Deck[18] and others have pointed out, the assumption of extreme forward peaking of the t_{13} amplitude leads the 2-3 c.m. projections of Fig. 1 to be peaked in the low-s region, producing, or quite possibly distorting, low-mass enhancements in the 2-3 channel. Further work on rescattering corrections to the Deck mechanism is desirable.

[18] R. T. Deck, Phys. Rev. Letters **13**, 169 (1964).

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Threshold Electropion Production from Current Algebra and Partially Conserved Axial-Vector Current

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Threshold electropion production on nucleons, $e+N\rightarrow e+N+\pi$, is studied by current-algebra techniques using the hypothesis of partially conserved axial-vector current, which have proved useful in describing low-energy meson-baryon elastic scattering and photopion production on nucleons. The electric and longitudinal multipole moments E_{0+} and L_{0+} are calculated at threshold in terms of the form factors of the electromagnetic and weak axial-vector currents. The experimental upper bounds on the slope of the differential cross section as a function of $|\mathbf{q}|$, the momentum in the πN c.m. system, i.e., $(1/|\mathbf{q}|)(d^2\sigma/d\Omega dS_{20}{}^L)$, where $S_{20}{}^L$ is the laboratory energy of the final electron, are sufficiently strong to relate the form factors for various values of $-k^2$, the momentum transfer squared of the electrons. More precisely, in this way one can relate the neutron charge form factor $G_e{}^n(k^2)$ to normalized axial-vector form factor $F_A(k^2)$. If one takes $F_A(k^2)$ to have the dipole form $F_A(k^2)=(1+k^2/M_A{}^2)^{-2}$ with $M_A{}^2=1.42$ BeV^2, which is given by arguments based on chiral $SU(2)\times SU(2)$ and consistent with recent neutrino experiments, then the resulting values of $G_e{}^n(k^2)$ in the range considered, $0.2\leq k^2\leq 0.6$ BeV^2, are consistent with information about $G_e{}^n(k^2)$ from electron-deuteron and thermal-neutron–electron scattering.

I. INTRODUCTION

IN the past few years much activity in elementary-particle physics has been devoted to the complete exploitation of the principle that the equal-time commutators of the weak and electromagnetic currents of the strongly interacting particles form a chiral $SU(2)\times SU(2)$ algebra.[1] One of the most fruitful branches of these researches has been the investigation of low-energy processes involving these currents. The current algebra together with the hypothesis of partially conserved axial-vector current (PCAC) leads to simple models which compare remarkably well with the presently available data on meson-baryon scattering.[2] These methods have also been applied to the case of pion photoproduction.[3] Here we consider the extension to pion electroproduction.

Electroproduction provides an interesting problem both theoretically and experimentally, and has received considerable attention. Earlier analysis utilized the static model and stressed the importance of the electroproduction process in describing the nucleon form factors.[4] Fubini, Nambu, and Wataghin[4] (FNW) noted

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[1] For a complete review of the references on current algebras, see S. L. Adler and R. F. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968); and B. Renner, *Current Algebras and Their Applications* (Pergamon Press, Inc., New York, 1968).

[2] A. P. Balachandran, M. G. Gundzik, and F. Nicodemi, Nuovo Cimento **44**, 1257 (1966); Y. Tomazawa, *ibid.* **46**, 707 (1966); K. Raman and E. C. G. Sudarshan, Phys. Letters **21**, 450 (1966); Phys. Rev. **154**, 1499 (1967); S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[3] A. P. Balachandran, M. G. Gundzik, P. Narayanaswami, and F. Nicodemi, Ann. Phys. (N.Y.) **45**, 339 (1967); M. S. Bhatia and P. Narayanaswami, Phys. Rev. (to be published).

[4] G. F. Chew, F. Low, M. L. Goldberger, and Y. Nambu, Phys. Rev. **106**, 1345 (1957); S. Fubini, Y. Nambu, and A. Wataghin, *ibid.* **111**, 329 (1958); R. Blankenbecler, S. Gartenhaus, R. Huff, and Y. Nambu, Nuovo Cimento **42**, 775 (1960); P. Dennery, Phys. Rev. **124**, 2000 (1961).

that the scattering amplitude could be made manifestly gauge-invariant. Further, by examining the lowest-order perturbation theory of strong interactions, they found that the subtraction terms for their dispersion treatment of this problem are associated with the pion pole-diagrams.

The PCAC and current-algebra hypothesis have very definite implications about the structure of the electroproduction process. There are at present two ways to utilize these new theoretical structures. In one approach, Adler and Gilman[5] and Riazuddin and Lee[6] compare the standard dispersion treatment of electroproduction which utilizes pseudoscalar pion-nucleon coupling with the PCAC-current-algebra approach, which in effect utilizes pseudovector pion-nucleon coupling. Equating these approaches and using a zero-pion-mass argument to eliminate the continuum contributions in the latter, they derive sum rules. As is the case in all electroproduction calculations, these sum rules place restrictions on the form factors.

In our approach, we consider the PCAC and current-algebra hypotheses as supplying the entire structure of the electroproduction process near threshold. This implies that in this region the only possible coupling in the Born amplitudes is the effective pseudovector coupling. There is no use of the equivalence theorem which characterizes the previous approaches. We then apply the methods of Ref. 3 to derive threshold multipole moments for the electroproduction process. Utilizing these results as well as the experimental data, we derive restrictions among the electromagnetic and pseudovector weak form factors. Assuming a resonable structure for the pseudovector weak form factor, an analysis of the neutron electric form factor for $0.2 \leqslant k^2 \leqslant 0.6$ BeV^2 is carried out.

