, Physics **1**, 229 (1965).
[10] A. K. Mann and H. Primakoff, Phys. Rev. Letters **20**, 32 (1968).
[11] M. Fitelson and E. Kazes, Phys. Rev. Letters **20**, 304 (1968).

the coefficients of $\delta(\mu^2-m_V^2)$, $\delta(\mu^2-m_A^2)$, $\delta(\mu^2-m_p^2)$, and $\delta(\mu^2-m_\sigma^2)$ in $\rho^V(\mu^2)$, $\rho^A(\mu^2)$, $\rho_p{}^A(\mu^2)$, and $\rho_\sigma{}^V(\mu^2)$, respectively, the suffixes p and σ representing the pseudoscalar and scalar mesons. If we now make use of the Weinberg identity[12] and the well-known current commutation relations, we obtain, as in RSF,

$$-\frac{f_\pi f_K}{\sqrt{2}}m_\pi^2 m_K^2\frac{R_\lambda}{(k^2+m_K^2)(p^2+m_\pi^2)}$$
$$=k_\mu p_\nu M_{\mu\nu\lambda}+R_\lambda{}^\sigma-\tfrac{1}{4}i(p-k)_\nu\tilde{\Delta}_{\nu\lambda}{}^{V(K)}(q)$$
$$+\tfrac{1}{4}ip_\mu\tilde{\Delta}_{\lambda\mu}{}^{A(\pi)}(p)-\tfrac{1}{4}ik_\mu\tilde{\Delta}_{\lambda\mu}{}^{A(K)}(k)$$
$$+\tfrac{1}{2}\int d^4y\, e^{-ip\cdot y}\langle T\{A_\lambda{}^3(0)\partial_\nu A_\nu{}^3(y)\}\rangle_0$$
$$-\tfrac{1}{4}\int d^4x\, e^{-ik\cdot x}\langle T\{A_\lambda{}^{4+i5}(0)\partial_\mu A_\mu{}^{4-i5}(x)\}\rangle_0, \quad (2.9)$$

where $R_\lambda{}^\sigma$ is the familiar σ-type term and

$$\tilde{\Delta}_{\lambda\mu}{}^{A(K)}(k)=\int d^4x\, e^{-ik\cdot x}\langle T\{A_\lambda{}^{4+i5}(0)A_\mu{}^{4-i5}(x)\}\rangle_0, \quad (2.10)$$

and similarly for $\Delta_{\lambda\mu}{}^{A(\pi^0)}=\frac{1}{2}\Delta_{\lambda\mu}{}^{A(\pi^+)}$ and $\Delta_{\lambda\mu}{}^{V(K)}$. So far, no approximations have been made.

RSF took the soft-meson limits k^2, p^2, $k\cdot p\to 0$ so that the term $k_\mu p_\nu M_{\mu\nu\lambda}$ on the right-hand side of (2.9) vanishes; the left-hand side of (2.9) was then evaluated in these limits. In the present paper we shall take the mesons on the mass shell, so that $p^2=-m_\pi^2$ and $k^2=-m_K^2$, and we shall also retain the term $k_\mu p_\nu M_{\mu\nu\lambda}$. Then the left-hand side of (2.9) has singularities in these limits. We therefore substitute the definitions (2.2)–(2.4) for $a=4-i5$, $b=3$, and $c=4+i5$ in (2.9) and then, equating the coefficients of each order of singularities from both sides, we obtain

$$R_\lambda=i\Gamma_\lambda(k,-p), \quad (2.11)$$

$$\tfrac{1}{2}i\sqrt{2}g_{K_A}{}^{-1}f_\pi C_A{}^K k_\mu\Gamma_{\mu\lambda}=\tfrac{1}{2}i\sqrt{2}f_\pi f_K\Gamma_\lambda-\tfrac{1}{2}f_\pi{}^2p_\lambda, \quad (2.12)$$

$$\tfrac{1}{2}i\sqrt{2}g_{A_1}{}^{-1}f_K C_A{}^\pi p_\mu\Gamma_{\mu\lambda}=\tfrac{1}{2}i\sqrt{2}f_\pi f_K\Gamma_\lambda+\tfrac{1}{2}f_K{}^2k_\lambda, \quad (2.13)$$

