Three-Particle Final States in the Quark Model

SUTAPA DAS GUPTA
Department of Physics and Astrophysics, University of Delhi, Delhi 7, India
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Scattering processes at high energies involving the production of a single meson are studied in the framework of a $SU(6)\times O(3)$ quark model which was recently proposed for high-energy hadron-hadron processes. A large number of interesting sum rules, including some weaker predictions of $SU(3)$, are obtained for the nonresonant production cross sections of pp, $\pi^{\pm}p$, and $K^{\pm}p$ processes. A comparison of the results with the available experimental information for these reactions indicates fair agreement for many of these predictions.

1. INTRODUCTION

THE success of the independent quark model for high-energy scattering[1] has recently led to the applications of the model to certain inelastic processes, which include inelastic meson-baryon processes,[2] baryon-antibaryon annihilation at high energies,[3] multipion photoproduction,[4] joint decay correlations in resonance production,[5] and so on. The additive hypothesis for the scattering amplitude has been subjected to a detailed study and the possibility of nonadditive contributions to high-energy cross sections has been considered by several authors.[6,7] These analyses show that the experimental data for hadron-hadron collisions can be explained within 10% if only nonannihilation production amplitudes are assumed to obey the additivity assumption.

In view of this, several investigations of nonannihilation production reactions have been recently carried out in the additive quark model.[8,9] In particular, a quark concept has been proposed in the additive model for multipion-production processes[9] ($NN\rightarrow Ns\pi$) which are taken as the sum of two-quark interactions leading to the production of quarks and antiquarks which rearrange into s pions

$$QQ'\rightarrow \bar{Q}\bar{Q}'+s\pi.$$

These additive QQ' amplitudes have been used to connect the production processes $\sigma(\pi N\rightarrow N(s+1)\pi)$ and $\sigma(NN\rightarrow Ns\pi)$ with remarkable success.

In this paper, we wish to consider scattering processes with three-particle final states in an $SU(6)\times O(3)$ quark model of baryons and resonances.[10] Several interesting results have already been derived from this model for various phenomena such as strong decays of (70,3) baryon resonances,[11] photoproduction, and inelastic hadron-hadron reactions.[12] One feature of this model is that the meson, as a tightly bound state of Q-$\bar{Q}$ pair, is treated as "elementary" relative to the loose $3Q$ baryon composites. Using mesons as probes, the baryon structures can thus be investigated through individual meson-quark (ΠQ) interactions. The meson-baryon processes $\Pi B\rightarrow \Pi\,\Pi B$ and $\Pi B\rightarrow B_{10}{}^*\Pi\,\Pi$ (where B, $B_{10}{}^*$ denote a baryon octet and decuplet, respectively, and Π is a meson octet) are taken as a sum of $SU(3)$-spin-invariant meson-quark interactions with the production of a meson at each baryon quark,

$$\Pi+Q\rightarrow \Pi+\Pi+Q.$$

The $SU(3)$-spin-invariant structure of this basic amplitude can be easily recognized by counting the number of $SU(3)$ singlets in the direct product of irreducible representations of $SU(3)$, one for each particle, which is of the form

$$\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8},$$

where $\mathbf{3}$ is the quark representation and $\mathbf{8}$ is the pseudoscalar meson octet (no quark-antiquark structure). The $SU(3)$-spin degree of freedom of the basic amplitude is evaluated for appropriate initial and final $3Q$ baryon wave functions. This procedure gives a number of relations for the nonresonant production cross sections of the $(B_8\Pi\,\Pi)$ and $(B_{10}{}^*\Pi\,\Pi)$ final states. The elementary meson concept however, does not allow us to connect the production processes $\sigma(NN\rightarrow \Pi BB)$ and $\sigma(\Pi N\rightarrow \Pi\,\Pi B)$ as has been considered in the additive model.

