Multiple Scattering in the Quark Model

DAVID R. HARRINGTON AND ANTONIO PAGNAMENTA
Department of Physics, Rutgers, The State University, New Brunswick, New Jersey
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We show that a systematic analysis of multiple-scattering contributions in the quark model can explain the detailed structure of the hadronic differential cross sections at high energies. As an extension of previous work on multiple-scattering effects in the quark model, we fit the high-energy differential cross sections for nucleon-nucleon, nucleon-antinucleon, and pion-nucleon scattering, assuming simple forms for the quark-quark scattering amplitudes and the baryon and meson form factors. Of the ten parameters involved, seven are determined by measurements at forward or near-forward angles, leaving the entire wide-angle cross sections to be fitted by the remaining three parameters. Reasonably good fits are obtained to the present data, and calculations at higher t values which may become experimentally accessible in the near future are presented. The parameters found provide some information about the interactions of quarks (should they exist) at high energy and are used to calculate double-scattering corrections to total cross sections and the phase of the nucleon-nucleon amplitude as a function of momentum transfer.

I. INTRODUCTION

ALTHOUGH most of its results have alternative derivations, the nonrelativistic quark model[1] has been quite successful in providing simple explanations for the properties of hadrons and the nature of their interactions. Among the more striking results is the set of sum rules[2–5] relating meson-baryon and baryon-baryon total cross sections, obtained using an "additive" assumption for high-energy collisions. It has recently been pointed out,[6,7] however, that if the quark model is taken seriously there must be corrections to these sum rules from the multiple scattering of the quarks. In this paper we wish to investigate the possibility that the same multiple scattering might provide a simple explanation for the structure seen in hadron-hadron[8] differential cross sections at high energy.[9,10] This possibility immediately suggests itself when one considers the very similar structure[11] seen in hadron-nucleus collisions, structure which has been quite satisfactorily explained as due to multiple scattering on the constituent nucleons.[12]

It seems to be generally true both in hadron-nucleus scattering, and (as we show below) in hadron-hadron scattering in the quark model, that the $(N+1)$th-order scattering amplitude is much smaller than the Nth-order scattering amplitude near the forward direction, but decreases more slowly with increasing momentum transfer, eventually becoming the larger of the two. One therefore expects, and in most cases finds, the differential cross section to be divided into regions of increasingly gradual decrease, with perhaps some additional structure in the regions where the slope changes, due to interference between two amplitudes.

In this paper we present a more quantitative discussion of multiple-scattering effects in the quark model using the high-energy multiple-scattering expansion developed by Glauber.[13] We are, of course, hampered by the absence of any direct information on the quark quark scattering amplitude and are forced to guess its properties from the shape of the near-forward differential cross sections, the total cross sections, and the ratio of real to imaginary parts of the forward amplitudes for hadron-hadron scattering. We attempt to minimize this difficulty by fitting NN, $\bar{N}N$, and πN scattering simultaneously, reducing the ratio of the number of free parameters to the amount of experimental information. We find that quite satisfactory fits are possible, and in the process obtain some information about the interaction between quarks at high energy.

We begin with a brief derivation of the multiple-scattering expansion for the scattering of two composite particles, obtaining the scattering amplitudes in terms of quark-quark scattering amplitudes and the generalized form factors of the composite hadrons. This deriva-

[1] For a recent review with a large number of references, see R. H. Dalitz, *1966 Tokyo Summer Lectures in Theoretical Physics* (W. A. Benjamin, Inc., New York, 1967), Vol. II, p. 56.
[2] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu, 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].
[3] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).
[4] H. J. Lipkin, Phys. Rev. Letters 16, 1015 (1966).
[5] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42, 711 (1966).
[6] V. Franco, Phys. Rev. Letters 18, 1159 (1967).
[7] M. V. Barnhill, Phys. Rev. 163, 1735 (1967).
[8] We shall refer to the "elementary" hadrons (e.g., nucleons and pions) simply as hadrons. The "more elementary" strongly interacting particles will be called quarks, the less elementary called nuclei.
[9] Some preliminary results have been presented in D. R. Harrington and A. Pagnamenta, Phys. Rev. Letters 18, 1147 (1967).
[10] Related work has been done by C. H. Woo, Phys. Rev. 150, 1372 (1966); see also A. Deloff, Nucl. Phys. B2, 597 (1967), who considers πN scattering with only the nucleon composite, and E. Schrauner, Bull. Am. Phys. Soc. 13, 49 (1968), who considers the point-quark limit.
[11] E. Coleman, R. M. Heinz, O. E. Overseth, and D. E. Pellet, Phys. Rev. Letters 16, 761 (1966); H. Palevsky, J. L. Friedes, R. J. Sutter, G. W. Bennet, G. J. Igo, W. D. Simpson, G. C. Phillips, D. M. Corley, N. S. Wall, R. L. Sterns, and B. Gottschalk, *ibid.* 18, 1200 (1967).
[12] V. Franco and E. Coleman, Phys. Rev. Letters 17, 827 (1966); W. Czyż and L. Leśniak, Phys. Letters 24B, 227 (1967); R. H. Bassel and C. Wilkin, Phys. Rev. Letters 18, 871 (1967); H. K. Lee and H. McManus, *ibid.* 20, 337 (1968).
[13] R. J. Glauber, *Lectures in Theoretical Physics* (Interscience Publishers, Inc., New York, 1959), p. 315; see also the references given in Ref. 12 and V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

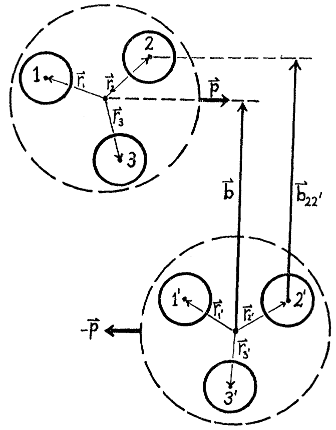

FIG. 1. Collision of two composite systems, showing the internal coordinate $\mathbf{r}_i$ and $\mathbf{r}_i'$ and the relation between the impact parameters $\mathbf{b}$ and $\mathbf{b}_{ij'}$.

tion is a straightforward application of the Glauber method and is included here to introduce our notation and to make clear the nature of the approximations made. In Sec. III we assume simple forms for the quark-quark amplitudes and the pion and nucleon form factors and obtain fits to the experimental cross sections. The parameters found are used to estimate the corrections to total cross sections in Sec. IV. We conclude with a discussion of our results and suggest additional experimental and theoretical tests for the quark model.