$$-\tfrac{1}{2}i\sqrt{2}g_{K_A}{}^{-1}g_{A_1}{}^{-1}C_A{}^KC_A{}^\pi k_\mu p_\nu\Gamma_{\mu\nu\lambda}=\tfrac{1}{2}i\sqrt{2}g_{K_A}{}^{-1}f_\pi C_A{}^Kk_\mu\Gamma_{\mu\lambda}+\tfrac{1}{2}i\sqrt{2}g_{A_1}{}^{-1}f_KC_A{}^\pi p_\mu\Gamma_{\mu\lambda}-\tfrac{1}{2}i\sqrt{2}f_\pi f_K\Gamma_\lambda-\tfrac{1}{4}(p-k)_\nu$$
$$\times\{\Delta_{\nu\lambda}{}^{K^*}(q)+[q^\nu q^\lambda/(q^2+m_\kappa{}^2)]f_\kappa{}^2-\eta_\mu\eta_\nu(C_V{}^\kappa+f_\kappa{}^2)\}+\tfrac{1}{4}[C_A{}^\pi p_\lambda+f_\pi{}^2p_\lambda-p\cdot\eta\eta_\lambda(C_A{}^\pi+f_\pi{}^2)]$$
$$-\tfrac{1}{4}[C_A{}^Kk_\lambda+f_K{}^2k_\lambda-k\cdot\eta\eta_\lambda(C_A{}^K+f_K{}^2)]+R_\lambda{}^\sigma, \quad (2.14)$$

where we have made use of the following expressions for the propagators:

$$\int d^4x\, e^{ik\cdot x}\langle T\{V_\mu{}^a(x)V_\nu{}^b(0)\}\rangle_0=-\tfrac{1}{2}i\delta_{ab}\{\Delta_{\mu\nu}{}^{V(a)}(k)+[k_\mu k_\nu/(k^2+m_\kappa{}^2)]f_\kappa{}^2-\eta_\mu\eta_\nu(C_V{}^\kappa+f_\kappa{}^2)\}, \quad (2.15)$$

$$\int d^4x\, e^{ik\cdot x}\langle T\{A_\mu{}^a(x)A_\nu{}^b(0)\}\rangle_0=-\tfrac{1}{2}i\delta_{ab}\{\Delta_{\mu\nu}{}^A(k)+[k_\mu k_\nu/(k^2+m_p{}^2)]f_p{}^2-\eta_\mu\eta_\nu(C_A{}^p+f_p{}^2)\}, \quad (2.16)$$

with

$$\eta_\mu=\{0,0,0,1\}.$$
$$p_\nu\Delta_{\nu\lambda}{}^V(p)=C_Vp_\lambda, \quad (2.17)$$
$$p_\nu\Delta_{\nu\lambda}{}^A(p)=C_AP_\lambda, \quad (2.18)$$

and

$$C_V=\int\rho^V(\mu^2)\mu^{-2}d\mu^2, \quad (2.19a)$$

$$C_A=\int\rho^A(\mu^2)\mu^{-2}d\mu^2. \quad (2.19b)$$

First, we note that the Schwinger terms (coefficients of η_λ) cancel out in (2.14) because of Weinberg first-sum rules:

$$C_V{}^\kappa-C_A{}^\pi=f_\pi{}^2-f_\kappa{}^2, \quad (2.20a)$$
$$C_A{}^K-C_V{}^\kappa=f_\kappa{}^2-f_K{}^2. \quad (2.20b)$$

[12] S. Weinberg, Phys. Rev. Letters 17, 336 (1966).