The proton-proton reactions with three-particle final states may be considered separately in the present model as proceeding via the $SU(3)$-spin-invariant

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[12] See G. C. Joshi, V. S. Bhasin, and A. N. Mitra, [Phys. Rev. **156**, 1572 (1967)] for meson-baryon processes; S. Das Gupta and A. N. Mitra [*ibid.* **156**, 1581 (1967)] for photoproduction; S. Das Gupta and A. N. Mitra, [*ibid.* **159**, 1285 (1967)] for baryon-baryon processes, which is referred to as BB in text; A. N. Mitra, [*ibid.* **167**, 1382 (1968)] for a $QQ\bar{Q}$ model for meson-baryon processes.

process

$$Q+Q\rightarrow Q+Q+\Pi,$$

with the direct emission of a meson in two-quark interactions, without suggesting any rearrangement of final-state quarks and antiquarks into mesons as in the additive model. The essential calculation is similar to that developed in a quark-model formalism for baryon-baryon processes (which we refer to as BB). On proceeding on analogous lines with BB, one can distinguish between two types of QQ interactions acting in the pp system. A strong, short-ranged QQ force acts between the quarks of the same baryon, and a second type of QQ force (weaker and long-ranged) acts between the quarks of different hadrons. The production amplitude is generated by the weak, long-ranged force. These two types of forces have nonoverlapping domains of validity' so that the quarks of different baryons have no chance of exchanging positions. The pp system may be thus regarded not as a "six-body" symmetry, but as two separate three-body systems with the symmetry confined to each $3Q$ system. The basic $QQ\rightarrow QQ\Pi$ amplitude can have the $SU(3)$-invariant form which is recognized from the direct product $\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\mathbf{8}$ to be of the types, $\lambda_i^{(1)}\cdot\Pi$, $\lambda_j^{(2)}\cdot\Pi$, $[\lambda_{i\alpha}^{(1)},\lambda_{j\beta}^{(2)}]_- f_{\alpha\beta\gamma}\Pi_\gamma$, and $[\lambda_{i\alpha}^{(1)},\lambda_{j\beta}^{(2)}]_+ d_{\alpha\beta\gamma}\Pi_\gamma$, where $\lambda_{i\alpha}^{(1)}$ is the Gell-Mann λ matrix for the quark (i) of baryon (1). The $SU(3)$ structure of the basic $QQ\rightarrow QQ\Pi$ amplitude is now evaluated to give a set of relations for the amplitudes which correspond to various ($BB\Pi$) final states of $pp\rightarrow BB\Pi$. The term $[\lambda_{i\alpha}^{(1)},\lambda_{j\beta}^{(2)}]_- f_{\alpha\beta\gamma}\Pi_\gamma$ in the amplitude does not contribute, owing to the assumption that $SU(3)$ symmetry is confined separately to each $3Q$ baryon states.

The present model may be taken as complementary to the "resonant" mechanism $pp\rightarrow B_{10}{}^*B\rightarrow BB\Pi$ which proceeds via an intermediate resonant ($B_{10}{}^*B$) state. An experimental comparison for the present analysis is thus restricted exclusively to the nonresonant background amplitude.

In Sec. 2, we consider the necessary formalism and evaluation of the $pp\rightarrow BB\Pi$ amplitudes and obtain certain sum rules which include the relation predicted from $SU(3)$. In Sec. 3, we discuss the structure and evaluation of the $\Pi B\rightarrow \Pi\,\Pi B$ amplitude. A number of interesting relations are derived for these amplitudes. In Sec. 4, we consider the production of a meson in association with a decuplet in $\pi^\pm p$ and $K^\pm p$ collisions, ($\Pi B\rightarrow B_{10}{}^*\Pi\,\Pi$). Finally, Sec. 5 deals with the experimental comparison and discussion of the results which are obtained only for the nonresonant background contributions to the production cross sections. A comparison with the available experimental data indicates fair agreement for some of these predictions.

