These may be recast in the form

$$I_1=\frac{\partial}{\partial\alpha}\frac{\partial}{\partial\lambda^2}D^+(x+\alpha r,\lambda^2)\,, \tag{B3}$$

$$I_2=-\frac{\partial^2}{\partial\alpha\partial\alpha}\frac{\partial^2}{\partial\lambda\partial\lambda}iD^+(x+\alpha r,\lambda^2)\,, \tag{B4}$$

where only terms which do not vanish when α and λ go to zero are kept. We find

$$I_1=-x\cdot r\left[\frac{i}{8\pi^2(x^2-i\epsilon)}+\frac{\delta(x^2)\epsilon(x^0)}{8\pi}\right], \tag{B5}$$

$$I_2=-\frac{i}{32\pi}\epsilon(x^0)[\delta(x^2)2(x\cdot r)^2+\theta(x^2)m^2]$$
$$+\frac{1}{32\pi^2}\left[\frac{2(x\cdot r)^2}{x^2-i\epsilon}+m^2\left(\ln\frac{\lambda^2|x^2|}{4}+2C\right)\right]. \tag{B6}$$

With these results F_c and σ_1 can be evaluated as before.

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Unitarity Corrections to Pole Dominance in Spectral-Function Sum Rules*

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Unitarity corrections to pole dominance in Weinberg's spectral-function sum rules and the corresponding sum rules for asymptotic SU_3 and $SU_3\times SU_3$ are evaluated in a model in which we use the field-current identities for the vector and axial-vector currents, and in which the propagators of the vector, axial-vector, and pseudoscalar mesons are evaluated in a chain approximation. This gives sum rules relating the various coupling constants and masses. It is pointed out that although the unitarity corrections to the coefficients of λ_ρ^2, $\lambda_{A_1}^2$, etc., in the spectral-function sum rules are within 20 to 30%, they can give rise to significant deviations from the pole-dominance predictions for decay rates and mass ratios.

I. INTRODUCTION

SEVERAL interesting results have been obtained recently, starting with a broken chiral symmetry or a chiral symmetry that is manifested only in some limit, such as the asymptotic limit of a two-point function. In particular, sum rules were obtained by Weinberg for the spectral functions of the isotopic vector and axial-vector currents[1]; these were later extended to the corresponding octet currents.[2] In most of this work the spectral functions have been approximated by the pole contributions, and this pole-dominance assumption has been found to give some remarkably good results. An extension of these methods to n-point functions, using phenomenological Lagrangians, has been carried out in the same spirit; one starts with a chiral Lagrangian and computes all the "tree" diagrams for an amplitude with given external lines.[3]

An important question arises in this connection. The pole-dominance assumption or the tree approximation does not preserve unitarity, and one is therefore led to look for simple approximation schemes in which unitarity is guaranteed.

In this paper we consider this question for the two-point functions.[4] We first remark that although the pole-dominance assumption has given some good results, in order to examine the validity of Weinberg's sum rules it is necessary to evaluate the spectral functions in a better approximation and examine how corrections to the pole-dominance approximation affect the results obtained from the sum rules.[5] In particular, the validity of Weinberg's second sum rule has been questioned.[6] However, this criticism was based on results obtained using pole dominance. As the second Weinberg sum rule is not expected, *a priori*, to be well saturated with low-lying states (in contrast to the first sum rule, which has an integrand that is damped faster at infinity), it is of interest to examine whether (nonpole) unitarity corrections to the second sum rule improve the results following from it.

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[1] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967).

[2] T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **18**, 761 (1967); S. L. Glashow, H. J. Schnitzer, and S. Weinberg, *ibid.* **19**, 139 (1967).

[3] See, for instance, B. W. Lee and H. T. Nieh, Phys. Rev. **166**, 1507 (1968).

[4] A brief summary of part of the results of this work has been given by W. S. Lam and K. Raman, Nuovo Cimento (to be published).