Then we get from Eqs. (2.11)–(2.14), by eliminating $k_\mu\Gamma_{\mu\lambda}$ and $p_\mu\Gamma_{\mu\lambda}$,

$$\tfrac{1}{2}\sqrt{2}f_K f_\pi R_\lambda=\tfrac{1}{2}i\sqrt{2}f_\pi f_K\Gamma_\lambda=-\tfrac{1}{2}i\sqrt{2}g_{K_A}{}^{-1}g_{A_1}{}^{-1}C_A{}^K C_A{}^\pi k_\mu k_\nu\Gamma_{\mu\nu\lambda}-\tfrac{1}{4}C_A{}^\pi p_\lambda+\tfrac{1}{4}C_A{}^K k_\lambda+\tfrac{1}{4}f_\pi{}^2 p_\lambda-\tfrac{1}{4}f_K{}^2 k_\lambda$$
$$+\tfrac{1}{4}(p-k)_\nu\{\Delta_{\nu\lambda}{}^{K^*}(q)+[q_\nu q_\lambda/(q^2+m_\kappa{}^2)]f_\kappa{}^2\}-R_\lambda{}^\sigma. \quad (2.21)$$

Before we proceed further, let us first evaluate the term $\Gamma_{\mu\nu\lambda}(k,-p)$ in Eq. (2.21). Following SW, by invoking the meson dominance we make the further assumption that the current $V_\lambda{}^{4+i5}(0)$ is conserved, which means that in a relation for $\Gamma_{\mu\nu\lambda}(k,-p)$ like (2.24) of SW, we neglect terms like

$$-i\int d^4x d^4y\, e^{-ik\cdot x}e^{-iq\cdot y}\langle T\{\partial_\mu A_\mu{}^{4-i5}(x)\partial_\nu A_\nu{}^3(0)\partial_\lambda V_\lambda{}^{4+i5}(y)\}\rangle_0$$
$$=-\frac{f_\kappa m_\kappa{}^2}{q^2+m_\kappa{}^2}\int d^4x\, e^{-ik\cdot x}\langle 0|T(A_\mu{}^{4-i5}(x)A_\nu{}^3(0))|\kappa^+(-q)\rangle.$$

This is probably not a bad approximation, since the decay constant f_κ, obtained from the algebra of currents and the spectral-function sum rules and also from the broken chiral dynamics, is quite small, so that the $SU(3)$-violating effect from the κ meson is negligible. We have assumed that the main contribution to $SU(3)$-violating effects comes from the m_K-m_π mass difference, so that when we multiply $\Gamma_{\mu\nu\lambda}(k,-p)$ by $k_\nu p_\mu$, we put the physical masses m_K and m_π in the final expression. Then, making use of Eq. (3.5) of SW and symmetrizing it, following Fenster and Hussain,[13] we obtain (with our normalization)

$$\Gamma_{\mu\nu\lambda}(k,-p)\simeq(\tfrac{1}{2}i\sqrt{2})\tfrac{1}{2}(g_{K_A}/g_{A_1}+g_{A_1}/g_{K_A})\{\delta_{\mu\nu}(k-p)_\lambda+(\delta_{\mu\lambda}q_\nu-\delta_{\nu\lambda}q_\mu)(2+\delta)+\delta_{\mu\lambda}p_\nu-\delta_{\nu\lambda}k_\mu\}, \quad (2.22)$$

where δ is a dimensionless parameter, so that

$$k_\mu p_\nu\Gamma_{\mu\nu\lambda}(k,-p)=(\tfrac{1}{2}i\sqrt{2})\tfrac{1}{2}(g_{K_A}/g_{A_1}+g_{A_1}/g_{K_A})[(p-k)_\lambda\tfrac{1}{4}(1+\delta)(2q^2)+q_\lambda\tfrac{1}{4}(1+\delta)2(p^2-k^2)]. \quad (2.23)$$

Substituting (2.23) in (2.21) and making use of the relations (2.20), we obtain, on using (2.1),

$$\tfrac{1}{2}f_\pi f_K[f_+(q^2)(p-k)_\lambda+f_-(q^2)q_\lambda]=\tfrac{1}{4}(p-k)_\lambda[(f_K{}^2+f_\pi{}^2-f_\kappa{}^2)-g_{K^*}{}^2q^2/m_{K^*}{}^2(q^2+m_{K^*}{}^2)+2\times(g_{K_A}/g_{A_1}+g_{A_1}/g_{K_A})$$
$$\times g_{K_A}{}^{-1}g_{A_1}{}^{-1}C_A{}^K C_A{}^\pi\tfrac{1}{4}(1+\delta)q^2]+\tfrac{1}{4}q_\lambda\{(f_K{}^2-f_\pi{}^2)-(p^2-k^2)g_{K^*}{}^2/m_{K^*}{}^2(q^2+m_{K^*}{}^2)$$
$$+2\times(g_{K_A}/g_{A_1}+g_{A_1}/g_{K_A})g_{K_A}{}^{-1}g_{A_1}{}^{-1}C_A{}^K C_A{}^\pi(p^2-k^2)\tfrac{1}{4}(1+\delta)+[(p^2-k^2)/(q^2+m_\kappa{}^2)]f_\kappa{}^2\}-R_\lambda{}^\sigma. \quad (2.24)$$

