Some Remarks about Dynamical Asymmetry

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Model equations are studied for the purpose of gaining some understanding about the possible origins of strong $SU(3)$ breaking. The equations are response equations in the octet space of $SU(3)$. They have full $SU(3)$ symmetry apart from electromagnetic and weak driving terms. A class of equations is described for which, amongst others, solutions exist such that (1) they are tilted with respect to the plane defined by the driving forces, thereby generating the formal equivalent of a Cabibbo angle, and (2) large asymmetries occur in a direction which is the one of hypercharge up to electromagnetic corrections.

I. INTRODUCTION

It is the purpose of this paper to make some comments on the often asked question whether the strong symmetry breaking of $SU(3)$ could have a dynamical origin, in contradistinction to the weak and electromagnetic symmetry breaking. By "dynamical origin" is meant that the input forces are fully $SU(3)$-symmetric, apart from weak and electromagnetic violations, and that the strong breaking emerges as an output, namely, as the consequence of the existence of preferred asymmetric solutions of equations generated by this input. The motivation for this question is twofold. First, the success of mass formulas is hard to understand, thus far, from any perturbative point of view. Secondly, no distinct physical attributes can be ascribed, thus far, to the strong breaking, in contradistinction to the electromagnetic and weak breakings. It cannot be asserted, however, that either motivation is entirely compelling.

Of practical necessity, investigations of dynamical symmetry and of dynamical asymmetry most often deal with truncated systems. One attempts to reduce the problem to a finite set of algebraic c-number equations which span one or more representation spaces of the internal symmetry group at hand. Examples are the following:

(1) Bootstrap equations, generally obtained from multichannel dispersion relations with unitarity cuts replaced with a finite set of poles. The bootstrap equations appear as internal consistency conditions on the truncated equations. As a typical example, bootstraps for masses m_i and coupling constants g_i take the form

$$m_i=\sum_j \alpha_{ij}m_j(m_k,g_k)\,, \quad g_i=\sum_j \beta_{ij}g_j(m_k,g_k)\,. \qquad (1.1)$$

The summation may be over one or over more multiplets. Depending on the input, such equations have been used either to implement a dynamical symmetry[1] or a dynamical asymmetry.[2]

(2) The saturation of current-algebra relations with a finite number of states. Here, too, consistency conditions appear.[3] Such equations have been used in attempts to implement dynamical symmetries.

(3) Lagrangian models with truncated equations for sets of Green's functions, or for scalar multiplets with a nonvanishing static limit.[4–6] Again consistency conditions appear. In these Lagrangian models, one has to face the occurrence of Goldstone particles.[4–6] This complication does not necessarily arise in the bootstrap or in the saturation approach.[5]

It should be recalled that the consistency conditions which appear in all of these approaches are, in general, nonlinear and have, in general, more than one admissible solution. One has to seek for stability criteria which may lie outside the set of equations itself to distinguish those solutions further.

In order to solve equations like (1.1), one has to find whether there exists an intersection of a number of hypersurfaces. Imagine that one could use the equations for the g_i to eliminate the latter from the m_i equations, and, furthermore, that in the set of m_i equations, one could eliminate all but one multiplet in favor of the remaining one. In this way, one would obtain consistency conditions within one representation space only. It is this latter kind of consistency condition which forms the subject of the present investigation. We shall consider the specific and limited case where one deals with the octet space of $SU(3)$ in which one *real* (self-adjoint) c-number octet y_i appears, $i=1,\cdots,8$. In the absence of weak and electromagnetic effects, a typical structure for the consistency condition on the y_i is

$$y_i=\sqrt{3}d_{ijk}y_jy_k, \qquad (1.2)$$

an equation which has been studied by several authors.[4–7] The d_{ijk} are the well-known totally sym-

[1] See, for example, E. Abers, F. Zachariasen, and C. Zemach, Phys. Rev. **132**, 1831 (1963).

[2] See, for example, S. L. Glashow, Phys. Rev. **130**, 2132 (1963); R. E. Cutkosky and P. Tarjanne, *ibid.* **132**, 1354 (1963).

[3] For examples, see A. Pais, Phys. Rev. Letters **18**, 17 (1967); M. A. B. Bég and A. Pais, Phys. Rev. **160**, 1479 (1967).

[4] G. Domokos and R. Suranyi, Yadern. Fiz. **2**, 501 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 361 (1966).

[5] R. Brout, Nuovo Cimento **47A**, 932 (1967).

[6] P. di Mottoni and E. Fabri, Nuovo Cimento **54A**, 42 (1968).

[7] N. Cabibbo, Nota Interna 141, Instituto di Fisica Marconi, Rome University, 1967 (unpublished); lecture delivered at Erice in the Proceedings of the International Summer School "Ettore Majorana," 1967 (unpublished); see also *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy, 1968*, edited by A. Perlmutter, C. A. Hurst, and B. Kursunoglu (W. A. Benjamin, Inc., New York, 1968), p. 339.

metric quantities which make $d_{ijk}y_iy_jy_k$ $SU(3)$-invariant. The twofold, linear and quadratic, y dependence in Eq. (1.2) corresponds to the rank of $SU(3)$. It expresses the fact that both sides of Eq. (1.2) have the same (octet) behavior under $SU(3)$ transformations. Equation (1.2) can of course be generalized so as to include coefficients which depend on y_l^2 and on $d_{klm}y_ky_ly_m$. To this possibility we shall return below.

Equation (1.2) has been studied in connection with the problem of spontaneous breakdown of $SU(3)$ symmetry, which does not concern us here. Rather, we shall consider response equations with octet driving forces a_i like

$$y_i=\sqrt{3}d_{ijk}y_jy_k+a_i\,, \tag{1.3}$$

and generalizations thereof. The purpose of this work is to examine further the following pair of questions, which has been broached, with varying emphasis, by several authors[5,7,8] in their discussion of Eq. (1.3):

(a) Consider a system with exact $SU(3)$ symmetry acted on by electromagnetic and weak driving terms. Can these forces generate a large breaking of $SU(3)$ symmetry, thereby obviating the need for the introduction of a separate "medium-strong interaction"?

