Sources and Magnetic Charge

JULIAN SCHWINGER
Institute for Theoretical Physics, State University of New York, Stony Brook, New York*
(Received 8 April 1968)

A beginning is made on a phenomenological reconstruction of the theory of magnetic charge. The concept is introduced by reference to a new kind of photon source. It is shown that photon exchange between different source types is relativistically invariant. The space-time generalization of this coupling involves an arbitrary vector. The only way to remove a corresponding arbitrariness of physical predictions is to recognize the localization of charge and impose a charge quantization condition. The consideration of particles that carry both kinds of charge loosens the charge restrictions. The great strength of magnetic attraction indicated by $g^2/4\pi=4(137)$ suggests that ordinary matter is a magnetically neutral composite of magnetically charged particles that carry fractional electric charge. There is a brief discussion of such a magnetic model of strongly interacting particles, which makes contact with empirical classification schemes. Additional remarks on notation, and on the general nature of the source description, are appended.

THE concept of magnetic charge has great theoretical appeal, since it provides a beautiful explanation of the observed quantization of electric charge.[1] It has received some recent attention from the standpoint of operator quantum field theory.[2] Owing to the singular nature of operator field products, the resulting formalism, which makes liberal use of limiting processes, has a delicate and tentative aspect. This must diminish considerably the impact of the assertion that magnetic charge is a physical possibility, and certainly makes the operator theory ill-suited for quantitative application. It is our intention here to use the phenomenological and nonoperator approach of source theory[3] to provide a new foundation for the idea of magnetic charge, and to develop its implications sufficiently that one recognizes the existence of phenomenological charge quantization.

Sources. We first review the description of photon emission and absorption by sources, $J^\mu(x)$, which are introduced as idealizations of realistic mechanisms. Complete processes of emission and absorption are contained in the vacuum amplitude

$$\langle 0_+|0_-\rangle^J =\exp\left[i\frac{1}{2}\int(dx)(dx')J^\mu(x)D_+(x-x')J_\mu(x')\right], \quad (1)$$

where

$$D_+(x-x')=D_+(x'-x),$$

$$x^0>x^{0\prime}:\quad D_+(x-x')=i\int d\omega_k e^{ik(x-x')}, \quad (2)$$

and

$$d\omega_k=\frac{(d\mathbf{k})}{(2\pi)^3}\frac{1}{2k^0},\quad k^0=|\mathbf{k}|. \quad (3)$$

The vectorial source must obey the conservation condition

$$\partial_\mu J^\mu(x)=0. \quad (4)$$

The consideration of a causal arrangement, with emission source $J_2{}^\mu$ and detection source $J_1{}^\mu$, gives

$$\langle 0_+|0_-\rangle^J=\langle 0_+|0_-\rangle^{J_1}\times\exp\left[\int d\omega_k iJ_1{}^\mu(-k)g_{\mu\nu}iJ_2{}^\nu(k)\right]\langle 0_+|0_-\rangle^{J_2}, \quad (5)$$

where

$$J^\mu(k)=\int(dx)e^{-ikx}J^\mu(x) \quad (6)$$

and

$$k_\mu J^\mu(k)=0. \quad (7)$$

Polarization vectors are introduced by writing

$$g^{\mu\nu}=\sum_{\lambda=1,2}e_{k\lambda}{}^\mu e_{k\lambda}{}^\nu+(k^\mu\bar{k}^\nu+\bar{k}^\mu k^\nu)/(k\bar{k}), \quad (8)$$

in which $\bar{k}^\mu$ is obtained from k^μ by reflecting the spatial components, in some coordinate frame. In that frame, the two $e_{k\lambda}{}^\mu$, $\lambda=1,2$, are unit orthogonal spatial vectors that are perpendicular to the photon momentum $\mathbf{k}$. As a result

$$\int d\omega_k iJ_1{}^\mu(-k)g_{\mu\nu}J_2{}^\nu(k) =\int d\omega_k\sum_\lambda i\mathbf{J}_1(-k)\cdot\mathbf{e}_{k\lambda}\mathbf{e}_{k\lambda}\cdot i\mathbf{J}_2(k), \quad (9)$$

and the subsequent analysis into multiparticle states, through the causal decomposition

$$\langle 0_+|0_-\rangle^J=\sum_{\{n\}}\langle 0_+|\{n\}\rangle^{J_1}\langle\{n\}|0_-\rangle^{J_2}, \quad (10)$$

yields

$$\langle\{n\}|0_-\rangle^J=\langle 0_+|0_-\rangle^J\prod_{k\lambda}(iJ_{k\lambda})^{n_{k\lambda}}/(n_{k\lambda}!)^{1/2},$$

$$\langle 0_+|\{n\}\rangle^J=\langle 0_+|0_-\rangle^J\prod_{k\lambda}(iJ_{k\lambda}{}^*)^{n_{k\lambda}}/(n_{k\lambda}!)^{1/2}, \quad (11)$$

* Permanent address: Harvard University, Cambridge, Mass.
[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **A133**, 60 (1931); Phys. Rev. **74**, 817 (1948).
[2] J. Schwinger, Phys. Rev. **144**, 1087 (1966).
[3] J. Schwinger, Phys. Rev. **152**, 1219 (1966); **158**, 1391 (1967).

where

$$J_{k\lambda}=(d\omega_k)^{1/2}\mathbf{e}_{k\lambda}\cdot\mathbf{J}(k). \quad (12)$$

Given one pair of polarization vectors $\mathbf{e}_{k\lambda}$, another pair is produced by rotation about $\mathbf{k}$ through 90°, as expressed by

$$^*\mathbf{e}_{k\lambda}=(\mathbf{k}/k^0)\times\mathbf{e}_{k\lambda}. \quad (13)$$