The work is divided as follows. In Sec. II we present the necessary kinematics and expressions for the multipole moments in terms of invariant amplitudes. In Sec. III we discuss the method of calculating the threshold moments and the results are presented in Sec. IV. The comparison of our results to the pion-electroproduction data near threshold and the resultant restrictions among the electromagnetic and pseudovector weak form factors are presented in Sec. V. Finally, we summarize our work in Sec. VI.

II. KINEMATICS

We assume the process $e+N \to e+N+\pi$ to second order in the electromagnetic couplings, but all orders in the strong couplings, proceeds via the diagram in Fig. 1. The initial (final) four-momenta of the electrons and nucleons in the c.m. system of the final πN system are $S_1(S_2)$ and $p_1(p_2)$, respectively. Also in this c.m. system, the virtual photon of four-momentum $k=(k_0,\mathbf{k})$ has a mass $\lambda^2=-k^2$ and the final meson has four-

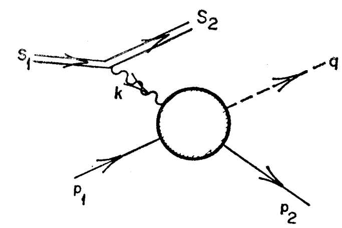

FIG. 1. Diagram for the process $e+N \to e+N+\pi$.

momentum $q=(q_0,\mathbf{q})$. The external nucleon, electron, and pion masses are designated M, M_e, and μ. The invariant energy and momentum transfer variables are defined by

$$\nu=-(p_1+p_2)\cdot k/2M\,,\quad \nu_B=q\cdot k/2M\,,$$

and

$$\nu_B-\nu=(W^2-M^2)/2M\,,$$

where W is the invariant mass of the final $\pi+N$ system.

The T matrix, defined by

$$S_{fi}=\delta_{fi}+(2\pi)^4\delta^4(p_2+q+k_2-p_1-k_1) \times(M^2M_e^2/p_{10}p_{20}S_{10}S_{20}2q_0)^{1/2}T_{fi}\,, \quad (2.1)$$

can be shown to have the form[4]

$$T_{fi}=\epsilon^\mu F_\mu\,,$$

where

$$\epsilon^\mu=e\bar{u}(S_2)\gamma_\mu u(S_1)/(S_2-S_1)^2 \quad (2.2)$$

and

$$F_\mu=\langle qp_2|j_\mu|p_1\rangle. \quad (2.3)$$

F_μ contains the electromagnetic current of the strongly interacting particles. Conservation of this current implies the condition

$$k_\mu F^\mu=0.$$

Further the photon polarization ϵ^μ satisfies the condition

$$k_\mu\epsilon^\mu=0.$$

Following Adler and Gilman,[5] we factor the amplitude into its isospin parts as

$$F_\lambda=a^{(+)}V_\lambda^{(+)}+a^{(-)}V_\lambda^{(-)}+a^{(0)}V_\lambda^{(0)}\,, \quad (2.4)$$

where

$$a^{(\pm)}=\chi_f^*\psi_c^*\tfrac{1}{4}(\tau_c\tau_3\pm\tau_3\tau_c)\chi_i\,,$$
$$a^{(0)}=\chi_f^*\psi_c^*\tfrac{1}{2}\tau_c\chi_i\,,$$

with ψ_c, χ_f, and χ_i as the isospinors of the final pion, final nucleon, and initial nucleon, respectively. The space-spin part of the amplitude can be expanded in terms of six invariants as

$$\epsilon^\lambda V_\lambda^{(\pm,0)}=\sum_{i=1}^{6} V_i^{(\pm,0)}(\nu,\nu_B,\lambda^2)\bar{u}(p_2)O(V_i)u(p_1)\,, \quad (2.5)$$

[5] S. L. Adler and F. J. Gilman, Phys. Rev. 152, 1460 (1966).
[6] Riazuddin and B. W. Lee, Phys. Rev. 146, 1202 (1966).

where

$$O(V_1)=\tfrac{1}{2}i\gamma_5\{\gamma,\gamma\}, \qquad \eta_1=1 \tag{2.6a}$$

$$O(V_2)=i\gamma_5\{p_1+p_2,q\}, \qquad \eta_2=1 \tag{2.6b}$$

$$O(V_3)=\gamma_5\{\gamma,q\}, \qquad \eta_3=-1 \tag{2.6c}$$

$$O(V_4)=\gamma_5\{\gamma,p_1+p_2\}-iM\gamma_5\{\gamma,\gamma\}, \quad \eta_4=1 \tag{2.6d}$$

$$O(V_5)=i\gamma_5\{k,q\}, \qquad \eta_5=-1 \tag{2.6e}$$

$$O(V_6)=\gamma_5\{k,\gamma\}, \qquad \eta_6=-1 \tag{2.6f}$$

with $\{A,B\}\equiv A\cdot\epsilon B\cdot k-A\cdot kB\cdot\epsilon$. The η_i are used to specify the crossing properties of the associated amplitudes

$$V_j^{(\pm,0)}(\nu,\nu_S,\mu^2\lambda^2)=(\pm,+)\eta_j V_j^{(\pm,0)}(-\nu,\nu_B,\mu^2,\lambda^2). \tag{2.7}$$