[5] K. Dietz and H. Pietschmann [Universität Bonn Report, 1967 (unpublished)] have taken into account two-particle states in obtaining a sum rule for ω-ϕ mixing.

[6] See, for instance, T. Das, V. S. Mathur, and S. Okubo, Phys. Rev. Letters **19**, 470 (1967); J. J. Sakurai, *ibid.* **19**, 803 (1967).

In order to go beyond pole dominance and obtain a unitary approximation, one needs to consider specific models.

The simplest approximation would seem to be to include the contributions of two-particle states (in addition to that of the pole) in the unitarity relations for the spectral functions. When the pole corresponds to a bound state, this would be a valid approximation. However, the single-particle states occurring in the spectral functions of the vector and axial-vector currents are unstable particles. If these are regarded as elementary particles (which exist independently of the interaction in the two-particle scattering channels), then it would still be valid to add the two-particle contributions to the pole terms.[5] On the other hand, if these 1^- and 1^+ mesons are regarded as dynamical resonances (generated in part by the two-particle interaction), then adding two-particle scattering states to the resonant states would imply double counting. We would then be led to consider more elaborate approximations in which the resonant "pole" contributions are themselves generated by the model; the spectral function in such a model would include corrections to the pole-dominance approximation, without double counting.

In this paper we consider a simple soluble model of this type. In this model, we first approximate the vector current by a linear combination of renormalized vector-meson field operators, as suggested by the theory of Kroll, Lee, and Zumino[7]; a similar approximation is considered for the axial-vector current. To evaluate Weinberg's sum rules, one then has to obtain the complete renormalized propagators of the vector, axial-vector, and pseudoscalar mesons. In our model we evaluate these in a chain approximation (or Zachariasen-Thirring type of model),[8] in which 1^-, 1^+, 0^-, and 0^+ mesons are kept in the intermediate states in the kernel of the chain integral equation. This gives a unitary approximation for the spectral functions which are then substituted into Weinberg's sum rules, resulting in sum rules for the masses and coupling constants.

We discuss the details of our model in Sec. II. In Secs. III and IV we discuss the relations following in our model from the spectral-function sum rules for $SU_2\times SU_2$ and SU_3, respectively. In Sec. V we give the $SU_3\times SU_3$ sum rules, and in Sec. VI we summarize our conclusions.

II. MODEL

We define the spectral functions of the vector and axial-vector currents in the usual manner by assuming the following (unsubtracted) representation (for the axial-vector current):

$$\bar{\Delta}_A{}^{\mu\nu}(p)\equiv(2\pi)^{-4}\int d^4x\, e^{ip\cdot x}\langle 0|T(\mathcal{A}^\mu(x)\mathcal{A}^\nu(y))|0\rangle$$
$$=i\int\frac{ds'}{s'-s-i0}\left\{\left[g^{\mu\nu}-\frac{p^\mu p^\nu}{s'}\right]\rho_{(1)}{}^A(s')+p^\mu p^\nu\rho_{(0)}{}^A(s')\right\}$$
$$+ig^{\mu 0}g^{\nu 0}\int ds'\left[\frac{\rho_{(1)}{}^A(s')}{s'}+\rho_{(0)}{}^A(s')\right],\quad(2.1)$$

with $s=p^2$, and a similar representation for the vector current. For the conserved components of the octet vector current, one has, of course, $\rho_{(0)}{}^V(s)=0$.

