general conditions (i), (ii), and (iii) (Sec. 2) hold, as well as unitarity. Since Eqs. (19) imply that the two amplitudes have equal moduli, the corresponding angular distributions should be equal.

This conclusion contradicts Törnqvist's relation, and therefore we conclude that the only possible way out is that Eq. (1) is valid only in a part of the angular interval. We are comforted by the fact that the phases of the $\frac{1}{2}$ and $\frac{3}{2}$ amplitudes constructed with the phenomenological πN phase shifts do not satisfy the Törnqvist relation.[5] Of course, our conclusion was reached only for the spinless case, but we have little doubt that the proof could be carried out also in the physically interesting case.

[5] V. Grecchi and G. Turchetti (private communication).

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Double Bootstrap of the ρ and f^0

ALLEN E. EVERETT*
Department of Physics, Tufts University, Medford, Massachusetts
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We obtain a "double bootstrap" of the ρ and f^0 mesons, in which the input force consists of both ρ and f^0 exchange, and both particles are obtained as ouptut resonances, with corresponding input and output values of masses and widths being equal to within about 1%. The values obtained for the resonance parameters are $m_\rho = 750$ MeV, $\Gamma_\rho = 162$ MeV, $m_f = 1240$ MeV, and $\Gamma_f = 100$ MeV, in reasonable agreement with experiment. The ρ width is somewhat larger than most measured values, but much smaller than generally obtained in bootstrap calculations in which only ρ exchange is included as an input force. Our calculations are carried out using the equivalent-potential method, and are free of arbitrary parameters. The model yields a second $I=0$, $j=2$ resonance, presumably to be associated with the $f^{0\prime}$, and also a broad second P-wave resonance. The latter may correspond to the ρ' Regge trajectory hypothesized in several Regge-pole analyses of high-energy data, especially the polarization in πN charge exchange. The parameters of the predicted resonance do not agree with those of any known resonance, but it might be difficult to observe because of its width. The output Regge trajectories predicted by the model are roughly linear. The ρ trajectory has a slope about half the generally accepted experimental value of 1 BeV^{-2}. We comment in passing that general considerations, based only on the crossing matrix, make it somewhat difficult to reconcile the latter value with the absence of an $I=0$ D-wave resonance at an energy less than 1250 MeV.

A VIGOROUS attack has been made in recent years on the problem of determining the parameters of the ρ resonance in $\pi\pi$ scattering by a "bootstrap" calculation, in which one first assumes that the dominant force producing the ρ is ρ exchange, and then seeks self-consistent values for the mass and width. A variety of such calculations have been published.[1–3] One finds in general that one can obtain the ρ from these calculations at more or less the experimental value of the mass, though in many of these calculations this is due to the freedom one has to adjust a cutoff parameter which enters because of the exchange of a vector particle. However, one invariably obtains a theoretical value for the width which is much too large. Most searches for solutions to this problem have tended in the direction of including inelastic effects, either through the inclusion of one or more inelastic channels explicitly in a multichannel calculation, or through the inclusion of an inelasticity parameter in a one-channel calculation. Examples are given in Ref. 2. While the inclusion of inelastic effects has tended to bring the theoretical and experimental widths into better agreement, the theoretical values remain too large.[2] It is possible that if one could include further inelastic channels in a correct way, the theoretical width would be further improved, though in practice this will be very difficult to do.

A second possible effect which might contribute to narrowing the theoretical resonance widths has been suggested by Chew.[4] Chew works in the context of the "new form of the strip approximation," in which it is assumed that the generalized potential (that part of the amplitude not containing direct-channel poles) can

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[2] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962); L. A. P. Balázs, *ibid.* **132**, 867 (1963); J. R. Fulco, G. L. Shaw, and D. Y. Wong, *ibid.* **137**, B1242 (1965); R. Atkinson, III, and A. E. Everett, *ibid.* **154**, 1430 (1967).

