V. CONCLUSIONS

We have shown that the Feynman amplitude of a pole diagram mediated by a particle (j,k) at $s=0$ is the same as the result of the $O(4)$ symmetry, and that, in addition, (j,k) corresponds in a one-to-one manner to (n,M) introduced by Toller, Freedman, and Wang. With these considerations we have concluded that it is difficult to assign the π meson to the class III.

Another point we want to stress is the fact that the pole terms appear in more than one partial wave, even if a single particle is exchanged. This fact indicates that there are poles in the S matrix which do not correspond to a "real" particle. We call this pole a "shadow pole." The fact that there is a pole which does not correspond to a "real" particle forces one to modify the usual assumption that a pole in the S matrix corresponds to a real particle.

The relations between shadow poles and abnormal solutions of the Bethe-Salpeter equation, and the phenomenological effects of shadow poles will be discussed in subsequent papers.

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Rescattering Model Applied to Y^* Production

C. P. Singh and B. K. Agarwal
Physics Department, Allahabad University, Allahabad, India

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The production angular distributions for $Y_1^*(1385)$, $Y_0^*(1520)$, and $Y_1^*(1660)$ in K^--p scattering are characterized by forward and backward peakings. As the single-particle-exchange model is unable to account for these features, we have attempted to explain them by considering rescattering square diagrams. We find that the use of a coincident-pole method leads to a simple prescription for evaluating the production angular distributions. Our results show agreement with the observed data when spin-parity assignments are $\frac{3}{2}^+$ for $Y_1^*(1385)$ and $\frac{3}{2}^-$ for $Y^*(1520, 1660)$.

1. INTRODUCTION

In K^--p scattering, the following quasi-two-body final states have been observed[1-3]:

$$K^-+p \to Y_1^{*+}(1385)+\pi^-, \tag{1}$$

$$K^-+p \to Y_1^{*-}(1385)+\pi^+, \tag{2}$$

$$K^-+p \to Y_0^{*0}(1520)+\pi^0, \tag{3}$$

$$K^-+p \to Y_1^{*0}(1660)+\pi^0. \tag{4}$$

In all these processes, a characteristic feature of the center-of-mass production angular distributions for the various Y^* is that there is an approximate symmetry at about 90° due to the presence of both forward and backward peakings. Such a characteristic defies explanation in terms of either the one-meson-exchange model or the one-baryon-exchange model. For reaction (1) only $\bar{K}^{*0}$ can be exchanged, for (2) only the nucleon can be exchanged, and for (3) and (4) both can be exchanged. Therefore, for (1) and (2) we cannot hope to get all the observed features from a one-particle-exchange model.[4] For reactions (3) and (4), one may combine the two single-particle-exchange diagrams and use *ad hoc*, drastic, form factors to obtain the observed structure. We have shown[5] that in such cases the rescattering square diagrams can offer a natural explanation. The purpose of this paper is to consider such diagrams for reactions (1)–(4), to explain the structure of the production angular distributions and thereby to fix the spin-parity assignments.

2. METHOD OF CALCULATION

The rescattering diagram for the general process $A+B \to C+D$ is shown in Fig. 1. The various momenta have been labeled in the diagram. In Table I, we summarize the intermediate states possible for the reactions (1)–(4). The invariant amplitude for the diagram shown

[1] Birmingham-Glasgow-London (I.C.)-Oxford-Rutherford Collaboration, Phys. Rev. **152**, 1148 (1966).

[2] W. A. Cooper, H. Filthuth, A. Fridman, E. Malamud, H. Schneider, E. S. Gelsema, J. C. Kluyver, and A. G. Tenner, in *Proceedings oj the Sienna International Conference on Elementary Particles and High-Energy Physics, 1963*, edited by G. Bernardini and G. P. Puppi (Società Italiana di Fisica, Bologna, 1963), p. 160.

[3] R. P. Ely, S. Y. Fung, G. Gidal, Y. L. Pan, W. M. Powell, and M. S. White, Phys. Rev. Letters **7**, 461 (1961).

[4] Y. M. Gupta and B. K. Agarwal, Nuovo Cimento **40**, 434 (1966).

[5] C. P. Singh and B. K. Agarwal, Nuovo Cimento **54A**, 497 (1968).