2. PRODUCTION PROCESSES $NN\rightarrow \Pi BB$

The $SU(3)$-invariant $BB\rightarrow BB\Pi$ amplitude is given by

$$\langle\psi_f(1)\psi_f(2)|\sum_{i=1}^{3}\sum_{j=1}^{3}A_{ij}|\psi_i(1)\psi_i(2)\rangle.$$

$$A_{ij}=a_{ij}\lambda_i^{(1)}\cdot\Pi+b_{ij}\lambda_j^{(2)}\cdot\Pi+c_{ij}[\lambda_{i\alpha}^{(1)},\lambda_{j\beta}^{(2)}]_+ d_{\alpha\beta\gamma}\Pi_\gamma,\quad(2.1)$$

where the $SU(3)$ indices α,β,γ run from 1, 2, $\cdots$, 8 and have the appropriate spin-spatial structure to give the correct matrix element. The $3Q$ baryon wave functions $\psi(1)$, $\psi(2)$ are explicitly listed elsewhere as[10]

$$\psi=(1/\sqrt{2})\psi^s(\chi'\phi'+\chi''\phi''),\quad(2.2)$$

where ψ^s is the spatial wave function, and (χ',χ'') and (ϕ',ϕ'') are mixed-symmetric wave functions in spin and $SU(3)$ space, respectively. To evaluate the $SU(3)$ degree of freedom of (2.1), the following sixteen types of terms are calculated for each (ij) pair of quarks in exact analogy with BB,

$$M_{ij}=\langle(\phi',\phi'')_{(1)}\times(\phi',\phi'')_{(2)}|\times A_{ij}|(\phi',\phi'')_{(1)}\times(\phi',\phi'')_{(2)}\rangle.\quad(2.3)$$

An $SU(3)$-type relation is obtained for the various amplitudes $BB\rightarrow BB\Pi$ when the same sum rule is satisfied on comparing M_{ij} separately for each (ij) value of quark indices. We obtain, for the physically interesting process $pp\rightarrow BB\Pi$, the following sum rules for the amplitudes, which we denote by $(BB\Pi|pp)$:

$$\sqrt{2}(\Sigma^0pK^+|pp)+(\Sigma^+pK^0|pp)=-2(\Sigma^+nK^+|pp),\quad(2.4)$$

$$\sqrt{3}(np\eta|pp)=(np\pi^+|pp)+2(\Sigma^+pK^0|pp).\quad(2.5)$$

We also recover an $SU(3)$ prediction which was obtained by Bar-Nir and Harari,[13] and is of the form

$$\sqrt{2}(np\pi^+|pp)+\sqrt{2}(p\Sigma^+K^0|pp)+\sqrt{3}(\Lambda^0pK^+|pp)+(\Sigma^0pK^+|pp)=0.\quad(2.6)$$

Only a crude comparison of Eq. (2.4) with experiment is possible with present available data for nonresonant background cross sections of proton-proton reactions at 6 BeV/c,[14] and is in reasonable agreement as has been shown in Sec. 5. In the next section, we consider the slightly more involved case of meson production in meson-nucleon collisions.

3. STRUCTURE AND EVALUATION OF $\Pi N\rightarrow \Pi\Pi B$ AMPLITUDE

The $SU(3)$-invariant form for the basic meson-quark amplitude $\Pi_\beta{}^aQ_i\rightarrow\Pi_\gamma{}^b\Pi_\delta{}^cQ_i$ can be recognized from

[13] I. M. Bar-Nir and H. Harari, Phys. Rev. **144**, 1363 (1966).
[14] W. Chinowsky *et al.*, Phys. Rev. **165**, 1466 (1968).

the $SU(3)$ singlets in the direct product of the irreducible representations of $SU(3)$, one for each particle,

$$\mathbf{3}\otimes\mathbf{8}\otimes\bar{\mathbf{3}}\otimes\mathbf{8}\otimes\mathbf{8},$$

where **3** is the quark representation and **8** the meson octet (no quark-antiquark structure). These are of the types

