Radiative Pion Capture and the Pion-Nucleus Vertex Function*

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We use the Kroll-Ruderman theorem and the elementary-particle treatment of the nucleus to calculate s-wave capture rate of pions by He^3 and Li^6. The results are compared with previous estimates and experiment. We also use the observed Panofsky ratio to study the πHe^3H^3 vertex function.

I. INTRODUCTION

THE Kroll-Ruderman[1] theorem relates the transition matrix element for the photoproduction of pions from nucleons near threshold to the nucleon-to-nucleon matrix element of the axial-vector hadron weak current in the limit $m_\pi/m_p \to 0$ (m_π and m_p being the pion and nucleon masses). Recent derivations[2] have simplified the proof considerably.

The theorem has been generalized to nuclear systems and applied to photoproduction of pions from He^3,[3,4] B^{11}, and O^{16}.[5] The nuclei involved are treated as elementary particles[6] so that the corresponding axial-vector matrix elements are calculated in a model-independent way.

In Sec. II we apply the method to the s-wave capture of pions by He^3 and Li^6. Capture by He^3 has also been discussed by Ericson and Figureau.[4] We compare our results with experiment and previous theoretical estimates. The πHe^3H^3 vertex function is discussed in Sec. III.

II. CAPTURE RATES

A. Calculation

We can write the Kroll-Ruderman theorem for the lowest-order s-wave amplitude for the process $\pi^- + N_i \to N_f + \gamma$ in the form[3,5]

$$\langle N_f, \gamma | j_\pi^{(-)}(0) | N_i \rangle \cong i\frac{(4\pi)^{1/2}e}{a_\pi m_\pi}\epsilon_\mu^{(\lambda)}\langle N_f | A_\mu^{(-)}(0) | N_i\rangle, \quad (1)$$

where m_π, $a_\pi(=0.95)$, and $j_\pi^{(-)}(0)$ are, respectively, the mass, the decay coupling constant, and the source of the pion field. $\epsilon_\mu^{(\lambda)}$ is the photon polarization vector and $A_\mu^{(-)}(0)$ is the axial-vector hadron weak current. The corrections to Eq. (1) are of order m_π/m, where m is the nuclear mass ($\sim 5\%$ for He^3, 2.5% for Li^6). If we use the elementary-particle description of nuclei, the matrix elements of the axial-vector current can be written as

$$\langle H^3 | A_\mu^{(-)}(0) | He^3\rangle = \bar{u}(H^3)[\gamma_\mu\gamma_5 F_A(q^2; He^3 \leftrightarrow H^3) + (i2m/m_\pi^2)q_\mu\gamma_5 F_P(q^2; He^3 \leftrightarrow H^3)]u(He^3), \quad (2a)$$

$$\langle He^6 | A_\mu^{(-)}(0) | Li^6\rangle = \sqrt{2}m[\xi_\mu F_A(q^2; Li^6 \leftrightarrow He^6) + (\xi\cdot q/m_\pi^2)q_\mu F_P(q^2; Li^6 \leftrightarrow He^6)], \quad (2b)$$

$$q_\mu = (p_f - p_i)_\mu,$$

where $u(N)$ is a Dirac spinor and ξ_μ is spin-1 polarization vector. $F_A(q^2; N_i \leftrightarrow N_f)$ and $F_P(q^2; N_i \leftrightarrow N_f)$ are the *nuclear* axial-vector and induced pseudoscalar form factors which in this treatment contain the complexity of the nuclear structure. The pions are mostly captured from the s orbit[4] and can be treated as if they were at rest. In this approximation the momentum transfer $|\mathbf{q}|$ is equal to the momentum of the photon $|\mathbf{k}|$, and the F_P terms in Eq. (2) do not contribute.