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be represented by the high-lying Regge trajectories in the crossed channels.[5] He then shows that if the contribution from isospin-0 exchange in the crossed channels is represented by the P and P' trajectories, one obtains an effective long-range repulsion from the exchange of $I=0$, $j=0$ pion pairs in addition to the attraction due to ρ exchange; such a long-range repulsion would be expected to have the effect of narrowing resonance widths. There are theoretical questions concerning this attractive idea, however, arising from the fact that one is representing the crossed-channel absorptive parts by a Regge formula including only the P and P' contributions even though one is far from working at asymptotic values of s. In particular, as Chew points out, the force from the exchange of systems with $4\mu^2 \leqq t \leqq 16\mu^2$ (as usual, t is the c.m. energy squared in the crossed channel; μ is the pion mass) must be attractive, as follows from crossed-channel unitarity, so that the representation of the generalized potential by the P and P' contributions alone cannot be correct for these values of t. For larger values of t, the t discontinuities of the amplitude and of the generalized potential need no longer have the same sign, and it is possible that the net effect may be a repulsion, although this has not yet been established.

It is the purpose of this paper to report on a simultaneous or "double bootstrap" calculation of the parameters of the ρ and f^0 mesons, which yields self-consistent parameters for both particles in reasonable agreement with experiment. The value obtained for the ρ width, 161 MeV, is somewhat wider than most, but not all, of the experimental measurements, while the masses and the f^0 width are in agreement with the experimental values. (The reader is referred to Ref. 6, unless otherwise stated, for data on particles and resonances used in this work.) Our calculation also predicts a second $I=0$, $j=2$ resonance, presumably to be associated with the $f^{0\prime}$ meson; the agreement with experiment is not good, but this is not surprising in view of the important contribution of other channels to the $f^{0\prime}$. We find, in addition, a broad second $I=1$, P-wave resonance, with a mass of 1455 MeV and width of 280 MeV. This might be associated with the ρ', the second trajectory with the quantum numbers of the ρ, which has been hypothesized in several Regge-pole analyses of high-energy data, especially to account for the existence of polarization in πN charge-exchange scattering.[7] The predicted ρ' parameters do not agree with any presently known resonance, as a result either of inadequacies in the approximations used in our calculation, discussed below, or possibly of the difficulty of observing such a broad resonance. We also obtain the approximate behavior of the Regge trajectories. The ρ trajectory is qualitatively reasonable, although probably too flat, while it is difficult to draw conclusions about the $I=0$ trajectories because of uncertainty in the experimental picture.

We have employed the equivalent potential method developed by Balázs and further studied by Finkelstein.[3] Balázs shows that one may obtain an exact solution for the $\pi\pi$ scattering amplitude by solving the nonrelativistic Schrödinger equation

$$(\nabla^2+k^2-V)\psi(r)=0 \tag{1}$$

with an appropriate local energy-dependent potential. The exact construction of this potential requires an iteration procedure which is as difficult as the exact iterative solution for the amplitude. However, an approximate form for the potential, under the assumption that ρ exchange is the only force which needs to be considered, may be obtained. For the state with isospin equal to 1 it is given by[3]

$$V(r,q^2)=-24\beta_{11}\Gamma_{\text{in}}s^{-1/2}(s+2q_r^2)me^{-mr}/mr. \tag{2}$$

In Eq. (2), β_{11} is the element of the $\pi\pi$ crossing matrix connecting two states of isotopic spin 1, s is the square of the c.m. energy in the direct channel, m is the mass of the ρ, and $q_r^2=\frac{1}{4}m^2-\mu^2$. Γ_{in} is the input reduced width of the ρ in the crossed channel; in terms of Γ_t, the half-width in the energy-squared variable, it is given by $\Gamma_{\text{in}}=m\Gamma_t/8q_r^3$.

The equivalent potential method has two advantages over the more conventional N/D technique. In the first place, there is no arbitrary cutoff parameter. Secondly, the N/D method underestimates the strength of an attractive force, since it ignores the attractive contributions to the left-hand cut coming from the higher-order Born terms. In contrast, the use of the Schrödinger equation rather than the N/D procedure to enforce unitarity is equivalent to carrying out the entire Mandelstam iteration procedure,[8,9] although it is true that only the first step in the iteration is treated exactly. Consequently, the resulting amplitude takes into account, albeit in an approximate way, the attractive contributions of the higher-order terms in the Born series as well as of the first term.