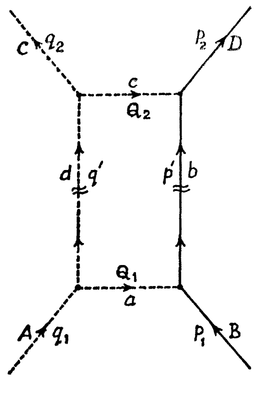

FIG. 1. Rescattering diagram for a general process $A+B\rightarrow C+D$.

in Fig. 1 is given by

$$T_4=\int\frac{d^4p'}{(2\pi)^4}g_{Dbc}g_{Ccd}g_{Bba}g_{Aad}\bar{\psi}(p_2)\left(\frac{p_\sigma'}{m_c}\right)^{s-1/2}$$
$$\times\Gamma\left\{\frac{-i\gamma\cdot p'+m_b}{p'^2+m_b^2}\right\}\gamma_5\frac{1}{(Q_1^2+m_a^2)}i(q_1+Q_1)_\lambda\epsilon_\lambda$$
$$\times\frac{1}{(q'^2+m_d^2)}i(q_2-Q_2)_\nu\epsilon_\nu\frac{1}{(Q_2^2+m_c^2)}u(p_1),\quad(5)$$

where ϵ is the polarization vector of the ρ meson, the g's are the coupling constants, Γ is 1 (γ_5) for Y^* spin-parity assignment $\frac{3}{2}^+$ ($\frac{3}{2}^-$) and $\frac{1}{2}^-$ ($\frac{1}{2}^+$), $\psi(p_2)$ is the Rarita-Schwinger wave function $U_\sigma(p_2)$ for Y^* spin $\frac{3}{2}$ and the Dirac wave function $u(p_2)$ for Y^* spin $\frac{1}{2}$, s is the spin of the Y^* particle, and the m's are the masses of the particles concerned. Equation (5) requires an integration over the closed-loop variable which is very difficult to perform. If we put b and d states on the mass shell and apply the method of coincident poles,[6] then we can reduce the fourth-order matrix element to a product of two matrix elements corresponding to two successive second-order processes:

$$\langle q_2,p_2|T_4|q_1,p_1\rangle$$
$$=\tfrac{1}{2}\sum_{q',p'}\langle q_2,p_2|T_2|q',p'\rangle\langle q',p'|T_2|q_1,p_1\rangle,\quad(6)$$

where the summation (effected by a three-momentum integration) is over all real particle states q', p' which are consistent with the conservation of energy and momentum required by T_2. The extra factor $\frac{1}{2}$ before summation arises from the presence of a term $(\pi i)(2\pi i)^{n-1}$ rather than $(2\pi i)^n$ in front of the δ-function product corresponding to n coincident poles. Assuming that only the real states of Fig. 1 are contributing, we can cut off the summation after one term and rewrite the invariant amplitude as[6a]

$$T_4=\frac{1}{2}\frac{(2\pi i)^2}{(2\pi)^4}g_{Dbc}g_{Ccd}g_{Bba}g_{Aad}\bar{\psi}(p_2)\left(\frac{p_\sigma'}{m_c}\right)^{s-1/2}$$
$$\times\Gamma\frac{1}{(Q_2^2+m_c^2)}i(q_2-Q_2)_\nu\epsilon_\nu u(p')\bar{u}(p')\gamma_5$$
$$\times\frac{1}{(Q_1^2+m_a^2)}i(q_1+Q_1)_\lambda\epsilon_\lambda u(p_1).\quad(7)$$

Taking the usual sum for polarization and spin states, we find the differential cross section to be

$$\left(\frac{d\sigma}{d\Omega}\right)_s^{\pm}=\frac{0.38935m_Bm_D}{4(2\pi)^48(2\pi)^2W^2}\left(\frac{q_f}{q_i}\right)$$
$$\times\frac{g_{Dbc}^2g_{Ccd}^2g_{Bba}^2g_{Aad}^2}{4m_b^2(Q_1^2+m_a^2)^2(Q_2^2+m_c^2)^2}XYZ^{\pm}\text{ mb/sr},\quad(8)$$