From Eqs. (1) and (2) we obtain the transition rates

$$\lambda(\pi^- + He^3 \to H^3 + \gamma) \cong \frac{2\alpha}{a_\pi^2}\frac{E_\gamma}{m_\pi^3}\left(\frac{m}{m+E_\gamma}\right)C(He^3)\frac{Z^3\alpha^3 m_\pi^3}{\pi}\left(\frac{1}{1+m_\pi/m}\right)^3 \times [F_A(q^2; He^3 \leftrightarrow H^3)]^2, \quad (3a)$$

$$E_\gamma = 136 \text{ MeV}, \quad q^2 = -m_\pi^2 + 2m_\pi E_\gamma = 0.95 m_\pi^2,$$

$$\lambda(\pi^- + Li^6 \to He^6 + \gamma) \cong \frac{\alpha}{3a_\pi^2}\frac{E_\gamma}{m_\pi^3}\left(\frac{m}{m+E_\gamma}\right)C(Li^6)\frac{Z^3\alpha^3 m_\pi^3}{\pi}\left(\frac{1}{1+m_\pi/m}\right)^3 \times [F_A(q^2; Li^6 \leftrightarrow He^6)]^2, \quad (3b)$$

$$E_\gamma = 134 \text{ MeV}, \quad q^2 = -m_\pi^2 + 2m_\pi E_\gamma = 0.92 m_\pi^2,$$

where the nonpoint character of the charge distribution of the nucleus gives rise to a correction factor $C(N_i)$.

We now determine the numerical value of $F_A \times (q^2; He^3 \leftrightarrow H^3)$ and $F_A(q^2; Li^6 \leftrightarrow He^6)$ from the appropriate experimental data without using a nuclear

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[1] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).
[2] S. Okubo, Nuovo Cimento **41**, A586 (1966); P. de Baenst, M. Konuma, and J. Weyers, *ibid.* **45**, 501 (1966); S. Ragusa, *ibid.* **51**, 118 (1967); G. W. Gaffney, Phys. Rev. **161**, 1599 (1967). See also N. Cabibbo, in *Proceedings of the Thirteenth Annual International Conference on High-Energy Physics, Berkley, California, 1966* (University of California Press, Berkley, 1967), p. 29 and references therein.
[3] D. Griffiths and C. W. Kim, Nucl. Phys. **B4**, 309 (1968).
[4] M. Ericson and A. Figureau, CERN Report No. TH. 847 (unpublished).
[5] D. Griffiths and C. W. Kim, **B6**, 49 (1968).
[6] C. W. Kim and H. Primakoff, Phys. Rev. **139**, B1447 (1965).

model. From Eq. (2) we can relate $F_A(0;N_i \leftrightarrow N_f)$ to the ft value for the corresponding β decay;

$$ft(H^3 \to He^3 + e^- + \bar{\nu}) = \frac{2\pi^3 \ln 2}{G^2 m_e^5\{1+3[F_A(0;He^3 \leftrightarrow H^3)]^2\}}, \tag{4}$$

$$ft(He^6 \to Li^6 + e^- + \bar{\nu}) = \frac{2\pi^3 \ln 2}{G^2 m_e^5 \frac{3}{2}[F_A(0;Li^6 \leftrightarrow He^6)]^2}, \tag{5}$$

where $G=10^{-5}/m_p^2$. Inserting the experimental ft values, $ft(H^3 \to He^3+e^-+\bar{\nu})=1137$ sec and $ft(He^6 \to Li^6+e^-+\bar{\nu})=808$ sec, gives[7]

$$|F_A(0;He^3 \leftrightarrow H^3)|=1.22, \tag{6a}$$

$$|F_A(0;Li^6 \leftrightarrow He^6)|=2.26. \tag{6b}$$

The momentum-transfer dependence of the form factors can be obtained[8] from elastic e-He^3, e-H^3,[9] and inelastic e-Li^6 [10] scattering data, and at the value of q^2 appropriate to capture we find[11]

$$[F_A(q^2=0.95m_\pi^2;He^3 \leftrightarrow H^3)]^2 = 0.587[F_A(0;He^3 \leftrightarrow H^3)]^2, \tag{7a}$$

$$[F_A(q^2=0.92m_\pi^2;Li^6 \leftrightarrow He^6)]^2 = 0.208[F_A(0;Li^6 \leftrightarrow He^6)]^2. \tag{7b}$$