We have already evaluated the σ contribution $R_\lambda{}^\sigma$ in RSF and can express it in the form

$$R_\lambda{}^\sigma=[(p^2-k^2)/(q^2+m_\kappa{}^2)]f_\kappa{}^2, \quad (2.25)$$

so that it cancels the last term occurring in the curly brackets on the right-hand side of (2.24). We then obtain from (2.24)

$$f_+(0)=(f_K{}^2+f_\pi{}^2-f_\kappa{}^2)/2f_\pi f_K. \quad (2.26)$$

Since, because of the Ademollo-Gatto theorem,[14] $f_+(0)=1$, to first order in $SU(3)$-symmetry breaking Eq. (2.26) predicts

$$f_\kappa{}^2=(f_K-f_\pi)^2, \quad (2.27)$$

which gives for the total width of the κ meson about 1 MeV for $\kappa\to K\pi$ with $m_\kappa=725$ MeV, which is indeed small. This justifies the approximation of taking $V_\lambda{}^{4+i5}$ to be conserved in the evaluation of $\Gamma_{\mu\nu\lambda}{}^{abc}$ in the form (2.22). If we now make use of the relations

$$C_A{}^\pi=g_{A_1}{}^2/m_{A_1}{}^2,\quad C_A{}^K=g_{K_A}{}^2/m_{K_A}{}^2, \quad (2.28)$$

and

$$g_{A_1}{}^2=g_\rho{}^2,\quad g_{K_A}{}^2=g_{K^*}{}^2,\quad m_{A_1}{}^2=2m_\rho{}^2, \quad (2.29)$$

$$C_A{}^K-C_A{}^\pi\equiv\frac{g_{K_A}{}^2}{m_{K_A}{}^2}-\frac{g_{A_1}{}^2}{m_{A_1}{}^2}=f_\pi{}^2-f_K{}^2, \quad (2.30)$$

obtained from the Weinberg spectral-function sum rules,[15] and also

$$g_{K^*}{}^2/g_\rho{}^2=m_{K^*}{}^2/m_\rho{}^2, \quad (2.31)$$

obtained from the asymptotic $SU(3)$ symmetry,[16] we obtain from (2.24) the expressions for $f_\pm(q^2)$ in the form (with the pion and kaon on the mass shell)

$$f_+(q^2)\simeq1-\frac{f_\pi}{f_K}\frac{q^2}{q^2+m_{K^*}{}^2}+\frac{1+\delta}{4}\frac{m_{K^*}{}^2+m_\rho{}^2}{2m_\rho{}^2}\times\left(\frac{2f_\pi}{f_K}-\frac{f_K}{f_\pi}\right)\frac{q^2}{m_{K^*}{}^2}, \quad (2.32)$$

[13] S. Fenster and F. Hussain, Phys. Rev. **169**, 1314 (1968).

[14] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 264 (1964).

[15] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, *ibid.* **19**, 139 (1967); S. K. Bose and R. Torgerson, *ibid.* **19**, 115 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **19**, 470 (1967); P. P. Divakaran and L. K. Pandit, *ibid.* **19**, 539 (1967).