(b) If the hadron currents which appear in semileptonic decays are octet currents, one can define a "weak hypercharge" which is conserved in leptonic decays in the presence of $SU(3)$-invariant strong interactions and electromagnetic interactions but in the absence of strong $SU(3)$-symmetry breaking.[9] This argument can also be extended to nonleptonic decays.[10] In this situation, one has a zero Cabibbo angle. Is it possible that a fixed nonzero Cabibbo angle θ comes about as a result of weak and electromagnetic driving action, so that, once again, a separate medium-strong interaction need not be introduced from the outset? This question has especially been raised by Cabibbo.[7]

As was done in the cited papers, these questions will be examined here in terms of nonlinear finite systems of c-number equations. Such simplistic model equations may perhaps be of use to get some insight into the algebraic structure of the problem. Obviously, it would be rash to jump from such models to any general conclusions about what happens in the real world. Nevertheless, it seems of some interest to note that models do exist in which both questions can be answered affirmatively, as will be shown in this paper.

It is known[7] that Eq. (1.3) has no solutions which correspond[11] to a fixed nonzero θ. In spite of this negative answer to question (b), Eq. (1.3) is nevertheless important for what follows. We shall call it the special response equation. Its solutions will be discussed in Sec. II, with particular emphasis on completeness. We shall next use Eq. (1.3) to state the two problems just mentioned in a concise form which will also be applicable to more general response equations which are to follow.

We shall suppose that in Eq. (1.3), "8" is the (weak) hypercharge direction, while "1,2,3" span the isospin directions, and we shall say that this is the equation in the y language. The a_i represent the electromagnetic and weak driving forces. In particular,

$$a_i=0\,,\quad i=1,2,4,5,6,7 \tag{1.4}$$

while a_3, a_8 are generally nonzero and will further be specified presently.

Equations (1.3) and (1.4) have certain *formal* covariance properties. For example, the transformation

$$y_1=-q_6\,,\quad y_2=q_7\,,\quad y_4=q_4\,,\quad y_5=q_5\,,\quad y_6=q_1\,,\quad y_7=-q_2\,,$$
$$y_3=\tfrac{1}{2}(q_3-q_8\sqrt{3})\,,\quad y_8=-\tfrac{1}{2}(q_8+q_3\sqrt{3}) \tag{1.5}$$

brings Eq. (1.3) in the form

$$q_i=\sqrt{3}d_{ijk}q_jq_k+b_i\,, \tag{1.6}$$

$$b_3=\tfrac{1}{2}(a_3-a_8\sqrt{3})\,,\quad b_8=-\tfrac{1}{2}(a_8+a_3\sqrt{3}) \tag{1.7}$$

(all other $b_i=0$). The y language being fixed by convention, the q language then corresponds to "8" in the electric-charge direction, while "1,2,3" span the U-spin directions. The formal equivalence between Eqs. (1.3) and (1.4) and Eqs. (1.6) and (1.7) should, of course, not be considered as a physical covariance.

The case of a purely electromagnetic driving force is characterized by

$$a_3=a_8\sqrt{3}\quad \text{or}\quad b_3=0\,. \tag{1.8}$$

What is the scale of b_8 in this case, compared with the coefficients of order unity in the terms of Eq. (1.6)? We shall suppose that[12] $b_8=O(e^2)$, i.e.,

$$b_8\approx 10^{-2}\,. \tag{1.9}$$

The weak-interaction driving force, conserving the weak hypercharge, can be expressed[7,10] as a contribution to a_3. What is the order of this contribution relative to the electromagnetic order of b_8? On dimensional grounds, one will suppose that the weak driving force is $O(GM^2)$ $=O[10^{-5}(M/m)^2]$, where G is the Fermi constant, M is a characteristic mass, and m is the nucleon mass. Thus the relative order in question depends on the unknown M. If M is not much different from m, then the purely electromagnetic relation $b_3=0$ is replaced with

$$b_3\ll b_8\,. \tag{1.10}$$

In what follows, the numerical values of b_3 and b_8 will play no role. Rather, they will be considered as

[8] L. Michel and L. A. Radicati, *Proceedings of the Fifth Coral Gables Conference on Symmetry Principles at High Energy, 1968*, edited by A. Perlmutter, C. A. Hurst, and B. Kursunoglu (W. A. Benjamin, Inc., New York, 1968), p. 19.

[9] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[10] N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964).

[11] Cabibbo shows in Ref. 7 that either $\tan 2\theta=0$ (corresponding to the indistinguishable values $\theta=0$, $\frac{1}{2}\pi$) or else θ is undetermined. These results will be reproduced in Sec. II

[12] Here we follow Ref. 7. While Eq. (1.9) seems plausible, it remains an assumption, nevertheless, which can only be justified by a dynamical derivation of Eq. (1.6).

parameters on which the answers will depend. However, it will be assumed that both b_3 and b_8 are $\ll 1$.

It may be recalled that the pair of quantities (b_3,b_8) in general define a plane in the octet space, if neither quantity vanishes. There are two exceptions to this, namely, for $b_3=\pm b_8\sqrt{3}$, in which case only an axis is defined. It will be assumed that the b_3/b_8 ratio does not take on either of these exceptional values.

After these preliminaries, the questions (a) and (b) will now be stated more precisely. One asks if there exist solutions for q_i which satisfy the following conditions:

$$q_3=C_1\sqrt{3}+C_2b_3\,, \tag{1.11}$$

$$q_8=C_1+C_3b_8+Db_3\,, \tag{1.12}$$

$$(q_1^2+q_2^2)^{1/2}=C_4b_3\,, \tag{1.13}$$

$$q_4=q_5=q_6=q_7=0\,, \tag{1.14}$$

where C_1,C_2,C_3,D are numbers (independent of b_3,b_8) of the general order of unity. In explanation of the Eqs. (1.11)–(1.14), the following should be noted:

(1) The C_1 terms represent the leading order, and correspond to a large symmetry breaking in the response. The fact that $q_3\approx q_8\sqrt{3}$ up to $O(b_8)$ corresponds to the fact that this large symmetry breaking should be in the hypercharge direction to within electromagnetic accuracy. According to a general argument,[8] the equality $q_3=q_8\sqrt{3}$ is unrealizable. However, it is not obvious why such a strict equality should be required physically.

(2) q_8 may be allowed to have an electromagnetic component C_3b_8. It may also have a weak component Db_3. If $D=0$, the strangeness-nonchanging components have $\Delta U=1$, hence $\Delta I=0,1$. If $D=-C_2\sqrt{3}$, one has $\Delta I=1$ only.

(3) A nonvanishing $(q_1^2+q_2^2)^{1/2}=(y_6^2+y_7^2)^{1/2}$ represents a response component of the strangeness-changing $\Delta I=\frac{1}{2}$ kind. Clearly, then, for $(q_1^2+q_2^2)^{1/2}\neq 0$, a θ would be generated characteristic for the response orientation in octet space. θ may be identified in terms of $(q_1^2+q_2^2)^{1/2}$ and the weak component of q_3.

$$\tan 2\theta=C_4/C_2\,, \tag{1.15}$$

whence the requirement that the right-hand side of Eq. (1.14) is $O(b_3)$.