From Eqs. (3), (6), and (7) we then obtain[12]

$$\lambda(\pi^- + He^3 \to H^3 + \gamma) = 2.32\times10^{15}\ \text{sec}^{-1}, \tag{8a}$$

$$\lambda(\pi^- + Li^6 \to He^6 + \gamma) = 1.65\times10^{15}\ \text{sec}^{-1}. \tag{8b}$$

We can also calculate[8] the rate $\lambda(\mu^-+Li^6 \to He^6+\nu)$ from the same experimental data used in estimating $\lambda(\pi^-+Li^6 \to He^6+\gamma)$ in Eq. (8b). In this case,

$$q^2 = -m_\mu^2 + 2m_\mu E_\nu = 0.91\ m_\mu^2,$$

$$[F_A(q^2=0.91m_\mu^2;He^6 \leftrightarrow Li^6)]^2 = 0.39[F_A(0;He^6 \leftrightarrow Li^6)]^2, \tag{9}$$

from which one finds

$$\lambda(\mu^- + Li^6 \to He^6 + \nu) = 1.33\times10^3\ \text{sec}^{-1}. \tag{10}$$

[7] The value of Eq. (6b) is twice the value given in Ref. 8. This is due to the different normalization used for the nuclei.

[8] C. W. Kim and H. Primakoff, Phys. Rev. **140**, B566 (1965).

[9] H. Collard, R. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. **138**, B57 (1956).

[10] J. Goldemberg, W. C. Barber, F. H. Lewis, Jr., and J. D. Walecka, Phys. Rev. **134**, B1022 (1964).

[11] The numberical coefficient in Eq. (7b) is based on the recently observed value $\Gamma(Li^{6*}(0^+) \to Li^6)=4.7$ eV as reported by W. C. Barber, J. Goldemberg, G. A. Peterson, and Y. Torizuka, Nucl. Phys. **14**, 461 (1963). This remark also applied to Eq. (10).

[12] We have used the values of $C(N_i)$ given in Ref. 8. A small correction due to the difference of the pion and muon masses is neglected.

B. Discussion

Deutsch *et al.*[13] measured the μ-capture rate of Li^6 and found

$$\lambda(\mu^- + Li^6 \to He^6 + \nu) = (1.6_{-0.13}^{+0.33})\times10^3\ \text{sec}^{-1}. \tag{11}$$

This disagrees with our result in Eq. (10) and would seem to indicate that there is a discrepancy between the electron-scattering data[10,11] and the muon-capture data.[13] More experiments might clarify this point.

The rates $\lambda(\pi^-+He^3 \to H^3+\gamma)$ and $\lambda(\pi^-+Li^6 \to He^6+\gamma)$ have not been directly measured. In the He^3 case, impulse-approximation calculations have given values ranging from 0.97×10^{15} sec^{-1} [14] to 8.32×10^{15} sec^{-1},[15] and on the basis of a treatment similar to ours Ericson and Figureau[4] made an estimate of 3.46×10^{15} sec^{-1}.

Deutsch *et al.* combined the relation[13]

$$[F_A(q^2=0.92m_\pi^2;He^6 \leftrightarrow Li^6)]^2 = 0.44[F_A(q^2=0.91m_\mu^2;He^6 \leftrightarrow Li^6)]^2 \tag{12}$$

and their experimental data on muon capture to obtain

$$\lambda(\pi^- + Li^6 \to He^6 + \gamma) = (2.0_{-0.2}^{+0.4})\times10^{15}\ \text{sec}^{-1}. \tag{13}$$