II. MULTIPLE-SCATTERING EXPANSION

In the quark model the scattering of two hadrons becomes the scattering of two composite systems, each described by an internal wave function. We assume that this can be accurately described by the eikonal method, as developed by Glauber,[13] which has been applied to elementary-particle nucleus scattering with considerable success.[12] In this section we develop the straightforward extension of this method to the scattering of two composite systems. The analysis would be the same for nucleus-nucleus scattering but, for definiteness, we shall use the language of the quark model.

We consider the process

$$A(\mathbf{p})+B(-\mathbf{p})\to A(\mathbf{p}+\boldsymbol{\Delta})+B(\mathbf{p}-\boldsymbol{\Delta}), \quad (1)$$

where the hadrons A and B are assumed to be composed of n_A and n_B quarks, respectively, with internal wave functions $\psi_A(\{\mathbf{r}\})$ and $\psi_B(\{\mathbf{r}'\})$. The amplitude for this process will depend upon $f_{ii'}(\boldsymbol{\Delta})$, the amplitude for the scattering of the two free quarks i and i', for which we chose the impact-parameter representation

$$f_{ii'}(\boldsymbol{\Delta})=\pi^{-1}\int d^2b\,\exp(-i\boldsymbol{\Delta}\cdot\mathbf{b})\,\gamma_{ii'}(\mathbf{b}). \quad (2)$$

The function $\gamma_{ii'}(\mathbf{b})$ is given by the inverse transform

$$\gamma_{ii'}(\mathbf{b})=(4\pi)^{-1}\int d^2\Delta\,\exp(i\boldsymbol{\Delta}\cdot\mathbf{b})\,f_{ii'}(\boldsymbol{\Delta}) \quad (3)$$

and is equivalent to $(2i)^{-1}[\exp(2i\delta_l)-1]$, where δ_l is the phase shift for angular momentum $l\approx pb$, so that

$$\exp(2i\delta_{ii'})=1+2i\gamma_{ii'}(\mathbf{b}_{ii'}). \quad (4)$$

We assume that each hadron can be reasonably well described as a composite of quarks with nonrelativistic internal motions. The eikonal method in potential theory then suggests that for high energies and small angles the complete phase shift for AB scattering is simply the sum of the corresponding quark-quark phase shifts:

$$\delta_{AB}(\mathbf{b};\{\mathbf{r}\},\{\mathbf{r}'\})=\sum_{ii'}\delta_{ii'}(\mathbf{b}_{ii'}), \quad (5)$$

where, as shown in Fig. 1,

$$\mathbf{b}_{ii'}\cdot\boldsymbol{\Delta}=(\mathbf{b}-\mathbf{r}_i+\mathbf{r}_{i'})\cdot\boldsymbol{\Delta} \quad (6)$$

relates $\mathbf{b}_{ii'}$ to $\mathbf{b}$, $\mathbf{r}_i$, and $\mathbf{r}_{i'}$. If, in analogy with (3), we write for the full amplitude

$$F_{AB}(\boldsymbol{\Delta})=\pi^{-1}\int d^2b\,\exp(-i\boldsymbol{\Delta}\cdot\mathbf{b})\Gamma_{AB}(\mathbf{b}), \quad (7)$$

then, using the internal wave functions to average over the positions of the quarks, we have

$$\Gamma_{AB}(\mathbf{b})=\int d\tau_A d\tau_B \psi_A^\dagger(\{\mathbf{r}_i\})\psi_B^\dagger(\{\mathbf{r}_{i'}\})$$
$$\times\Gamma_{AB}(\mathbf{b};\{\mathbf{r}_i\},\{\mathbf{r}_{i'}\})\psi_A(\{\mathbf{r}_i\})\psi_B(\{\mathbf{r}_{i'}\}), \quad (8)$$

where, using (5) and (4),

$$\Gamma_{AB}(\mathbf{b};\{\mathbf{r}_i\},\{\mathbf{r}_{i'}\})=(2_i)^{-1}\left\{\prod_{ii'}[1+2i\gamma_{ii'}(\mathbf{b}_{ii'})]-1\right\}$$
$$=\sum_{ii'}\gamma_{ii'}+2i\sum_{ii'jj'}{}'\gamma_{ii'}\gamma_{jj'}+\cdots+(2i)^{n_An_B-1}\prod_{ii'}\gamma_{ii'}. \quad (9)$$

The prime on the sum in the second term in (9) indicates that each product occurs only once, and that the two factors are never the same. The terms in the expansion correspond to single, double, $\cdots$, and n_An_Bth-order scattering, the number of nth order scattering terms being given by the binomial coefficient

$$\binom{n_An_B}{n}.$$

With our normalization for the scattering amplitude $F_{AB}(\Delta)$,

$$(d\sigma/d\Delta^2)(AB)=\pi|F_{AB}(\Delta)|^2 \quad (10)$$

and

$$\sigma(AB)=4\pi\,\mathrm{Im}F_{AB}(0). \quad (11)$$

Combining (7)–(9), and (2) we find the following expansion for F_{AB} in powers of the quark-quark scattering amplitudes:

$$F_{AB}(\mathbf{\Delta})=\sum_{ii'}\int d^2\Delta_1\delta^2(\mathbf{\Delta}-\mathbf{\Delta}_1)S_{A,i}(\mathbf{\Delta}_1)f_{ii'}(\mathbf{\Delta}_1)S_{B,i'}(\mathbf{\Delta}_1)+\frac{2i}{4\pi}\sum_{ii',jj'}{}'\int d^2\Delta_1 d^2\Delta_2\delta^2(\mathbf{\Delta}-\mathbf{\Delta}_1-\mathbf{\Delta}_2)$$
$$\times S_{A,ij}(\mathbf{\Delta}_1,\mathbf{\Delta}_2)f_{ii'}(\mathbf{\Delta}_1)f_{jj'}(\mathbf{\Delta}_2)S_{B,i'j'}(\mathbf{\Delta}_1,\mathbf{\Delta}_2)+\left(\frac{2i}{4\pi}\right)^2\sum_{ii',jj',kk'}{}'\int d^2\Delta_1 d^2\Delta_2 d^2\Delta_3\delta^2(\mathbf{\Delta}-\mathbf{\Delta}_1-\mathbf{\Delta}_2-\mathbf{\Delta}_3)$$
$$\times S_{A,ijk}(\mathbf{\Delta}_1,\mathbf{\Delta}_2,\mathbf{\Delta}_3)f_{ii'}(\mathbf{\Delta}_1)f_{jj'}(\mathbf{\Delta}_2)f_{kk'}(\mathbf{\Delta}_3)S_{B,i'j'k'}(\mathbf{\Delta}_1,\mathbf{\Delta}_2,\mathbf{\Delta}_3)+\cdots. \quad (12)$$