Let us suppose that a different type of photon source exists, designated by $^*J^\mu(x)$, such that the effective component for the emission of the photon labeled $k\lambda$ is not along $\mathbf{e}_{k\lambda}$, but is in the perpendicular direction specified by $^*\mathbf{e}_{k\lambda}$:

$$^*J_{k\lambda}=(d\omega_k)^{1/2}\,{}^*\mathbf{e}_{k\lambda}\cdot{}^*\mathbf{J}(k). \quad (14)$$

The two types of sources are not intrinsically distinguishable through the act of photon exchange since

$$\sum_\lambda \mathbf{e}_{k\lambda}\mathbf{e}_{k\lambda}=\sum_\lambda {}^*\mathbf{e}_{k\lambda}\,{}^*\mathbf{e}_{k\lambda}, \quad (15)$$

but they can be contrasted by exchanging a photon between the two varieties. If we arrange that $^*J^\mu$ emits and J^μ absorbs, the vacuum amplitude is

$$\langle 0_+|0_-\rangle^{J,{}^*J}=\langle 0_+|0_-\rangle^J \times\exp\left[\int d\omega_k\sum_\lambda i\mathbf{J}(-k)\cdot\mathbf{e}_{k\lambda}\,{}^*\mathbf{e}_{k\lambda}\cdot i\,{}^*\mathbf{J}(k)\right] \times\langle 0_+|0_-\rangle^{{}^*J}, \quad (16)$$

where

$$\sum_\lambda \mathbf{e}_{k\lambda}\,{}^*\mathbf{e}_{k\lambda}\cdot{}^*\mathbf{J}(k)=\sum_\lambda \mathbf{e}_{k\lambda}\mathbf{e}_{k\lambda}\cdot{}^*\mathbf{J}(k)\times\mathbf{k}/k^0 ={}^*\mathbf{J}(k)\times\mathbf{k}/k^0, \quad (17)$$

since $^*\mathbf{J}\times\mathbf{k}$ only has components along the two directions represented by $\mathbf{e}_{k\lambda}$.

Invariance. It is now vital to recognize that, despite its three-dimensional appearance,

$$\mathbf{J}(-k)\times{}^*\mathbf{J}(k)\cdot\mathbf{k}/k^0 \quad (18)$$

is a Lorentz scalar, so that the description of photon-mediated coupling between different source types is Lorentz-invariant. This property is a consequence of the conserved nature of the sources,

$$\mathbf{k}\cdot\mathbf{J}(k)=k^0J^0(k),\qquad \mathbf{k}\cdot{}^*\mathbf{J}(k)=k^0\,{}^*J^0(k), \quad (19)$$

which is an aspect of the masslessness of the photon,

$$k^0=|\mathbf{k}|. \quad (20)$$

We examine the response to the infinitesimal Lorentz transformation indicated by

$$\delta\mathbf{J}=\delta\mathbf{v}J^0=\delta\mathbf{v}(\mathbf{k}/k^0)\cdot\mathbf{J}, \quad (21)$$

which, with a similar equation for $^*\mathbf{J}$, gives

$$\delta[\mathbf{J}\times{}^*\mathbf{J}\cdot\mathbf{k}/k^0]=-[(\mathbf{k}/k^0)\times(\delta\mathbf{v}\times\mathbf{k}/k^0)]\cdot\mathbf{J}\times{}^*\mathbf{J} +\delta(\mathbf{k}/k^0)\cdot\mathbf{J}\times{}^*\mathbf{J}. \quad (22)$$

But, since

$$\delta\mathbf{k}=\delta\mathbf{v}k^0,\quad \delta k^0=\delta\mathbf{v}\cdot\mathbf{k}, \quad (23)$$

we have

$$\delta(\mathbf{k}/k^0)=\delta\mathbf{v}-(\mathbf{k}/k^0)\delta\mathbf{v}\cdot\mathbf{k}/k^0 =(\mathbf{k}/k^0)\times(\delta\mathbf{v}\times\mathbf{k}/k^0), \quad (24)$$

invoking explicitly the zero mass of the photon, and the statement is verified.

Space-Time Description. The logical connection between the photon and the long-range Coulomb interaction of static charges is introduced by performing a space-time extrapolation of the source coupling that is first established by considering causal arrangements involving propagating photons. Preparatory to carrying out such a space-time generalization in the present situation, we observe that

$$\mathbf{J}(-k)\times{}^*\mathbf{J}(k)\cdot\mathbf{k}/k^0=\epsilon^{\mu\nu\lambda\kappa}J_\mu(-k)f_\nu(k)ik_\lambda\,{}^*J_\kappa(k), \quad (25)$$

where $f_\nu(k)$ is *any* vector that obeys

$$ik^\nu f_\nu(k)=1 \quad (26)$$

and $\epsilon^{\mu\nu\lambda\kappa}$ is the totally antisymmetrical symbol normalized by

$$\epsilon^{0123}=+1. \quad (27)$$

The coupling under discussion can be regarded as operating between J^μ and the effective source

$$J^\mu(k)|_{\text{eff}}=\epsilon^{\mu\nu\lambda\kappa}f_\nu(k)ik_\lambda\,{}^*J_\kappa(k), \quad (28)$$

which correctly obeys

$$k_\mu J^\mu(k)|_{\text{eff}}=0. \quad (29)$$

The space-time description of this effective source is

$$J^\mu(x)|_{\text{eff}}=\epsilon^{\mu\nu\lambda\kappa}\int(dx')f_\nu(x-x')\partial_\lambda'\,{}^*J_\kappa(x'), \quad (30)$$

where

$$\partial_\nu f^\nu(x-x')=\delta(x-x'). \quad (31)$$

A space-time transcription of the vacuum amplitude is now at hand. We write it as

$$\langle 0_+|0_-\rangle^{J,{}^*J}=\exp[iW(J,{}^*J)], \quad (32)$$

with

$$W(J,{}^*J)=\frac{1}{2}\int(dx)(dx')J^\mu(x)D_+(x-x')J_\mu(x') +\frac{1}{2}\int(dx)(dx')\,{}^*J^\mu(x)D_+(x-x')\,{}^*J_\mu(x') +\int(dx)(dx')(dx'')J^\mu(x)\epsilon_{\mu\nu\lambda\kappa}f^\nu(x-x') \times D_+(x'-x'')\partial''^\lambda\,{}^*J^\kappa(x''), \quad (33)$$

which makes use of the equivalence

$$\int (dx')D_+(x-x')f^\nu(x'-x'') = \int (dx')f^\nu(x-x')D_+(x'-x''). \quad (34)$$