We make the following approximation for the octet vector current $\mathcal{V}^\mu$ (as suggested by the theory of Kroll, Lee, and Zumino[7]) and the axial-vector current $\mathcal{A}^\mu$ [9]:

$$\mathcal{V}^\mu(x)=\lambda_V\phi_V{}^\mu(x),\quad \mathcal{A}^\mu(x)=\lambda_A\phi_A{}^\mu(x)+F_P\partial_\mu\phi_P(x).\quad(2.2)$$

Here, $\phi_V{}^\mu$, $\phi_A{}^\mu$, and ϕ_P are the renormalized 1^-, 1^+, and 0^- meson field operators. λ_V is defined by

$$\langle 0|\mathcal{V}^\mu(0)|V(q)\rangle=[2q_0]^{-1/2}\lambda_V\epsilon^\mu,$$

where V is a vector meson. λ_A is defined by a similar equation. F_P is defined by

$$\partial_\mu\mathcal{A}^\mu(x)=F_P\mu_P{}^2\phi_P(x).$$

We now obtain

$$\langle 0|T(\mathcal{V}^\mu(x)\mathcal{V}^\nu(y))|0\rangle=\lambda_V{}^2\langle 0|T(\phi_V{}^\mu(x)\phi_V{}^\nu(y))|0\rangle\quad(2.3)$$

and

$$\langle 0|T(\mathcal{A}^\mu(x)\mathcal{A}^\nu(y))|0\rangle=\lambda_A{}^2\langle 0|T(\phi_A{}^\mu(x)\phi_A{}^\nu(y))|0\rangle$$
$$+F_P{}^2\left\{\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\langle 0|T(\phi_P(x)\phi_P(y))|0\rangle+Z_P{}^{-1}ig^{\mu 0}g^{\nu 0}\delta(x-y)\right\}$$
$$+F_P\left\{\frac{\partial}{\partial x^\mu}\langle 0|T(\phi_P(x)\phi_A{}^\nu(y))|0\rangle+\frac{\partial}{\partial y^\nu}\langle 0|T(\phi_A{}^\mu(x)\phi_P(y))|0\rangle\right\}.\quad(2.4)$$

[7] N. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. **157**, 1376 (1967).

[8] F. Zachariasen, Phys. Rev. **121**, 1851 (1961); W. Thirring, in *Theoretical Physics* (International Atomic Energy Agency, Vienna, 1963). Mass relations for 0^- and 1^- mesons were obtained from such a model by H. Pietschmann, Phys. Rev. **139**, B447 (1965). We remark that here we regard the Zachariasen-Thirring model as an approximation to the propagator in a complete theory, and not as a field theory by itself of the type considered by W. Thirring, Phys. Rev. **126**, 1209 (1962). We recall that in such an approximation the scattering amplitude is not crossing-symmetric; however, unitarity is preserved.

[9] Assumptions of this nature were probably first used by M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961). In connection with the assumption about the axial-vector current, see J. J. Sakurai, Report No. EFINS 67-64, University of Chicago, 1967 (unpublished); and Phys. Letters **24B**, 619 (1967).

In obtaining these we have assumed the commutation relations $[\phi_P(x),\phi_P(y)]=iZ_P^{-1}\Delta(x-y)$ and $[\phi_P(x),\phi_A{}^\mu(y)]\delta(x_0-y_0)=0$. We define the wave-function renormalization constants Z_V, Z_A, and Z_P by[10]

$$\phi_V{}^\mu(x)=Z_V{}^{-1/2}\phi_V{}^{\mu(0)}(x)\,, \tag{2.5}$$

etc., where $\phi_{V,A}{}^{\mu(0)}$ and $\phi_P{}^{(0)}$ are the unrenormalized field operators. The last pair of terms in (2.4) gives rise to a mixing between 1^+ and 0^- mesons.