Here the functions S are generalized form factors for the hadrons, determined by their internal wave functions. For example,

$$S_{A,ijk}(\mathbf{\Delta}_1,\mathbf{\Delta}_2,\mathbf{\Delta}_3)=\int d\tau_A\psi_A^{\dagger}(\{\mathbf{r}_i\})$$
$$\times\exp[-i(\mathbf{\Delta}_1\cdot\mathbf{r}_i+\mathbf{\Delta}_2\cdot\mathbf{r}_j+\mathbf{\Delta}_3\cdot\mathbf{r}_k)]\psi_A(\{\mathbf{r}_i\}). \quad (13)$$

Note, however, that i, j, and k need not be all different, and, since ψ_A is an internal wave function,

$$d\tau_A=\prod_{i=1}^{nA} d^3r_i\delta^3\left[n_A^{-1}\left(\sum_i \mathbf{r}_i\right)\right]$$

(assuming all quark masses equal). A typical term in (12) is represented graphically in Fig. 2.

The application of (12) to hadron-hadron scattering in the quark model will differ from its application to nucleus-nucleus scattering mainly in that we have no direct information on quark-quark scattering, whereas nucleon-nucleon scattering has been studied extensively. We shall have to attempt to guess the properties of $f_{ii'}(\mathbf{\Delta})$ from the observed properties of hadron-hadron scattering.

Note also that the above formulas are true only if there is no spin or isospin flip in quark-quark scattering. They can be modified to include these effects, at least approximately, and this would certainly be necessary in order to treat such inelastic processes[14,15] as $\pi+N\rightarrow\rho+\Delta$. It seems likely, however, that the probabilities for spin and isospin flip are small at high energies and, furthermore, we want to avoid introducing a large number of parameters at this early stage. In the calculations below, we shall therefore proceed as if the spin and isospin of the individual quarks were strictly conserved.

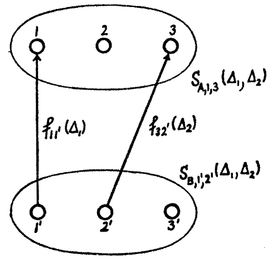

FIG. 2. Graphical representation of a double-scattering term in the multiple-scattering expansion (11), with $i=1$, $j=3$, $i'=1'$, and $j'=2'$.

III. DIFFERENTIAL CROSS SECTIONS

In this section we want to see whether the detailed structure of the differential cross sections for hadron collisions can be explained consistently by taking into account multiple-scattering effects in the quark model if, in the absence of any direct information, we make the simplest assumptions for the forms of the quark-quark scattering amplitudes and the form factors. To reduce the ratio of the number of parameters to the amount of experimental data we consider nucleon-nucleon (NN), nucleon-antinucleon ($\bar{N}N$), and pion-nucleon scattering (πN) simultaneously.

For NN and $\bar{N}N$ scattering we have $n_A=n_B=3$, and therefore nine first-order, 36 second-order, and 84 third-order terms in the expansion (12). For πN scattering $n_A=2$ and $n_B=3$, giving six, 15, and 20 first-, second-, and third-order terms, respectively. The nature of the various terms contributing to a given order of scattering can be quite different depending on the choice of values for the dummy indices ii', jj', and kk'. Among the triple-scattering contributions to NN scattering, for example, the choice $i=j=k=1$ and $i'=1$, $j'=2$, $k'=3$ corresponds to one quark in A scattering successively from each of the three quarks in B, while $i=i'=1$, $j=j'=2$ and $k=k'=3$ corresponds to a process in which each quark in A scatters from a different quark in B.

To clarify this point, and as an aid to keeping the

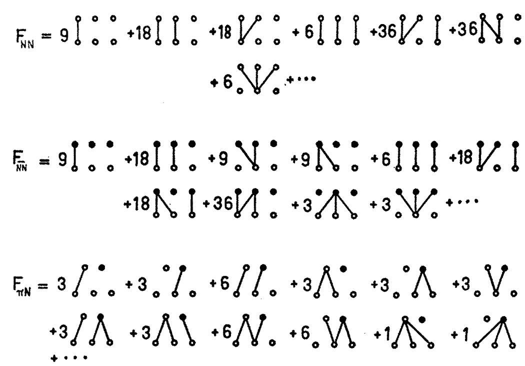

FIG. 3. Diagrams representing the multiple-scattering expansions for the NN, $\bar{N}N$, and πN scattering amplitudes. The integers indicate the number of equivalent terms of a given type.

[14] C. Itzykson and M. Jacob, Nuovo Cimento 48, 909 (1967).
[15] J. L. Friar and J. S. Trefil, Nuovo Cimento 49, 642 (1967).

variables straight, we have introduced the graphs in Fig. 3 to represent the NN, $\bar{N}N$, and πN amplitudes. Each circle in the upper line represents a quark in hadron A, while the circles in the lower line correspond to quarks in B. The quarks and antiquarks are represented by open and solid circles, respectively. The integer in front of each graph indicates the number of equivalent terms of that type contributing to the total amplitude. A line between two quarks indicates a collision between them, transferring momentum $\boldsymbol{\Delta}_i$, and is to be associated with a scattering amplitude $f(\Delta_i)$ (for quark-quark) or $\bar{f}(\Delta_i)$ (for quark-antiquark). The particular version of the form factors S_A and S_B to be used can be determined from the number of lines attached to each quark and the corresponding momentum transferred to it. Note that because backward scattering is ignored no pair of quarks is connected more than once.

The explicit expressions for the hadron-scattering amplitudes are

$$F_{NN}(\boldsymbol{\Delta})=9f(\Delta)S_N^2(\boldsymbol{\Delta},0,0)+\frac{2i}{4\pi}\int d^2\Delta_1 d^2\Delta_2\delta^2(\boldsymbol{\Delta}-\boldsymbol{\Delta}_1-\boldsymbol{\Delta}_2)f(\boldsymbol{\Delta}_1)f(\boldsymbol{\Delta}_2)[18S_N^2(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,0)+18S_N(\boldsymbol{\Delta},0,0)S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,0)]$$
$$+\left(\frac{2i}{4\pi}\right)^2\int d^2\Delta_1 d^2\Delta_2 d^2\Delta_3\delta^2(\boldsymbol{\Delta}-\boldsymbol{\Delta}_1-\boldsymbol{\Delta}_2-\boldsymbol{\Delta}_3)f(\boldsymbol{\Delta}_1)f(\boldsymbol{\Delta}_2)f(\boldsymbol{\Delta}_3)[6S_N^2(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)+36S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)S_N(\boldsymbol{\Delta}_1+\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3,0)$$
$$+36S_N(\boldsymbol{\Delta}_1+\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3,0)S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2+\boldsymbol{\Delta}_3,0)+6S_N(\boldsymbol{\Delta},0,0)S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)]$$
$$+\text{terms from fourth- and higher-order scattering}, \qquad (14)$$