(4) Equation (1.6), as well as related equations to follow, will contain information about $q_1^2+q_2^2$ only, not about q_1,q_2 separately. This freedom (which does not affect the definability of θ) corresponds precisely to the following. Consider all $SU(3)$ transformations which keep b_3 and b_8 separately fixed and which maintain Eq. (1.14). These transformations are just the rotations in the (q_1,q_2) plane, all else fixed.

(5) We are now ready to state a more delicate question concerning the present problem. Does not the existence of a response with a nonvanishing component in the q_1,q_2 plane violate the symmetry of the input (namely, the conservation of charge and of weak hypercharge) *unless* the solution found is only one of a continuum of equivalent solutions? The answer to this question is that there does exist such a continuum, but that they are not equivalent, for the following reason. If there exists a solution which satisfies Eqs. (1.11)–(1.14), then, for example, there must also exist a formal solution obtained from Eqs. (1.11)–(1.14) by the *substitutions* $q_i\rightarrow y_i$, $b_i\rightarrow a_i$ given in Eqs. (1.5) and (1.7). But this new formal solution is *not* the transform of the actual solution (1.11)–(1.14) to another frame of reference. For example, if our initial solution satisfies Eq. (1.10), then the new solution will satisfy $a_3\ll a_8$, which is clearly not the transform of Eq. (1.10). In other words, the new solution is distinct from the initial one. In general, what has happened is therefore the following: By the identification of the 3 and 8 directions *in the q language* with the weak and electromagnetic driving directions, we have chosen a boundary condition which selects (to within rotations in the 1,2 plane) an absolute frame of reference from among a continuum of equivalent ones.

(6) Because of the arbitrariness in the (1,2) plane (perhaps connected with the CP question), one can eliminate q_2 in favor of q_1 by a rotation; equivalently, y_7 in favor of y_6. This will be done for simplicity, so that Eq. (1.13) becomes

$$q_1=C_4b_3. \tag{1.16}$$

As was stated earlier, the preceding discussion is rather academic inasfar as Eq. (1.6) is concerned. In Sec. III, we shall discuss the solutions of a more general class of response equations which does yield a $(q_1\neq 0)$-type solution. The equation is

$$\rho q_i=\alpha\sqrt{3}d_{ijk}q_jq_k+b_i+\sqrt{3}d_{ijk}q_jb_k'\,, \tag{1.17}$$

$$b_3'=\lambda b_3\,,\quad b_8'=\mu b_8. \tag{1.18}$$

Here λ represents the relative scale of the weak interaction as it enters in $d_{ijk}q_jb_k'$, as compared with the weak interaction in b_i. In a similar way, μ refers to the electromagnetic-interaction ratio. The quantities λ and μ will be supposed to be of the general order of unity, so that all weak driving forces will have a common order, and likewise all electromagnetic driving forces. Note that, whereas all $b_i=0$ for $i\neq 3$ or 8, b_i' and b_i belong to the same octet. It will be shown in Sec. III that for given ρ,α,λ,μ this equation has essentially only one kind of solution with $q_1\neq 0$, provided that ρ,α,λ,μ are not all numerical constants. It is in accord with the covariance content of Eq. (1.17) that ρ,α,λ,μ may depend on the bilinear and trilinear scalar products formed from q's and b's.

As will be seen in Sec. III, one of the equations which determine the particular solution at hand is

$$b_3(1+\lambda q_8)=0. \tag{1.19}$$

This equation demonstrates the curious factorization which, time and again, plays a role in this problem. If the small parameter b_3 were zero, Eq. (1.19) would be without content. But if this small parameter is nonzero, then $q_8=-1/\lambda$, independent of the value of b_3. In fact, from the way λ is introduced one sees that the large value of q_8 ($\lambda\sim 1$) is a *ratio* of weak interactions—a nonperturbative result.

Thus q_8 has the desired order of magnitude. It is then shown in Sec. III that the solution at hand, has furthermore, the following property: If $q_1\sim b_3$, then

$$3q_8^2-q_3^2=O(b_8). \tag{1.20}$$

Equation (1.20) is in accord with Eqs. (1.11) and (1.12). The remaining question is then if it is indeed true that $q_1\sim b_3$. This will be seen to be dependent on more detailed properties of ρ,α,λ,μ, and, at least by example, it is shown that $q_1\sim b_3$ is indeed possible. It seems unsatisfactory that the relation (1.16) does not appear in a very transparent way.

Apart from this particular solution of Eq. (1.17), one should ask for all its solutions. A method to classify those is given in Sec. III B. For this purpose, it turns out to be useful to have an explicit derivation of the complete set of the solutions of Eq. (1.6). This problem is discussed in Sec. II. Once the characterization of all solutions of Eq. (1.17) is found, one may ask for the stability of the solutions other than the particular one of interest here. A few remarks are made about this question in Sec. III B, but no complete answers are derived.

If ρ,α,λ,μ are all constants, we shall see in Sec. III A that something remarkable happens too. It will be found that then there exists a special solution to Eq. (1.17) which is possible only for a prescribed value of b_8 in terms of the constants of the equation, namely,

$$\mu b_8=\rho-2\alpha/\lambda. \tag{1.21}$$

Thus nonlinear equations of the type considered here may have, under circumstances, special solutions only for prescribed values of "coupling constants." (This situation is not interesting for our present purpose, since it turns out to yield $q_1=0$.)

It is, of course, a far cry from the models studied here to a full dynamics. Nevertheless, there is one quite qualitative question which one is tempted to ask at once: If weak driving forces (along with electromagnetic ones) are really important to generate a large $SU(3)$-breaking, then why is the induced large breaking not strongly parity-violating? The present considerations are simply not rich enough in structure to answer this question with any definiteness. Yet, one may perhaps see in the following consideration a clue to the fact that such a catastrophe need not necessarily happen. Imagine that one replaces b_3 everywhere in the foregoing with $b_3^{(+)}+b_3^{(-)}$ and that there exists a transformation $b_3^{(\pm)}\rightarrow \pm b_3^{(\pm)}$ which defines an "even" and an "odd" part of b_3. Such a transformation is the closest we can come to representing a parity operation in this model. From Eq. (1.19) one sees that $q_8=-\lambda^{-1}$ is invariant under this transformation, and so is the relation (1.20). Thus, if the large $SU(3)$-breaking is associated with a weak-interaction ratio, as in the present case, large parity violations need not necessarily occur.

