Polarization in $\pi^- p \to \eta n$, the Double-Pole A_2, and a Possible Pion Conspirator*

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The $\pi^- p \to \eta n$ differential-cross-section data are analyzed within the Regge framework, using two A_2 trajectories which are constrained to reproduce the double-pole characteristics at $J=2$, $t=(1.3 \text{ GeV})^2$ of the A_2 mass distribution. The best fit yields an upper trajectory degenerate with that for the ρ and a lower-lying trajectory degenerate with that for the pion at $t=0$ (a possible pion conspirator). The calculated neutron polarization is in reasonable agreement with new measurements, although these data were not employed in the fitting.

By considering a degenerate mass matrix for two mesons with the A_2 quantum numbers $I^G(J^P)C = 1^-(2^+)+$, Lassila and Ruuskanen[1] were able to explain the two-peak resonance structure of the $A_2(1300)$ meson established by the CERN missing-mass spectrometer group.[2] Recent measurements[3] of the neutron polarization in the reaction

$$\pi^- p \to \eta n \quad (1)$$

yield nonzero results for incident pion lab momenta of 3.2 and 3.47 GeV/c [$(40\pm15)\%$ and $(27\pm14)\%$, respectively] and possibly nonzero results at 5.0 and 11.2 GeV/c [$(-4\pm15)\%$ and $(2\pm7)\%$, respectively, all near $t=-0.2$ (GeV/c)^2], lending credibility to the two-A_2 hypothesis since the A_2 is the only known particle which can be exchanged in the t-channel reaction.

The amplitudes obtained from the meson-mixing formalism were Reggeized, and a least-squares fit to the averaged differential-cross-section data of Guisan *et al.*[4] and Wahlig and Mannelli[5] for reaction (1) was obtained. The polarization calculated with the Regge parameters yielding the best fit to the cross-section data is in good agreement with the recent measurements[3]; however, the most interesting results are the parameters of the two Regge trajectories involved. The leading trajectory is essentially degenerate with the ρ trajectory, a result required by the "exchange degeneracy" model of Arnold.[6] The lower trajectory is degenerate with the pion trajectory at $t=0$ and has all the properties required of the pion conspirator discussed recently in connection with pion photoproduction[7] and n-p charge exchange.[8] This formalism also eliminates the need for a nonlinear $R(A_2)$ trajectory required in previous fits to reaction (1).[9]

The amplitudes are Reggeized by hypothesizing two A_2-like mesons [e.g., the (5,27) and (5,8) $Y=0$, $I=1$ components of the 405 representation of $SU(6)$[10]] which are mixed near $t=t_{\text{res}}$, giving rise to a mass matrix M in the denominator of the propagator function; the correspondence

$$\Delta^2 I - \begin{pmatrix} M_1^2 & M_{12} \\ M_{12} & M_2^2 \end{pmatrix} \Rightarrow JI - \begin{pmatrix} \alpha_1 & V_{12} \\ V_{12} & \alpha_2 \end{pmatrix} \quad (2)$$

is made, where I = unit matrix and

$$M_i^2 = m_i^2 - i\gamma_i m_i, \quad m_i \text{ and } \gamma_i \text{ real}$$
$$\alpha_i = 2 + \alpha_i'(t - m_i^2) + i\beta_i, \quad \alpha_i \text{ and } \beta_i \text{ real.} \quad (i=1,2) \quad (3)$$

In order to reproduce the symmetric twin-peak resonance structure seen in

$$\pi + N \to \text{missing mass} + N \quad (4)$$

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[4] O. Guisan, J. Kirz, P. Sonderegger, A. V. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Variant, B. Amblard, C. Caversaiso, J. P. Guillaud, and M. Yvert, Phys. Letters **18**, 200 (1965).

[5] M. A. Wahlig and I. Mannelli, Phys. Rev. **168**, 1515 (1968).

[6] R. C. Arnold, Phys. Rev. Letters **14**, 657 (1965); Phys. Rev. **153**, 1506 (1967); A. Ahmadzadeh, *ibid.* **134**, B633 (1964); Phys. Rev. Letters **16**, 952 (1966); **20**, 1125 (1968).

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[9] W. Rarita and M. Schwarzschild, Phys. Rev. **162**, 1378 (1967); R. J. N. Phillips and W. Rarita, *ibid.* **139**, B1336 (1965).