Let us rewrite Eq. (2) in the form

$$V(r,q^2)=-V_0f(q^2)e^{-mr}/mr, \tag{3}$$

where

$$f(q^2)=m(s+2q_r^2)/s^{1/2}(m^2+2q_r^2), \tag{4}$$

so that $f(q_r^2)=1$. Finkelstein[3] obtained an approximately self-consistent solution by taking $m=750$ MeV, the experimental value, and $\Gamma_{\text{in}}=0.46$, in contrast to the experimental reduced width of about 0.17–0.21, where the two numbers quoted correspond to taking

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the experimental width of the ρ to be about 100 or 125 MeV, respectively; Balázs[3] had earlier obtained a self-consistent solution with a set of parameters which were similar but in somewhat worse agreement with experiment. Finkelstein's parameters correspond to a value of $227\mu^2$ for V_0, while the values of V_0 corresponding to taking Γ_{in} to be 0.17 and 0.21 are 83.4 and $104\mu^2$, respectively. The large value of Γ_{in} has nothing to do with the requirement of self-consistency. It results from the fact that a force of this general strength is required in order to produce a P-wave resonance. This is evidenced by the results we obtained by repeating the calculation with $V_0=104\mu^2$. With this potential, the value of the $I=1$, P-wave phase shift at the energy of the ρ is only about 9°. Even at double the energy of the ρ, the phase shift has only just passed through 50°. Thus it appears that the force due to the exchange of a ρ with the correct physical parameters is not nearly strong enough, at least in the effective-potential approximation, to reproduce a resonance as output in the direct channel.

The foregoing suggests that additional attractive forces may, perhaps, be important in producing the ρ. The most obvious candidate for such a force is the exchange of the $I=0$, $j=2$ resonance (the f^0) with mass 1250 MeV and width of about 100 MeV. f^0 exchange leads to a contribution to the potential in the $I=1$ state of the form

$$V_f(r,s)=-(V_{f0}P_2(x)/0.772\sqrt{s})e^{-8.92r}/8.92r\,, \quad (5)$$

where $x=1+2s/(m^2-4)$. Written in the above way, V_f reduces to $-V_{f0}e^{-8.92r}/8.92r$ at $s=m^2$. For a purely elastic f^0 of width 100 MeV, $V_{f0}=293\,\mu^2$. One must, of course, use the appropriate crossing matrix element, $\beta_{10}=\frac{1}{3}$, rather than β_{11} in computing V_{f0}.

Before discussing the results of calculations employing the potential of Eq. (5), we should point out one theoretical objection which can be raised to it. Chew,[4] again using the assumption that the crossed-channel absorptive part is correctly given by one or a few leading Regge trajectories, has argued that the force due to the exchange of higher angular momentum states in the crossed channels will be very much reduced in strength as compared with what one would expect for the exchange of an elementary particle, because of the presence of what he refers to as a form factor. This means, essentially, that if the Regge representation is correct, then the t discontinuity in the generalized potential for large t must be repulsive and of such nature as to strongly reduce the effective attraction due to the exchange of higher partial waves in the crossed channel over what would be predicted from simply looking at low-lying particles and resonances, i.e., at the nearby singularities. If Chew's argument is correct, then Eq. (5) overstates the strength of the f^0 exchange by about an order of magnitude, and the f^0 exchange force is negligible. As already noted, however, there are questions concerning the validity of the Regge representation for the potential at nonasymptotic values of s. In view of the argument which we have made as to the possible importance of attractive forces in addition to ρ exchange, we wish here to explore the consequences of abandoning the Regge representation of the generalized potential at the relevant values of s, and using Eq. (5) as it stands. The rather reasonable results which we shall obtain would seem to give additional justification to this procedure.