However, from the electron-scattering data alone, i.e., from Eqs. (7b) and (9), one can see that the *experimental* value of the coefficient in Eq. (12) is 0.535 rather than 0.44. This would change the value of Eq. (13) to $(2.44_{-0.24}^{+0.49})\times10^{15}$ sec^{-1}. As already mentioned, the disagreement between this value and our result, Eq. (8b), may be attributed to the discrepancy between the electron-scattering data and the muon-capture data. Combining the capture rate $\lambda(\pi^-+Li^6 \to He^6+\gamma)$ and the observed branching ratio[13]

$$\frac{\lambda(\pi^- + Li^6 \to He^6 + \gamma)}{\lambda(\pi^- + Li^6)_{\text{total}}} = 0.010 \pm 0.001, \tag{14}$$

we can find the total capture rate $\lambda(\pi^+ + Li^6)_{\text{total}}$ which is approximately equal to the width Γ_{1s} of the 1s orbit of the pion. The results

$$\Gamma_{1s} = 0.159\ \text{keV} \quad \text{from } \lambda(\pi^- + Li^6 \to He^6 + \gamma) = 2.44\times10^{15}\ \text{sec}^{-1},$$
$$\Gamma_{1s} = 0.107\ \text{keV} \quad \text{from } \lambda(\pi^- + Li^6 \to He^6 + \gamma) = 1.65\times10^{15}\ \text{sec}^{-1}, \tag{15}$$

are both consistent with a recent measurement,[16] $\Gamma_{1s}=0.15\pm0.05$ keV.

III. π-NUCLEUS VERTEX FUNCTION

Although $\lambda(\pi^-+He^3 \to H^3+\gamma)$ has not been directly measured, the Panofsky ratio, which is defined as

$$P = \lambda(\pi^- + He^3 \to H^3 + \pi^0)/\lambda(\pi^- + He^3 \to H^3 + \gamma),$$

[13] J. P. Deutsch *et al.*, Phys. Letters **26B**, 315 (1968).

[14] P. P. Divakaran, Phys. Rev. **139**, B387 (1965).

[15] A. Fujii and D. J. Hall, Nucl. Phys. **32**, 102 (1962).

[16] R. J. Harris, Jr., W. B. Shuler, M. Eckhause, R. T. Siegel, and R. E. Welsh, Phys. Rev. Letters **20**, 505 (1968).

has an observed value of 2.28±0.18.[17] Treating the nuclei as elementary particles, we can calculate the rate $\lambda(\pi^-+\mathrm{He}^3\to\mathrm{H}^3+\pi^0)$ in terms of the scattering lengths. The result is

$$\lambda(\pi^-+\mathrm{He}^3\to\mathrm{H}^3+\pi^0)=\frac{8\pi}{9}|a_{1/2}-a_{3/2}|^2\frac{p_0}{m_\pi}\left(1+\frac{m_\pi}{m}\right)\times C(\mathrm{He}^3)\frac{Z^3\alpha^3m_\pi^3}{\pi}\left(\frac{1}{1+m_\pi/m}\right)^3,\quad(16)$$

where $a_{1/2}$ and $a_{3/2}$ are the π^--He^3 scattering lengths in states of isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$ and $p_0(=36.9$ MeV) is the momentum of the outgoing π^0. From Eqs. (3a) and (16),

$$P=\frac{4\pi a_\pi^2}{9\alpha}\frac{p_0}{E_\gamma}\left(1+\frac{E_\gamma}{m}\right)\left(1+\frac{m_\pi}{m}\right)\times\frac{m_\pi^2|a_{1/2}-a_{3/2}|^2}{[F_A(q^2=0.95m_\pi^2;\mathrm{He}^3\leftrightarrow\mathrm{H}^3)]^2}=59m_\pi^2|a_{1/2}-a_{3/2}|^2,\quad(17)$$

and using the experimental value of P we obtain[18]

$$|a_{1/2}-a_{3/2}|=0.196/m_\pi.\quad(18)$$

The relation between the physical $\pi\mathrm{He}^3\mathrm{H}^3$ coupling constant $f(-m_\pi^2;\pi\,\mathrm{He}^3\mathrm{H}^3)$ and the scattering lengths is given in Ref. 19 as