We also expand our amplitudes in terms of the usual c.m. frame amplitudes. These are defined as

$$\epsilon_\lambda V_\lambda^{(\pm,0)}=\sum_{j=1}^{6}\mathfrak{F}_j^{(\pm,0)}\chi_f^*\textstyle\sum_j\chi_i, \tag{2.8}$$

where χ_f and χ_i are the nucleon spinors and the $\sum_j$'s are those of Ref. 5. The multipole expansions are given by

$$M_{l+}=\frac{1}{2(l+1)}\int_{-1}^{1}dy\left[\mathfrak{F}_1P_l(y)-\mathfrak{F}_2P_{l+1}(y)-\mathfrak{F}_3\frac{(1-y^2)P_l'(y)}{l(l+1)}\right], \tag{2.9a}$$

$$M_{l-}=\frac{1}{2l}\int_{-1}^{1}dy\left[-\mathfrak{F}_1P_l(y)+\mathfrak{F}_2P_{l-1}(y)+\mathfrak{F}_3\frac{(1-y^2)}{l(l+1)}P_l'(y)\right], \tag{2.9b}$$

$$E_{l+}=\frac{1}{2(l+1)}\int_{-1}^{1}dy\left[\mathfrak{F}_1P_l(y)-\mathfrak{F}_2P_{l+1}(y)+\mathfrak{F}_3\frac{(1-y^2)}{(l+1)}P_l'(y)+\mathfrak{F}_4\frac{(1-y^2)}{l+2}P_{l+1}'(y)\right], \tag{2.9c}$$

$$E_{l-}=\frac{1}{2l}\int_{-1}^{1}dy\left[\mathfrak{F}_1P_l(y)-\mathfrak{F}_2P_{l+1}(y)-\mathfrak{F}_3\frac{(1-y^2)}{l}P_l'(y)-\mathfrak{F}_4\frac{(1-y^2)}{(l-1)}P_{l-1}'(y)\right], \tag{2.9d}$$

$$L_{l+}=\frac{1}{2(l+1)}\int_{-1}^{1}dy[k_0\mathfrak{F}_5P_l(y)+k_0\mathfrak{F}_6P_{l+1}(y)], \tag{2.9e}$$

and

$$L_{l-}=\frac{1}{2l}\int_{-1}^{1}dy[k_0\mathfrak{F}_5P_l(y)+k_0\mathfrak{F}_6P_{l-1}(y)]. \tag{2.9f}$$

The index $l\pm$ of the multipole specifies the orbital angular momentum l and the total angular momentum $J=l\pm\frac{1}{2}$ of the final pion-nucleon system. The linear relationship between the $\mathfrak{F}_i$'s and the V_i's is given in Ref. 5.

III. METHOD

The method that we apply is a direct application of the approach to pion-nucleon elastic scattering and pion photoproduction developed by Balachandran *et al.*[3,7] It is based on the observation that the threshold values of the physical multipole moments of the process under study are reached by a smooth extrapolation from those found at zero-pion mass. We discuss the off-mass-shell (OMS) extrapolation of the electroproduction amplitude and then the expansion of the OMS multipole moments in energy (ϵ) and κ in the region $|\kappa|\leqslant\mu_\pi$ and $\epsilon\simeq0$. The quantities κ and ϵ are defined as $\kappa=(-q^2)^{1/2}$ and $\epsilon=q_0-\kappa$.

We begin by defining an amplitude $G(\epsilon,\kappa)$ which satisfies the requirement

$$G_\lambda(\epsilon,\kappa)|_{\kappa=\mu}=F_\lambda\equiv\langle qp_2|j_\lambda|p_1\rangle. \tag{3.1}$$

This amplitude is defined by means of the usual reduction formula to be

$$G_\lambda(\epsilon,\kappa)=i(q^2+\mu^2)\psi_c^*\int d^4x\,\theta(x_0)e^{-iqx}+\langle p_2|[\phi_c(x),j_\lambda(0)]|p_1\rangle, \tag{3.2}$$

where ϕ_c is the Hermitian pion field with isospin index α. The reduction formula can be used to guarantee (3.1), but it does not insure the uniqueness of (3.2). Having noted this possible ambiguity which characterizes all OMS extrapolations, we study $G(\epsilon,\kappa)$ as our extrapolated amplitude. The next step is the replacement in (3.2) of the pion field with the divergence of the axial-vector current.

$$\partial^\mu A_\mu{}^\alpha=C\phi^\alpha, \tag{3.3}$$

where $|C|=\sqrt{2}|C_+|$, $|C_+|$ being the PCAC constant for the charged pion field, $|C_+|=0.935\mu^3$.

Integrating by parts, we obtain

$$\begin{aligned}G_\lambda(\epsilon,\kappa)=&-\frac{(q^2+\mu^2)}{C}\psi_\alpha^*\int d^4y\,e^{-iqy}\delta(y_0)\\&\times\langle p_2|[A_0{}^\alpha(y),j_\lambda(0)]|p_1\rangle-\frac{(q^2+\mu^2)}{C}q^\mu\psi_\alpha^*\\&\times\int d^4y\,e^{-iqy}\theta(y_0)\langle p_2|[A_\mu{}^\alpha(y),j_\lambda(0)]|p_1\rangle\\=&E_\lambda(\epsilon,\kappa)+R_\lambda(\epsilon,\kappa),\end{aligned} \tag{3.4}$$

[7] A. P. Balachandran, M. G. Gundzik, and F. Nicodemi, Nucl. Phys. (to be published); *Lectures in Theoretical Physics* (Gordon and Breach Science Publishers, Inc., New York, 1967), Vol. IXB, p. 361.

where $E_\lambda(\epsilon,\kappa)$ is given by the covariant parts of the equal-time commutator and $R_\lambda(\epsilon,\kappa)$ contains all the rest. In the usual spirit of current algebras, we assume that the covariant part of the equal-time commutator is given by the quark-model commutation relations. The Schwinger terms of the equal-time commutator and the retarded commutator, both of which are separately noncovariant, combine in $R_\lambda(\epsilon,\kappa)$ to form a covariant total.[8] We are now in a position to discuss these two covariant parts separately.