TABLE I. The input ρ and f^0 widths, Γ_ρ and Γ_f, are given for two self-consistent calculations of the ρ, together with the resulting upper, lower, and total output widths for the ρ. In each case the input and output ρ masses are equal to each other and to the physical value of 750 MeV. The first line describes the results reported here, in which the f^0 exchange force given by Eq. (5), with the f^0 mass and width set equal to their physical value, is included in addition to the ρ exchange potential of Eq. (3). For comparison, the second line gives the self-consistent solution of Finkelstein (Ref. 3) including ρ exchange alone. Γ_u and Γ_d are, respectively, the energy intervals in which the predicted $\pi\pi$ P-wave cross section falls to half its maximum value above and below the resonance, and $\Gamma=\Gamma_u+\Gamma_d$. All quantities are given in MeV. For self-consistency, Γ should equal Γ_ρ.

Γ_ρ	Γ_f	Γ_u	Γ_d	Γ
162	100	93	68	161
270	0	155	115	270

We now consider the solution of the Schrödinger equation with a potential given by the sum of the ρ- and f^0-exchange potentials of Eqs. (3) and (5). Taking $V_0=104$ (henceforth we take $\mu=1$ unless otherwise specified) corresponding to a ρ width of 125 MeV, and $V_{f0}=293$, we find a resonance in the $I=1$, $j=1$ partial wave at an energy of about 960 MeV with a width of about 180 MeV. Thus, while the f^0 exchange potential coupled with the exchange of a physical ρ is enough to produce a P-wave resonance, in contrast to the situation with physical ρ exchange alone, the quantitative agreement with experiment is poor. To remedy the situation, we increase the input ρ width until the output ρ is produced at the proper mass, 750 MeV. It turns out that this requires an input of 162 MeV, corresponding to $V_0=135$. The results are given in Table I, along with, for comparison, those obtained by Finkelstein, which were reproduced by our own program. It will be seen that this is an exact bootstrap for the ρ in that not only does the output mass equal the input, but the output ρ width turns out to be 161 MeV, essentially identical to the input width. While the ρ width obtained in this calculation is somewhat wider than indicated in the bulk of the experiments, the agreement is quite good, and indeed some experimental groups have obtained ρ widths in the vicinity of 160 MeV.[6] Moreover, the inclusion of inelastic effects, which are totally absent in this calculation, might be expected to further decrease the theoretical value of the width.

It may be of use to indicate briefly the qualitative reasons why the inclusion of f^0 exchange narrows the theoretical value of the ρ width by more than 100 MeV.

TABLE II. The first four columns give the output parameters of the remaining resonances, in addition to the ρ, in the $\pi\pi$ system which are predicted by the ρ plus f^0 exchange model of Table I up to masses of about 2.2 BeV. I, j, $m_{\rho f}$, and $\Gamma_{\rho f}$ are the predicted isotopic spin, spin, mass, and width of the resonances. The last two columns, m_ρ and Γ_ρ, give the mass and width of the corresponding resonance, if obtained, as given by Finkelstein for the model in which only ρ exchange is included. All masses and widths are given in MeV.

I	j	$m_{\rho f}$	$\Gamma_{\rho f}$	m_ρ	Γ_ρ
0	2	1230	100	1070	120
0	2	1680	200	1900	700
1	3	2180	160	a	···
1	1	1455	280	b	···

[a] Finkelstein (Ref. 3) reports obtaining Regge recurrences at masses greater than 2400 MeV.
[b] Using Finkelstein's potential, we found no second P-wave resonance in a search that extended up to 2200 MeV.

The input width required to produce a resonance at the correct mass is, of course, lowered just because of the additional attractive force present. That the output width is also decreased is due to a combination of two effects. In the first place, it appears that the resonance width is smaller when the resonance is produced by a combination of ρ exchange and a shorter-range force than when it is produced by either separately. We verified this by an additional calculation. We fixed V_0 and m in the input potential at the values corresponding to the physical ρ, and then studied the effect of introducing a shorter-range force into the Schrödinger equation; ranges of $1/2m$ and $1/3m$ were considered. We then found that, when the strength of the short-range force was adjusted to produce an output resonance at 750 MeV, the width of the resonance was considerably narrower than the value of 270 MeV obtained when the resonance is produced by a stronger ρ exchange force alone; the amount of this reduction in width was greater when the short-range force was chosen to have the smaller of the two ranges studied, i.e., $(3m)^{-1}$. That the reduction in width is due to the interference between the two potentials of different range is shown by the fact that when the ρ exchange force was set equal to zero, and the strength of the short-range force increased to produce a 750-MeV resonance by itself, the resulting resonance was actually somewhat wider than when produced by ρ exchange alone. Hence one expects a resonance produced by a combination of ρ and f^0 exchange to be narrower than one due only to a strong ρ-exchange force. In addition to this effect, the output width is further reduced by the more rapid increase of the f^0-exchange force with energy, which results from the f^0 having spin 2, so that $P_2(x)$ appears in the potential of Eq. (5); this helps to cause the phase shift to rise more rapidly as a function of energy.