$$\begin{aligned} A&=\sum_{m=1}^{8} a_m O_m,\\ O_1&=\mathrm{Tr}[\Pi_\beta{}^a,\Pi_\gamma{}^b]_+\Pi_\delta{}^c,\\ O_2&=\mathrm{Tr}[\Pi_\beta{}^a,\Pi_\gamma{}^b]_-\Pi_\delta{}^c,\\ O_3&=\mathrm{Tr}\lambda_\alpha\Pi_\delta{}^c\delta_{\beta\gamma},\\ O_4&=\mathrm{Tr}[\lambda_\alpha\Pi_\gamma{}^b\delta_{\beta\delta}+\lambda_\alpha\Pi_\beta{}^a\delta_{\gamma\delta}],\\ O_{5,6}&=\mathrm{Tr}\lambda_\alpha[\Pi_\delta{}^c,[\Pi_\beta{}^a,\Pi_\gamma{}^b]_+]_\pm,\\ O_{7,8}&=\mathrm{Tr}\lambda_\alpha[\Pi_\delta{}^c,[\Pi_\beta{}^a,\Pi_\gamma{}^b]_-]_\pm, \end{aligned} \tag{3.1}$$

where $\Pi_\beta{}^a$ ($\Pi_\gamma{}^b,\Pi_\delta{}^c$) is the octet representation for the pseudoscalar meson a (b,c) with $SU(3)$ index β (γ,δ) and λ_α is the Gell-Mann matrix for the baryon quark. The $SU(3)$ indices α, β, γ, and δ run from 1, 2, $\cdots$, 8. The coefficients a_m denote the spin and spatial structures of the amplitude. The $\Pi N\to\Pi\,\Pi B$ amplitude is obtained as

$$M=\sum_{i=1}^{3} M_i=\sum_{i=1}^{3}\langle\psi_f|A_i|\psi_i\rangle, \tag{3.2}$$

where ψ_f, ψ_i are baryon wave functions of Eq. (2.2). The cases of physical interest are single-meson production in $\pi^\pm p$ and $K^\pm p$ collisions. An evaluation of the $SU(3)$ degree of freedom in Eq. (3.2) is sufficient to obtain $SU(3)$-type relations among the amplitudes of these processes. Four types of amplitudes are obtained for each baryon quark i:

$$\langle(\phi',\phi'')|A_i|(\phi',\phi'')\rangle. \tag{3.3}$$

A comparison of M_i for the processes $\Pi N\to\Pi\,\Pi B$ gives a set of sum rules. These relations are obtained separately for each baryon quark i, which gives for the total $(\Pi\,\Pi B|\Pi N)$ amplitudes, the following $SU(3)$ predictions obtained by Bar-Nir and Harari[13]

$$\begin{aligned}(\pi^-\pi^0p|\pi^-p)&+\sqrt{3}(\pi^-\eta p|\pi^-p)\\&=\sqrt{2}(K^-K^0p|\pi^-p)+\sqrt{2}(\pi^-K^0\Sigma^+|\pi^-p),\end{aligned} \tag{3.4}$$

$$\begin{aligned}\sqrt{2}(\pi^-\pi^+n|\pi^-p)&+\sqrt{3}(\pi^-K^+\Lambda^0|\pi^-p)\\&=\sqrt{2}(K^-K^+n|\pi^-p)+(\pi^-K^+\Sigma^0|\pi^-p),\end{aligned} \tag{3.5}$$

$$\begin{aligned}\sqrt{2}(\pi^-\pi^+n|\pi^-p)&+(\pi^-\pi^+\Sigma^0|K^-p)\\&+\sqrt{3}(K^-K^+\Lambda^0|K^-p)=\sqrt{3}(\pi^-\pi^+\Lambda^0|K^-p)\\&+(K^-K^+\Sigma^0|K^-p)+\sqrt{2}(K^-K^+n|\pi^-p),\end{aligned} \tag{3.6}$$

$$\begin{aligned}(\pi^+\pi^0p|\pi^+p)&+\sqrt{3}(\pi^+\eta p|\pi^+p)\\&=\sqrt{2}(K^+K^0\Sigma^+|K^+p)+\sqrt{2}(K^+\bar{K}^0p|\pi^+p),\end{aligned} \tag{3.7}$$

$$\begin{aligned}(K^+\pi^0p|K^+p)&+\sqrt{2}(K^+K^0\Sigma^+|K^+p)\\&=\sqrt{2}(\pi^+K^0p|K^+p)+\sqrt{3}(K^+\eta p|K^+p).\end{aligned} \tag{3.8}$$