The $E_\lambda(\epsilon,\kappa)$ term is given completely by the quark-model commutation relations. The oddness in the isospin indices of the commutation relations require that the $E_\lambda(\epsilon,\kappa)$ be zero for the isospin components (0) and (+). We also note that the extrapolated amplitude $G_\lambda(\epsilon,\kappa)$ does not satisfy a divergence-zero condition in its isospin (−) part. The commutation relations are consistent with attributing this nonvanishing divergence to both $E_\lambda^{(-)}(\epsilon,\kappa)$ and $R_\lambda^{(-)}(\epsilon,\kappa)$ parts of the amplitude. The total amplitude can be made divergence-free by means of the trick of FNW.[4] In this case, a new extrapolated amplitude is defined in terms of the present extrapolation by means of the formula

$$G_\lambda'(\epsilon,\kappa)=G_\lambda(\epsilon,\kappa)-k_\lambda k^\mu G_\mu(\epsilon,\kappa)/k^2. \tag{3.5}$$

Obviously since the (0) and (+) isospin components are already divergenceless the only modification arises in the (−) isospin part. Since $k\cdot\epsilon=0$, this added part makes no contribution to the extrapolated scattering amplitude

$$\mathcal{T}_{fi}(\kappa)\big|_{q^2=\mu^2}=T_{fi}, \tag{3.6}$$

where

$$\mathcal{T}_{fi}(\epsilon,\kappa)\equiv\epsilon\cdot G'(\epsilon,\kappa)=\epsilon\cdot G(\epsilon,\kappa). \tag{3.7}$$

The $R_\lambda(\epsilon,\kappa)$ term is a covariant with the usual analytic properties of a retarded commutator. The noncovariant Schwinger terms and seagull graphs[8,9] cancel to allow the usual analysis of $R_\lambda(\epsilon,\kappa)$ into a covariant pole term and continuum. The pole term can be computed exactly. The continuum contributions are the unknowns which limit the range of validity of the ϵ,κ expansion. Following the analysis of Balachandran *et al.*,[3] we find that the continuum will have contributions to the lowest nonvanishing order in κ and ϵ in the lowest moments. Therefore, we find that by neglecting the continuum contributions we can calculate the lowest moments exactly. This result is reminiscent of the low-energy theorem of Low[10] and of Gell-Mann and Goldberger.[11] In their case, they found that the low-energy or Thompson limit of the Compton scattering amplitude was given exactly by the pole contributions and that the continuum or excited states contributed only in the next order in the expansion parameter. In our case, we find that the single-nucleon-pole diagrams give the entire contribution to the lowest-order parts of E_{0+} and L_{0+} for the (+) and (0) amplitudes while for the (−) amplitude the poles and equal-time commutator give the total contribution.

It was noted in Ref. 3 that the combination of multipoles which is free of the threshold unitarity cut is the proper function to use for defining an OMS. This function is a regular function of κ at threshold. Thus in analogy with the treatment of photoproduction we define the on-mass-shell quantities with $q^2=2\kappa\epsilon+\epsilon^2$ as

$$\tfrac{1}{2}[E_{0+}(\epsilon,\mu)+E_{0+}{}^{II}(\epsilon,\mu)]=e_1(\mu)+f_1(\mu)q^2+O(q^4)$$
$$=e_1(\mu)+2\mu f_1(\mu)\epsilon+O(q^4), \tag{3.8a}$$

$$\tfrac{1}{2}[L_{0+}(\epsilon,\mu)+L_{0+}{}^{II}(\epsilon,\mu)]=l_1(\mu)$$
$$+2\mu m_1(\mu)\epsilon+O(q^4), \tag{3.8b}$$

where $E_{0+}{}^{II}$ $(L_{0+}{}^{II})$ is the continuation of E_{0+} (L_{0+}) to the second sheet reached by encircling the threshold unitarity cut. For simplicity, we neglect the elastic unitarity cut. Then, the functions with superscript *II* are identical to those without it, and we can replace the left-hand side of the above equations by the first-sheet functions. Using the method of Ref. 7, the effect of this simplification can be estimated and found to introduce an error of less than about 15%. Thus we define the extrapolation

$$e_1(\kappa)=E_{0+}(0,\kappa),\quad f_1(\kappa)=\frac{1}{2\mu}\frac{\partial E_{0+}(\epsilon,\kappa)}{\partial\epsilon}\bigg|_{\epsilon=0},$$

$$l_1(\kappa)=L_{0+}(0,\kappa),\quad m_1(\kappa)=\frac{1}{2\mu}\frac{\partial L_{0+}(\epsilon,\kappa)}{\partial\epsilon}\bigg|_{\epsilon=0},$$

where $E_{0+}(\epsilon,\kappa)$ and $L_{0+}(\epsilon,\kappa)$ are the OMS moments calculated from the amplitude discussed above. Denoting the separation into continuum (C) and noncontinuum (N) parts, e_1 and l_1, e.g., have the form

$$e_1(\kappa)=e_1{}^N(\kappa)+e_1{}^C(\kappa),$$
$$l_1(\kappa)=l_1{}^N(\kappa)+l_1{}^C(\kappa).$$

Using the method of Ref. 3 with $k^2\neq0$ for estimating the continuum contributions, we find

$$e_1{}^C(\kappa)=O(\kappa),\quad f_1{}^C(\kappa)=O(1)$$
$$l_1{}^C(\kappa)=O(\kappa),\quad m_1{}^C(\kappa)=O(1).$$

The OMS extrapolation of the higher moments, i.e., M_{1-}/q, E_{1+}/q, and M_{1+}/q, as defined in Ref. 3, are found to be $O(1)$.