We now turn to the predictions of our model for other aspects of the $\pi\pi$ system. Table II gives the masses and widths of the remaining resonances predicted up to a mass in the vicinity of 2 BeV; the last two columns of the table again give Finkelstein's results for comparison. Turning first to the $I=0$, $j=2$ partial wave, we see that we have a double bootstrap in that there is an output f^0 whose parameters agree almost exactly with the input parameters, and hence with experiment. We also find a second resonance which presumably corresponds to the $f^{0\prime}$. The latter has an experimental mass of 1500 MeV and width of about 80 MeV, so that the agreement of the model with experiment is not particularly good in this case. This is not surprising, since the $f^{0\prime}$ is strongly coupled to channels other than two pions, which are completely ignored in our calculation. The calculation using ρ exchange alone yields a mass for the f^0 which is in considerably worse agreement with experiment and a width which is, assuming a q^{2j+1} energy dependence for the width, about double the experimental value.

We obtain two other resonances which are also of some interest. In the $I=1$, $j=3$ partial wave we find a resonance at 2180 MeV, which is the $j=3$ recurrence of the ρ trajectory. Experimentally, this is presumably to be identified with the g meson at 1630 MeV.[10] Such an identification yields a slope of about 1 BeV^{-2} for the ρ trajectory as a function of s, which is consistent with the value obtained in the negative-t region by fits to high-energy cross-section data,[11] while a $j=3$ recurrence at 2180 MeV corresponds to a slope of roughly 0.5 BeV^{-2}, or about half the accepted experimental value.

Again it is not surprising that, in view of its lack of inelasticity, the model should not be particularly good at these energies. Note, however, that it will be difficult for any theoretical calculation of a conventional nature to fit the mass of the f^0 and simultaneously to yield a linear ρ trajectory passing through the g. The reason is simply that such a trajectory passes through $j=2$ below the f^0 mass. On the other hand, the crossing matrix is such that the ρ-exchange force (and that due to the exchange of any other $I=1$ system) is twice as strong, and the $I=0$ exchange force has the same strength, in $I=0$ as in $I=1$ states. Thus one is led by general considerations to expect that there should be an $I=0$ D-wave resonance at an energy less than that at which the ρ trajectory passes through $j=2$. Perhaps the relative ordering of the ρ and f^0 trajectories at $j=2$ could be reversed by a large difference in the inelastic effects in the two states, due, for example, to the fact that one couples to the $\pi\omega$ channel while the other does not. Any model, however, which depends primarily on the relative strengths of the exchange forces and in which the ρ trajectory is mainly coupled to the 2π channel will not be able to fit our present experimental picture of the ρ and f^0 trajectories. The alternative to a departure from a comparatively simple theoretical picture is that our understanding of the experimental

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situation is incomplete. It could be that the slope of the ρ trajectory for $t>0$ is less than believed. In this case, the g meson would not be a recurrence of the ρ, but perhaps the $j=1$ intercept of the hypothesized ρ' trajectory. Our model also predicts the existence of a ρ', as we shall discuss below, but suggests that it will be too wide to be associated with the g. The only other alternative would be the existence of a thus far undetected $I=0$ D-wave resonance at a mass less than that of the f^0. If such a resonance were sufficiently narrow, it might possibly have escaped detection just from having a very small number of events under its peak in effective-mass plots; one would expect a D-wave resonance lying appreciably lower in energy than the f^0 to be very narrow.