One would obtain such large violations if one were to replace b_3' in Eq. (1.18) with $\lambda_1 b_3^{(+)}+\lambda_2 b_3^{(-)}$, $\lambda_1\neq\lambda_2$. Thus an intrinsic weak-interaction property should be invoked to prohibit this [such as, for example, a "chiral" substitution invariance under $b_3^{(+)}\leftrightarrow b_3^{(-)}$].

Next, the question may be asked how such a rather complicated driving force-response coupling as the last term in Eq. (1.17) could come about. The following example may indicate that the occurrence of such a term is not necessarily too far-fetched. Consider, along with q_i, a second octet x_i. Let q_i, x_i mutually drive each other and let both be driven by weak and electromagnetic forces, as follows:

$$q_i=d_{ijk}q_jq_k+Ad_{ijk}x_jq_k+b_i\,, \tag{1.22}$$

$$x_i=Bd_{ijk}q_jq_k+b_i'. \tag{1.23}$$

As discussed in connection with Eq. (1.1), eliminate x_i. One gets

$$(1-\tfrac{1}{3}ABq_k^2)q_i=d_{ijk}q_jq_k+b_i+Ad_{ijk}q_jb_k'\,, \tag{1.24}$$

which illustrates that both the coupling in question and also nonconstant coefficients can readily come about as multichannel effects.

In deriving iterates like Eq. (1.24) and related relations, a number of identities have been found useful, which are recorded in the Appendix.

As a final question, one must ask if Eq. (1.17) is the most general form of a response equation for one octet q_i driven by b_3, b_8. The answer is no, for two reasons.

(1) One may include a term of the type $\epsilon\sqrt{3}d_{ijk}b_jb_k$. This does not qualitatively change the nature of the special solution, because this quadratic driving term lies itself again in the 3 and 8 directions only. Note that in this case, Eq. (1.19) becomes

$$b_3(1+\lambda q_8+2\epsilon b_8)=0\,. \tag{1.25}$$

This generalizes a previous remark: What is nth order in general, is actually $(n-1)$th order for the special solution. See, further, Sec. III A.

(2) Just as one has two types of terms, linear and quadratic, depending on q only [see Eq. (1.2)], so one has two possible distinct couplings of the type $d_{ijk}\xi_jb_k'$, namely, either $\xi_j=q_j$ or $\xi_j=d_{jkl}q_kq_l$. With the help of the identities given in the Appendix one shows that this exhausts the possibilities. As an example of this general situation, consider an alternative to Eqs. (1.22) and (1.23), namely,

$$q_i=d_{ijk}x_jx_k+Ad_{ijk}x_jq_k+b_i\,, \tag{1.26}$$

$$x_i=Bd_{ijk}q_jq_k+b_i'\,, \tag{1.27}$$

which yields, with the help of Eqs. (A1)–(A7) of the Appendix,

$$(1-\tfrac{1}{3}ABq_k^2-\tfrac{2}{3}B^2d_{klm}q_kq_lq_m-\tfrac{4}{3}Bq_kb_k')q_i$$
$$=-\tfrac{1}{3}q_m^2d_{ijk}q_jq_k-\tfrac{4}{3}Bq_m^2b_i'+b_i+Ad_{ijk}b_j'q_k+2Bd_{ijk}d_{jlm}q_lq_mb_k'+d_{ijk}b_j'b_k'. \quad (1.28)$$

The general response equation is briefly discussed in Sec. IV. The equation can have special solutions quite similar to the one discussed for Eq. (1.17).

The present exploration raises many new questions. To name but some: Is there a specific model which will give the physically correct Cabibbo angle? By what criteria could a solution with a tilt out of the 3,8 plane be preferred over a solution without tilt? What are the consistency conditions for representations other than octet? If strong $SU(3)$-breaking is dynamical, then how uniform is this breaking for different processes or vertices? At present, we shall not speculate whether satisfactory answers to all such questions exist. Even so, it is hoped that the present considerations may perhaps be instructive in showing what may happen in nonlinear systems, where common intuition fails.

II. SPECIAL RESPONSE EQUATION

For definiteness, we work in the y language and therefore start with Eq. (1.3). The driving terms are a_3 and a_8. The set of eight equations is invariant for simultaneous rotations in the "4,5" plane and in the "6,7" plane over an equal angle. For any solution y_i we can therefore arrange it so that by such a rotation one of the four quantities y_4, y_5, y_6, y_7 is made to vanish. We choose to do this for y_4. The solutions with $y_4\neq 0$ can then be found by rotating back in the manner indicated above.

With $y_4=0$, Eq. (1.2) yields the following set[13]:

$$y_1(1-2y_8)=\sqrt{3}y_5y_7, \quad (2.1)$$

$$y_2(1-2y_8)=\sqrt{3}y_5y_6, \quad (2.2)$$

$$y_3(1-2y_8)-a_3=\tfrac{1}{2}\sqrt{3}(y_5^2-y_6^2-y_7^2), \quad (2.3)$$

$$y_1y_6=y_2y_7, \quad (2.4)$$

$$y_5(1+y_8-y_3\sqrt{3})=\sqrt{3}(y_1y_7+y_2y_6), \quad (2.5)$$

$$y_6(1+y_8+y_3\sqrt{3})=\sqrt{3}y_2y_5, \quad (2.6)$$

$$y_7(1+y_8+y_3\sqrt{3})=\sqrt{3}y_1y_5, \quad (2.7)$$

$$y_8=-y_8^2+(y_1^2+y_2^2+y_3^2)-\tfrac{1}{2}(y_5^2+y_6^2+y_7^2)+a_8. \quad (2.8)$$

Case 1. If $y_5\neq 0$ and $1+y_8\pm y_3\sqrt{3}\neq 0$, then there are solutions if and only if $a_3=a_8=0$.

Proof. Multiply Eq. (2.5) by the (nonzero quantity) $1+y_8+y_3\sqrt{3}$ and use Eqs. (2.6) and (2.7). This yields

$$(1+y_8)^2=3(y_1^2+y_2^2+y_3^2). \quad (2.9)$$

[13] The nonvanishing d's are fixed by $d_{118}=d_{228}=d_{338}=-2d_{448}=-2d_{558}=-2d_{668}=-2d_{778}=-d_{888}=1/\sqrt{3}$; $d_{146}=d_{157}=-d_{247}=d_{256}=d_{344}=d_{355}=-d_{366}=-d_{377}=\tfrac{1}{2}$.