IV. CALCULATION OF E_{0+} AND L_{0+} AT THRESHOLD

To obtain E_{0+} and L_{0+} using the method described in Sec. III we first evaluate (3.4) taking the covariant

[8] L. S. Brown, Phys. Rev. **150**, 1338 (1966).
[9] S. L. Adler and Y. Dothan, Phys. Rev. **151**, 1267 (1966).
[10] F. E. Low, Phys. Rev. **96**, 1428 (1954).
[11] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **96**, 1433 (1954).

single nucleon pole and equal-time commutator contributions. We find that the $V_i^{(\pm,0)}$ of (2.2) then have the following form:

$$V_1^{(+,0)} = -\tfrac{1}{2}eN(q^2)\Big[\mathcal{G}(q^2)F_1^{(V,S)}(k^2)\Big(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\Big) - (1/M)G_1(q^2)F_2^{(V,S)}(k^2)\Big], \quad (4.1a)$$

$$V_1^{(-)} = -\tfrac{1}{2}eN(q^2)\Big[\mathcal{G}(q^2)F_1^{(V)}(k^2) \times\Big(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\Big)\Big], \quad (4.1b)$$

$$V_2^{(\pm,0)} = (e/4M\nu_B)N(q^2) \times\Big[\mathcal{G}(q^2)F_1^{(V,S)}(k^2)\Big(\frac{1}{\nu_B-\nu}\pm\frac{1}{\nu_B+\nu}\Big)\Big], \quad (4.1c)$$

$$V_3^{(\pm,0)} = (e/4M)N(q^2) \times\Big[\mathcal{G}(q^2)F_2^{(V,S)}(k^2)\Big(\frac{1}{\nu_B-\nu}\mp\frac{1}{\nu_B+\nu}\Big)\Big], \quad (4.1d)$$

$$V_4^{(\pm,0)} = (e/4M)N(q^2) \times\Big[\mathcal{G}(q^2)F_2^{(V,S)}(k^2)\Big(\frac{1}{\nu_B-\nu}\pm\frac{1}{\nu_B+\nu}\Big)\Big], \quad (4.1e)$$

$$V_5^{(+,0)} = V_6^{(+,0)} = 0, \quad (4.1f)$$

$$V_5^{(-)} = (e/k^2)N(q^2)\Big[\frac{\mathcal{G}(q^2)}{\nu_B}F_1^V(k^2)+G_2(t)\Big], \quad (4.1g)$$

$$V_6^{(-)} = (e/k^2)N(q^2)[G_1(q^2)F_1^V(k^2)-G_1(t)]. \quad (4.1h)$$

In the above, the (+) and (0) parts have the same signs and we have made the following replacements:

$$\mathcal{G}(q^2) \equiv G_1(q^2)-(q^2/2M)G_2(q^2),$$
$$N(q^2) \equiv (\mu^2+q^2)/C.$$

Here $t=-(q-k)^2$, G_1 and G_2 are the axial-vector and induced pseudoscalar form factors with $G_1(0)=g_A=1.18$, $F_1^{(V,S)}$ and $F_2^{(V,S)}$ refer to the isovector (V) and isoscalar (S) electromagnetic charge and magnetic form factors; they are normalized as follows.

$$F_1^{(V)}(0)=F_1^S(0)=1, \quad (4.2)$$

$$F_2^V(0)=\kappa^p-\kappa^n=3.70, \qquad F_2^S(0)=\kappa^p+\kappa^n=-0.12. \quad (4.3)$$

κ^p and κ^n are the anomolous proton and neutron magnetic moments.

The V_i's of Eqs. (4.1) can now be used with the results of Ref. 5 relating them to the $\mathcal{F}_i$'s of Sec. II and then to the multipole moments of equations (2.9). Taking the noncontinuum part of the multipoles as an approximation to the nonvanishing physical-threshold multipole moments, we find

$$E_{0+}^{(+,0)}(0,\mu) \simeq e_1^{(+,0)}(\kappa=0) = \frac{eN(0)g_A\sigma k^2}{4M(2M^2+k^2)} \times[F_1^{(V,S)}(k^2)+F_2^{(V,S)}(k^2)], \quad (4.4a)$$

$$L_{0+}^{(+,0)}(0,\mu) \simeq l_1^{(+,0)}(\kappa=0) = \frac{eN(0)g_A\sigma k^2}{4M(2M^2+k^2)} \times\Big[F_1^{(V,S)}(k^2)-\frac{k^2}{4M^2}F_2^{(V,S)}(k^2)\Big], \quad (4.4b)$$

$$E_{0+}^{(-)}(0,\mu) \simeq e_1^{(-)}(\kappa=0) = \frac{-eN(0)\sigma g_A}{2M} \times\Big[G_{1,0}(k^2)-\frac{4M^2F_1^V(k^2)}{(2M^2+k^2)}-\frac{3k^2}{2(2M^2+k^2)} \times[F_1^V(k^2)-\tfrac{1}{3}F_2^V(k^2)]\Big], \quad (4.4c)$$

$$L_{0+}^{(-)}(0,\mu) \simeq l_1^{(-)}(\kappa=0) = \frac{-eN(0)\sigma g_A}{2M} \times\Big[2G_{1,0}(k^2)-\frac{F_1^V(k^2)(2k^2+5M^2)}{(2M^2+k^2)} - \frac{F_2^V(k^2)k^4}{8(2M^2+k^2)M^2}\Big], \quad (4.4d)$$

where

$$\sigma=(4M^2+k^2)^{1/2},$$

$$G_{1,0}(k^2)=G_{1,0}(\epsilon=0,\kappa=0)=\tfrac{1}{2}\int_{-1}^{1}G_1(t)dz, \quad (4.5)$$

with z the cosine of the angle between $\mathbf{k}$ and $\mathbf{q}$ in the πN c.m. system.