The following new predictions are also obtained in the present model:

$$(\pi^-K^0\Sigma^+|\pi^-p)=(K^+K^0\Sigma^+|K^+p), \tag{3.9}$$

$$(\pi^+K^0\Sigma^+|\pi^+p)=(K^-K^0\Sigma^+|K^-p), \tag{3.10}$$

$$\begin{aligned}(\pi^-\pi^+\Sigma^0|K^-p)&=\sqrt{2}(K^-K^+n|\pi^-p)\\&+\sqrt{3}(\pi^-\pi^+\Lambda^0|K^-p),\end{aligned} \tag{3.11}$$

$$(\pi^-\pi^+n|\pi^-p)=(K^-\pi^+n|K^-p)+(K^-K^+n|\pi^-p), \tag{3.12}$$

$$\begin{aligned}(K^-\pi^0p|K^-p)+\sqrt{3}(K^-\eta p|K^-p)&=2(\pi^-\pi^0p|\pi^-p)\\&+2\sqrt{2}(\pi^-K^0\Sigma^+|\pi^-p),\end{aligned} \tag{3.13}$$

$$\begin{aligned}(\pi^-K^+\Sigma^0|\pi^-p)&=\sqrt{3}(\pi^-K^+\Lambda^0|\pi^-p)\\&+\sqrt{2}(K^-\pi^+n|K^-p),\end{aligned} \tag{3.14}$$

$$\begin{aligned}(\pi^+K^+\Sigma^0|\pi^+p)&=\sqrt{3}(\pi^+K^+\Lambda^0|\pi^+p)\\&+\sqrt{2}(K^+\pi^+n|K^+p),\end{aligned} \tag{3.15}$$

$$\begin{aligned}(K^+K^+\Sigma^0|K^+p)&=\sqrt{2}(\pi^+\pi^+n|\pi^+p)\\&+\sqrt{3}(K^+K^+\Lambda^0|K^+p).\end{aligned} \tag{3.16}$$

On considering the additional symmetry of the two final-state mesons, one obtains $O_2=O_{7,8}=0$ in Eq. (3.1). We obtain the following additional relations:

$$(\pi^+\pi^0p|\pi^+p)=(\pi^-\pi^0p|\pi^-p), \tag{3.17}$$

$$\begin{aligned}(\pi^-K^+\Lambda^0|\pi^-p)&+(\pi^-\pi^+\Lambda^0|K^-p)\\&=(K^-K^+\Lambda^0|K^-p),\end{aligned} \tag{3.18}$$

$$\begin{aligned}(\pi^-K^+\Sigma^0|\pi^-p)&+(\pi^-\pi^+\Sigma^0|K^-p)\\&=(K^-K^+\Sigma^0|K^-p),\end{aligned} \tag{3.19}$$

$$\begin{aligned}\sqrt{2}(\pi^0K^+\Sigma^+|\pi^+p)&+(\pi^+K^0\Sigma^+|\pi^+p)\\&=(\pi^+K^+\Sigma^0|\pi^+p).\end{aligned} \tag{3.20}$$

4. PRODUCTION PROCESSES $\Pi N\to\Pi\,\Pi B_{10}{}^*$

The production of a decuplet in association with mesons in meson-nucleon collisions can be considered in this simple model by evaluating Eq. (3.2) for an initial proton and a final baryon resonance $B_{10}{}^*$. This case is simpler since the $SU(3)$ matrix elements to be evaluated are of two types for each baryon quark:

$$\mu_i'=\langle\phi^s|\sum_{m=3}^{8}a_m{}^iO_m{}^i|\phi'\rangle, \tag{4.1}$$

$$\mu_i''=\langle\phi^s|\sum_{m=3}^{8}a_m{}^iO_m{}^i|\phi''\rangle. \tag{4.2}$$

An $SU(3)$-type relation is obtained when the same relation is separately satisfied by each of μ_i', μ_i'' for the various processes $\Pi N\to\Pi\,\Pi B_{10}{}^*$. For the quark Q_1 ($i=1$), we note that $\mu_1'=0$. A comparison of μ_1'' for these processes gives the following relations for the amplitudes which we denote as $(\Pi\,\Pi B_{10}{}^*|\Pi B)$:

