processes are related through isospin invariance[2]:

$$(\pi^- N^{*+}|\pi^- p) = -(\pi^+ N^{*+}|\pi^+ p) = \sqrt{2}(\pi^0 N^{*0}|\pi^- p) = (\sqrt{\tfrac{2}{3}})(\pi^0 N^{*++}|\pi^+ p). \quad (5.5)$$

Thus a comparison of Eq. (3.17) may be made in the form

$$\sigma_{\text{tot}}(\pi^+\pi^0 p|\pi^+ p) - \sigma(\rho^+ p|\pi^+ p) - \tfrac{2}{3}\sigma(\pi^0 N^{*++}|\pi^+ p) = \sigma_{\text{tot}}(\pi^-\pi^0 p|\pi^- p) - \sigma(\rho^- p|\pi^- p). \quad (5.6)$$

The right-hand side of Eq. (5.6) has been obtained at 2.75 BeV/c by the Saclay-Orsay-Berlin-Bologna collaboration[17] as 2.05±0.1 mb and at 4 BeV/c by Bondar *et al.*[18] as 1.86 mb. The corresponding values of the left-hand sides at these energies are obtained by Armenise *et al.*[19] as 1.8±0.2 mb and by Aderholz *et al.*[20] as 1.87 mb. The remaining predictions cannot be tested due to lack of suitable data on nonresonant production cross sections.

One limitation of the model is that it is inadequate for many processes which involve Σ^- or Ξ production amplitudes with two units of charge or strangeness transfer and are experimentally observable. However, the present model has an advantage over the additivity model in that it can be easily extended to study the group structures of higher baryon resonances through such nonresonant mechanisms for production of mesons.

ACKNOWLEDGMENTS

The author would like to thank Professor A. N. Mitra for suggesting the problem and for valuable discussions, and Professor F. C. Auluck for his kind interest in this work.

Financial assistance from the Council of Scientific and Industrial Research, India, is also acknowledged.

[17] Saclay-Orsay-Berlin-Bologna Collaboration, Nuovo Cimento **35**, 713 (1965).
[18] L. Bondar *et al.*, Nuovo Cimento **31**, 729 (1964).
[19] N. Armenise *et al.*, Phys. Letters **13**, 341 (1964).
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PHYSICAL REVIEW VOLUME 173, NUMBER 5 25 SEPTEMBER 1968

Ratio of Axial-Vector to Vector Contributions in $K \to l\nu\gamma$ Decay

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Sum rules are obtained by using the dispersion technique of Fubini in the radiative weak decay $K \to l\nu\gamma$. The relevant ratio of the axial-vector to vector contributions is then predicted by saturating these sum rules with a few low-lying resonance states, and its relationship with the question of subtraction in the dispersion relation of certain amplitudes in $K \to l\nu\gamma$ is also discussed.

The radiative decays of the charged pions and kaons are of considerable interest with reference to the structure of weak interactions. Das, Mathur, and Okubo,[1] and also Riazuddin and Fayyazuddin,[2] have already considered the radiative decay of the charged pions. In the present paper, we discuss the radiative decay of the charged kaons. In the case of $\pi^\pm \to l^\pm\nu\gamma$, the relevant vector contributions can be related[3] to the matrix element of the decay, $\pi^0 \to 2\gamma$, by means of the conserved vector current (CVC) hypothesis. Instead, one can use the ρ-dominance model to evaluate this vector contribution and obtain the CVC result within a few percent.[4] In the case of $K \to l\nu\gamma$, we do not have the CVC, but we can use the K^*-dominance model to calculate the relevant vector contribution. The above estimate of the vector contributions in $K \to l\nu\gamma$ is expected to be reliable, as we are encouraged by the results of the ρ-dominance model in the case of $\pi \to l\nu\gamma$, and also as we shall be able to use the values of the relevant coupling constants with the symmetry-breaking effects taken into account. A sum rule for the structure-dependent axial-vector part of the radiative decay $K \to l\nu\gamma$ is obtained using the dispersion technique of Fubini.[5] Now, saturating the sum rules by a few long-lying resonance states, we predict the ratio $r_K(0)$ of the structure-dependent axial-vector to the vector contributions, which can be checked experimentally in $K \to l\nu\gamma$ decay. In addition, the question of subtraction in the relevant axial-vector amplitudes of $K \to l\nu\gamma$ is discussed, and its effect on the ratio r_K is then evaluated.

