

previously which have been used in this paper are applicable to other atoms and other atomic properties. However, in other calculations of hyperfine structure we may find that many of the near cancellations of diagrams found in this paper do not occur; and we may also find that other diagrams are important.

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Measurement of the Sodium *D*-Line Absolute Oscillator Strengths by the Roschdestvenskii Hook Method

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The absolute oscillator strengths for the sodium *D* lines have been measured using the Roschdestvenskii hook method, a technique which can provide an accurate means for measuring the product of the population and *f* value, *Nf*, for a gas. The experiment was performed under well-defined conditions, with a systematic variation of temperature, fringe angle, and fringe spacing. It was found that the *f* values were independent of these variations in accordance with expectations. The measured *f* values were $f_{D_2} = 0.677 \pm 0.007$ and $f_{D_1} = 0.341 \pm 0.009$. These results are in excellent agreement with the quantum-mechanical calculations which employ Bates-Damgaard or Hartree-Fock methods.

The work presented here represents the first phase of a program to apply spectral interferometry to the study of atomic properties and behavior

of shock-heated gases. A well-known and particularly powerful technique in this area is the Roschdestvenskii "hook method," which yields the prod-

uct Nf of a resonant transition, where N is the number density of atoms in the lower state, and f is the absorption oscillator strength associated with the transition.

In what follows, we describe the application of the hook method to sodium vapor heated by a suitable furnace arrangement with the object of obtaining improved values for the oscillator strengths for the D_1 and D_2 lines of sodium. The study was motivated, in part, by the fact that a recent tabulation¹ of f values for these lines vary in value as much as 25% from quantum-mechanical calculations using the Coulomb or hydrogenic approximations in the Bates-Damgaard or Hartree-Fock techniques.^{2,3}

The first measurements of f values were not absolute, but rather ratios of f values or relative f values. From such information, absolute f values can be determined if one of the absolute f values can be computed from quantum-mechanical calculations.

Absolute f values for the sodium D lines were first obtained experimentally in 1931 by Weingeroff⁴ using the technique of magnetorotation and Zehden⁵ using the absorption method. Weingeroff reported values of $f_{D_2} = 0.67$ and $f_{D_1} = 0.33$, which agree well with absolute f values computed by quantum-mechanical calculations. Zehden gave values of $f_{D_2} = 0.532$ and $f_{D_1} = 0.267$, which differ by about 20% from the commonly accepted f values as obtained from quantum mechanics.

Kvater⁶ in 1947 first used the hook method to measure absolute f values, and he chose sodium as his test medium. He reported f values of $f_{D_2} = 0.822$ and $f_{D_1} = 0.415$. These values differ by 25% from theoretical quantum-mechanical calculations mentioned above, but it has been reported by Penkin⁷ that the vapor-pressure-temperature data used by Kvater were significantly in error.

Ostrovskii and Penkin⁸ have used a variation of the Roschdestvenskii method to measure the f values of sodium and report values of $f_{D_2} = 0.76$ and $f_{D_1} = 0.39$. These values were later corrected by Mazing and Penkin⁹; this led to values of $f_{D_2} = 0.68$ and $f_{D_1} = 0.35$. Brehm *et al.*¹⁰ used a lifetime measurement of the particular excited states to determine the f values for sodium and found them to be $f_{D_2} = 0.654$ and $f_{D_1} = 0.328$.

The experimental results of the present work¹¹ are in excellent agreement with predictions of quantum-mechanical calculations. Comparisons with other experiments and with theory may be made from the listing in Table I.

EXPERIMENTAL METHOD

The experimental technique was that of the classical Roschdestvenskii hook-method experiment. The fringes formed by a Mach-Zehnder interferometer from a continuous spectrum source are focused onto the slit of a high-dispersion spectrograph. The experimental apparatus is shown in Fig. 1.

The sodium vapor was produced by heating American Chemical Society reagent sodium in a permanently sealed vapor tube. The tube was kept evacuated to a pressure of 10^{-7} mm Hg using 1-liter/sec ion pumps turned on between experimental runs.

TABLE I. Experimental and theoretical f values for the sodium D -line doublet.

INVESTIGATION	f_{D_2}	f_{D_1}	f_{D_2}/f_{D_1}	$\sum f = f_{D_1} + f_{D_2}$	METHOD
Goldberg <i>et al.</i> ¹	0.623	0.311	2.00	0.934	Bates-Damgaard
Goldberg <i>et al.</i> ¹	0.65	0.35	1.93	1.00	f sum rule
Weingeroff ⁴	0.67	0.33	2.02	1.00	Magnetorotation
Zehden ⁵	0.532	0.267	1.99	0.799	Absorption
Kvater ⁶	0.822	0.415	1.99	1.237	Hook method
Ostrovskii <i>et al.</i> ⁸	0.76	0.39	1.98	1.15	Penkin, Total Absorption
Brehm <i>et al.</i> ¹⁰	0.654	0.328	1.99	0.98	Lifetime
Forbrich	0.677	0.341	1.99	1.02	Hook method