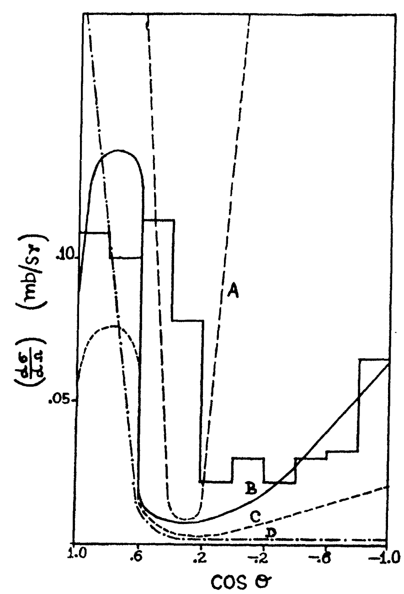

FIG. 2. Production angular distribution of $Y_1^{*+}(1385)$ in reaction (1). [$\cos\theta=\hat{Y}_1{}^{*+}{}_{\text{out}}\cdot\hat{p}_{\text{in}}$.] The present calculation from Eq. (8) is shown by curves A ($J^P=\frac{3}{2}^-$), B ($J^P=\frac{3}{2}^+$), C ($J^P=\frac{1}{2}^+$), and D [$J^P=(\frac{1}{2})^-$]. The histogram represents the experimental data of Cooper *et al.* (Ref. 2).

TABLE I. Possible intermediate states for the reactions (1)–(4).

$A+B\rightarrow C+D$	a	b	c	d
(1) $K^-+p\rightarrow\pi^-+Y_1^{*+}(1385)$	K^-	Λ^0	π^+	ρ^0
(2) $K^-+p\rightarrow\pi^++Y_1^{*-}(1385)$	K^-	Λ^0	π^-	ρ^0
(3) $K^-+p\rightarrow\pi^0+Y_0^{*0}(1520)$	$\bar{K}^0$	Σ^+	π^-	ρ^-
(4) $K^-+p\rightarrow\pi^0+Y_1^{*0}(1660)$	$\bar{K}^0$	Σ^+	π^-	ρ^-

[6] J. Hamilton, Proc. Cambridge Phil. Soc. **48**, 640 (1952).

[6a] *Note added in proof.* Equation (6) implies that reactions (1)–(4) are two-step processes of the type $A+B\rightarrow b+d\rightarrow C+D$. Strictly speaking, one should use the propagators in (7) to perform an angular integration contained in (6). We have, however, fixed the angle as stated in the paragraph following Eq. (8). We are grateful to Professor J. Hamilton for drawing our attention to these points.

where q_f (q_i) is the final (initial) center-of-mass momentum, W is the center-of-mass total energy, and

$$X=\left[4q_1\cdot q_2+q'^2-2q'\cdot q_1-2q_2\cdot q' +\frac{(2q_2\cdot q'-q'^2)(2q_1\cdot q'-q'^2)}{m_d^2}\right]^2,$$

$$Y=\left\{\frac{2}{3m_c^2}\left[p'^2+\frac{1}{m_D^2}(p'\cdot p_2)^2\right]\right\}^{s-1/2},$$

$$Z^{\pm}=\mp(2/m_Bm_D)[(m_Dm_b\mp p_2\cdot p')(m_Bm_b+p_1\cdot p')].$$

Here Z^+ corresponds to $J^P=\frac{3}{2}^+$, $\frac{1}{2}^-$ and Z^- to $J^P=\frac{3}{2}^-$, $\frac{1}{2}^+$.

To evaluate the differential cross section (8), we shall have to make some assumption about the production angle of the intermediate d state. We make the drastic approximation that d is produced in the incoming A direction in the over-all center-of-mass system, which is not incompatible with the experimental data.[1]

3. RESULTS

Results of our calculations using the values of g's given in Table II are shown in Figs. 2–5. The calculated curves for the production angular distribution of $Y_1^{*+}(1385)$ are given in Fig. 2. It shows our predictions for the different spin-parity assignments $\frac{3}{2}^{\pm}$, $\frac{1}{2}^{\pm}$ for Y_1^{*+}. We find that the curve corresponding to $\frac{3}{2}^+$ is of the right order of magnitude and is in close agreement with the experimental data[2] at a K^- momentum of 1.46 GeV/c. Thus the rescattering square diagram seems to account for the observed production angular distribution and to select out the $\frac{3}{2}^+$ spin-parity assignment. Although the $\frac{1}{2}^+$ assignment is not as glaringly excluded as $\frac{3}{2}^-$ and $\frac{1}{2}^-$, we can omit it in view of the observed[7] decay distribution of Y_1^*, which gives $J>\frac{1}{2}$. Figure 3 gives similar results for Y_1^{*-}.