Fields. Auxiliary quantities—fields—are introduced as measures of the effects experienced by additional weak sources. This is conveyed by the differential expression

$$\delta W(J,{}^*J) = \int (dx)[\delta J^\mu(x)A_\mu(x) + \delta\,{}^*J^\mu(x)B_\mu(x)], \quad (35)$$

where

$$A_\mu(x) = \partial_\mu\lambda(x) + \int (dx')D_+(x-x')J_\mu(x') + \int (dx')(dx'')f^\nu(x-x')D_+(x'-x'') \times {}^*(\partial_\mu''\,{}^*J_\nu(x'') - \partial_\nu''\,{}^*J_\mu(x'')) \quad (36)$$

and

$$B_\mu(x) = \partial_\mu\,{}^*\lambda(x) + \int (dx')D_+(x-x')\,{}^*J_\mu(x') + \int (dx')(dx'')f^\nu(x'-x)D_+(x'-x'') \times {}^*(\partial_\mu''J_\nu(x'') - \partial_\nu''J_\mu(x'')). \quad (37)$$

We have used a notation for dual tensor, in accordance with the definition

$${}^*A^{\mu\nu} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\kappa}A_{\lambda\kappa}. \quad (38)$$

The arbitrary gauge functions $\lambda(x)$, ${}^*\lambda(x)$ appear in consequence of the conservation restrictions on the sources.

A particularly advantageous gauge choice is

$$\lambda(x) = -\int (dx')(dx'')f^\nu(x-x')D_+(x'-x'')J_\nu(x''),$$
$${}^*\lambda(x) = \int (dx')(dx'')f^\nu(x'-x)D_+(x'-x'')\,{}^*J_\nu(x''), \quad (39)$$

for, then,

$$A_\mu(x) = -\int (dx')f^\nu(x-x')F_{\mu\nu}(x'),$$
$$B_\mu(x) = \int (dx')f^\nu(x'-x)\,{}^*F_{\mu\nu}(x'), \quad (40)$$

where

$$F_{\mu\nu}(x) = \int (dx')D_+(x-x')[\partial_\mu'J_\nu(x') - \partial_\nu'J_\mu(x') - {}^*(\partial_\mu'\,{}^*J_\nu(x') - \partial_\nu'\,{}^*J_\mu(x'))], \quad (41)$$

and (${}^{**}A_{\mu\nu} = -A_{\mu\nu}$),

$${}^*F_{\mu\nu}(x) = \int (dx')D_+(x-x')[\partial_\mu'\,{}^*J_\nu(x') - \partial_\nu'\,{}^*J_\mu(x') + {}^*(\partial_\mu'J_\nu(x') - \partial_\nu'J_\mu(x'))]. \quad (42)$$

The identity

$$\partial_\nu\,{}^*(\partial^\mu A^\nu - \partial^\nu A^\mu) = \epsilon^{\mu\nu\lambda\kappa}\partial_\nu\partial_\lambda A_\kappa = 0 \quad (43)$$

leads immediately to

$$\partial_\nu F^{\mu\nu}(x) = J^\mu(x), \qquad \partial_\nu\,{}^*F^{\mu\nu}(x) = {}^*J^\mu(x), \quad (44)$$

which justifies the description of the two source varieties as electric and magnetic currents. The relations

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = \epsilon_{\mu\nu\lambda\kappa}\partial_\alpha\,{}^*F^{\alpha\kappa} = \epsilon_{\mu\nu\lambda\kappa}\,{}^*J^\kappa \quad (45)$$

and

$$\partial_\mu\,{}^*F_{\nu\lambda} + \partial_\nu\,{}^*F_{\lambda\mu} + \partial_\lambda\,{}^*F_{\mu\nu} = -\epsilon_{\mu\nu\lambda\kappa}J^\kappa \quad (46)$$

are used in deriving

$$\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) = F_{\mu\nu}(x) - {}^*\left[\int (dx')(f_\mu(x-x')\,{}^*J_\nu(x') - f_\nu(x-x')\,{}^*J_\mu(x'))\right],$$
$$\partial_\mu B_\nu(x) - \partial_\nu B_\mu(x) = {}^*F_{\mu\nu}(x) - {}^*\left[\int (dx')(f_\mu(x'-x)J_\nu(x') - f_\nu(x'-x)J_\mu(x'))\right]. \quad (47)$$

Action. In view of the linear relation between fields and sources, the quantity W is given by

$$W = \frac{1}{2}\int (dx)[J^\mu(x)A_\mu(x) + {}^*J^\mu(x)B_\mu(x)]. \quad (48)$$