$$
\begin{aligned}
(K^+K^0Y^{*+}|K^+p) &= -(K^+\pi^+N^{*0}|K^+p)\\
&= -(\pi^+\pi^+N^{*0}|\pi^+p)\\
&= (1/\sqrt{2})(\pi^+K^+Y^{*0}|\pi^+p)\\
&= (\pi^-K^0Y^{*+}|\pi^-p)\\
&= -\tfrac{1}{4}[(\pi^-\pi^0N^{*+}|\pi^-p)+(\pi^+\pi^0N^{*+}|\pi^+p)], \quad (4.3)
\end{aligned}
$$

$$
\begin{aligned}
(K^-K^0Y^{*+}|K^-p) &= (\pi^+K^0Y^{*+}|\pi^+p)\\
&= -(K^-\pi^+N^{*0}|K^-p)\\
&= -\sqrt{2}(\pi^-K^+Y^{*0}|\pi^-p), \quad (4.4)
\end{aligned}
$$

$$
\begin{aligned}
(K^-K^0N^{*+}|\pi^-p) &= 2\sqrt{2}(K^+\pi^0Y^{*+}|\pi^+p)\\
&= -(K^+\bar{K}^0N^{*+}|\pi^+p). \quad (4.5)
\end{aligned}
$$

On considering the symmetry of the two final-state mesons, $O_{7,8}=0$ and we obtain the additional relations:

$$
\begin{aligned}
(\pi^-\pi^0N^{*+}|\pi^-p) &= (\pi^+\pi^0N^{*+}|\pi^+p)\\
&= -2(\pi^-K^0Y^{*+}|\pi^-p)\\
&= 2(\pi^+\pi^+N^{*0}|\pi^+p)\\
&= (K^+\pi^0N^{*+}|K^+p)\\
&= -2(K^+K^0Y^{*+}|K^+p). \quad (4.6)
\end{aligned}
$$

Equations (4.3)–(4.6) are within easy reach of experiment, although a comparison is not possible with the present experimental data for such processes.

5. EXPERIMENTAL COMPARISON OF RESULTS AND DISCUSSION

It is important to note that the calculations based on the present model are true only for:

(i) nonresonant background amplitudes,

(ii) at high energies when the mass-splitting effects within the baryon and meson octets are negligible.

These present serious difficulties in comparing the predictions with experiment. For most of these processes, the experimental data are available for the limited momentum range of 1–4 BeV/c, which by (ii) is not a suitable range of comparison for the model. Also the experimental information is obtained, except for a limited case, only for the total three-particle final states of these processes. In a U-spin analysis of three-particle final states, Bar-Nir and Harari[13] have obtained sum rules for the background amplitudes, which are also predicted by the present model. These authors have shown that by considering an appropriate phase-space correction, the total cross sections can be compared at the same values of total energy E for a comparison of these predictions.

Within the present model, however, the corrections due to mass-splitting and phase space cannot be considered without further assumptions on the dynamics of quarks. Thus the nonresonant background cross sections must be considered for a discussion of these results. In this respect, one may note that an estimation for the relative contributions of resonances in the three-particle final states has also been obtained in recent experiments. In these analysis, the values for the mass, widths, and the relative amounts of well-known resonances which best explain the mass distribution plots of $(B\Pi\,\Pi)$ states have been estimated by a Breit-Wigner fit for the resonance processes. The nonresonant contributions to the $(B\Pi\,\Pi)$ states can be obtained by subtracting the appropriate resonant contributions from the total cross sections for the three-particle states.