Multiply Eqs. (2.6) and (2.7) by $y_5\sqrt{3}$ and use Eqs. (2.1) and (2.2). This yields

$$y_2(1-2y_8)(1+y_8+y_3\sqrt{3})=3y_2y_5^2, \quad (2.10)$$

$$y_1(1-2y_8)(1+y_8+y_3\sqrt{3})=3y_1y_5^2. \quad (2.11)$$

If $y_1=y_2=0$, then Eqs. (2.1) and (2.2) yield $y_6=y_7=0$, since $y_5\neq 0$. Then Eq. (2.5) would become $y_5(1+y_8-y_3\sqrt{3})=0$, which is against the hypothesis. Thus y_1 and y_2 cannot both be zero. Then Eqs. (2.10) and (2.11) yield

$$(1-2y_8)(1+y_8+y_3\sqrt{3})=3y_5^2. \quad (2.12)$$

From Eqs. (2.9), (2.3), and (2.8) we get

$$3y_5^2=(1+y_8+y_3\sqrt{3})(1-2y_8)+3a_8-a_3\sqrt{3}, \quad (2.13)$$

$$3(y_6^2+y_7^2)=(1+y_8-y_3\sqrt{3})(1-2y_8)+3a_8+a_3\sqrt{3}. \quad (2.14)$$

From Eqs. (2.12) and (2.13),

$$a_3=a_8\sqrt{3}. \quad (2.15)$$

Multiply Eq. (2.13) by $1+y_8-y_3\sqrt{3}$ and Eq. (2.14) by $1+y_8+y_3\sqrt{3}$; use Eq. (2.15) and subtract. This gives

$$3y_5^2(1+y_8-y_3\sqrt{3})=(1+y_8+y_3\sqrt{3})\times[3(y_6^2+y_7^2)+3a_8+a_3\sqrt{3}]. \quad (2.16)$$

Multiply Eq. (2.5) by y_5 and use Eqs. (2.6) and (2.7). This gives

$$y_5^2(1+y_8-y_3\sqrt{3})=(y_6^2+y_7^2)(1+y_8+y_3\sqrt{3}). \quad (2.17)$$

From Eqs. (2.16) and (2.17) one finds that $a_3=-a_8\sqrt{3}$. Together with Eq. (2.15), this proves the result.

Case 2. If $y_5=0$, there are three possibilities:

Case 2a.

$$y_4=y_5=y_6=y_7=0, \quad (2.18)$$

$$y_1^2+y_2^2+y_3^2=\tfrac{3}{4}-a_8, \quad (2.19)$$

$$y_8=\tfrac{1}{2}. \quad (2.20)$$

This is possible if and only if

$$a_3=0. \quad (2.21)$$

Case 2b.

$$y_1=y_2=y_4=y_5=y_6=y_7=0, \quad (2.22)$$

$$y_3(1-2y_8)=a_3, \quad (2.23)$$

$$y_8=-y_8^2+y_3^2+a_8. \quad (2.24)$$

Case 2c.

$$y_1=y_2=y_4=y_5=0, \quad (2.25)$$

$$1+y_8+y_3\sqrt{3}=0, \quad (2.26)$$

$$(1+y_8)(1-2y_8)+3a_8=\tfrac{3}{2}(y_6^2+y_7^2). \quad (2.27)$$

Case 2c is possible if and only if

$$a_3=a_8\sqrt{3}. \quad (2.28)$$

Proof. Since $y_5=0$, Eq. (2.5) gives

$$y_1y_7+y_2y_6=0. \quad (2.29)$$

From this and from Eq. (2.4) we have the following:

Cases 2a and 2b. $y_4=y_5=y_6=y_7=0$. Equations (2.1)–(2.3) and (2.8) then reduce to

$$y_1(1-2y_8)=0\,, \tag{2.30}$$

$$y_2(1-2y_8)=0\,, \tag{2.31}$$

$$y_3(1-2y_8)=a_3\,, \tag{2.32}$$

$$y_8=-y_8^2+y_1^2+y_2^2+y_3^2+a_8\,. \tag{2.33}$$

Case 2a. If y_1 and/or $y_2\neq 0$, then $y_8=\frac{1}{2}$, which is possible if and only if $a_3=0$.

Case 2b. If $y_8\neq\frac{1}{2}$, then $y_1=y_2=0$.

Case 2c. If y_6 and y_7 are nonzero, then from Eqs. (2.5) and (2.29), $y_1^2+y_2^2=0\rightarrow y_1=y_2=0$, since we only allow real solutions. Then Eqs. (2.3) and (2.6)–(2.8) become

$$y_3(1-2y_8)-a_3=-\tfrac{1}{2}\sqrt{3}(y_6^2+y_7^2)\,, \tag{2.34}$$

$$1+y_8+y_3\sqrt{3}=0\,, \tag{2.35}$$

$$y_8=-y_8^2+y_3^2-\tfrac{1}{2}(y_6^2+y_7^2)+a_8\,. \tag{2.36}$$

Substituting Eq. (2.35) into Eqs. (2.34) and (2.36), one obtains Eqs. (2.27) and (2.28).

Remark. The starting point for case 2 was the pair of equations (2.4) and (2.29). Other ways to satisfy this pair are checked to be included in the above three subdivisions.

Case 3. If $y_5\neq 0$ and $1+y_8+y_3\sqrt{3}\neq 0$, but

$$1+y_8-y_3\sqrt{3}=0\,, \tag{2.37}$$

then

$$y_1=y_2=y_4=y_6=y_7=0\,, \tag{2.38}$$

$$(1+y_8)(1-2y_8)+a_8=\tfrac{3}{2}y_5^2 \tag{2.39}$$

if and only if

$$a_3=-a_8\sqrt{3}\,. \tag{2.40}$$

Remark. The existence of case 3 is a consequence of the existence of case 2c if one notes the invariance of Eq. (1.3) for the substitutions $y_4\leftrightarrow y_7$, $y_5\leftrightarrow y_6$, $y_3\rightarrow -y_3$, $a_3\rightarrow -a_3$.

Proof. Equations (2.4) and (2.29) are again both valid. $y_6=y_7=0$ now implies $y_1=y_2=0$ because of Eqs. (2.6) and (2.7), as $y_5=0$. Conversely, $y_1=y_2=0\rightarrow y_6=y_7=0$ as $1+y_8+y_3\sqrt{3}\neq 0$; so we have Eq. (2.30). Substitute this information in Eqs. (2.3) and (2.8), and use Eq. (2.37). The result follows.