To proceed further, we define a covariant two-point amplitude $\tilde{\Delta}^{\mu\nu}(p)$ by the following representation:

$$\tilde{\Delta}_A{}^{\mu\nu}(p)=i\int\frac{ds'}{s'-s-i0}\times\left\{\left[g^{\mu\nu}-\frac{p^\mu p^\nu}{s'}\right]\rho_{(1)}{}^A(s')+p^\mu p^\nu\rho_{(0)}{}^A(s')\right\}\,, \tag{2.6}$$

and a similar covariant amplitude $\tilde{\Delta}_V{}^{\mu\nu}(p)$. We have defined these covariant two-point amplitudes in such a way that they have the same analytic structure as the time-ordered amplitudes $\bar{\Delta}^{\mu\nu}(p)$ for finite s and differ only in their asymptotic behavior.[11] Further, their asymptotic behavior is defined to be such that[12]

$$\tilde{\Delta}_{T,L}{}^{V,A}(s)\to 0 \quad\text{as}\quad s\to\infty\,, \tag{2.7}$$

where $\tilde{\Delta}_{T,L}{}^{V,A}(s)$ are defined by

$$\tilde{\Delta}_{V,A}{}^{\mu\nu}(p)=\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)\tilde{\Delta}_T{}^{V,A}(s)+\frac{p^\mu p^\nu}{p^2}\tilde{\Delta}_L{}^{V,A}(s)\,. \tag{2.8}$$

The functions $\tilde{\Delta}_{(j)}{}^A(s)$, $j=0, 1$, defined by

$$\tilde{\Delta}_{(j)}{}^A(s)=i\int ds'\frac{\rho_{(j)}{}^A(s')}{s'-s-i0}\,, \tag{2.9}$$

are related to $\tilde{\Delta}_{T,L}{}^A(s)$ by

$$\tilde{\Delta}_{(1)}{}^A(s)=\tilde{\Delta}_T{}^A(s),\quad \tilde{\Delta}_{(0)}{}^A(s)=(1/s)[\tilde{\Delta}_L{}^A(s)-\tilde{\Delta}_T{}^A(0)]\,. \tag{2.10}$$

[10] For both stable and unstable particles, we assume a convention for Z that corresponds to choosing $Z=Z_1$ in the notation of Ref. 7. This would give the usual definitions $\langle 0|\mathcal{V}_3{}^\mu(0)|\rho_0(q)\rangle=(2q_0)^{-1/2}\epsilon_\mu\lambda_\rho$, etc.

[11] The covariant two-point amplitude defined here may be obtained by starting with the Fourier transform of a covariant current-correlation function [defined in a manner analogous to L. S. Brown, Phys. Rev. **150**, 1338 (1966)] and subtracting from it the parts that do not vanish as $s\to\infty$. (The current-correlation function would in general have, in addition to the terms in (2.6), terms of the form $ig^{\mu\nu}\int ds\,[s^{-1}\rho_{(1)}{}^A(s)+\rho_{(0)}{}^A(s)]$.)

[12] The reason for requiring this is that the covariant two-point amplitudes $\tilde{\Delta}_{T,L}{}^{V,A}(s)$ in our model vanish as $s\to\infty$.

We note that Weinberg's sum rules may be written in the form

$$\tilde{\Delta}_{(1)}{}^V(0)-\tilde{\Delta}_{(1)}{}^A(0)=-\lim_{s\to\infty}s[\tilde{\Delta}_{(0)}{}^V(s)-\tilde{\Delta}_{(0)}{}^A(s)]\,, \tag{2.11a}$$

$$\lim_{s\to\infty}s[\tilde{\Delta}_{(1)}{}^V(s)-\tilde{\Delta}_{(1)}{}^A(s)]=0\,. \tag{2.11b}$$

To evaluate the quantities entering into these sum rules, we need to evaluate the covariant parts of the Fourier transforms of the two-point functions occurring on the right-hand sides of (2.3) and (2.4). [These covariant parts are the covariant (renormalized) vector, axial-vector, and pseudoscalar-meson propagators.] We denote the corresponding unrenormalized covariant propagators by $\Delta'_V{}^{\mu\nu}(p)$, $\Delta'_A{}^{\mu\nu}(p)$, and $\Delta'_P(p^2)$; in our model these are evaluated in a chain approximation. As an approximation we shall here neglect the mixing terms between the 1^+ and 0^- mesons given by the last pair of terms in (2.4). We hope to consider consequences of such a mixing elsewhere.