Finally, we note from Table II that an additional $I=1$, $j=1$ resonance is predicted at a mass of 1455 MeV and with a width of 280 MeV. As we have mentioned, there are theoretical arguments in favor of the existence of a ρ' Regge trajectory.[7] Possible candidates for the first resonance on such a trajectory are the δ meson at 965 MeV, or the g, although the experimental data in Ref. 10 somewhat favor spin 3 for the latter particle. The mass of the ρ' predicted by our calculation does not coincide with that of any known experimental possibility. It is not clear whether this is because the theory is substantially in error as regards the mass of the ρ', if indeed such a particle exists, or whether the predicted resonance is simply so broad that it has not been observed. The question of the existence of a ρ' serves to distinguish this calculation from that of Finkelstein based on ρ exchange alone; we found that the latter predicts no $j=1$ resonance other than the ρ, at least up to a mass of 2200 MeV.

Table III summarizes the information about the Regge trajectories predicted by the model. It will be observed that the ρ trajectory is close to being linear; its average slope is the same to within about 20% in the intervals $1\leqq j\leqq 2$ and $2\leqq j\leqq 3$. We have already noted that the predicted slope is about half the accepted experimental value. The $f^{0\prime}$ trajectory is, like the ρ, linear to within about 20%. The f^0 trajectory is roughly parallel to it in the region between $j=2$ and $j=3$; its $j=1$ intercept is below threshold and is therefore not obtainable with the present version of the program that we were using. One can assume that the f^0 trajectory remains approximately linear and parallel to the $f^{0\prime}$ in order to obtain a rough estimate of its value at $s=0$; as shown in the table, the resulting value is

TABLE III. Summary of the predicted properties, in the ρ plus f^0 exchange model, of the three Regge trajectories passing through the particles indicated in column 1. α_{12}' and α_{23}' are the average values, in BeV^{-2}, of the real parts of the trajectory slopes $d\alpha/ds$ in the intervals $1\leqq j\leqq 2$ and $2\leqq j\leqq 3$, respectively; $\alpha(0)$ is the value of the trajectory, as estimated by linear extrapolation, at $s=0$.

Trajectory	α_{12}'	α_{23}'	$\alpha(0)$
ρ	0.53	0.42	0.7
f^0	...	0.47	1.1[a]
$f^{0\prime}$	0.57	0.43	0.36

[a] As explained in the text, the value of s for which $\alpha(s)=1$ is below threshold and cannot be obtained with our program. $\alpha(0)$ is obtained by extrapolating from $j=2$ using the value α_{12}' for the $f^{0\prime}$ trajectory, and hence is less reliable than the estimates of $\alpha(0)$ for the other trajectories.

close to 1, which would suggest that the f^0 lies on the Pomeranchuk trajectory. It is by no means clear that this should be taken seriously, however. If the model underestimates the slope of the ρ trajectory, it may well do the same thing for the f^0, thus meaning that the latter has an intercept less than 1 and should be associated with the P' trajectory. The values obtained for the slopes and intercepts of the P and P' from scattering data[11] are both inconsistent with the f^0; the slope of the former is too small (and there is speculation that it may even be zero), while that of the latter is too large to pass through the f^0.

The trajectories obtained by Finkelstein with ρ exchange only are roughly comparable to those obtained here. The former are slightly less linear and have more tendency to flatten out, so that the higher recurrences are found at larger masses; these recurrences are probably quite sensitive to inelastic effects, so that it is not clear that either calculation is very relevant.

In conclusion, then, we have found that it is possible, in the effective-potential approximation, to obtain a double bootstrap of the ρ and f^0. The resulting theoretical values of the masses and widths are, on the whole, in good agreement with experiment. In particular, the predicted ρ width, though still somewhat wider than the bulk of the measured values, is much less than what has generally been obtained in theoretical calculations, suggesting that the inclusion of f^0 exchange may be important in understanding the ρ quantitatively. We note again the absence of inelastic effects in our calculation; their inclusion would be expected to further narrow the ρ width. The possibility of obtaining a double bootstrap, and the quality of the resulting agreement with experiment, seems to us to be encouraging.