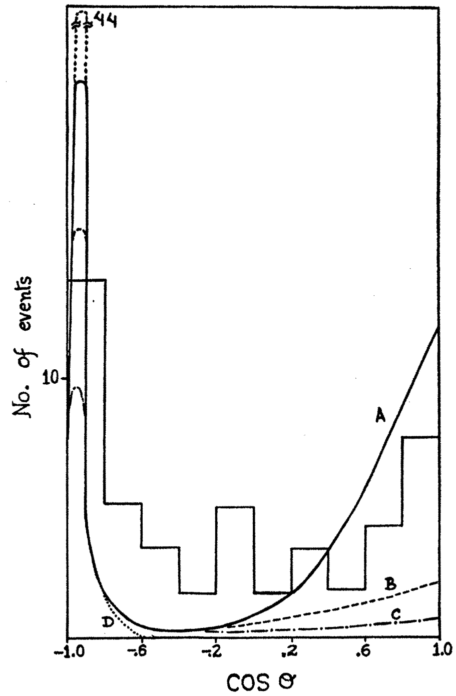

FIG. 4. Production angular distribution of $Y_0^{*0}(1520)$ in reaction (3). [$\cos\theta={}_0\hat{Y}^{*0}{}_{\rm out}\cdot K^-{}_{\rm in}$.] The present calculation from Eq. (8) is shown by curves A ($J^P=\frac{3}{2}^-$), B ($J^P=\frac{3}{2}^+$), C ($J^P=\frac{1}{2}^+$), and D ($J^P=\frac{1}{2}^-$). The histogram represents the experimental data in Ref. 1. The curves have been multiplied by a constant factor.

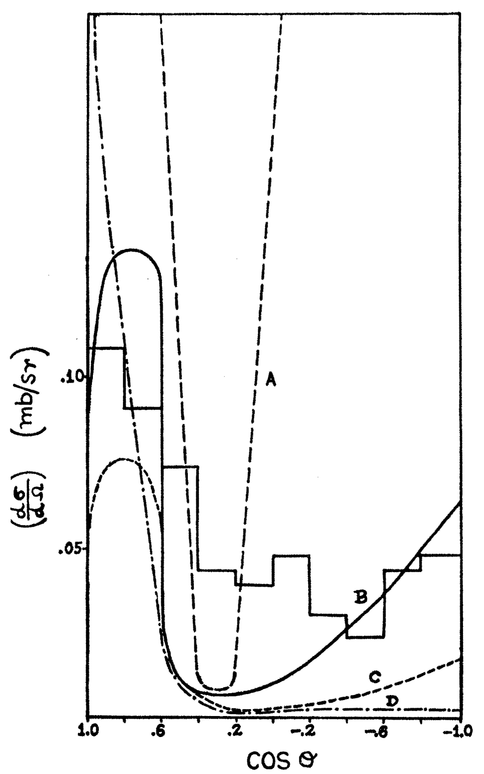

FIG. 3. Production angular distribution of $Y_1^{*-}(1385)$ in reaction (2). [$\cos\theta=\hat{Y}_1^{*-}{}_{\rm out}\cdot\hat{p}_{\rm in}$.] The present calculation from Eq. (8) is shown by curves A ($J^P=\frac{3}{2}^-$), B ($J^P=\frac{3}{2}^+$), C ($J^P=\frac{1}{2}^+$), and D ($J^P=\frac{1}{2}^-$). The histogram represents the experimental data of Cooper *et al.* (Ref. 2).

TABLE II. Values of $g_{Dbc}^2/4\pi$ calculated from the known decay widths using Eqs. (2.17) and (3.6) of Graham *et al.*[a] for $\frac{3}{2}^{\pm}$ and the formula $\Gamma=(g^2/4\pi)(E\mp m_b)p/m_D$ for $\frac{1}{2}^{\pm}$ in the usual notation. Other well-known values are: $g_{KK\rho}^2/4\pi=1.2$, $g_{\rho\pi\pi}^2/4\pi=2.4$, $g_{Kp\Lambda}^2/4\pi=4.8$, $g_{Kp\Sigma}^2/4\pi=0.3$.