[16] See, e.g., S. Okubo, "Asymptotic Symmetry and Algebra of Currents," lectures delivered at the Institute of Physics, University of Islamabad, 1967 (unpublished).

$$f_-(q^2)=\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)-\frac{f_\pi}{f_K}\frac{m_K{}^2-m_\pi{}^2}{q^2+m_{K^*}{}^2}+\frac{m_K{}^2-m_\pi{}^2}{m_{K^*}{}^2}$$

$$\times\left(\frac{m_{K^*}{}^2+m_\rho{}^2}{2m_\rho{}^2}\right)\frac{1+\delta}{4}\left(\frac{2f_\pi}{f_K}-\frac{f_K}{f_\pi}\right). \quad (2.33)$$

The difference between expressions (2.32) and (2.33) for $f_\pm(q^2)$ and those given by Lee[3] from chiral dynamics should be noted. We believe that this difference arises from the specific form of the chiral-dynamical Lagrangian for $K^*K\pi$ used by Lee, and also from some other approximations, which are not explicit in Ref. 3. Numerically, this difference is not very significant for $f_+(q^2)$, but for $f_-(q^2)$ it may be significant, as indicated below.

III. NUMERICAL RESULTS

The linear approximation to $f_+(q^2)$ of Eq. (2.39) gives

$$f_+(q^2)\approx 1-\lambda_+q^2/m_\pi{}^2, \quad (3.1)$$

with

$$\lambda_+=\frac{m_\pi{}^2}{m_{K^*}{}^2}\left[\frac{f_\pi}{f_K}-\frac{1+\delta}{4}\frac{m_{K^*}{}^2+m_\rho{}^2}{2m_\rho{}^2}\left(\frac{2f_\pi}{f_K}-\frac{f_K}{f_\pi}\right)\right] \quad (3.2)$$

$$=0.0166, \quad \text{for} \quad \delta=0 \quad (3.3)$$

$$=0.0172, \quad \text{for} \quad \delta=-\tfrac{1}{3} \quad (3.4)$$

where we have used $f_K/f_\pi=1.28$. The results (3.3) and (3.4) are in agreement with those quoted by Lee.[3] Now parametrizing $f_-(q^2)$ in the usual form

$$f_-(q^2)=\xi(1-\lambda_-q^2/m_\pi{}^2), \quad (3.5)$$

we have from (2.33)

$$\xi=\tfrac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)-\frac{m_K{}^2-m_\pi{}^2}{m_{K^*}{}^2}$$

$$\times\left[1-\frac{m_{K^*}{}^2+m_\rho{}^2}{2m_\rho{}^2}\left(\frac{2f_\pi}{f_K}-\frac{f_K}{f_\pi}\right)\tfrac{1}{4}(1+\delta)\right], \quad (3.6)$$

$$\xi\lambda_-=-[(m_K{}^2-m_\pi{}^2)/m_{K^*}{}^2]f_\pi/f_K. \quad (3.7)$$

The relations (3.6) and (3.7) predict

$$\xi=0.050, \quad (3.8)$$

$$\lambda_-=-0.107, \quad \text{for} \quad \delta=0 \quad (3.9)$$

and

$$\xi=\ 0.042, \quad (3.10)$$

$$\lambda_-=-0.128, \quad \text{for} \quad \delta=-\tfrac{1}{3}. \quad (3.11)$$

All along we have used the mean meson masses $m_K=495.84$ MeV and $m_\pi=137.28$ MeV. It is worthwhile to note that the results (3.8)-(3.11) for ξ and λ_- are not very sensitive to the values of δ. This is due to the fact that the terms containing δ in (2.32) and (2.33) are suppressed by the factor $2f_\pi/f_K-f_K/f_\pi$.

The experimental value[10] of λ_+, obtained from the pion momentum spectrum in K_{e3}, is (with our normalization)

$$\lambda_+=0.019\pm0.006, \quad (3.12)$$

which is in good agreement with (3.3) and (3.4). The branching ratio $K_{\mu3}/K_{e3}$ measurement yields $\xi_{\rm br}=0.6\pm0.3$, while the muon polarization measurements gives $\xi_{\rm pol}=-1.0\pm0.2$. No trustworthy value of λ_- is available as yet from experiments. Accurate experimental values of ξ and λ_- are, therefore, needed to test the above predictions from the current algebra as well as those from chiral dynamics and other approaches.

After we finished this work, we were informed by Dr. Fayyazuddin that the technique of Schnitzer and Weinberg was also recently applied to the calculation of the K_{l3} form factors by Ueda at Toronto.