The reduced T-matrix element for the process $K^+(p) \to l^+(p_1) + \nu(p_2) + \gamma(k)$ is given by

$$T = T_{\text{LB}} + T', \quad (1)$$

where T_{LB} is the lepton internal bremsstrahlung term,

[1] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 859 (1967).
[2] Riazuddin and Fayyazuddin, Phys. Rev. **171**, 1428 (1968).
[3] V. G. Vaks and B. L. Ioffe, Nuovo Cimento **10**, 342 (1958).
[4] The ρ dominance gives $a_\pi(0) = 2.17 \times 10^{-4}$ MeV^{-1}, compared to $a_\pi(+\frac{1}{2}m_\pi^2) = (1.91_{-0.7}^{+0.04}) \times 10^{-4}$ MeV^{-1} obtained from the assumption of CVC and the experimental value of $\pi^0 \to 2\gamma$. It is to be noted that in the latter case, $a_\pi(\nu)$ is given at $\nu = +\frac{1}{2}m_{\pi^0}^2$, so it is then assumed that $a_\pi(\nu)$ is more or less constant in the range $\nu = 0$ to $\nu = +\frac{1}{2}m_{\pi^0}^2$.
[5] S. Fubini, Nuovo Cimento **43**, 475 (1966).

and T' is the structure-dependent term given by

$$T'=\frac{iG\sin\theta}{\sqrt{2}}\epsilon_\nu\frac{1}{(2\pi)^{9/2}}\left(\frac{m_l m_\nu}{4p_{10}p_{20}k_0}\right)^{1/2}\bar{u}(p_1)\gamma_\lambda(1+\gamma_5)u(p_2)$$
$$\times ie\int d^4x\, e^{-ik\cdot x}\langle 0|T\{j_\nu^{\text{e.m.}}(x),$$
$$\times[V_\lambda^{4-i5}(0)+A_\lambda^{4-i5}(0)]\}|K^+(p)\rangle. \quad (2)$$

For the vector contribution in Eq. (2), we define the amplitude $a_K(\nu)$ as

$$\int d^4x\, e^{-ik\cdot x}\langle 0|T\{j_\nu^{\text{e.m.}}(x),V_\lambda^{4-i5}(0)\}|K^+(p)\rangle$$
$$=\frac{a_K(\nu)}{(2\pi)^{1/2}(2p_0)^{1/2}}\epsilon_{\nu\lambda\rho\mu}p_\rho k_\mu, \quad (3)$$

where ν is defined by $\nu=-p\cdot k$. To evalute $a_K(\nu)$, we assume that it satisfies an unsubtracted dispersion relation, and saturate it with single-particle states. The absorptive part of $a_K(\nu)$ is obtained from (3):

$$[a_K(\nu)]_{\text{abs}}=i(2\pi)^4\sum_n[\delta^4(k-n)\langle 0|j_\nu^{\text{e.m.}}(k)|n\rangle$$
$$\times\langle n|V_\lambda^{4-i5}(0)|K^+(p)\rangle-\delta^4(p-k-n)$$
$$\times\langle 0|V_\lambda^{4-i5}(0)|n\rangle\langle n|j_\nu^{\text{e.m.}}(k)|K^+(p)\rangle. \quad (4)$$

The first term on the right-hand side of (4) does not contribute simply because of energy-momentum conservation; for the second term we take only the K^* single particle for the intermediate state. We then obtain

$$a_K(0)=\frac{f_{K^*}G_{K^{*+}K^+\gamma}}{(m_{K^*}^2-m_K^2)}, \quad (5)$$

where the coupling constants f_{K^*} and $G_{K^{*+}K^+\gamma}$ are defined by

$$(2q_0)^{1/2}\langle 0|V_\lambda^{4-i5}(0)|K^{*+}(q)\rangle=f_{K^*}\epsilon_\lambda, \quad (6)$$

$$(4q_0p_0)^{1/2}\langle K^+(p)|j_\nu^{\text{e.m.}}(k)|K^{*+}(q)\rangle$$
$$=G_{K^*K\gamma}\epsilon_{\nu\beta\gamma\sigma}p_\beta p_\gamma\epsilon_\sigma. \quad (7)$$

To evaluate explicitly expression (5), we can use the $SU(3)$-symmetric results[6]:

$$G_{K^{*+}K^+\gamma}=\tfrac{1}{3}G_{\omega\pi\gamma}\simeq 2/3m_\rho, \quad (8)$$

and, further, from current algebra[7] as well as from the asymptotic $SU(3)$ symmetry[8]:

$$f_{K^*}^2/f_\rho^2=m_{K^*}^2/m_\rho^2, \quad (9)$$

where, in Eq. (9), the symmetry-breaking effect is incorporated.