Remarks. (1) The case $y_5\neq 0$, $1+y_8+y_3\sqrt{3}=0$ is readily checked to be a special form of case 3, since these two conditions imply Eq. (2.37). (2) The case $y_5=1+y_8\pm y_3\sqrt{3}=0$ is a special form of case 2c. (3) The rotation back to general y_4 leaves all answers unchanged, except for the right side of Eq. (2.39), which becomes $\frac{3}{2}(y_4^2+y_5^2)$.

Thus we have now found all solutions by considering y_5, $1+y_8+y_3\sqrt{3}$, $1+y_8-y_3\sqrt{3}$, whether zero or not, in all combinations. We translate the classification of the solutions in the q language, noting that $y_5=q_5$, $1+y_8+y_3\sqrt{3}=1-2q_8$, $1+y_8-y_3\sqrt{3}=1+q_8-q_3\sqrt{3}$.

Case 1. If $q_5\neq 0$, $q_8\neq\frac{1}{2}$, $1+q_8-q_3\sqrt{3}\neq 0$, then there are solutions if and only if $b_3=b_8=0$.

Case 2a. If and only if

$$b_3=b_8\sqrt{3}\,, \tag{2.41}$$

we have

$$q_1=q_2=q_4=q_5=0\,, \tag{2.42}$$

$$1+q_8+q_3\sqrt{3}=0\,, \tag{2.43}$$

$$(1+q_8)(1-2q_8)+3b_8=\tfrac{3}{2}(q_6^2+q_7^2)\,. \tag{2.44}$$

Case 2b.

$$q_1=q_2=q_4=q_5=q_6=q_7=0\,, \tag{2.45}$$

$$q_3(1-2q_8)=b_3\,, \tag{2.46}$$

$$q_8=-q_8^2+q_3^2+b_8\,. \tag{2.47}$$

Case 2c.

$$q_4=q_5=q_6=q_7=0\,, \tag{2.48}$$

$$q_1^2+q_2^2+q_3^2=\tfrac{3}{4}-b_8\,, \tag{2.49}$$

$$q_8=\tfrac{1}{2}\,. \tag{2.50}$$

Case 2c is possible if and only if

$$b_3=0\,. \tag{2.51}$$

Case 3. If $q_5=0$, $q_8\neq\frac{1}{2}$, but

$$1+q_8-q_3\sqrt{3}=0\,, \tag{2.52}$$

then

$$q_1=q_2=q_4=q_6=q_7=0\,, \tag{2.53}$$

$$(1+q_8)(1-2q_8)+b_8=\tfrac{3}{2}q_5^2\,, \tag{2.54}$$

if and only if

$$b_3=-b_8\sqrt{3}\,. \tag{2.55}$$

Thus Cabibbo's conclusion[7] that the special response equation does not yield a prescribed component in the (q_1,q_2) plane is clear: In the cases 2a, 2b, and 3, one has $q_1=q_2=0$. In the case 2c, the problem is underdetermined, since Eq. (2.49) is the only equation which governs the behavior in the (q_1,q_2,q_3) subspace. For case 1, we have not written down the actual manifold of solutions, but it is known that in that case, the solutions suffer from either one or the other of the defects just mentioned.

III. A MORE GENERAL RESPONSE EQUATION

A. The Special Solution

We start from Eqs. (1.17) and (1.18) and rotate q_4 to zero, as described in Sec. II. The following set of equations results:

$$q_1(\rho-2\alpha q_8-\mu b_8)=\alpha\sqrt{3}q_5q_7\,, \tag{3.1}$$

$$q_2(\rho-2\alpha q_8-\mu b_8)=\alpha\sqrt{3}q_5q_6\,, \tag{3.2}$$

$$q_3(\rho-2\alpha q_8-\mu b_8)=b_3(1+\lambda q_8)+\tfrac{1}{2}\alpha\sqrt{3}(q_5^2-q_6^2-q_7^2)\,, \tag{3.3}$$

$$q_1q_6=q_2q_7\,, \tag{3.4}$$

$$q_5(\rho+\alpha q_8-\alpha q_3\sqrt{3})=\alpha\sqrt{3}(q_1q_7+q_2q_6)+\tfrac{1}{2}(\lambda b_3\sqrt{3}-\mu b_8)q_5\,, \tag{3.5}$$

$$q_6(\rho+\alpha q_8+\alpha q_3\sqrt{3})=\alpha\sqrt{3}q_2q_5+\tfrac{1}{2}(\lambda b_3\sqrt{3}+\mu b_8)q_6\,, \tag{3.6}$$

$$q_7(\rho+\alpha q_8+\alpha q_3\sqrt{3})=\alpha\sqrt{3}q_1q_5+\tfrac{1}{2}(\lambda b_3\sqrt{3}+\mu b_8)q_7\,, \tag{3.7}$$

$$\rho q_8=\alpha[-q_8^2+q_1^2+q_2^2+q_3^2-\tfrac{1}{2}(q_5^2+q_6^2+q_7^2)]+b_8+\lambda q_3b_3-\mu q_8b_8\,. \tag{3.8}$$

The special solution is obtained by putting

$$q_5=q_6=q_7=0\,, \tag{3.9}$$

so that we are left with

$$q_i(\rho-2\alpha q_8-\mu b_8)=0\,,\quad i=1,2 \tag{3.10}$$

$$q_3(\rho-2\alpha q_8-\mu b_8)=b_3(1+\lambda q_8)\,, \tag{3.11}$$

$$\rho q_8=\alpha[-q_8^2+q_1^2+q_2^2+q_3^2]+b_8+\lambda q_3b_3-\mu q_8b_8\,. \tag{3.12}$$

The solution is further characterized by

$$q_1,q_2 \text{ not both}=0\,, \tag{3.13}$$

which is possible, provided that

$$\rho-2\alpha q_8-\mu b_8=0\,, \tag{3.14}$$

$$b_3(1+\lambda q_8)=0\,. \tag{3.15}$$

Equations (3.12)–(3.15) specify this solution. Equation (3.15) was already referred to in Sec. I, Eq. (1.19). As described in Sec. I, we now rotate q_2 to zero in the (q_1,q_2) plane. With the help of Eqs. (3.14) and (3.15), Eq. (3.12) can then be written as

$$3q_8^2-\left(q_3+\frac{\lambda}{2\alpha}b_3\right)^2=q_1^2-\frac{\lambda^2}{4\alpha^2}b_3^2+\frac{\lambda+2\mu}{\lambda\alpha}b_8\,. \tag{3.16}$$

If ρ,α,λ,μ all were constants, then Eqs. (3.14) and (3.15) would determine b_8 in terms of these four numbers. This is the result mentioned in Eq. (1.21). On the other hand, if these four quantities are not all constant, then we have three equations for the three unknowns q_1,q_3,q_8 (unless ρ,α,λ,μ were such that functional dependence between the equations would arise). For example, if α,λ,μ are constants and ρ does not depend on q_1 (which is possible), then q_8 is fixed by Eq. (3.15), q_3 by Eq. (3.14), and q_1 by Eq. (3.16). Furthermore, it follows from Eq. (3.16) that, if $q_1\sim b_3$, we have Eq. (1.20).