V. COMPARISON WITH EXPERIMENT

We now consider the application of our above results for the threshold multipole moments to the analysis of pion electroproduction data near threshold. It is clear that if the experimental data are sufficiently well determined, then our expressions for E_{0+} and L_{0+} at threshold should provide us with information about the vector and axial-vector form factors.

First, we discuss the properties of the vector and axial-vector form factors as a function of k^2 and what is known of them from analysis of various experiments. Then, we consider the form for the differential cross section for pion electroproduction near threshold as a function of L_{0+} and E_{0+}.

We find that the experimental data near threshold place a strong restriction on the charge-neutron form factor $F_1{}^n(k^2)$ and the axial-vector form factor $G_1(k^2)/g_A \equiv F_A(k^2)$. Using a simple model for $F_A(k^2)$ which is consistent with the recent Argonne neutrino experiment,[12] we find that for values of k^2 between 0.2 and 0.6 BeV^2, $F_1{}^n(k^2)$ is small and consistent with zero.

A. Vector Form Factors

The electromagnetic form factors used above are more conveniently related to experiment through the various proton (p) and neutron (n) parts

$$F_{1,2}{}^V(k^2)=F_{1,2}{}^p(k^2)-F_{1,2}{}^n(k^2)\,, \qquad (5.1a)$$

$$F_{1,2}{}^S(k^2)=F_{1,2}{}^p(k^2)+F_{1,2}{}^n(k^2)\,, \qquad (5.1b)$$

by the form factors[13]

$$G_m{}^{p,n}(k^2)=F_1{}^{p,n}(k^2)+F_2{}^{p,n}(k^2)\,, \qquad (5.2a)$$

$$G_e{}^{p,n}(k^2)=F_1{}^{p,n}(k^2)-(k^2/4M^2)F_2{}^{p,n}(k^2)\,. \qquad (5.2b)$$

Experimentally, one finds that $G_e{}^p$, $G_m{}^n$, and $G_m{}^p$ are determined and are well represented by the dipole fit[14]

$$G_e{}^p(k^2)=\frac{G_m{}^p(k^2)}{\mu_p}=\frac{G_m{}^n(k^2)}{\mu_n}=\left(1+\frac{k^2}{M_V{}^2}\right)^{-2}, \qquad (5.3)$$

where $M_V{}^2=0.71$ BeV^2 and μ_p and μ_n are the proton and neutron magnetic moments, respectively;

$$\mu_p=2.79,\ \mu_n=-1.91\,. \qquad (5.4)$$

On the contrary, $G_e{}^n(k^2)$ is not very well known for a large range of values of k^2. The present evidence[14] indicates that $G_e{}^n$ is small and $\lesssim 0.2$ but not zero. Elastic electron-deuteron scattering below $k^2=0.2$ BeV^2 analyzed by an appropriate model[14] is found to be consistent with the slope

$$dG_e{}^n/dk^2|_{k^2=0}=+0.0193\pm0.0004\ \mathrm{F}^2\,,$$

determined from scattering of thermal neutrons on atomic electrons.[15] Inelastic and quasielastic electron-deuteron scattering experiments have also provided information about $G_e{}^n$. The Stanford data of Hughes *et al.*[16] lead to complex values for $G_e{}^n(k^2)$ for $k^2\leqslant 0.5$ BeV^2 and values consistent with zero for higher k^2.[17] The data of Stein *et al.*[16] leads to values of $G_e{}^n(k^2)$ between 0.1 and 0 below $k^2=0.6$ BeV^2. Finally, the data of Grossetete *et al.*[16] analyzed by Hasselmann and Kramer[17] give $G_e{}^n$ consistent with thermal neutron scattering but with rather large errors. The data of the last two experiments appear in Fig. 2.

Since in our range $0.2\leqslant k^2\leqslant 0.6$ the situation regarding the value of $G_e{}^n(k^2)$ is not clear, we leave this as a parameter to be determined by our analysis. The remaining electromagnetic form factors are given by the dipole fit (5.3).

B. Axial-Vector Form Factor

The axial-vector form factor is not very well known, though several measurements of it have been made in recent experiments[12,18] of neutrino elastic scattering, i.e., $\nu+n\rightarrow\mu^-+p$. In these experiments, the data are fitted using the form factor $F_A(t)$ in the form

$$F_A(t)\equiv G_1(t)/g_A=(1-t/M_A{}^2)^{-n}\,, \qquad (5.5)$$

where t is the momentum transfer squared and $n=1$ or 2 with correspondingly different values of $M_A{}^2$ determined.