The number density N of sodium atoms was then found from the sodium vapor-pressure-temperature data of Theile.¹²

When the fringes are focused on the spectrograph slit, the image photographed at the exit focal plane of the spectrograph appears as shown in Fig. 2a. The horizontal interference fringes change slope rapidly as a result of anomalous dispersion as they approach resonance wavelength. If the fringes are translated by means of inserting a plane glass plate in the compensation beam of the interferometer, the resulting image at the spectrograph focal plane is shown in Fig. 2b. The maxima and minima of the fringes are called the hooks. Theoretical analysis shows that Nf is given by^{11,13}

$$Nf = \pi K \Delta^2 / r_0 \lambda_0^3 L$$

for the case of a single absorption line, where Δ is the distance between hooks, L is the vapor-column length, r_0 is the classical electron radius e^2/mc^2 , λ_0 is the wavelength of the transition, and K is an experimental constant which depends on the experimental arrangement. To first order, K is given by the expression

$$K = p \lambda_0 / \Delta \lambda,$$

where p is the number of fringes in a distance $\Delta \lambda$ on the spectral interferogram in some region of the spectrum close to the line λ_0 .

For the case of two neighboring lines, as for the sodium doublet, the product Nf for the two lines is given by¹⁴

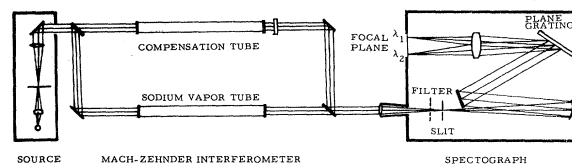


FIG. 1. Schematic diagram of experimental apparatus.

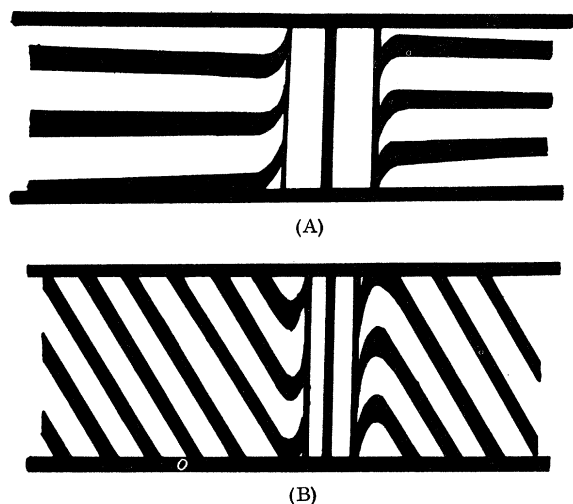


FIG. 2 Schematic fringe patterns observed at the spectrograph focal plane in the hook method.

$$N_1 f_1 = \frac{4\pi K_1}{\lambda_1^3 r_0 L} \delta_1^I \delta_2^I \left\{ 1 + \frac{\delta_3^{\text{II}} \delta_4^{\text{II}}}{\sigma^2 - \delta_1^I \delta_2^I - \delta_3^{\text{II}} \delta_4^{\text{II}}} \right\}^{-1},$$

$$N_2 f_2 = \frac{4\pi K_2}{\lambda_2^3 r_0 L} \delta_3^{\text{II}} \delta_4^{\text{II}} \left\{ 1 + \frac{\delta_1^I \delta_2^I}{\sigma^2 - \delta_1^I \delta_2^I - \delta_3^{\text{II}} \delta_4^{\text{II}}} \right\}^{-1},$$

where all quantities not previously defined are shown in Fig. 3.

EXPERIMENTAL DETAILS

As shown in Fig. 1, the experimental apparatus includes an absorption tube and compensation tube in the interferometer arms. The two tubes utilized ion pumps for evacuation. The sodium vapor tube shown schematically in Fig. 4 had a 6-g ampule of sodium metal sealed in the vapor tube during construction in a temporary reservoir. After the vapor tube had been sealed, outgassed by heating to 500°C, and evacuated to 10⁻⁷ mm Hg with ion pumps, the sodium ampule was cracked, and the sodium metal was distilled into the permanent reservoir. The temporary reservoir was then pinched off from the vapor tube leaving a permanently sealed, highly evacuated vapor tube. The

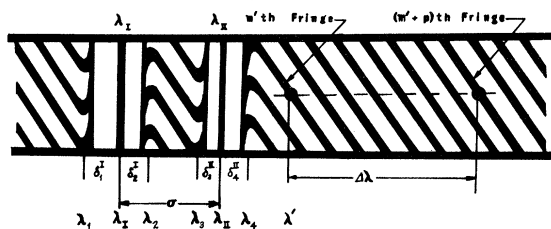


FIG. 3. Notation used in solving hook position equation for the doublet case.