$$F_{\bar{N}N}(\boldsymbol{\Delta})=F_{NN}(\Delta), \quad \text{with } f(\Delta_i)\to\bar{f}(\Delta_i), \qquad (15)$$

and

$$F_{\pi N}(\boldsymbol{\Delta})=3[f(\boldsymbol{\Delta})+\bar{f}(\boldsymbol{\Delta})]S_N(\boldsymbol{\Delta},0,0)S_\pi(\boldsymbol{\Delta},0)+\frac{2i}{4\pi}\int d^2\Delta_1 d^2\Delta_2\delta^2(\boldsymbol{\Delta}-\boldsymbol{\Delta}_1-\boldsymbol{\Delta}_2)$$
$$\times\{3f(\boldsymbol{\Delta}_1)\bar{f}(\boldsymbol{\Delta}_2)[2S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,0)+S_N(\boldsymbol{\Delta},0,0)]S_\pi(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2)+3[f(\boldsymbol{\Delta}_1)f(\boldsymbol{\Delta}_2)+\bar{f}(\boldsymbol{\Delta}_1)\bar{f}(\boldsymbol{\Delta}_2)]S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,0)S_\pi(\boldsymbol{\Delta},0)\}$$
$$+\left(\frac{2i}{4\pi}\right)^2\int d^2\Delta_1 d^2\Delta_2 d^2\Delta_3\delta^2(\boldsymbol{\Delta}-\boldsymbol{\Delta}_1-\boldsymbol{\Delta}_2-\boldsymbol{\Delta}_3)\{3[\bar{f}(\boldsymbol{\Delta}_1)f(\boldsymbol{\Delta}_2)f(\boldsymbol{\Delta}_3)+f(\boldsymbol{\Delta}_1)\bar{f}(\boldsymbol{\Delta}_2)\bar{f}(\boldsymbol{\Delta}_3)]$$
$$\times[S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)+2S_N(\boldsymbol{\Delta}_1+\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3,0)]S_\pi(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2+\boldsymbol{\Delta}_3)+[f(\boldsymbol{\Delta}_1)f(\boldsymbol{\Delta}_2)f(\boldsymbol{\Delta}_3)+\bar{f}(\boldsymbol{\Delta}_1)\bar{f}(\boldsymbol{\Delta}_2)\bar{f}(\boldsymbol{\Delta}_3)]$$
$$\times S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)S_\pi(\boldsymbol{\Delta},0)\}+\cdots. \qquad (16)$$

For convenience we use the notation

$$S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3)\equiv S_{N,123}(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3) \qquad (17)$$

and

$$S_\pi(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2)\equiv S_{\pi,12}(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2), \qquad (18)$$

where $S_{N,123}$ and $S_{\pi,12}$ are defined by (13). These form factors must be completely symmetric under interchange of their arguments, and translational invariance requires that

$$S_N(\boldsymbol{\Delta}_1+\mathbf{q},\boldsymbol{\Delta}_2+\mathbf{q},\boldsymbol{\Delta}_3+\mathbf{q})=S_N(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2,\boldsymbol{\Delta}_3) \qquad (19)$$

and

$$S_\pi(\boldsymbol{\Delta}_1+\mathbf{q},\boldsymbol{\Delta}_2+\mathbf{q})=S_\pi(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2). \qquad (20)$$

Note that (19) implies that the first triple-scattering contribution to F_{NN} in Fig. 3 has no Δ^2 dependence if $f(\Delta)$ is constant. Here each quark in each nucleon can receive the same momentum transfer and thus the nucleons can exchange arbitrarily large amounts of momentum without breaking up.

In order to test the expressions (14)–(16) against the available experimental data we must choose parametrizations for the scattering amplitudes and form factors. For the former we assume

$$f(\Delta)=f(0)\exp(-\tfrac{1}{2}b^2\Delta^2), \qquad (21)$$

$$\bar{f}(\Delta)=\bar{f}(0)\exp(-\tfrac{1}{2}\bar{b}^2\Delta^2), \qquad (22)$$

and take b^2 and $\bar{b}^2$ as well as $f(0)$ and $\bar{f}(0)$ to be complex numbers. In previous work[9] we found that it was impossible to avoid an interference minimum in the differential cross section and at the same time match the phase for the forward proton-proton amplitude if the phase of $f(\Delta)$ were constant. Giving b^2 and $\bar{b}^2$ imaginary parts is perhaps the simplest way to allow the phases of $f(\Delta)$ and $\bar{f}(\Delta)$ to change with Δ.[16]

For the pion and nucleon form factors we take the expressions

$$S_\pi(\boldsymbol{\Delta}_1,\boldsymbol{\Delta}_2)=\exp[-\tfrac{1}{4}a_\pi^2(\boldsymbol{\Delta}_1-\boldsymbol{\Delta}_2)^2] \qquad (23)$$

[16] This particular parametrization of the phase variation was suggested to the authors by Dr. V. Franco (private communication).

and

$$S_N(\mathbf{\Delta}_1,\mathbf{\Delta}_2,\mathbf{\Delta}_3)=\exp\{-\tfrac{1}{4}a_N^2[(\mathbf{\Delta}_1-\mathbf{\Delta}_2)^2+(\mathbf{\Delta}_2-\mathbf{\Delta}_3)^2+(\mathbf{\Delta}_3-\mathbf{\Delta}_1)^2]\}, \quad (24)$$

corresponding to Gaussian wave functions. Equation (24) arises from a completely symmetric spatial wave function, while the standard version of the quark model requires complete antisymmetry[1] and therefore a form factor which takes negative values. This point has also come up with regard to the electromagnetic form factor of the nucleon $F_N(\Delta)$.[17] In the quark model (leaving out inessential complications), we have

$$F_N(\Delta)=F_q(\Delta)S_N(\Delta,0,0), \quad (25)$$

where $F_q(\Delta)$ is the electromagnetic form factor of the quarks. Extensive experimental studies[18] have revealed no zeros in $F_N(\Delta)$ up to $\Delta^2\approx 25$ $(\text{GeV}/c)^2$, implying that if $S_N(\Delta,0,0)$ has any zeros they must be at quite large values of Δ.[19] Although the extrapolation to three variables is a bit risky, we believe that (24) is a reasonable way of parametrizing S_N when the arguments are not too large. Equations (14)–(16) show that for small Δ the differential cross sections go as S_N^4, which in the point-quark limit is equal[20] to F_N^4.