In our analysis we take $n=2$ for convenience. Then, $G_{1,0}(k^2)$ of (4.5) becomes

$$G_{1,0}(k^2)=\lim_{\epsilon=0,\kappa=0} g_A\left[\left(1+\frac{\kappa^2+k^2+2k_0q_0}{M_A{}^2}\right)^2-\frac{(2|\mathbf{k}|\,|\mathbf{q}|)^2}{M_A{}^4}\right]^{-1},$$

$$G_{1,0}(k^2)=g_A\left(1+\frac{k^2}{M_A{}^2}\right)^{-2}. \qquad (5.6)$$

The value of M_A can be determined in several ways. The simplest approach is to utilize again the properties of the chiral algebra. Assuming that the current algebra is a manifestation of an exact asymptotic $SU(2)\times SU(2)$ symmetry, Fayyazuddin and Hussain[19] show that

$$(M_A/M_V)^4=(\mu_p-\mu_n)/g_A=3.99\,, \qquad (5.7)$$

where M_V is the mass parameter used in the double-pole fit of the vector form factors; see Eq. (5.3). Schecter and Venturi[19] obtain the same result using the $SU(3)\times SU(3)$ chiral symmetry.

This result is in agreement with the preliminary

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Argonne neutrino elastic scattering results[12] where a dipole fit to the experimental data gives

$$M_A = 1.1 \pm 0.3\ \text{BeV}.$$

However, it does not agree so well with the results of the recent analysis of the CERN[18] data. A dipole fit to their experimental neutrino elastic scattering data gives

$$M_A = 0.81_{-0.20}^{+0.13}\ \text{BeV}.$$

Both numbers are obtained from normalization-independent fits.[20] This difference in the two experiments as well as other results are as yet unresolved.

C. Differential Cross Section for Electropion Production

The form of the differential electropion production cross section near threshold in terms of the multipole moments, using the notation of Sec. II, is given by[21]

$$H(k^2,S_{10}^L) \equiv \frac{1}{|\mathbf{q}|}\frac{d^2\sigma}{d\Omega dS_{20}^L},$$

$$H(k^2,S_{10}^L) = \frac{\alpha}{2(2\pi)^3}\frac{M}{W}\frac{S_{20}^L}{S_{10}^L}\frac{1}{k^2}\left\{\left(1-\frac{2m_e^2}{k^2}\right)|E_{0+}|^2 + \frac{k^2}{k_0^2\mathbf{k}^2}(\tfrac{1}{2}k^2 - 2S_{10}S_{20}) \times\left[\left(1-\frac{\mathbf{k}^2}{k^2}\right)|E_{0+}|^2 - |L_{0+}|^2\right]\right\}. \quad (5.8)$$

Here $\alpha = 1/137 = e^2/4\pi$ and *all* quantities are in the c.m. system of the final πN except those quantities with the (L) superscript which are in the laboratory system. We consider the slope of the differential cross section $H(k^2,S_{10}^L)$ as a function of the three-momentum $|\mathbf{q}|$.

In the pion electroproduction experiment[22] which we shall use, only the final electron is observed. Thus $H(k^2,S_{10}^L)$ is an incoherent sum of the π^+n and π^0p final states. Noting that the corresponding π^0p and π^+n multipoles are given by

$$E_{0+}^{\pi^0p} = E_{0+}^{(+)} + E_{0+}^{(0)},$$
$$E_{0+}^{\pi^+n} = \sqrt{2}(E_{0+}^{(-)} + E_{0+}^{(0)}) \quad (5.9)$$

and the same for $L_{0+}^{\pi^0p}$ and $L_{0+}^{\pi^+n}$, then in terms of the form factors of (5.1), (5.2), and (5.5) and using the results of (4.4) for E_{0+} and L_{0+}, we have the following:

$$E_{0+}^{\pi^0p} = \Omega G_m^p(k^2), \quad (5.10a)$$

$$L_{0+}^{\pi^0p} = \Omega G_e^p(k^2), \quad (5.10b)$$

$$E_{0+}^{\pi^+n} = \sqrt{2}\Omega\left\{\mu_n G_e^p(k^2) + \left[2\Delta F_1(k^2) - \left(1+\frac{k^2}{M_A^2}\right)^{-2}\right]\left(1+\frac{2M^2}{k^2}\right)\right\}, \quad (5.10c)$$

$$L_{0+}^{\pi^+n} = \sqrt{2}\Omega\left\{\left(1+\frac{k^2}{4M^2}(\mu_p-\mu_n)\right)G_e^p(k^2) - \Delta F_1(k^2)\left(1+\frac{k^2}{4M^2}\right) + \left[\tfrac{5}{2}\Delta F_1(k^2) - 2\left(1+\frac{k^2}{M_A^2}\right)^{-2}\right]\left(1+\frac{2M^2}{k^2}\right)\right\}. \quad (5.10d)$$

In the above, we have defined

$$\Delta F_1(k^2) \equiv F_1^p(k^2) - F_1^n(k^2), \quad (5.11a)$$

$$\Omega \equiv +\frac{k^2}{C}\frac{(\mu^2)}{2M}g_A\frac{(4M^2+k^2)^{1/2}}{(2M^2+k^2)}. \quad (5.11b)$$

The rather large systematic errors that occur in pion-electroproduction measurements near threshold because of the relatively large contribution due to the radiation losses of the electrons make it impossible to obtain accurate values of $H(k^2,S_{10}^L)$. The best one might hope for is an upper bound obtained by taking the measured points for $H(k^2,S_{10}^L)$ with the smallest systematic errors, as near to threshold as possible, and making a linear extrapolation to threshold. The resultant upper bound provides information only for those values of k^2 and S_{10}^L where the values of the kinematic coefficients in (5.8) place a strong restriction on L_{0+} and/or E_{0+}.