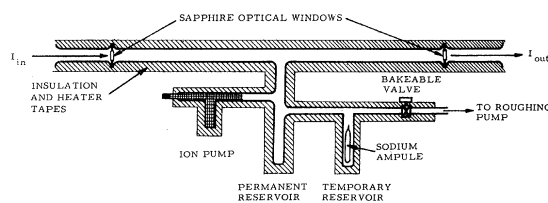


FIG. 4. Schematic diagram of sodium vapor tube.

optical windows were sapphire, although glass would have been all right for the present work in the visible spectrum. To produce sodium vapor, the tube was heated externally to a predetermined temperature. The temperature of the entire tube was maintained by a set of heat tapes arranged to produce a slight overheating at the windows to prevent sodium condensation. The temperature was measured and monitored by 13 calibrated iron-constantan thermocouples mounted on the tube. The temperature was varied from 438°K to 465°K. It was found at temperatures higher than 465°K that the fringes between the *D* lines became nearly vertical so that the quantities δ_2^I and δ_3^{II} could not be measured accurately. The length of the vapor column was determined accurately including the effects of expansion due to heating. The length of the vapor column at room temperature was 109.9 cm.

The spectrograph used was constructed with a Bausch & Lomb echelle grating having 300 lines/mm with a blaze angle of 63° 26'. This blaze angle gave maximum intensity in the 5th order with a dispersion of 4.22 Å/mm.

The interferometer was a 3×1-m Mach-Zehnder interferometer. The large size of the interferometer resulted in fringe drift. Fringe drift and defocusing resulting from convection heat currents in the room also occurred when heating the sodium vapor and compensation tubes. Fortunately, the fringe shifting does not affect the experimental results since only hook-to-spectral-line distances are used in the data reduction. The interferometer light source was a 1400-W continuous-feed carbon arc lamp having a fairly uniform spectral intensity over the range of interest of this experiment.

The experimental constant *K* was varied from values of about 5000 to about 16 000 by controlling the fringe adjustment and the thickness of the glass plate inserted in the compensation beam light path. It was found that the *f* values were independent of the temperature and also the value of *K*. This behavior was as expected and gave added confidence in the result.

EXPERIMENTAL RESULTS

The experimental results were obtained from 64 spectral interferograms representing the best third of those recorded in this study. The complete data are given elsewhere.¹¹ A histogram of the values of fD_1 and fD_2 are plotted in Fig. 5. Superimposed on the histogram is a plot of the Gaussian probability distribution for the data recorded. The

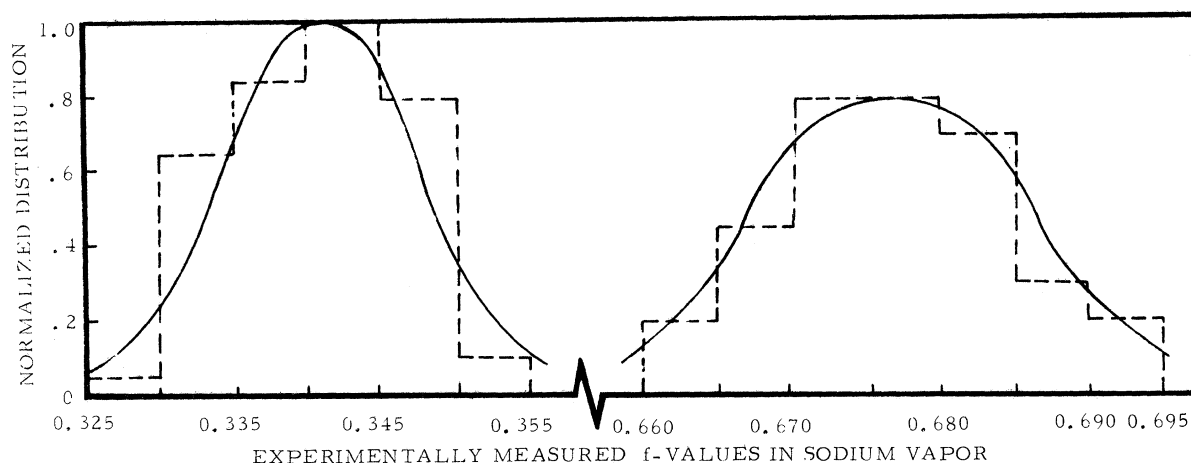


FIG. 5. Histogram of experimental data with Gaussian distribution curves superposed.

values of f_{D_1} and f_{D_2} were found to be $f_{D_1} = 0.341$ and $f_{D_2} = 0.677$. Using the standard deviation of the mean $\sigma/\sqrt{N_T}$ as the index of precision of the mean, where N_T is the total number of measurements, the f values for the sodium D lines can be given by

$$f_{D_2} = 0.677 \pm 0.007,$$

$$f_{D_1} = 0.341 \pm 0.009.$$

These results are in good agreement with the quantum-mechanical calculations as shown in Table I.

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