$$[f(-m_\pi^2;\pi\,\mathrm{He}^3\mathrm{H}^3)]^2\cong\tfrac{4}{3}\pi(1+m_\pi/m)\times[F_A(0;\mathrm{He}^3\leftrightarrow\mathrm{H}^3)]^2|a_{1/2}-a_{3/2}|,\quad(19)$$

provided that $\mathrm{Im}a_{1/2}=\mathrm{Im}a_{3/2}=0$. In general, π-N_i scattering lengths are complex, but for the He^3-H^3 system the imaginary parts of the scattering lengths are known to be negligible.[4] We thus find from Eqs. (6a), (18), and (19)

$$|f(-m_\pi^2;\pi\mathrm{He}^3\mathrm{H}^3)|=1.13,\quad(20)$$

which may be compared with the nucleon value[20]

$$|f(-m_\pi^2;\pi np)|=1.43.\quad(21)$$

Using the nuclear Goldberger-Treiman relations[6,19]

$$F_A(0;N_i\leftrightarrow N\mathrm{I})=a_\pi f(0;\pi N_iN_f)\equiv a_\pi f(-m_\pi^2;\pi N_iN_f)K(0;\pi N_iN_f),\quad(22)$$

and Eqs. (20) and (21), we find

$$|K(0;\pi np)|=0.87,\quad(23\mathrm{a})$$

$$|K(0;\pi\mathrm{He}^3\,\mathrm{H}^3)|=1.14.\quad(23\mathrm{b})$$

In the dispersion-theory treatment of the Goldberger-Treiman relation, the results of Eq. (23) would imply that while the sign of the contribution of states, other than the dominant single-pion pole, is negative for the n-p system, it is positive for the H^3-He^3 system.

It is generally expected[21] that while $f(-m_\pi^2;\pi N_iN_f)$ will vary considerably from nucleus to nucleus since it contains much information on nuclear structure, $K(0;\pi N_iN_f)$ will vary very smoothly. If we assume a relation of the form

$$K(0;\pi N_iN_f)=A^{1/n}K(0;\pi np),\quad(24)$$

where A is the mass number of the nucleus N_i, the values of Eq. (23) imply that $n\cong4$. This is not very inconsistent with the value $n\cong6$, which has been suggested on the basis of a nuclear Adler-Weissberger relation.[22]

[17] O. A. Zaimidoroga *et al.*, Zh. Eksperim. i Teor. Fiz. **48**, 1267 (1965) [English transl.: Soviet Phys.—JETP **21**, 848 (1965)].

[18] The corresponding value for the π-p scattering is $|a_{1/2}-a_{3/2}|=(0.259\pm0.090)/m_\pi$, as reported by J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[19] H. Primakoff, *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967).

[20] See, for example, Ref. 18.

[21] See remarks after Eqs. (19b) and (42b) in Ref. 19.

[22] A generalization of the Adler-Weisberger sum rule [S. Adler, Phys. Rev. **140**, B736 (1965); W. I. Weisberger, *ibid.* **143**, 1302 (1965)] to nuclear systems yields the following relation [see Eq. (36c) in Ref. 19]:

$$\mathcal{g}(0;\pi,N_f)\cong[K(0;\pi N_iN_f)]^2\mathcal{g}(-m_\pi^2;\pi,N_f),$$

$$\mathcal{g}(q^2;\pi,N_f)\equiv\frac{m_\pi^2}{\pi}\int_{m_\pi}^\infty\frac{dE_\pi(E_\pi^2-m_\pi^2)^{1/2}}{E_\pi^2}\times[\sigma(q^2;E_\pi;\pi^-,N_f)-\sigma(q^2;E_\pi;\pi^+,N_f)].$$

The left-hand side of the above relation is related to the ν-induced nuclear reaction cross sections [S. Adler, Phys. Rev. **143**, 1144 (1966)] so that it is proportional to A. On the other hand, $\mathcal{g}(-m_\pi^2;\pi,N_f)$ involves the $\pi^\pm-N_f$ total cross sections, which are proportional to $A^{2/3}$. Thus one expects $K(0;\pi N_iN_f)$ to be proportional to $A^{1/6}$.