In Table I we give the upper bounds for $H(k^2,S_{10}^L)$ at threshold in units of $\hbar = c = \mu = 1$ determined by the above method from the data of Lynch *et al.*[22] These values include their estimates for the systematic errors as well as the values of the kinematic coefficients that appear in (5.8) for various k^2 and S_{10}^L. In all cases except that for $k^2 = 0.1\ \text{BeV}^2$ and $S_{10}^L = 200$ MeV, we find that $|L_{0+}^{\pi^+n}|^2$ must be very nearly zero in order that the bound be satisfied. This restriction leads to various values for $\Delta F_1(k^2)$ given M_A^2 and vice versa. In

TABLE I. Experimental upper bound on $H(k^2,S_{10}^L)$ at threshold as well as coefficients of Eq. (5.8). Here we have $A \equiv \Omega^2[\alpha/2(2\pi)^3] \times (M/W)(S_{20}^L/S_{10}^L)(1/k^2)$ and $B \equiv [k^2/k_0^2(\mathbf{k})^2](\frac{1}{2}k^2 - 2S_{10}S_{20})$.

k^2 (BeV^2)	S_{10}^L (MeV)	$H(k^2,S_{10}^L)$ Upper bound at threshold ($\mu=1$)	A ($\mu=1$)	B ($\mu=1$)
0.1	200	11.0×10^{-7}	2.70×10^{-10}	-11.15
0.2	400	3.65×10^{-7}	6.35×10^{-10}	-230
0.3	200	1.05×10^{-7}	5.55×10^{-10}	-415
0.4	400	1.40×10^{-7}	9.18×10^{-10}	-322
0.6	400	0.27×10^{-7}	10.7×10^{-10}	-18.9

[20] In addition, the data are fitted in the CERN experiment assuming $F_1^n(k^2) \equiv 0$ to obtain $F_A(k^2)$, while in the Argonne experiment it is assumed that $G_e^n(k^2) = 0$.

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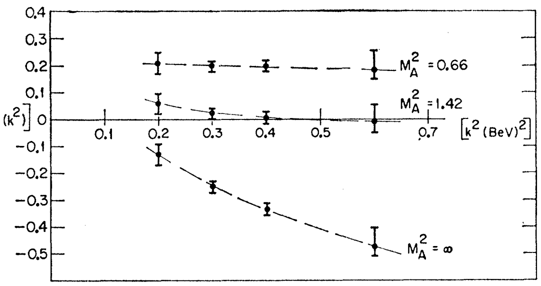

FIG. 2. Plot of $G_e^n(k^2)$ versus k^2 for various values of M_A^2 determined by analysis of $e+N \rightarrow e+N+\pi$ at threshold.

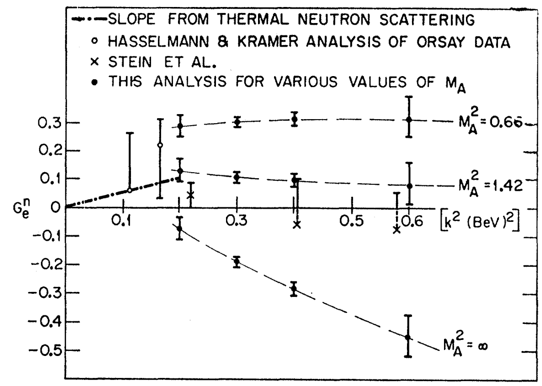

FIG. 3. Plot of $F_1^n(k^2)$ versus k^2 for various values of M_A^2 determined by analysis of $e+N \rightarrow e+N+\pi$ at threshold.

Figs. 2 and 3 we present the resulting values of $G_e^n(k^2)$ and $F_1^n(k^2)$, respectively, for different values of M_A^2. The error bars correspond to values allowed by the experimental upper bound on the threshold value of $H(k^2,S_{10}^L)$.

It is interesting to note that the result of the models of Ref. 19 which give $M_A^2=1.42$ BeV^2 lead to values of $G_e^n(k^2)$, which are positive and small and consistent with results of thermal neutron scattering (dotted line in Fig. 2) and other results[14,16,17] for $G_e^n(k^2)$ in the region $0.2 \leqslant k^2 \leqslant 0.6$ BeV. Furthermore, the values of M_A^2 from the Argonne experiment,[12] in terms of this analysis, are also consistent with what is known about G_e^n.

VI. CONCLUSIONS

In our approach we have related the vector and axial-vector nucleon form factors to the threshold pion-electroproduction amplitude by an OMS extrapolation using PCAC and the algebra of currents. The mass-shell electroproduction amplitude was then obtained by a suitable smooth extrapolation of the OMS amplitude back to the mass shell. Thus if one assumes that the form factor $G_e^n(k^2)$ defined above is approximately zero in the range $0.2 \leqslant k^2 \leqslant 0.6$ BeV^2 and that the axial-vector form factor is given by a dipole fit Eq. (5.5), then our analysis seems to suggest that $M_A \simeq 1.2-1.3$ BeV, which is consistent with other estimates[18] of M_A and present neutrino experiments.[12] The fact that such a technique has been successful in calculating threshold effects in meson-baryon scattering[2] as well as in pion photoproduction[3] suggests that this pion-electroproduction analysis provides a useful relation between $G_e^n(k^2)$ and $F_A(k^2)$ for values of k^2 where electroproduction data can provide a useful restriction.

The question of continuing these results to the $k^2=0$ limit of photoproduction is intimately related to the problem of gauge invariance in electroproduction. In our case, we have carefully maintained a gauge-invariant analysis of this process. The earlier calculations[4] of electroproduction retained the photoproduction limit by including the extra pion-pole subtraction terms which were implied by perturbation theory. These same terms made the entire set of pole terms gauge-invariant in the photoproduction limit. In the calculations using current algebras and PCAC[5,6] this has not been possible. We believe that range of values of k^2 considered here are sufficiently far from $k^2=0$ that this gauge-invariant approach provides a valid description of pion-electroproduction.

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