The final question is if there exist ρ,α,λ,μ such that $q_1\sim b_3$. These quantities may depend on the scalar products q_i^2, $d_{ijk}q_iq_jq_k$, q_3b_3, q_8b_8, $d_{ij3}q_iq_jb_3$, and $d_{ij8}q_iq_jb_8$. Let us assume that λ is a constant. Then it follows from Eq. (3.15) that, for the special solution, this set reduces to $q_1^2+q_3^2$, q_3b_3, b_8, and $q_3^2b_8$. If we also introduce scalar products quadratic in the b's, then all we get in addition is b_3^2. We shall give just one example to show that $q_1\sim b_3$ is indeed possible. Namely, one can easily verify that the ρ and α can be such that

$$3q_8^2-(q_3+\lambda b_3/2\alpha)^2=0\,, \tag{3.17}$$

so that $q_1\sim b_3$ if $\lambda=-2\mu$. Observe that it is possible to have Eq. (3.17), because it is admissible that $q_3^2b_8$ occurs separately. This would not have been possible for $b_8=0$! We also note that ρ and/or α should contain some second-order contribution in b_3 in order to get Eq. (3.17).

The inclusion of a term $\epsilon\sqrt{3}d_{ijk}b_jb_k$ on the right-hand side of Eq. (1.17) leads to the following modifications: Equation (3.15) becomes

$$b_3(1+\lambda q_8+2\epsilon b_8)=0\,, \tag{3.18}$$

which has been commented on previously [see Eq. (1.25)]. Furthermore, Eq. (3.14) remains unmodified, while Eq. (3.16) becomes

$$3\left(q_8^2+\frac{\lambda+2\mu}{3\alpha}b_8q_8+\frac{\epsilon}{\alpha}b_8^2\right)-\left(q_3+\frac{\lambda}{2\alpha}b_3\right)^2=q_1^2-\left(\frac{\lambda^2-4\epsilon\alpha}{4\alpha^2}\right)b_3^2\,. \tag{3.19}$$

An argument similar to the one used in connection with Eq. (3.17) readily yields parameter ranges for which $q_1\sim b_3$.

Note that q_1^2 rather than q_1 can possibly be determined, corresponding to the sign ambiguity for θ.

B. Characterization of All Solutions

Define Q_i and B_i as follows:

$$Q_i=\frac{\alpha}{\rho}q_i+\frac{1}{2\rho}b_i'\,, \tag{3.20}$$

$$B_i=\frac{\alpha b_i}{\rho^2}+\frac{1}{2\rho}b_i'-\frac{\sqrt{3}}{4\rho^2}d_{ijk}b_j'b_k'\,. \tag{3.21}$$

Equation (1.17) can be reexpressed as

$$Q_i=\sqrt{3}d_{ijk}Q_jQ_k+B_i\,, \tag{3.22}$$

which has the same formal structure as the special response equation (1.6). Of course, the B_i are now in general functions of the q's, while, furthermore, one will have to check from case to case whether the transformations (3.20) and (3.21) are properly nonsingular. Nevertheless, it is evident that we can begin to classify the solutions by means of the case distinctions of Sec. II. We shall use the results of Sec. II in the q language, Eqs. (2.41)–(2.55), and make the substitutions $q_i\to Q_i$, $b_i\to B_i$ in those equations.

First, let us recognize the special solution discussed in Sec. III A in terms of the new variables. One verifies that that solution is case 2c, Eqs. (2.49)–(2.51). Indeed, Eqs. (3.14)–(3.16) are equivalent to

$$Q_1^2+Q_2^2+Q_3^2=\tfrac{3}{4}-B_8, \tag{3.23}$$

$$Q_8=\tfrac{1}{2}, \tag{3.24}$$

$$B_3=0. \tag{3.25}$$

Of course, while Eq. (2.51) implies the absence of a weak driving term, Eq. (3.25) does not imply this at all!

Next, note that for the cases 2a, 2b, and 3, we have from Eqs. (2.42), (2.45), and (2.53) that[14]

$$Q_1=Q_2=0 \rightarrow q_1=q_2=0. \tag{3.26}$$

Thus none of these cases can generate a Cabibbo angle. There remains case 1, which is characterized by

$$B_3=B_8=0. \tag{3.27}$$

Since we are interested in the special-solution condition $B_3=0$, we must ask if this condition can imply that $B_8=0$ as well. From Eq. (3.21) one checks that, since b_3 and b_8 are linearly independent parameters, $B_8\neq 0$. Hence one can exclude case 1 altogether. Thus we have shown that all solutions with $q_1\neq 0$ are contained in the special solution.

Furthermore, cases 2a and 3 can also be thrown out, as long as $B_3=0$, by the same argument as just given. Hence the only alternative solution to the special solution is given by

$$q_1=q_2=q_4=q_5=q_6=q_7=0, \tag{3.28}$$

$$Q_3(1-2Q_8)=B_3, \tag{3.29}$$

$$Q_8=-Q_8^2+Q_3^2+B_8. \tag{3.30}$$

One can now ask the following question. Suppose a specific form for ρ, λ, α, and μ is given in terms of the scalar products. This form then governs the behavior of both the special and the alternative solution. Can this form be such that the requirements on the special solution are satisfied, while, at the same time, the alternative solution gives rise to a q_3,q_8 which are not both real? If this were the case, only the special solution would be stable, a desirable situation. We have no general answer to this question, but we have found some cases where the answer to the question is negative.

Thus solutions with $q_1\neq 0$ are class 2c. There may be more than one solution of this kind, depending on the structure of ρ, α, λ, and μ. If so, further distinguishing criteria will be necessary; similarly for the alternative solutions which are all class 2b.