[10] This is not the only possible mechanism; however, the fact that a four standard deviation $\rho^-\pi^-$ enhancement has been observed in the A_2 region by R. Vanderhagen *et al.* [Phys. Letters **24B**, 493 (1967)] provides support for the existence of particles belonging to the 27-dimensional representation of $SU(3)$. See also E. Fiorini, in *1966 International School of Physics "Ettore Majorana,"* edited by A. Zichichi (Academic Press Inc., New York, 1966).

near $\Delta^2 = m_{A_2}{}^2$, the following conditions are imposed:

$$m_1{}^2 = m_2{}^2 = m_{A_2}{}^2 = (1.3\text{ GeV})^2, \tag{5a}$$

$$4M_{12}{}^2 = (\gamma_1 m_1 - \gamma_2 m_2)^2 \Rightarrow 4V_{12}{}^2 = (\beta_1 - \beta_2)^2, \tag{5b}$$

$$\gamma_2 \approx 0 \Rightarrow \beta_2 \approx 0. \tag{5c}$$

The mixed trajectories are given by

$$\alpha_\pm = \tfrac{1}{2}(\alpha_1 + \alpha_2) \pm \tfrac{1}{2}[(\alpha_1 - \alpha_2)^2 + 4V_{12}{}^2]^{1/2}. \tag{6}$$

Hence, conditions (5) correspond to trajectories which cross at $t = t_{\text{res}}$, giving rise to the double-pole structure. The vanishing of the width γ_2 implies very weak coupling to the usual A_2 decay modes near t_{res} for this new meson, which would be the case for a particle belonging to an $SU(3)$ 27-plet[10] from empirical studies of exchange reactions.

To apply this formalism to the s-channel reaction (1), the simplest assumption is that the mesons decouple away from t_{res}, i.e., M_{12} (or V_{12}) $\to 0$ for $t < t_{\text{res}}$. The t-channel amplitudes in this region are simply the sums of two Regge amplitudes dominated by these two trajectories, which we assume are linear and cross at $\alpha_\pm(t_{\text{res}}) = 2$. We further assume that their residue functions have the same smooth t-dependence for $t < 0$ and that the ratios of the constant parts of their residue functions are the same for the nonflip and the flip amplitudes. In terms of Wang's[11] amplitudes, the cross section is

$$\frac{d\sigma}{dt} = \frac{1}{4\pi s P^2} |(t - 4M^2)^{-1/2}|^2 \times [|\sum_{i=1}^{2} (2\alpha_i + 1)\xi_i B_{00}{}^i(t) E_{00}{}^{\alpha_i}(\cos\theta_t)|^2 + \Phi(s,t)|\sum_{i=1}^{2}(2\alpha_i+1)\xi_i B_{10}{}^i(t) E_{10}{}^{\alpha_i}(\cos\theta_t)|^2], \tag{7}$$

where $i = 1, 2$,

$$\xi_i = [1 + \exp(-i\pi\alpha_i)]/\sin\pi\alpha_i \Gamma(\alpha_i + \tfrac{3}{2}),$$

and

$$B_{00}{}^i(t) = \alpha_i(\alpha_i + 1)(4p_t p_t'/4ME_0)^{\alpha_i} C_i \exp(Dt), \tag{8}$$

$$B_{10}{}^i(t) = [\alpha_i(\alpha_i + 1)]^{1/2}(4p_t p_t'/4ME_0)^{\alpha_i - 1} F_i \exp(Gt). \tag{9}$$

An 8-parameter fit was obtained, using $\alpha_1'(m^2)$, $\alpha_2'(m^2)$, C_1, D, F_1, G, E_0, and $R = C_1/C_2 = F_1/F_2$; 32 averaged data points at $P_{\text{lab}} = 5.9$, 9.8, 13.3, and 18.2 GeV/c were used in the calculation. The set of parameters giving the lowest value of χ^2 and the most reasonable polarization has $\chi^2 = 24.6$; the values for the $t = 0$ intercepts of the trajectories are $\alpha_1(0) = 0.61$ and $\alpha_2(0) = -0.001$. This suggested the degeneracy with the ρ and π trajectories. By constraining the trajectories to match the accepted values of the ρ and π intercepts at $t = 0$ and to cross at $t = (1.3\text{ GeV})^2$, we obtained a 6-parameter fit to 32 data points with $\chi^2 = 25.2$. The parameters are shown below; note that C and F implicitly contain the $\eta \to 2\gamma$ branching ratio.

$$C = 94.6, \quad D = 0.31, \quad F = 166.0,$$
$$G = 0.66, \quad E_0{}^{-1} = 0.61, \quad R = 0.66,$$
$$\alpha_1 = 0.58 + 0.83t, \quad \alpha_2 = -0.2 + 1.19t \quad \text{(input)}.$$

Some of the polarization values calculated with these parameters are shown in Figs. 1 and 2.

By assigning the α_2 trajectory to Class III of Freedman and Wang [$O(4)$ decomposition, which is the same as in Toller's $O(3,1)$ classification[12]], it becomes a

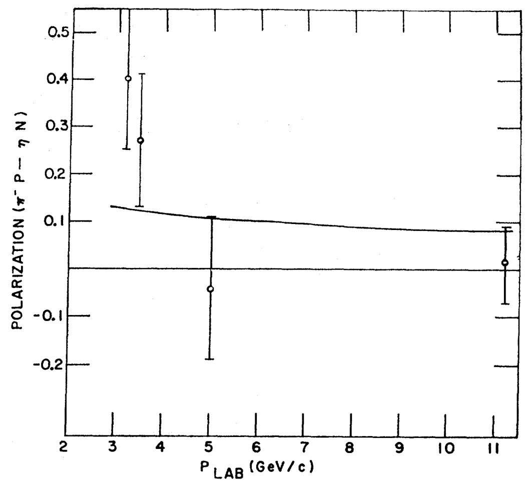

FIG. 1. Neutron polarization at $t \approx -0.2$ GeV^2 in the reaction $\pi^- p \to \eta n$ calculated with the parameters listed in the text. Data points from Ref. 3 were averaged over the range $0.14 < -t < 0.35$ GeV^2. P_L is the incident pion lab momentum.