The relevant matrix element for the axial-vector contribution in $K^+\to l^+\nu\gamma$ is

$$M_{\nu\lambda}=i\int d^4x\, e^{-ik\cdot x}$$
$$\times\langle 0|T\{j_\nu^{\text{e.m.}}(x), A_\lambda^{4-i5}(0)\}|K^+(p)\rangle. \quad (10)$$

This can also be written in terms of the amplitudes H^i, which are functions of ν only, as

$$M_{\nu\lambda}=i[H_1(\nu)p_\nu p_\lambda+H_2(\nu)p_\nu k_\lambda+H_3(\nu)k_\nu p_\lambda$$
$$+H_4(\nu)k_\nu k_\lambda+\delta_{\nu\lambda}H_5(\nu)]. \quad (11)$$

If one insists that the sum of $T_{\text{LB}}+T'$ should be gauge-invariant,[9] then one can show that

$$ik_\nu M_{\nu\lambda}=-i\langle 0|A_\lambda^{4-i5}(0)|K^+(p)\rangle$$
$$=-f_K p_\lambda. \quad (12)$$

If we separate out the Born contributions from the amplitudes H_i, then for the remainder parts $\tilde{H}_i$, we obtain from (12):

$$\tilde{H}_5(\nu)-\nu\tilde{H}_2(\nu)=f_K, \quad (13)$$

$$-f_K+\nu\tilde{H}_1(\nu)=-f_K. \quad (14)$$

Thus, we have $\tilde{H}_1(\nu)=0$ for $\nu\neq 0$ and, for $\tilde{H}_5$, the sum rule

$$\frac{1}{2\pi i}\int\frac{\tilde{h}_5(\nu')}{\nu'}d\nu'=f_K, \quad (15)$$

where in (15) $\tilde{h}_5(\nu)$ is the coefficient of $\delta_{\nu\lambda}$ to be picked out from

$$\tilde{m}_{\nu\lambda}=i(2\pi)^4\sum_n[\delta^4(k-n)\langle 0|j_\nu^{\text{e.m.}}(k)|n\rangle$$
$$\times\langle n|A_\lambda^{4-i5}(0)|K^+(p)\rangle-\delta^4(p-k-n)$$
$$\times\langle 0|A_\lambda^{4-i5}(0)|n\rangle\langle n|j_\nu^{\text{l.m.}}(k)|K^+(p)\rangle. \quad (16)$$

Only the K_A intermediate state contributes in the second term on the right-hand side of (16), and we obtain

$$\tilde{H}_5(\nu)=\frac{g_{K_A}f_{K_AK\gamma}}{2\nu+(m_{K_A}^2-m_K^2)}, \quad (17)$$

where g_{K_A} and $f_{K_AK\gamma}$ are defined by

$$((2q_0)^{1/2}\langle 0|A_\lambda^{4-i5}(0)|K_A^+(q)\rangle=g_{K_A}\eta_\lambda, \quad (18)$$

$$(4q_0p_0)^{1/2}\langle K^+(p)|j_\nu^{\text{l.m.}}(k)|K_A^+(q)\rangle$$
$$=if_{K_AK\gamma}\eta_\mu\left[\delta_{\nu\mu}+\frac{p_\mu(p+q)_\nu}{p^2-q^2}\right]. \quad (19)$$

For the physical problem ($k^2=0$), we identify the structure-dependent axial-vector form factor $b_K(\nu)$ in (11) as

$$H_5(\nu)=f_K+\nu b_K(\nu). \quad (20)$$

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[6] See, for example, M. Gourdan, *Unitary Symmetry* (North-Holland Publishing Co., Amsterdam, 1967), p. 100.

[7] Riazuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).

[8] See, for example, S. Okubo, Lectures delivered at the Institute of Physics, University of Islamabad, 1967 (unpublished).

[9] A. N. Kamal, Nuovo Cimento **50A**, 972 (1967).

Then the quantity of direct experimental interest in $K^+\to l^+\nu\gamma$ decay is the ratio

$$r_K(0)=b_K(0)/a_K(0). \quad (21)$$

From (13), (17), and (20), we get

$$f_K=\frac{g_{K_A}f_{K_AK\gamma}}{(m_{K_A}{}^2-m_K{}^2)}, \quad (22)$$

$$b_K(0)=\left.\frac{d\tilde{H}_5(\nu)}{d\nu}\right|_{\nu=0}=\frac{-2g_{K_A}f_{K_AK\gamma}}{(m_{K_A}{}^2-m_K{}^2)^2}$$

$$=-\frac{2f_K}{(m_{K_A}{}^2-m_K{}^2)}. \quad (23)$$

From (5), (8), (9), (21), and (23), we obtain

$$r_K(0)=-0.85, \quad (24)$$

where we have used $f_K=1.28f_\pi$ and also[7] $f_\rho{}^2=2f_\pi{}^2m_\rho{}^2$.