With the above expressions for the amplitudes and form factors, all the integrals involved in (14)–(16) can be evaluated analytically (we do not reproduce the lengthy expressions here). The differential cross sections for NN, $\bar{N}N$, and πN scattering are then determined by 10 real parameters:

$$f(0)\equiv f_R+if_I, \quad (26)$$

$$\bar{f}(0)\equiv \bar{f}_R+i\bar{f}_I, \quad (27)$$

$$b^2\equiv (b^2)_R+i(b^2)_I, \quad (28)$$

$$\bar{b}^2\equiv (\bar{b}^2)_R+i(\bar{b}^2)_I, \quad (29)$$

$$a_N^2, \text{ and } a_\pi^2.$$

We wish to emphasize that the parameters $f(0)$, $\bar{f}(0)$, b^2, and $\bar{b}^2$ in general will be energy-dependent. Judging from the behavior of hadron-hadron scattering, this dependence becomes weaker as the energy increases, but at present accessible energies cannot be ignored. This limits us, in principle, to treating NN and $\bar{N}N$ scattering at the same energy, and πN scattering at some definite related energy. Unfortunately, there is no convincing prescription relating the πN energy to the NN, $\bar{N}N$ energy, although the former should presumably be taken somewhat smaller than the latter.[21]

Our fitting procedure is roughly as follows: We first choose trial values for

$$R\equiv (b^2)_R/a_N^2, \quad (30)$$

$(b^2)_I$, and $(\bar{b}^2)_I$. Once these are fixed, we determine the remaining parameters from the experimental values for $\sigma(NN)$, $\sigma(\bar{N}N)$, $\alpha(NN)$, $\alpha(\bar{N}N)$, $B^2(NN)$, $B^2(\bar{N}N)$, and $B^2(\pi N)$, where

$$\alpha(AB)\equiv \text{Re}F_{AB}(0)/\text{Im}F_{AB}(0) \quad (31)$$

and $B^2(AB)$ is defined by

$$(d\sigma/d\Delta^2)(AB)\propto \exp[-B^2(AB)\Delta^2] \quad (32)$$

for small Δ^2. [The experimental values for $\alpha(\pi N)$ and $\sigma(\pi N)$ are of essentially no use in fixing R, $(b^2)_I$, and $(\bar{b}^2)_I$, but are reasonably well fitted by the calculated values.] If multiple-scattering effects were ignored, the relations used would be simply

$$f(0)=(36\pi)^{-1}\sigma(NN)[i+\alpha(NN)], \quad (33)$$

$$\bar{f}(0)=(36\pi)^{-1}\sigma(\bar{N}N)[i+\alpha(\bar{N}N)], \quad (34)$$

$$2a_N^2+(b^2)_R=B^2(NN), \quad (35)$$

$$2a_N^2+(\bar{b}^2)_R=B^2(\bar{N}N), \quad (36)$$

and

$$a_N^2+\tfrac{1}{2}[a_\pi^2+(b^2)_R+(\bar{b}^2)_R]=B^2(\pi N). \quad (37)$$

We begin with values determined by these relations and correct iteratively for multiple-scattering effects, typical corrections being of the order of 20%. The differential cross sections are then calculated on an electronic computer using complex arithmetic and are compared with experiment. By repeating this procedure for many sets of values for R, $(b^2)_I$, and $(\bar{b}^2)_I$ we determine which set gives the best over-all fit to the experimental data. We discuss below the problems associated with fitting the individual processes and present the results.

A. Proton-Proton Scattering

Since there seem to be no convenient data spanning a wide enough range of momentum transfer at a single energy, we have used an analysis given by Krisch,[22] who has shown that most of the presently available experimental points fall on a single curve when plotted against $\beta^2p^2\sin^2\theta\,(\beta=v/c)$, and when a reasonable adjustment for the effects of the identity of the particles is made. We have used this result to construct an "experimental" plot of $d\sigma/d\Delta^2$ versus Δ^2 (which we interpret as $p_\perp^2=p^2\sin^2\theta$) for $p_{\text{lab}}=15$ GeV/c. This is the highest momentum for which experimentally determined values

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[18] D. H. Coward, H. DeStaebler, R. A. Early, J. Litt, A. Minten, L. W. Mo, W. K. H. Panofsky, R. E. Taylor, M. Breidenbach, J. I. Friedman, W. H. Kendall, P. N. Kirk, B. C. Barish, J. Mar, and J. Pine, Phys. Rev. Letters **20**, 292 (1968).

[19] It has been shown by R. E. Kreps and J. J. de Swart, Phys. Rev. **162**, 1729 (1967) that antisymmetric wave functions exist which can give such form factors.

[20] T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965); T. T. Chou and C. N. Yang, in *High-Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967).

[21] A discussion of this point, with additional references, is contained in Ref. 7.

[22] A. D. Kirsch, Phys. Rev. Letters **19**, 1149 (1967). We have used Kirsch's compilation of the pp data and refer to this reference for the original sources.

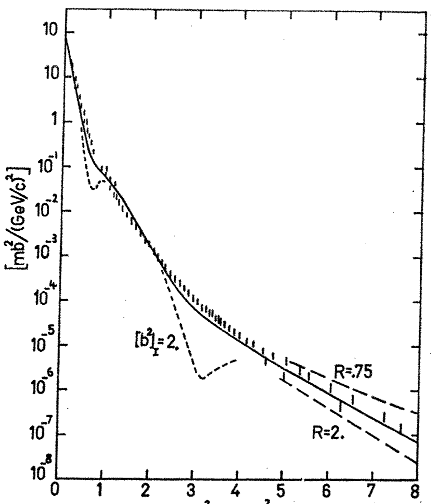

FIG. 4. Fit of the model to the pp scattering data collected and analyzed in Ref. 21. The experimental points reduced to $p_{\rm lab}=15$ GeV/c are represented by the vertical error bars. The short-dashed curve shows the influence of a variation in $(b^2)_I$, the long-dashed curves the influence of variation in $R=(b^2)_R/a_N^2$. The solid curve was drawn using the parameters given in the text.