With such a point of view, we can still look for a crude comparison of some of the predictions with experiment. Equation (2.3) requires

$$
\sigma^{1/2}(K^0\Sigma^+p|pp)+(2\sigma)^{1/2}(K^+\Sigma^0p|pp) = 2\sigma^{1/2}(K^+\Sigma^+n|pp). \quad (5.1)
$$

At 6-BeV/c incident momentum, from the data of Chinowsky *et al.*,[14] the cross sections $\sigma^{1/2}(K^0\Sigma^+p|pp)$ and $(2\sigma)^{1/2}(K^+\Sigma^0p|pp)$ are obtained as 5.09 and 5.83 μb, respectively, with unresolved possible resonant contributions due to a N^* resonance with $T=\frac{3}{2}$ and mass$=1900\,\text{MeV}/c^2$. From isospin invariance (see BB),[12]

$$
\sigma(N^*p|pp)=\tfrac{1}{3}\sigma(N^{*++}n|pp). \quad (5.2)
$$

The resonance contributions due to $N^{*++}(1920)$ have been found to account for $(38\pm5)\%$ of the total cross section $\sigma_{\text{tot}}(\Sigma^+K^+n|pp)$. From Eq. (5.2), this gives resonant effects of about 10% in $\sigma_{\text{tot}}(\Sigma^0K^+p|pp)$ and $\sigma_{\text{tot}}(\Sigma^+K^0p|pp)$ which lie within the experimental uncertainties. The nonresonant background contribution to $\sigma^{1/2}(\Sigma^+K^+n|pp)$ is obtained as 5.94 μb. These give the figures of 10.92 μb and 11.88 μb, respectively, for the left-hand side and right-hand side of Eq. (5.1), which is in reasonable agreement with the predictions.

Equation (3.9) gives

$$
\sigma(K^+K^0\Sigma^+|pp)=\sigma(\pi^-K^0\Sigma^+|\pi^-p). \quad (5.3)
$$

At 3-BeV/c incident momentum, the cross section $\sigma(K^+K^0\Sigma^+|K^+p)$ is obtained as 15 ± 5 μb by Joldersma *et al.*[15] with no appreciable evidence for resonances. The corresponding value of the total cross section $\sigma_{\text{tot}}(\pi^-K^0\Sigma^+|\pi^-p)$ is obtained by Dahl *et al.*[16] at 3 BeV/c. According to these authors, the three-particle state $(\pi^-K^0\Sigma^+)$ is dominated by 29% of $Y_1{}^{*0}(1385)$ and 14% of $Y_0{}^*(1517)$ for the momentum interval 2.9–3.3 BeV/c. This gives a value of 25 ± 5 μb for the nonresonant cross section $\sigma(\pi^-K^0\Sigma^+|\pi^-p)$, which is not inconsistent, within large experimental inaccuracies, with Eq. (3.9).

For a comparison Eq. (3.17) which requires

$$
\sigma(\pi^+\pi^0p|\pi^+p)=\sigma(\pi^-\pi^0p|\pi^-p), \quad (5.4)
$$

one notices that the resonant cross sections in these

[15] T. Joldersma *et al.*, Phys. Rev. Letters **17**, 716 (1966).
[16] O. Dahl *et al.*, Phys. Rev. **163**, 1377 (1967).

processes are related through isospin invariance[2]:

$$(\pi^- N^{*+}|\pi^- p) = -(\pi^+ N^{*+}|\pi^+ p) = \sqrt{2}(\pi^0 N^{*0}|\pi^- p) = (\sqrt{\tfrac{2}{3}})(\pi^0 N^{*++}|\pi^+ p). \quad (5.5)$$

Thus a comparison of Eq. (3.17) may be made in the form

$$\sigma_{\rm tot}(\pi^+\pi^0 p|\pi^+ p) - \sigma(\rho^+ p|\pi^+ p) - \tfrac{2}{3}\sigma(\pi^0 N^{*++}|\pi^+ p) = \sigma_{\rm tot}(\pi^-\pi^0 p|\pi^- p) - \sigma(\rho^- p|\pi^- p). \quad (5.6)$$

The right-hand side of Eq. (5.6) has been obtained at 2.75 BeV/c by the Saclay-Orsay-Berlin-Bologna collaboration[17] as 2.05 ± 0.1 mb and at 4 BeV/c by Bondar *et al.*[18] as 1.86 mb. The corresponding values of the left-hand sides at these energies are obtained by Armenise *et al.*[19] as 1.8 ± 0.2 mb and by Aderholz *et al.*[20] as 1.87 mb. The remaining predictions cannot be tested due to lack of suitable data on nonresonant production cross sections.