The identities

$$\int (dx)J^\mu A_\mu = \int (dx)\tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu),$$
$$\int (dx)\,{}^*J^\mu B_\mu = \int (dx)\tfrac{1}{2}\,{}^*F^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu), \quad (49)$$

and

$$\int (dx)\tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) = \int (dx)\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu} + \int (dx)\,{}^*J^\mu B_\mu\,,$$

$$\int (dx)\tfrac{1}{2}{}^*F^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) = \int (dx)\tfrac{1}{2}{}^*F^{\mu\nu}\,{}^*F_{\mu\nu} + \int (dx) J^\mu A_\mu\,, \tag{50}$$

enable one to rewrite W in equivalent forms that contain only fields. Thus,

$$W = \int (dx)[\tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}] = \int (dx)[\tfrac{1}{2}\,{}^*F^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) - \tfrac{1}{4}\,{}^*F^{\mu\nu}\,{}^*F_{\mu\nu}]\,, \tag{51}$$

from which we infer the additional equivalent forms

$$W = \int (dx)[J^\mu A_\mu + {}^*J^\mu B_\mu - \tfrac{1}{2}F^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}] \tag{52}$$

and

$$W = \int (dx)[J^\mu A_\mu + {}^*J^\mu B_\mu - \tfrac{1}{2}\,{}^*F^{\mu\nu}(\partial_\mu B_\nu - \partial_\nu B_\mu) + \tfrac{1}{4}\,{}^*F^{\mu\nu}\,{}^*F_{\mu\nu}]. \tag{53}$$

The last two structures have a significant property. Considered as a functional of appropriate fields, they are stationary with respect to variations of those fields. This gives the fundamental quantity W the character of an action. Indeed, the consequences of the stationary requirement comprise all the field equations necessary to eliminate the fields and express W in terms of the sources. For the first of these forms, the variables are A_μ and $F^{\mu\nu}$ with B_μ regarded as a functional of $F_{\mu\nu}$ that is given by

$$B_\mu(x) = \int (dx')f^\nu(x'-x)\,{}^*F_{\mu\nu}(x')\,. \tag{54}$$

The variational equations are

$$\partial_\nu F^{\mu\nu}(x) = J^\mu(x)\,,$$

$$\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) = F_{\mu\nu}(x) - {}^*\Big[\int (dx')(f_\mu(x-x')\,{}^*J_\nu(x') - f_\nu(x-x')\,{}^*J_\mu(x'))\Big]. \tag{55}$$

On differentiating the dual of the last equation, we get

$$\partial_\nu\,{}^*F^{\mu\nu}(x) = {}^*J^\mu(x)\,, \tag{56}$$

and enough information is available to reconstruct all the fields, within the gauge arbitrariness of A_μ. The second action form uses B_μ and ${}^*F^{\mu\nu}$ as variables, with

$$A_\mu(x) = -\int (dx')f^\nu(x-x')F_{\mu\nu}(x')\,. \tag{57}$$

The discussion is entirely analogous.

Charge Quantization. The action principles restate the differential dependence of W upon the sources:

$$\delta W = \int (dx)[\delta J^\mu A_\mu + \delta\,{}^*J^\mu B_\mu]. \tag{58}$$

To simplify the discussion let us hold the magnetic source fixed and choose

$$\delta J^\mu(x) = \partial_\nu \delta M^{\mu\nu}(x) = -\partial_\nu \delta M^{\nu\mu}(x)\,, \tag{59}$$

which satisfies identically the constraint

$$\partial_\mu \delta J^\mu(x) = 0\,. \tag{60}$$

This gives

$$\delta W = \int (dx)\tfrac{1}{2}\delta M^{\mu\nu}(x)[\partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)] = \int (dx)\tfrac{1}{2}\delta M^{\mu\nu}(x)F_{\mu\nu}(x) - \int (dx)(dx')\delta\,{}^*M^{\mu\nu}(x)f_\mu(x-x')\,{}^*J_\nu(x')\,. \tag{61}$$

The fundamental problem posed by the space-time extrapolation from the initial photon-mediated causal circumstances is now evident. There is an explicit dependence upon the physically undetermined function $f_\mu(x-x')$. The permissible restriction to the class of functions that only connect points in spacelike relation would remove that dependence when $\delta M_{\mu\nu}$ and ${}^*J_\nu$ are causally separated, but otherwise is of no avail.

One avenue of escape exists—that the f_μ-dependent part of δW is not continuously variable but is restricted to integer multiples of $2\pi(\hbar)$, for then the arbitrariness disappears from $\exp(i\delta W)$, which is the physically significant quantity. This possibility cannot be realized unless there is a discrete element intrinsic to the nature of photon sources. Sources are introduced to represent the common features of analogous mechanisms and to abstract from the vagaries of individual mechanisms. In this situation, we must conclude that charge is discretely localized. The idealizations involved in introducing sources cannot violate that general physical law if a consistent theory is to be constructed.

A point-charge representation of independent electron and magnetic sources is indicated by

$$J^\mu(x)=\sum_e e\int dx_e{}^\mu\delta(x-x_e),$$
$$*J^\mu(x)=\sum_g g\int dx_g{}^\mu\delta(x-x_g), \qquad (62)$$

where $x_e{}^\mu$ and $x_g{}^\mu$ are the coordinates of charge-bearing points. Since sources are present neither initially nor finally in the situation described by the vacuum amplitude, it may be supposed that focal points exist from which positive and negative charges appear and at which they eventually disappear. (Or, one can recognize that, in a description limited to a finite space-time region, charge can be introduced from the outside through the boundaries and then be subsequently withdrawn.) A variation in the paths of the electric charges, for example, leads to

$$\delta W=\sum_e e\delta\int dx_e{}^\mu A_\mu(x_e)$$
$$=\sum_e e\int \tfrac{1}{2}d\sigma_e{}^{\mu\nu}(\partial_\mu A_\nu-\partial_\nu A_\mu)(x_e) \qquad (63)$$

or

$$\delta W=\sum_e e\int \tfrac{1}{2}d\sigma_e{}^{\mu\nu}F_{\mu\nu}(x_e)$$
$$-\sum_{eg} eg\int d\,{}^*\sigma_e{}^{\mu\nu}f_\mu(x_e-x_g)dx_{g\nu}, \qquad (64)$$