for $\sigma(\bar{p}p)$, $\alpha(\bar{p}p)$, and $B^2(\bar{p}p)$ are available. For a given R and $(b^2)_I$ we choose the remaining parameters to reproduce[23–25] $\sigma(pp)=39$ mb, $B^2(pp)=10$ (GeV/c)$^{-2}$, and $\alpha(pp)=-0.2$. We had to take $(b^2)_I$ in the neighborhood of -6.0 (GeV/c)$^{-2}$ or $+12.0$ (GeV/c)$^{-2}$ to eliminate the dip due to interference between single and double scattering, and R had to be taken in the neighborhood of 1.2 to give the correct magnitude in the triple-scattering region. The weak-binding or point-quark limit $R=0$ gives a differential cross section which is very flat and too large in the triple-scattering region, while the strong-binding limit $R=\infty$ gives differential cross-section fits which are too small at large Δ^2. Our most satisfactory one is shown in Fig. 4. The parameters used were

$$f_I=1.1\ (\text{GeV}/c)^{-2},$$
$$f_R=-0.18\ (\text{GeV}/c)^{-2},$$
$$(b^2)_R=3.25\ (\text{GeV}/c)^{-2},$$
$$(b^2)_I=6.0\ (\text{GeV}/c)^{-2},$$

and

$$a_N^2=2.7\ (\text{GeV}/c)^{-2}.$$

Figure 5 shows the contributions to the differential cross section from the single-, double-, and triple-scattering amplitudes alone, indicating clearly how the

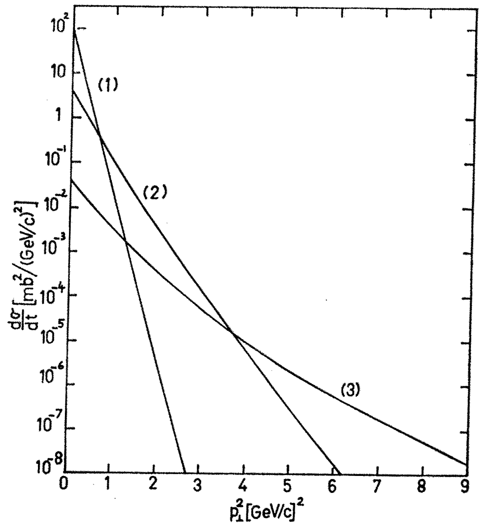

FIG. 5. The differential cross sections obtained from single (1), double (2), and triple (3) scattering terms taken separately.

resulting cross section of Fig. 4 is dominated first by the single-, then in turn by the double- and triple-scattering terms. The phase of the proton-proton scattering amplitude as given by the above set of parameters is shown in Fig. 6. It also reflects the contributions from the individual amplitudes. The initial slope of approximately -5.0 (GeV/c)$^{-2}$ flattens out considerably as the higher terms take over. Note that multiple-scattering analysis of pd scattering seem to require a similar dependence of the phase of the NN amplitude on Δ^2, at least at low momentum transfer.[26]

We should perhaps remind the reader here that our calculations ignore fourth- and higher-order scattering, and that, therefore, our curves should not be taken too

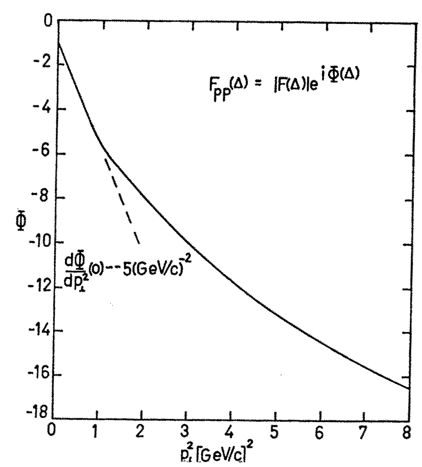

FIG. 6. Phase of the NN scattering amplitude as a function of $p_\perp^2$, calculated using the parameters given in the text.

[23] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. M. Phillips and A. L. Read, Phys. Rev. 138, B913 (1965).

[24] K. J. Foley, R. S. Gilmore, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. H. Willen, R. Yamada, and L. C. L. Yuan, Phys. Rev. Letters 15, 45 (1965).

[25] K. J. Foley, R. S. Jones, S. J. Lindenbaum, W. A. Love, S. Ozaki, E. D. Platner, C. A. Quarles, and E. H. Willen, Phys. Rev. Letters 19, 857 (1967).

[26] G. W. Bennett, J. L. Friedes, H. Palevsky, R. J. Sutter, G. J. Igo, W. D. Simpson, G. C. Phillips, R. L. Stearns, and D. M. Corley, Phys. Rev. Letters 19, 387 (1967); and V. Franco (private communication).

seriously at the highest momentum transfers shown. The qualitative features (in particular the dependence on the parameters) should be valid, however.

B. Proton-Antiproton Scattering

Unfortunately, no large-angle data are available at 15 GeV/c, so we begin by testing our double- and triple-scattering contributions at lower energies where such data are available. We keep a_N^2 fixed at 2.7 (GeV/c)$^{-2}$, the value which we found to give a good fit to the pp differential cross section. The parameters $\bar{f}_R$, $\bar{f}_I$, and $(\bar{b}^2)_R$ are chosen to fit $\sigma(\bar{p}p)$, $B^2(\bar{p}p)$, and $\alpha(\bar{p}p)$, respectively, at the appropriate energy. This leaves only $(\bar{b}^2)_I$ with which to fit the shape of the differential cross section away from the forward direction. The most satisfactory fit at $p_{\rm lab}=3.66$ GeV/c [27] is shown in Fig. 7. It is obtained with

$$\bar{f}_I=2.40\ ({\rm GeV}/c)^{-2},$$
$$\bar{f}_R=-0.53\ ({\rm GeV}/c)^{-2},$$
$$(\bar{b}^2)_R=5.1\ ({\rm GeV}/c)^{-2},$$

and

$$(\bar{b}^2)_I=-1.0\ ({\rm GeV}/c)^{-2}.$$

We also present a calculation for $p_{\rm lab}=6.0$ GeV/c, where we use the experimental values $\sigma(\bar{p}p)=50$ mb, $\alpha(\bar{p}p)=-0.05$, and $B^2(\bar{p}p)=9.0$ (GeV/c)$^{-2}$. We find

$$\bar{f}_I=1.59\ ({\rm GeV}/c)^{-2},$$
$$\bar{f}_R=-0.21\ ({\rm GeV}/c)^{-2},$$
$$(\bar{b}^2)_R=4.9\ ({\rm GeV}/c)^{-2},$$

and use

$$(\bar{b}^2)_I=0.0\,.$$

We have not quite enough data to fix $(\bar{b}^2)_I$ at 15 GeV/c in this way. If we use a value which allows a satisfactory fit to the πp data, we predict the wide-angle differential cross section shown as the dashed curve in Fig. 7, which is obtained with

$$\bar{f}_I=1.4\ ({\rm GeV}/c)^{-2},$$
$$\bar{f}_R=-0.47\ ({\rm GeV}/c)^{-2},$$
$$(\bar{b}^2)_R=1.59\ ({\rm GeV}/c)^{-2},$$

and

$$(\bar{b}^2)_I=-3.0\ ({\rm GeV}/c)^{-2}.$$

Note that the antishrinkage which is barely noticeable in the forward peak becomes very large in the triple-scattering region.