One limitation of the model is that it is inadequate for many processes which involve Σ^- or Ξ production amplitudes with two units of charge or strangeness transfer and are experimentally observable. However, the present model has an advantage over the additivity model in that it can be easily extended to study the group structures of higher baryon resonances through such nonresonant mechanisms for production of mesons.

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Ratio of Axial-Vector to Vector Contributions in $K \to l\nu\gamma$ Decay

A. Q. SARKER
Institute of Physics, University of Islamabad, Rawalpindi, Pakistan
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Sum rules are obtained by using the dispersion technique of Fubini in the radiative weak decay $K \to l\nu\gamma$. The relevant ratio of the axial-vector to vector contributions is then predicted by saturating these sum rules with a few low-lying resonance states, and its relationship with the question of subtraction in the dispersion relation of certain amplitudes in $K \to l\nu\gamma$ is also discussed.

THE radiative decays of the charged pions and kaons are of considerable interest with reference to the structure of weak interactions. Das, Mathur, and Okubo,[1] and also Riazuddin and Fayyazuddin,[2] have already considered the radiative decay of the charged pions. In the present paper, we discuss the radiative decay of the charged kaons. In the case of $\pi^\pm \to l^\pm\nu\gamma$, the relevant vector contributions can be related[3] to the matrix element of the decay, $\pi^0 \to 2\gamma$, by means of the conserved vector current (CVC) hypothesis. Instead, one can use the ρ-dominance model to evaluate this vector contribution and obtain the CVC result within a few percent.[4] In the case of $K \to l\nu\gamma$, we do not have the CVC, but we can use the K^*-dominance model to calculate the relevant vector contribution. The above estimate of the vector contributions in $K \to l\nu\gamma$ is expected to be reliable, as we are encouraged by the results of the ρ-dominance model in the case of $\pi \to l\nu\gamma$, and also as we shall be able to use the values of the relevant coupling constants with the symmetry-breaking effects taken into account. A sum rule for the structure-dependent axial-vector part of the radiative decay $K \to l\nu\gamma$ is obtained using the dispersion technique of Fubini.[5] Now, saturating the sum rules by a few long-lying resonance states, we predict the ratio $r_K(0)$ of the structure-dependent axial-vector to the vector contributions, which can be checked experimentally in $K \to l\nu\gamma$ decay. In addition, the question of subtraction in the relevant axial-vector amplitudes of $K \to l\nu\gamma$ is discussed, and its effect on the ratio r_K is then evaluated.

The reduced T-matrix element for the process $K^+(p) \to l^+(p_1) + \nu(p_2) + \gamma(k)$ is given by

$$T = T_{\rm LB} + T', \quad (1)$$

where $T_{\rm LB}$ is the lepton internal bremsstrahlung term,

[1] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 859 (1967).

[2] Riazuddin and Fayyazuddin, Phys. Rev. **171**, 1428 (1968).

[3] V. G. Vaks and B. L. Ioffe, Nuovo Cimento **10**, 342 (1958).

[4] The ρ dominance gives $a_\pi(0) = 2.17\times10^{-4}$ MeV^{-1}, compared to $a_\pi(+\frac{1}{2}m_\pi^2) = (1.91_{-0.7}^{+0.04})\times10^{-4}$ MeV^{-1} obtained from the assumption of CVC and the experimental value of $\pi^0 \to 2\gamma$. It is to be noted that in the latter case, $a_\pi(\nu)$ is given at $\nu = +\frac{1}{2}m_{\pi^0}{}^2$, so it is then assumed that $a_\pi(\nu)$ is more or less constant in the range $\nu = 0$ to $\nu = +\frac{1}{2}m_{\pi^0}{}^2$.

[5] S. Fubini, Nuovo Cimento **43**, 475 (1966).