IV. GENERAL RESPONSE EQUATION

As was mentioned at the end of Sec. I, there remains the discussion of the role of terms with the structure $d_{ijk}d_{jlm}q_lq_mb_k$. In order to be able to apply the arguments of Sec. III A, it is convenient to make use of the following identity, which is readily proved with the help of the formulas given in the Appendix:

$$d_{ijk}d_{jlm}q_lq_mb_k=-2d_{ijl}d_{jkm}q_lq_mb_k+\tfrac{1}{3}q_m^2b_i+\tfrac{2}{3}b_mq_mq_i. \tag{4.1}$$

This relation leads to the following standard form for the general response equations for one octet:

$$\rho q_i=\alpha\sqrt{3}d_{ijk}q_jq_k+b_i+\sqrt{3}d_{ijk}q_jb_k'+3d_{ijk}q_j(d_{klm}q_lb_m'')+3d_{ijk}b_j'''b_k'''. \tag{4.2}$$

The distinguishing primes on the b factors represent the independent freedom of scalings, as was discussed in connection with Eq. (1.18). ρ and α, as well as the scaling factors, may depend on the scalar products made from q's and b's.

Define Q_i and B_i as follows:

$$Q_i=(\alpha/\rho)q_i+(1/2\rho)(b_i'+c_i), \tag{4.3}$$

$$B_i=(\alpha/\rho^2)(b_i+3d_{ijk}b_j'''b_k''')+(1/2\rho)(b_i'+c_i)-(\sqrt{3}/4\rho^2)d_{ijk}(b_j'+c_j)(b_k'+c_k), \tag{4.4}$$

where

$$c_i=\sqrt{3}d_{ijk}q_jb_k''. \tag{4.5}$$

Then, once again, we get the formal structure of the special response equation

$$Q_i=\sqrt{3}d_{ijk}Q_jQ_k+B_i. \tag{4.6}$$

Equations (3.20)–(3.22) must now be read as special examples of Eqs. (4.3)–(4.5). With this understanding, the content of Sec. III B can now be applied without modification to Eqs. (4.3)–(4.5). The special solutions of Eq. (4.6) (special in the same sense as in Sec. III A) are again contained in Eqs. (3.23)–(3.25). Also, it remains true that none of the other cases can generate a Cabibbo angle. One small difference should be noted: Instead of Eq. (3.26), we now have

$$Q_1=Q_2=0\rightarrow q_1=q_2=0, \tag{4.7}$$

provided that

$$2\alpha+b_8''\neq 0. \tag{4.8}$$

It remains true that Eqs. (3.23)–(3.25) will in general be sufficient for the determination of q_1, q_3, q_8. However, one can easily verify that the simple factorization, as in Eq. (1.19) or Eq. (1.25), does not hold for the general case without further qualifications.

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APPENDIX

In the derivation of consistency conditions for coupled $SU(3)$ multiplets [as in Eqs. (1.22), (1.23),

[14] We exclude the uninteresting case $\alpha=0$.

(1.26), and (1.27)], the following identities are helpful:

$$3d_{ijk}d_{ilm}=\delta_{jl}\delta_{km}+\delta_{jm}\delta_{kl}-\delta_{jk}\delta_{lm}+f_{jli}f_{kmi}+f_{jmi}f_{kli}, \quad (A1)$$

$$f_{ijk}f_{ilm}=\tfrac{2}{3}[\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}]+d_{jli}d_{kmi}-d_{jmi}d_{kli}, \quad (A2)$$

$$3(d_{ikn}d_{lmn}+d_{imn}d_{kln}+d_{iln}d_{kmn})=\delta_{ik}\delta_{lm}+\delta_{im}\delta_{kl}+\delta_{il}\delta_{km}, \quad (A3)$$

$$d_{kal}d_{lbm}d_{mck}=-\tfrac{1}{2}d_{abc}, \quad (A4)$$

$$f_{kal}d_{lbm}d_{mck}=\tfrac{5}{6}f_{abc}, \quad (A5)$$

$$f_{kal}f_{lbm}d_{mck}=-\tfrac{3}{2}d_{abc}, \quad (A6)$$

$$f_{kal}f_{lbm}f_{mck}=-\tfrac{3}{2}f_{abc}. \quad (A7)$$

The d symbols have been defined previously.[13] The f_{ijk} are the totally antisymmetric structure constants with the normalization $f_{123}=1$. Note that the identity (A3) is to the d's what the Jacobi identity is to the f's.

Note added in proof. I am indebted to L. Michel for pointing out to me that such identities also occur in I. Ogievetskii and I. V. Polubarinov, Yadern. Fiz. **4**, 853 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 605 (1967)].

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Asymptotic Behavior of Amplitudes for Meson Photoproduction*

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The asymptotic behavior of amplitudes for meson photoproduction off fermions is investigated in a simple perturbative model. It is found that a fixed power behavior and hence a fixed J-plane pole may occur even if the produced meson is "composite." The implications of this result are discussed.

I. INTRODUCTION

Since the discovery that Fubini–Dashen–Gell-Mann sum rules lead to right-signature fixed poles in the complex angular momentum plane for Compton scattering,[1,2] there has been speculation that fixed J-plane poles may also occur in meson photoproduction.[3,4] Both processes, Compton scattering and meson photoproduction, have in common the fact that they do not satisfy quadratic but only linear unitarity relations. A fixed angular momentum pole for meson photoproduction could explain very naturally two features of this process[4]: (1) The forward peak[5] (or at least a large fraction of it) in π^+ photoproduction can be understood without taking resource to a conspirator or a large cut contribution.[6–10] (2) The fact that there is no shrinking but that $s^2(d\sigma/dt)$ is independent of s [for $|t|>0.1$ $(\text{GeV}/c)^2$] in the energy range above 5 GeV[5] (up to 16 GeV to date) would, of course, be a direct consequence of a fixed pole at $J=0$ in the angular momentum plane.

Model investigations of Bronzan, Gerstein, Lee, and Low[1] and Abarbanel, Low, Muzinich, Nussinov, and Schwarz[11] have made it plausible that the fixed poles are brought into the weak amplitudes (i.e., amplitudes not subjected to quadratic unitarity) by the s- and u-channel Born terms and are not cancelled by the strong final-state interaction in the t channel.

Rubinstein, Veneziano, and Virasoro[12] have investigated the relation between fixed poles and compositeness in a specific model. They consider a case where the elementary particles are scalar and find that for photoproduction off scalar particles a fixed pole occurs at $J=0$ if the produced (scalar) particle is elementary, but not if the produced particle is composite (composed of two scalar particles). The results of Rubinstein, Veneziano, and Virasoro apply also for the Born term alone, and therefore one is led to the following conjecture: A fixed pole occurs then and only then in a weak amplitude, if the s- and u-channel Born terms give rise to a fixed pole.

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