C. Pion-Proton Scattering

Having found these sets of parameters fitting pp and $p\bar{p}$ scattering at 15 GeV/c, we can use them to calculate the πp differential cross section. The only remaining free parameters are a_π^2 [which is chosen to fit the forward slope $B^2(\pi p)$ [24]] and $(\bar{b}^2)_I$; the latter is free only to

[27] W. M. Katz, B. Forman, and T. Ferbel, Phys. Rev. Letters **19**, 265 (1967).

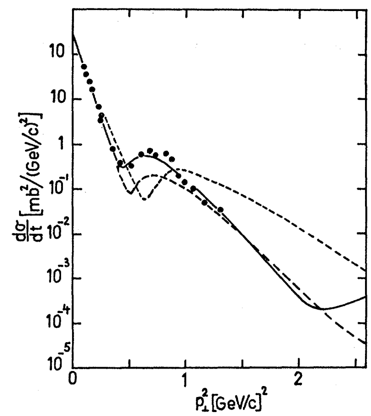

FIG. 7. The $\bar{p}p$ differential cross section as a function of $p_\perp^2$. The solid curve has been fitted to the data (solid circles) at $p_{\rm lab}=3.66$ GeV/c. The heavy dashed line shows a calculation for $p_{\rm lab}=6.0$ GeV/c with $(b^2)_I=0$. The light dashed curve is the prediction for $p_{\rm lab}=15$ GeV/c using $(\bar{b}^2)_I=-3.0$ (GeV/c)$^{-2}$.

the extent that we lack wide-angle $\bar{p}p$ data. The question immediately arises of what πp energy we should try to fit, and, as noted above, there seems to be no unambiguous answer (although it presumably must be less than 15 GeV/c). Fortunately $B^2(\pi N)$ is almost energy-independent above 4.0 GeV/c, and gives $a_\pi^2\simeq 4.5$ (GeV/c)$^{-2}$ with only a very weak dependence on $(b^2)_I$. Taking $(\bar{b}^2)_I=-3.0$ (GeV/c)$^{-2}$ gives the differential cross section shown in Fig. 7, along with some experimental data at 2.5, 2.0, 6.0,[28] and 8 GeV/c.[29] The plot indicates a secondary dip above $p_\perp^2=2.0$ (GeV/c)^2. These values also predict a total cross section $\sigma(\bar{p}p)=31.0$ mb, which is slightly too large.

Our calculations also seem to fit reasonably well some preliminary data for high $|t|$ values at 6.0 and 8.0 GeV/c.[30]

IV. TOTAL CROSS SECTIONS

If multiple-scattering effects are ignored, the quark model predicts a number of "additive" or linear sum rules for total cross sections. Multiple scattering produces corrections to each total cross section and therefore to the sum rules.[6,7] These corrections appear as a byproduct of our differential cross-section fits; we have only to use the optical theorem (3) and calculate $F_{AB}(0)$ using the parameters found. Since our parameters are not necessarily unique, we record here the formulas for the total cross sections with our parametrization includ-

[28] C. T. Coffin, N. Dikmen, L. Ettlinger, D. Meyer, A. Saulys, K. Terwilliger, and D. Williams, Phys. Rev. Letters **15**, 838 (1966); **17**, 458 (1966); D. D. Allen, G. P. Fisher, G. Godden, J. P. Kopelman, L. Marshall, and R. Sears, Phys. Letters **21**, 468 (1966).

[29] J. Orear, R. Rubinstein, D. B. Scarl, D. H. White, A. D. Krisch, W. R. Frisken, A. L. Read, and H. Ruderman, Phys. Rev. Letters **15**, 309 (1965).

[30] F. C. Peterson, J. Orear, D. P. Owen, A. L. Read, D. G. Ryan, D. H. White, A. Ashmore, C. J. S. Damerell, W. R. Frisken, and R. Rubinstein, Bull. Am. Phys. Soc. **12**, 1050 (1967).

ing all double-scattering corrections (the triple-scattering corrections are generally no more than 1%):

$$\sigma(NN)=36\pi f_I+4\pi\,\mathrm{Re}[f(0)]^2 \times[18(2b^2+6a_N^2)^{-1}+18(2b^2+3a_N^2)^{-1}], \quad (38)$$

$$\sigma(\bar{N}N)=36\pi\bar{f}_I+4\pi\,\mathrm{Re}[\bar{f}(0)]^2 \times[18(2\bar{b}^2+6a_N^2)^{-1}+18(2\bar{b}^2+3a_N^2)^{-1}], \quad (39)$$

$$\sigma(\pi N)=12\pi(f_I+\bar{f}_I) +4\pi\,\mathrm{Re}[6f(0)\bar{f}(0)(b^2+\bar{b}^2+2a_\pi^2+3a_N^2)^{-1} +3[f(0)]^2(2b^2+3a_N^2)^{-1} +3[\bar{f}(0)]^2(2\bar{b}^2+3a_N^2)^{-1} +3f(0)\bar{f}(0)(b^2+\bar{b}^2+2a_\pi^2)^{-1}]. \quad (40)$$

In the above formulas $[f(0)]^2$, $f(0)\bar{f}(0)$, and $[\bar{f}(0)]^2$ are close to being purely negative real numbers, and b^2 and $\bar{b}^2$ have nonzero imaginary parts. The latter generally decrease the magnitude of the double-scattering corrections by increasing the magnitude of the denominators and by changing the phase of the double-scattering amplitudes.

For our 15-GeV/c parameters, writing the results as the sum of single- and double-scattering contributions, we have

$$\sigma(NN)=48\text{ mb}-9\text{ mb}=39\text{ mb} \quad (41)$$

and

$$\sigma(\bar{N}N)=63\text{ mb}-13\text{ mb}=50\text{ mb}, \quad (42)$$

while for the corresponding πN energy

$$\sigma(\pi N)=37.7\text{ mb}-6.0\text{ mb}=31.7\text{ mb}, \quad (43)$$

the double scattering corrections reducing the first-order total cross sections by about 20%.[31] Our values for $\sigma(NN)$ and $\sigma(\bar{N}N)$ are taken from experiment at about 15 GeV/c, while $\sigma(\pi N)$ is calculated using the parameters which fit these 15-GeV/c NN and $\bar{N}N$ data. The value of 31.7 mb indicates that either the πN energy corresponding to 15 GeV/c laboratory momentum is very low, in the neighborhood of 3 or 4 GeV/c, or that our total cross-section results are in error by a few millibarns, perhaps because we have ignored corrections such as spin and isospin dependence.