J^P of Y^*	$\pi\Lambda Y_1^*(1385)$	$\pi\Sigma Y_0^*(1520)$	$\pi\Sigma Y_1^*(1660)$
$\frac{3}{2}^+$	0.14	0.017	0.01
$\frac{3}{2}^-$	17.4	1.48	0.44
$\frac{1}{2}^+$	12.3	1.66	1.05
$\frac{1}{2}^-$	0.11	0.018	0.025

[a] R. H. Graham, S. Pakvasa, and K. Raman, Phys. Rev. **163**, 1774 (1967).

[7] Janice B. Shafer, Joseph J. Murray, and D. O. Huwe, Phys. Rev. Letters **10**, 179 (1963).

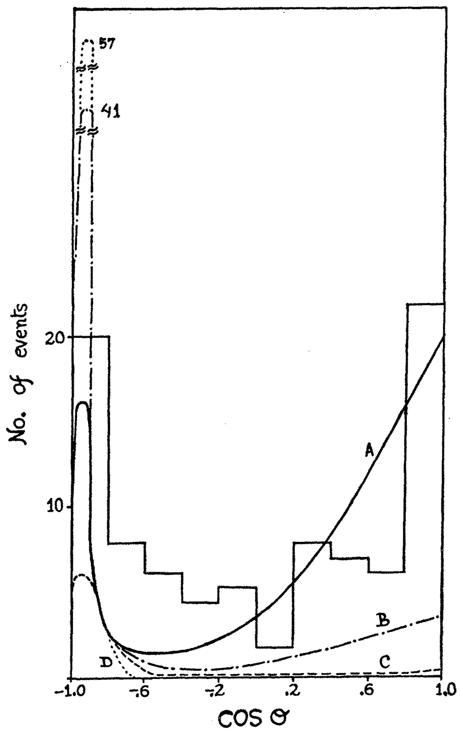

FIG. 5. Production angular distribution of $Y_1^{*0}(1660)$ in reaction (4). [$\cos\theta = \hat{Y}_1{}^{*0}{}_{\text{out}} \cdot K^-{}_{\text{in}}$.] The present calculation from Eq. (8) is shown by curves A ($J^P=\frac{3}{2}^-$), B ($J^P=\frac{3}{2}^+$), C ($J^P=\frac{1}{2}^+$), and D ($J^P=\frac{1}{2}^-$). The histogram represents the experimental data given in Ref. 1. The curves have been multiplied by the same constant factor as used in Fig. 4.

Figure 4 shows the predicted production angular distribution curves for $Y_0^{*0}(1520)$ for various spin-parity assignments $\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$ together with the experimental distribution[1] at a K^- momentum of 3.5 GeV/c. The actual curves have been drawn after multiplying the values due to Eq. (8) by a constant factor to compare with experimental data. We find that the assignment $\frac{3}{2}^-$ is distinctly favored by the experimental distribution. This agrees with the findings based on the elastic and charge-exchange differential cross sections.[8]

Figure 5 shows the predictions for $Y_1^{*0}(1660)$. The curves have been obtained by using the same constant multiplying factor as used in Fig. 4. The spin-parity assignment $\frac{3}{2}^-$ is again strongly favored by the experimental data[1] at a K^- momentum of 3.5 GeV/c. The experimental situation regarding the parity of $Y_1^*(1660)$ is ambiguous.[9] However, recent experimental data[10,11] appear to support the assignment $\frac{3}{2}^-$.

In conclusion, we can say that the rescattering diagrams are able to account for the observed experimental distributions and also lead to correct spin-parity assignments of the resonance states. Such diagrams have been found to play an important role in other high-energy processes[12,13] as well. We hope that the simple way of extracting a convergent contribution by the use of the coincident-pole approximation will be of use in these cases also.

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[8] R. D. Tripp, in *Proceedings of the International School of Physics "Enrico Fermi," Course 33* (Academic Press Inc., New York, 1966), p. 91.

[9] Ch. Peyrou, in *Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England, 1965* (Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England, 1966), p. 131.

[10] Y. Y. Lee, D. D. Reeder, and R. W. Hartung, Phys. Rev. Letters **17**, 45 (1966).

[11] Wesley M. Smart, Anne Kernan, George E. Kalmus, and Robert P. Ely, Jr., Phys. Rev. Letters **17**, 556 (1966).

[12] D. C. Peaslee, Nuovo Cimento **44A**, 784 (1966).

[13] S. Barshay, Phys. Rev. Letters **17**, 49 (1966).