where the two-dimensional surfaces are bounded by the initial and varied paths. The latter form is not restricted to small path changes. With respect to integration over the variable $x=x_e-x_g$, the product $d\,{}^*\sigma_e{}^{\mu\nu}dx_{g\nu}$ defines a three-dimensional directed surface area,

$$d\,{}^*\sigma_e{}^{\mu\nu}dx_{g\nu}=d\sigma^\mu. \qquad (65)$$

Just such a surface occurs in expressing a consequence of the differential equation

$$\partial_\mu f^\mu(x)=\delta(x), \qquad (66)$$

namely,

$$\int d\sigma_\mu f^\mu(x)=1, \qquad (67)$$

but this is a closed surface that surrounds the origin. We must now restrict f_μ to a class of functions such that $\int d\sigma_\mu f^\mu$ assumes only discrete values, for any integration surface. This indicates that the support domain for such a function on a regular surface enclosing the origin must be limited to a finite number of points.

If these points are sufficiently continuous in moving to neighboring surfaces, we can picture a number of filaments drawn out from the origin. Let there be ν of these, and let r_α, $\alpha=1,\cdots,\nu$ be the contribution of any portion of a closed surface that contains only the αth point, where

$$\sum_{\alpha=1}^{\nu} r_\alpha=1. \qquad (68)$$

We also recognize the possibility that a surface encounters the αth point on its boundary, and assign the integration value $\frac{1}{2}r_\alpha$ to this arrangement. The uniqueness of $\exp(i\delta W)$ then requires, for any pair of electric and magnetic charges, that

$$eg\tfrac{1}{2}r_\alpha=2\pi n_\alpha, \qquad (69)$$

where n_α is an integer. This is the strongest condition, the analogous one involving egr_α being satisfied as a consequence. On summing over the ν points, we reach the charge quantization relation

$$\tfrac{1}{2}eg=2\pi n, \qquad (70)$$

with

$$n=\sum_{\alpha=1}^{\nu} n_\alpha. \qquad (71)$$

The individual weights r_α are necessarily restricted to the rational form

$$r_\alpha=n_\alpha/n. \qquad (72)$$

If the ν points are all equivalent and $r_\alpha=1/\nu$, we have

$$n=n_\alpha\nu, \qquad (73)$$

which describes the situation where the minimum value assumed by a nonzero charge product $eg/4\pi$ is the integer ν.

We have now achieved the effective elimination of the arbitrary elements in δW, which can be replaced by

$$\delta W=\sum_e e\int \tfrac{1}{2}d\sigma_e{}^{\mu\nu}F_{\mu\nu}(x_e). \qquad (74)$$

But this expression is no longer the change of a quantity W, and the question of integrability arises. Consider, for a particular electric charge, a continuous deformation of paths that returns to the initial one and thereby defines a surface that encloses a three-dimensional volume. If there is to be no change in $\exp(iW)$, it is necessary that

$$e\oint \tfrac{1}{2}d\sigma_{\mu\nu}F^{\mu\nu}=e\int d\sigma_\mu\partial_\nu{}^*F^{\mu\nu}=e\int d\sigma_\mu{}^*J^\mu=2\pi n. \qquad (75)$$

An individual magnetic charge contributes the value g to this three-dimensional volume integration if it is in the interior, and $\frac{1}{2}g$ if it is on the surface of the volume.

Thus, the integrability requirement leads back to

$$\tfrac{1}{2}eg=2\pi n\,, \tag{76}$$

the general charge quantization condition.

Dual Charged Particles. The preceding discussion assumed that electric and magnetic charges are associated with different particles, which brings us to the interesting possibility that magnetically charged particles also carry electric charge. In this situation, a displacement of source particles produce a variation of both electric and magnetic sources, and we must write

$$\delta W=\sum_a\left[e_a\delta\int dx_a{}^\mu A_\mu(x_a)+g_a\delta\int dx_a{}^\mu B_\mu(x_a)\right], \tag{77}$$

where a is a particle label. This leads to

$$\delta W=\sum_a\left[e_a\int\tfrac{1}{2}d\sigma_a{}^{\mu\nu}F_{\mu\nu}(x_a)+g_a\int\tfrac{1}{2}d\sigma_a{}^{\mu\nu}\,{}^*F_{\mu\nu}(x_a)\right]$$
$$-\sum_{ab}\left[e_ag_b\int d\,{}^*\sigma_a{}^{\mu\nu}f_\mu(x_a-x_b)dx_{b\nu}\right.$$
$$\left.+g_ae_b\int d\,{}^*\sigma_a{}^{\mu\nu}f_\mu(x_b-x_a)dx_{b\nu}\right]. \tag{78}$$

Consider first the integrability condition that is required when the f_μ-dependent terms in δW are effectively eliminated. It is (${}^{**}F_{\mu\nu}=-F_{\mu\nu}$)

$$e_a\int d\sigma_\mu{}^*J^\mu-g_a\int d\sigma_\mu J^\mu=2\pi n\,, \tag{79}$$

and the presence of another particle on the surface of the three-dimensional volume gives the new charge quantization condition

$$\tfrac{1}{2}(e_ag_b-g_ae_b)=2\pi n_{ab}. \tag{80}$$

Before examining it, we must note the existence of a conflict with the independent quantization statement involving f_μ, unless

$$f_\mu(x-x')=-f_\mu(x'-x)\,, \tag{81}$$

which is compatible with the differential equation obeyed by the function. When $f_\mu(x-x')$ was introduced, the points x and x' referred to distinct electric and magnetic charge regions, respectively. In the present situation, with particles carrying both electric and magnetic charges, an additional symmetry property is required. It implies that every filament of $f_\mu(x)$ has its image, or that ν is even. The integer n of the charge quantization condition must also be even.