We now discuss the case when $H_5(\nu)$ should have a subtraction:

$$\tilde{H}_5(\nu)=f_K+\frac{\nu}{2\pi i}\int\frac{\tilde{h}_5(\nu')}{\nu'(\nu'-\nu)}d\nu'. \quad (25)$$

We then obtain from (16), (20), and (25),

$$b_K(\nu)=-\frac{2g_{K_A}f_{K_AK\gamma}}{(2\nu+m_{K_A}{}^2-m_K{}^2)(m_{K_A}{}^2-m_K{}^2)}. \quad (26)$$

In the present case, we do not have (22) to eliminate the parameters g_{K_A} and $f_{K_AK\gamma}$. However, we shall evaluate (26) by using the ρ-dominance model. We have

$$(4q_0p_0)^{1/2}\langle K^+(p)|j_\nu{}^{\text{e.m.}}(k)|K_A{}^+(q)\rangle$$
$$=i\{\alpha(-k^2)[-2(p\cdot k)\eta_\nu+2(\eta\cdot k)p_\nu]$$
$$+\beta(-k^2)[-k^2\eta_\nu+(\eta\cdot k)k_\nu]\}. \quad (27)$$

Current algebra, together with partially conserved axial-vector current (PCAC) gives

$$2\alpha(m_{K_A}{}^2)+\beta(m_{K_A}{}^2)=\frac{g_{K_A}}{f_Km_{K_A}{}^2}. \quad (28)$$

If we now define the s- and d-wave coupling constants G_s and G_d for the $K_A\to K^*\pi$ vertex as

$$(4q_0k_0)^{1/2}\langle K^{*i}(k)|j_\pi{}^j|K_A{}^l(q)\rangle$$
$$=i\epsilon_{ijl}i[G_s\eta\cdot\epsilon+2G_d(\eta\cdot k)(\epsilon\cdot p)] \quad (29)$$

we obtain, from the assumption of ρ dominance and on comparison of (29) and (27),

$$\alpha(-k^2)=-\frac{f_\rho G_d}{\sqrt{2}(k^2+m_\rho{}^2)}, \quad (30a)$$

$$\beta(-k^2)=-\frac{f_\rho}{\sqrt{2}(k^2+m_\rho{}^2)m_\rho{}^2}$$
$$\times[G_s-(m_{K_A}{}^2+m_\rho{}^2)G_d], \quad (30b)$$

and

$$f_{K_AK\gamma}=\frac{2f_\rho G_d}{\sqrt{2}m_\rho{}^2}(m_{K_A}{}^2-m_K{}^2). \quad (31)$$

The expression for G_d also can be obtained from the consideration of the processes $K^*\to K\pi$ and $K_A\to K^*\pi$ and is given by Srivastava[10] in the form

$$G_d=\frac{(1-\delta)}{\sqrt{2}f_\pi}\left[-\frac{1}{4}+\frac{1-(1+\delta)f_\pi/f_K}{(1-\delta)^2}\right], \quad (32)$$

where $\delta=(m_{K_A}{}^2-2m_{K^*}{}^2)/m_{K_A}{}^2$. In deriving the relation (32), the following results from the Weinberg sum rules[10] have been used:

$$g_{K_A}=f_{K^*},\quad f_K{}^2=\frac{f_{K^*}{}^2}{m_{K^*}{}^2}-\frac{g_{K_A}{}^2}{m_{K_A}{}^2}. \quad (33)$$

Substituting Eqs. (31) and (32) in (26), we obtain

$$b_K(0)=4\frac{m_{K^*}}{m_\rho}\frac{f_\pi(1-\delta)}{(m_{K_A}{}^2-m_K{}^2)}\left[\frac{1}{4}-\frac{1-(1+\delta)f_\pi/f_K}{(1-\delta)^2}\right], \quad (34)$$

where we have also made use of (8) and (9). Thus in the case of one subtraction in the dispersion relation for $\tilde{H}_5(\nu)$, we predict from (34) and (15)

$$r_K(0)=0.05,\quad \text{for}\quad \delta=0 \quad (35a)$$
$$=0.11,\quad \text{for}\quad \delta=0.09. \quad (35b)$$

The values of $r_K(0)$ in (35) are quite small because of the cancellation of two terms in the square bracket of (34). It is also worthwhile to note the difference of signs in (23) and (34). Further consequences of this in $K\to Kl\nu\gamma$ and $K\to\pi l\nu\gamma$ will be considered in a separate publication[11]:

It is, therefore, desirable to have an experimental value for $r_K(0)$, so that the above predictions can be tested; in addition, further understanding of the strangeness-changing weak vertex may also be gained.

The author would like to thank Dr. Riazuddin for discussions.

[10] P. P. Srivastava, Phys. Letters **26B**, 233 (1968).

[11] If one includes the $\omega-\phi$ mixing, the results (8), (24), and (35) are expected to change by about 10%. We do not take this into account explicitly, since the various other approximations and approximate formulas used in our calculations have the same order of accuracy.