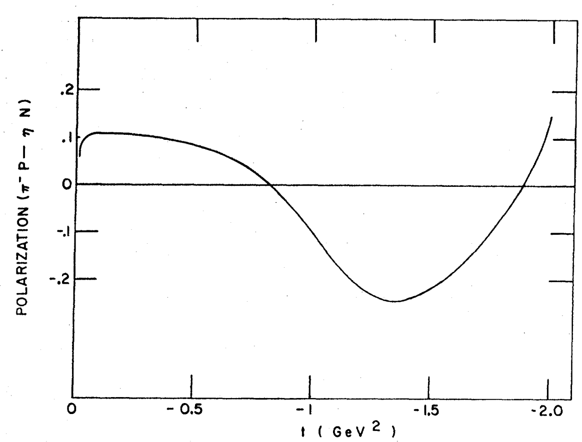

FIG. 2. Prediction of neutron polarization in the reaction $\pi^- p \to \eta n$ calculated with the parameters listed in the text as a function of t at $P_L = 5.9$ GeV/c.

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suitable pion conspirator, having $I^G(J^P)C=1^-(0^+)+$ at $t\cong 0\cong(m_\pi)^2$. We have used the Gell-Mann[13] ghost-killing mechanism, which implies that the sense-sense residue function vanishes at $\alpha_2=0$ (i.e., the trajectory chooses nonsense), preventing a physical manifestation of a 0^+ particle at the pion mass.[7,8]

A similar treatment may provide other conjectured conspirators, e.g., the K conspirator[7] may be the (5,27) [10] counterpart of the $K^*(1400)$. That these new mesons have small widths was assumed[1] in order to reproduce the symmetric two-peak structure in the case of the A_2,[1,2] and also explains why these mesons may be hard to detect experimentally. Application of this formalism to other relevant reactions involving V_2 exchange is now underway and will be reported in a later paper.

[13] M. Gell-Mann, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 539; M. Gell-Mann, M. Goldberger, F. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964).

A general dipole[14] (i.e., the amplitude is obtained by differentiating the single Regge pole form for all t) fit to reaction (1) was also tried, as well as some more complicated forms of the mixed-meson amplitudes. The best fit we were able to obtain for the general dipole had $\chi^2=67$, with 7 parameters and 32 data points. This rather poor quality fit supports the common view expressed in Ref. 1 that multiple poles in the S matrix are produced by the (probably accidental) coalescence of single poles and that therefore in the Regge picture trajectories crossing at $J=2$, $M=1.3$ GeV would be responsible for the double pole character of the experimental[2] mass distribution.

The authors are especially grateful to Vesa Ruuskanen for useful discussion and suggestions.

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Asymptotic Behavior of Form Factors for Some Composite Models

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The asymptotic behavior of electromagnetic form factors is examined for bound states treated by means of the Bethe-Salpeter equation in the ladder approximation. Results are found which depend on the behavior of the interaction at small distances, and the models examined are accordingly divided into regular and singular cases. For spin-0 and spin-$\frac{1}{2}$ bound states with regular interaction, the form factors go to zero as $(1/q^2)^2$ (apart from logarithmic factors). For singular cases (e.g., a spinless N-$\bar{N}$ bound state) it is shown that the asymptotic behavior is worse and depends on the strength of the interaction. In all cases a behavior more convergent than $1/q^2$ seems to occur, and to be related to the compositeness of the system rather than to the structure of the interaction.

I. INTRODUCTION

There has recently been some interest in the asymptotic behavior of electromagnetic form factors.[1] This is in part due to the experimental result[2] that the nucleon form factors decrease as $(1/q^2)^2$ for large four-momentum transfer, q.

The fact that the form factor goes to zero indicates that the bare electromagnetic charge is zero, while the fact that there is no $1/q^2$ term suggests that the bare strong-interaction coupling constant also vanishes. As this shows the nonelementary nature of the particle in question, it seems worthwhile to examine simplified composite models which can be treated rigorously.

The nonrelativistic potential model for s-wave bound states gives results which depend on the behavior of the potential at the origin. If the potential diverges as $1/r$, it follows in a straightforward way that the behavior is $(1/q^2)^2$. If the potential goes like $(-\lambda/r^2)$, then the behavior depends on the strength of the singular part and is given by $(1/q^2)^{1+\sqrt{(\frac{1}{4}-\lambda)}}$ $(0<\lambda<\frac{1}{4})$.[3]

A more realistic model is given by the ladder Bethe-

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[3] For potentials with hard core, also exponentially falling behaviors can be obtained. Other conditions for exponential falloff in nonrelativistic models have been examined by S. D. Drell, A. C. Finn, and M. H. Goldhaber [Phys. Rev. **157**, 1402 (1967)].