The charge quantization situation changes significantly on considering particles with both kinds of charge. This can be appreciated by examining the specific possibility that e_a/g_a is a fixed constant independent of a; there is *no* restriction on the individual products e_ag_b. But no conflict with the earlier discussion exists. If we consider a group of particles which are magnetically neutral as a whole, $\sum g_a=0$, their total electric charge, $e=\sum e_a$, satisfies the previous charge quantization condition:

$$\tfrac{1}{2}eg_b=2\pi n\,,\quad n=\sum_a n_{ab}. \tag{82}$$

In the special circumstance we have described, the total electric charge is also zero. As another example, if there are particles with a common magnetic charge g but different electric charges, $e_1\neq e_2$, the new condition asserts that

$$\tfrac{1}{2}(e_1-e_2)g=2\pi n\,, \tag{83}$$

but does not limit the individual electric charges.

Composite Particles. The formal symmetry between electric and magnetic charge is violated in nature by the great disparity between the charge units. If we use the smallest even integer to relate them, we conclude from

$$e^2/4\pi=1/137 \tag{84}$$

that

$$g^2/4\pi=4(137). \tag{85}$$

The enormously strong forces of attraction that must operate between positive and negative charges are consistent with the fact that the units of matter thus far observed are magnetically neutral. This opens the possibility that ordinary matter may be composed of magnetically charged constituents, which carry electric charges of values different from those characteristic of magnetically neutral matter.[4] We now discuss briefly a model of matter based on this idea, which has some contact with empirical classification schemes.

Consider a set of particles with two choices of magnetic charge, $-g_0$, $(N-1)g_0$, and two analogous choices of electric charge, $-e_0$, $(N-1)e_0$. The integer N may be $2,3,\cdots$. The various charge quantization conditions are all satisfied if

$$Ne_0g_0/4\pi=2\,, \tag{86}$$

which assumes that the smallest even integer is realized. If that is also true for the charge unit e of magnetically neutral particles,

$$eg_0/4\pi=2\,, \tag{87}$$

we conclude that

$$e_0=e/N. \tag{88}$$

The individual electric charges of the magnetically charged particles are thus $-1/N$ and $(N-1)/N$ in units of e. They differ by a unit charge.

[4] This approach to composite structure has nothing in common with attempts to describe observed particles in terms of the nonrelativistic behavior of weakly interacting constituents.

The minimum number of constituents required to produce a neutral combination, without using antiparticles, is N. Let us assume that the pattern of magnetic charge required for neutrality, $-g_0$ repeated $N-1$ times with $(N-1)g_0$ occurring once, is duplicated in the electric charges exhibited by each of the N particles, namely, $-e_0=-(1/N)e$ is repeated $N-1$ times and $(N-1)e_0=[(N-1)/N]e$ occurs once. The outcome is a set of magnetically neutral particles displaying integer electric charges that range from $N-1$ to -1. To the extent that electric charge and other unspecified properties are of secondary dynamical importance, this set, or some subset of it, may be recognizable as a particle multiplet with a broken mass spectrum. The analogous multiplet that is constructed completely from antiparticles is a different one, if $N>2$. That is an empirical characteristic of baryons, which are Fermi-Dirac particles. If we assume that all magnetically charged particles are fermions, it is necessary that N be odd. The first possibility is $N=3$.

The resulting baryon model composes these particles from three constituents, of magnetic charge $2g_0$, $-g_0$, $-g_0$, each with three choices of electric charge $\frac{2}{3}$, $-\frac{1}{3}$, $-\frac{1}{3}$, in units of e. This pattern of fractional electric charges is familiar in an empirical model, based on the symmetry group SU_3, which was introduced[5] without reference to the magnetic charge concept[6] that makes fractional electric charge understandable and physically acceptable. The consequences of the simplest meson model, comprising a magnetically charged electric triplet and an antiparticle triplet, are also familiar empirically in various nonuplet realizations.

Note added in proof. These remarks are made in response to a referee's comments.

(1) "The notation D_+ for the function defined in (2) is unfortunate, since $D_+(x-x')$ is universally used for the integral in (2) for *all* $x^0-x^{0\prime}$." It is also stated that D_F, D_c, or D_{1R} are the usual notations for the symmetric function defined in (2).

In fact, there has been no such historical consensus concerning the positive frequency function, notations such as $D^{(+)}$ being not uncommon. Since 1949, I have consistently used the symbol D_+ to designate a function of definition equivalent to (2). Apart from a different choice of factors and principal symbol, this has also been Feynman's usage. An advantage over notations such as D_F and D_c (the symbol D_{1R} is unfamiliar to me) is the uniformity with which one represents the alternative boundary conditions of outgoing waves (D_+) or incoming waves (D_-).

(2) "... for the case of point sources (62), the action (48) diverges due to Coulomb field singularities calling into question the theory based on (48)."

Presumably the *action* expression that is meant is (52) or (53). This would be a serious charge indeed if source theory were based on the field concept. It is not. The discussion in the text began with known results for distributed sources. It was eventually recognized that localization of charge must be introduced to reach a consistent theory of electric and magnetic charge. At this point I should have reconstructed the theory to deal with the new circumstance. I did not do so in the full knowledge that nothing I intended to discuss would be changed thereby. But I do concede that a more complete discussion is instructive for, in contrast with operator field theory, self-action holds no terrors for the sourcerer.