Finally, we record the quark-quark and quark-antiquark total cross sections from the 15-GeV/c parameters:

$$\sigma(qq)=4\pi f_I=5.4\text{ mb}$$

and

$$\sigma(\bar{q}q)=4\pi\bar{f}_I=7.0\text{ mb}.$$

As we would expect from comparing the NN and $\bar{N}N$ total cross sections, $\sigma(\bar{q}q)$ is somewhat larger than $\sigma(qq)$, presumably because of the large number of annihilation channels.

[31] Although his original estimates (Ref. 6) were somewhat larger, corrections of about this size are now also found by V. Franco (private communication).

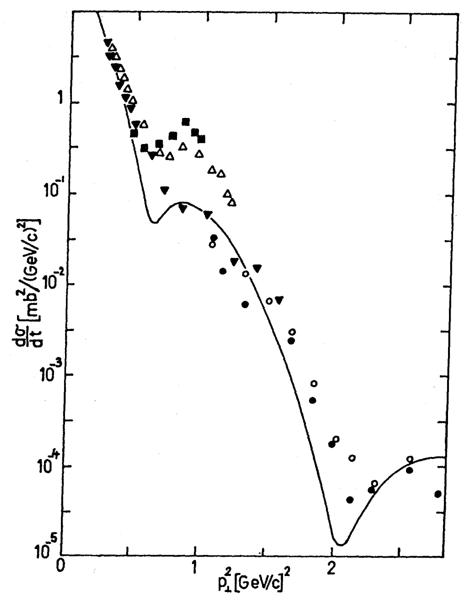

FIG. 8. Differential cross section for πp scattering as a function of $p_\perp^2$. The data are for π^-p at 2.5 GeV/c (open triangles), at 3.0 GeV/c (squares), at 6.0 GeV/c (solid triangles) and at 8.0 GeV/c (solid circles); for π^+p at 8.0 GeV/c (open circles). The curve is the prediction of the model with a_π^2 taken to fit the initial slope, $(b^2)_I=3.0$ (GeV/c)$^{-2}$, and the remaining parameters taken from pp and $\bar{p}p$ data at 15 GeV/c. It predicts a second dip above $p_\perp^2=2.0$ (GeV/c)^2.

V. DISCUSSION

By assuming that nucleons, antinucleons, and pions are composed of three quarks, three antiquarks, and a quark-antiquark pair, respectively, and including contributions from single, double, and triple scattering, we have been able to obtain rather satisfactory fits to the pp, $\bar{p}p$, and πp differential cross sections over a wide range of momentum transfer. Our formulas contain ten free parameters, but seven of these can be determined from the total cross sections, the ratio of real to imaginary part of the forward amplitudes, and the initial slopes of the differential cross sections. The complete structures of the three differential cross sections away from the forward direction are therefore fitted and predicted by three parameters. A glance at Figs. 4, 7, and 8 should convince the reader that this is not a trivial result.

In view of the very hypothetical character of quarks, the lack of some crucial experimental data, and the simplifying approximations we have made, we have not attempted a systematic search for the best possible set of parameters. The parameters given in Sec. III provide reasonably good fits to the available data, but we do not guarantee that another set giving comparable, or even better, fits to the data cannot be found, especially if higher-order scattering is included. Furthermore, we believe that it would be premature to say that our fits exclude composite hadrons with more than three constituents. The following conclusions, however, seem fairly unambiguous:

(1) It is consistent to assume that near the forward direction single scattering dominates, since with the parameters determined under this assumption the multiple-scattering corrections are of the order of 20%.

(2) To avoid a sharp dip in the pp differential cross section the phase of the qq amplitude must vary quite rapidly with momentum transfer. This produces a corresponding rapid variation in the phase of the pp amplitude, roughly consistent with values found in analyses of pd scattering.

(3) If the nucleon is composed of three quarks the "loose-binding" or "point-quark" limit (i.e., $a_N^2 \gg |b^2|$) is not consistent with experiment in the triple-scattering region. In this limit our theory would predict a flat, nearly energy-independent differential cross section for $\Delta^2 > 4.0$ $(\mathrm{GeV}/c)^2$, in contradiction with experiment.

(4) Although the case is not quite as strong as above, it seems difficult to have a large enough triple-scattering cross section in the opposite "tight-binding" limit, $|b^2| \gg a_N^2$. An intermediate situation seems to be required in order to fit the differential cross section for high momentum transfer.

While a detailed analysis of the differential cross sections may well be the most promising way to convince us of the existence of the quarks, the present results do not prove, of course, that the quark model is correct. There are many alternative explanations[32] for the structure in the differential cross sections but most of these other models address themselves to a single structural detail. Our general conclusion is that the quark model with multiple scattering provides a very simple explanation for the structure of high-energy differential cross sections, and thus far we have run into no serious contradictions with experiment. If further work proves as successful, the quark model must at least be regarded as a very convenient device for correlating a large amount of experimental information.

Extensions of these ideas are fairly obvious. The most direct, of course, would be the calculation of the $\pi\pi$ cross section using the parameters determined above. We have not done this in this paper because of the complete absence of direct experimental information. Of more immediate interest, the effects of spin- and isospin-flip amplitudes should be investigated to see if they might effect the shape of the elastic differential cross sections, and to see if they can consistently explain single- and double-flip inelastic processes such as

$$\pi^- p \to \rho^0 + n$$

and

$$\pi^- p \to \pi^+ + \Delta^-.$$

These processes should also be studied experimentally, especially at large momentum transfers. Returning to elastic scattering, we should like to make a special plea here for data covering a wide range of momentum transfer at a single high energy. Care should be taken to cover the region of the breaks carefully, and to ascertain that small-angle and large-angle measurements have consistent normalization by using overlapping regions. With these experimental data the quark model could be tested more rigorously and the parameters determined more precisely.

ACKNOWLEDGMENTS

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[32] R. Serber, Rev. Mod. Phys. **36**, 649 (1964); L. Van Hove, *ibid.* **36**, 655 (1964); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento **42**, 711 (1966), Phys. Letters **25B**. 228 (1967); Nucl. Phys. **B1**, 169 (1967); A. D. Krisch, Phys. Rev. **135**, B1456 (1964); H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. Letters **20**, 280 (1968); also, see Ref. 20.