The theory starts from a description of the exchange of photons between distinct, casually arranged sources, which is in no way dependent upon the details of the charge distributions. On considerlng point electric charges and placing them all on the same footing we arrive at the proper time structure

$$W=\frac{1}{2}\sum_{a\neq b} e_a e_b \int ds ds' v_a^{\mu}(s) v_{\mu b}(s') \times D_+[x_a(s)-x_b(s')]+\sum_a W_a,$$

where $v^{\mu}=dx^{\mu}/ds$, and W_a characterizes a single charge. The principle of source unity requires that W_a have the same structure as the mutual coupling terms. But we must also note that the individual charged particles will have been described already, under physical conditions of non-interaction. This is represented by the action term $-m_a\int ds_a$, where m_a is the observed mass of the particle. It is the essence of the phenomenological source theory that physical parameters, once identified under restricted physical conditions, do not alter their meaning when more general circumstances are examined. Hence we cannot include the electromagnetic self-action associated with

$$\mathrm{Re}D_+(x-x')=(1/4\pi)\delta[(x-x')^2],$$

for it would change the already correctly assigned mass m_a. This is a simple example of mass *normalization*. One can say that the observed mass already includes any inertial electromagnetic effect, which should not be counted twice. With this attitude we clearly separate particle phenomenology from speculations about inner particle structure. (That was also the intention of relativistic renormalization theory, as it was originally formulated in 1947, but the methods then used were too cumbersome and this simple idea was displaced by the regulators and counter terms that came into fashion). We conclude that

$$W_a=\frac{1}{2}e_a^2\int ds ds' v_a^{\mu}(s) v_{\mu a}(s') i\,\mathrm{Im}D_+[x_a(s)-x_a(s')],$$

which is exactly what is needed to give a consistent

[5] M. Gell-Mann, Phys. Letters 8, 214 (1964).

[6] The magnetic model of matter will be elaborated elsewhere.

quantum mechanical account of emission and absorption of photons by a single accelerated charge.

Now that photon sources are endowed with the more specific point structure, it is important to be remined that sources are never objects of explicit physical interest. The source is an instrument—a practical calculational tool on the one hand and, on the other, a device for initiating theory at a particularly simple level by utilizing idealizations of realistic particle behavior. Such idealizations may not violate general physical laws, however, and, conversely, provide a means for apprehending the existence of such laws when they appear as necessary restrictions on the nature of sources. That was our attitude in deducing, from the hypothesized existence of both electric and magnetic sources, the localization and quantization of charge.

We add a remark on the intrinsic symmetry between electricity and magnetism, which is not limited to the discrete substitution

$$F_{\mu\nu}\rightarrow {}^*F_{\mu\nu}, \qquad {}^*F_{\mu\nu}\rightarrow -F_{\mu\nu},$$
$$J_\mu\rightarrow {}^*J_\mu, \qquad {}^*J_\mu\rightarrow -J_\mu,$$

but is more fully expressed by the rotation

$$J_\mu\rightarrow J_\mu\cos\vartheta+{}^*J_\mu\sin\vartheta$$
$${}^*J_\mu\rightarrow -J_\mu\sin\vartheta+{}^*J_\mu\cos\vartheta,$$

with analogous equations for $F_{\mu\nu}$, ${}^*F_{\mu\nu}$. This property underlies the statement that no charge quantization would exist if all particles had the same g/e ratio for, by a suitable rotation in the e, g space, the situation is reduced to that of pure electricity. The charge rotational symmetry is inherent in the photon exchange definition of electric and magnetic sources since it is equivalent to a rotation of all polarization vectors. In order that the theory erected on this foundation [Eq. (33)] maintain that symmetry, it is necessary that $f_\mu(x-x')$ have the symmetry property given in Eq. (81). (Of course, the e, g rotational symmetry is only partial since the distinction between electricity and magnetism is an absolute one in the real world. That implies the existence of nonelectromagnetic but charge-dependent interactions of which the so-called weak interactions are a known example.) We can now recognize that the self-action of particles carrying both electric and magnetic charges introduces nothing new since e_a^2 is replaced by the invariant combinaton $e_a^2+g_a^2$ while no e_ag_a term can appear.

The unsymmetrical treatment of mutual and self actions is admittedly awkward when the source description is translated into field language. But no new physics is involved and elaborate formal devices are unnecessary. It suffices to proceed as in the text, with the understanding that the real parts of self-action terms are to be struck out when the fields are eliminated and attention returned to the sources. Nevertheless, in the interests of completeness we shall indicate that this mental process could be realized by a well-defined mathematical procedure. For simplicity we consider only electrically charged particles. The point charge form of W, which contains no real self-action terms, is unaltered if $D_+(x-x')$ is replaced by $D_+(x-x'+\lambda)$, where $\lambda^\mu\rightarrow 0$ through space-like values. But, for any finite λ^μ, the real self-action terms exist and could be added and substracted to give the form

$$W(\lambda)=\frac{1}{2}\int(dx)J^\mu(x)A_\mu(x+\lambda)-w_{\text{self}}(\lambda),$$

where $A_\mu(x)$ retains its original meaning in terms of the total source $J_\mu(x)$. In the following we shall understand that a symmetrization between λ^μ and $-\lambda^\mu$ is used. Then

$$W(\lambda)=\frac{1}{4}\int(dx)F^{\mu\nu}(x)F_{\mu\nu}(x+\lambda)-w_{\text{self}}(\lambda)$$
$$=\int(dx)[J^\mu(x)A_\mu(x+\lambda)-\tfrac{1}{4}F^{\mu\nu}(x)F_{\mu\nu}(x+\lambda)]-w_{\text{self}}(\lambda),$$

and the latter form has the stationary action property:

$$\delta_AW(\lambda)=\int(dx)\delta A_\mu(x+\lambda)[J^\mu(x)-\partial_\nu F^{\mu\nu}(x)]=0,$$

since $(\lambda^\mu\rightarrow-\lambda^\mu)$

$$\int(dx)\delta A_\mu(x)\partial_\nu F^{\mu\nu}(x+\lambda)\rightarrow\int(dx)\delta A_\mu(x)\partial_\nu F^{\mu\nu}(x-\lambda)$$
$$=\int(dx)\delta A_\mu(x+\lambda)\partial_\nu F^{\mu\nu}(x).$$

Analogous λ-generalizations of Eqs. (52) and (53) give the action expressions for the field descriptions of sources composed of dual charged particles.

Finally, a word about the charge quantization condition for dual charged particles when the λ process is used. If unphysical elements are not to appear during the limiting operation, a charge quantization condition must hold for almost all λ^μ. It is

$$(e_ag_b-e_bg_a)\frac{1}{2}\left(\int_{\sigma(\lambda)}+\int_{\sigma(-\lambda)}\right)d\sigma_\mu f^\mu(x)=2\pi n,$$

where $\sigma(\lambda)$ is obtained from the arbitrary three-dimensional surface σ by the rigid displacement λ^μ, and the necessary symmetrization between λ^μ and $-\lambda^\mu$ is made explicit. The critical situation occurs when one of the even number of filaments comprising $f^\mu(x)$ pierces $\sigma(\lambda)$ once but does not intersect $\sigma(-\lambda)$. This is not an exceptional possibility, but refers to a λ-domain of nonzero measure. The implied quantization condition is

$$\tfrac{1}{2}(e_a g_b - e_b g_a) = 2\pi n_{ab}, \quad n_{ab} \text{ even}.$$

This way of obtaining Eq. (80) differs from that of the text where the factor of $\frac{1}{2}$ derives from an exceptional geometrical situation, although the definition for that circumstance clearly depends on the same limiting process we have just described.

PHYSICAL REVIEW VOLUME 173, NUMBER 5 25 SEPTEMBER 1968

Extracting the Phase of Scattering Amplitudes by Means of the Glauber Formula*

J. C. VANDER VELDE
The University of Michigan, Ann Arbor, Michigan 48104
(Received 10 May 1968)

We present a method for extracting information on the phases of πp and πn scattering amplitudes by comparing data on πd scattering with the prediction of the Glauber formula. The essential feature of the analysis is our ability to write down the phase of the double-scattering term in terms of the phases, and their first derivatives, of the free π-nucleon amplitudes. The method is equally applicable to other beam particles $(K,p,\cdots)$ scattering off deuterons. A correction to the standard interpretation of the Glauber formula as applied to total cross-section defects is also given.

I. INTRODUCTION

During the past few years several experiments have been reported[1,2] which measure the high-energy elastic scattering of beam particles (π,K,p) on deuterons. It is customary to try to understand the results in terms of the Glauber high-energy approximation[3] which takes into account single and double scattering off the individual deuteron nucleons and the interference between the single and double scattering. In general, it is found that the calculated differential cross-section $d\sigma/dt$ agrees with the data in the region $-t<0.3$, where the single scattering dominates [t is the four-momentum transfer in $(\text{BeV}/c)^2$], and in the region $-t>0.5$, where the double scattering dominates. However, it is a quite common feature that the theory predicts a dip in the interference region ($-t\approx 0.3$ to 0.5) which is not generally observed.

The theoretical dip is caused by destructive interference of the single and double terms and the fact that the input, free-scattering amplitudes are mostly imaginary. If they are pure imaginary, then the cancellation will be complete for some $-t$. The theoretical curve can be brought into better agreement with the data by allowing the phases of the free amplitudes to vary from their (known) values at $t=0$. This has been tried, in one form or another, by most of the authors.[1,2] It is not at all obvious, however, just what parametrization one should choose for the phase variation, and the use of the Glauber formula with t-dependent phases can lead to quite lengthy calculations.

In the course of such calculations in analyzing the Michigan[1] data, we have devised a method to extract the maximum information on the phase change by directly comparing the data to the simple constant-phase Glauber calculation. The basic equations of the method have already been presented in the paper of Hsiung *et al.*[1] Our purpose here is to derive the equations and amplify slightly on their consequences.

In the interest of clarity we will use the Glauber formula in its simplest form, neglecting the corrections of Wilkin.[3] We make no attempt to assess the magnitude of possible other corrections to the Glauber formula (and therefore to our analysis) due to multiple-scattering effects, spin dependence, off-the-mass-shell effects, double spin-flip contributions, and the like.[4] Presumably any corrections which are well understood can be taken into account, while retaining the basic ideas of the analysis which we present. In order to be definite, we will consider the case of elastic π-d scattering. Our method can be taken over directly for other "simple" beam particles $(p,n,K,\cdots)$. For "compound" beams $(d,\alpha,\cdots)$ or more complicated targets, this general type of analysis should still be applicable, although we have made no study of this question.

* Work supported in part by the U. S. Atomic Energy Commission.

[1] H. Hsiung *et al.*, Phys. Rev. Letters **21**, 187 (1968).

[2] V. Franco and E. Coleman, Phys. Rev. Letters **17**, 827 (1966); G. W. Bennett *et al.*, *ibid.* **19**, 387 (1967).

[3] V. Franco and R. J. Glauber, Phys. Rev. **142**, 1195 (1966); C. Wilkin, Phys. Rev. Letters **17**, 561 (1966).

[4] The amplitudes in (1) refer to the non-spin-flip part of π-nucleon scattering. The relative smallness of the spin-flip amplitudes means that double spin-flip contributions can probably be safely neglected in high-energy π-d scattering. The same can be said of the off-mass-shell effects. [Jon Pumplin, University of Michigan (private communication) and Phys. Rev. **173**, 1651 (1968)]. This paper, along with a recent one by R. H. Bassel and C. Wilkin [Brookhaven National Laboratory Report No. 12430, 1967 (unpublished)] and our Ref. 3 contain discussions of the approximations and possible corrections to the Glauber formula and cite many references to other works on the subject.