We define the chain approximation to a spin-1 propagator as the solution to the following equation (in momentum space):

$$\Delta'^{\mu\nu}(p)=\Delta^{\mu\nu}(p)+\Delta^{\mu\lambda}(p)\Sigma^{\lambda\sigma}(p)\Delta'^{\sigma\nu}(p)\,; \tag{2.12}$$

a similar equation is written for a spin-0 propagator. In (2.12), $\Delta^{\mu\nu}(p)$ is given by

$$\Delta^{\mu\nu}(p)=-i(g^{\mu\nu}-p^\mu p^\nu/m_0{}^2)(p^2-m_0{}^2)^{-1}\,, \tag{2.13}$$

and $\Delta'^{\mu\nu}$ is the complete unrenormalized propagator. In (2.13), m_0 is the bare mass of the spin-1 field. $\Sigma^{\lambda\sigma}(p)$ and the corresponding function $\Sigma(p^2)$ for a spin-0 propagator are given by sums over a class of proper self-energy diagrams with $(P+P)$, $(S+S)$, $(P+S)$, $(P+V)$, and $(P+A)$ intermediate states (when these are not forbidden); here P, V, and A are 0^-, 1^-, and 1^+ mesons, and S is a possible scalar meson.

We define the transverse and longitudinal projections $\Sigma_{T,L}(s)$ of $\Sigma^{\mu\nu}(p)$ and similar projections of $\Delta'_V{}^{\mu\nu}$ and $\Delta'_A{}^{\mu\nu}$ by equations analogous to (2.8). Equation (2.12) may be solved to give

$$\Delta'_T(s)=(-i)[s-m_0{}^2+i\Sigma_T(s)]^{-1}\,; \tag{2.14a}$$

$$\Delta'_L(s)=i[m_0{}^2-i\Sigma_L(s)]^{-1}\,. \tag{2.14b}$$

The equation analogous to (2.12) for the spin-0 propagator gives in a similar manner

$$\Delta'_P(s)=i[s-(m_0{}^P)^2-i\Sigma_P(s)]^{-1}\,, \tag{2.14c}$$

where $m_0{}^P$ is the bare mass of the 0^- meson.

Using the condition that the real parts of $[\Delta'_T{}^V(s)]^{-1}$, $[\Delta'_T{}^A(s)]^{-1}$, and $[\Delta'_P(s)]^{-1}$ have zeros at the physical meson masses $s=m_V^2$, $s=m_A^2$, and $s=m_P^2$, respectively, we may eliminate the bare masses from Eqs. (2.14) and obtain

$$\Delta'_T{}^A(s)=(-i)\{s-m_A{}^2+i\Sigma_T{}^A(s)-\mathrm{Re}[i\Sigma_T{}^A(m_A{}^2)]\}^{-1}, \quad (2.15a)$$

$$\Delta'_L{}^A(s)=i\{m_A{}^2-i\Sigma_L{}^A(s)+\mathrm{Re}[i\Sigma_T{}^A(m_A{}^2)]\}^{-1}, \quad (2.15b)$$

$$\Delta_P(s)=i\{s-m_P{}^2-i\Sigma_P(s)+\mathrm{Re}[i\Sigma_P(m_P{}^2)]\}^{-1}, \quad (2.15c)$$

and analogous expressions for $\Delta'_{T,L}{}^V(s)$. Writing (once-subtracted) dispersion representations for $\Sigma_{T,L}{}^{V,A}(s)$ and $\Sigma_P(s)$, we finally obtain the following expressions[13]:

$$[i\Delta'_T{}^A(s)]^{-1}=D_A(s)\times[(s-m_A{}^2)+\tfrac{1}{2}iD_A{}^{-1}(s)\sum_j B_j{}^A(s)\theta(s-s_j)], \quad (2.16a)$$

$$[-i\Delta'_L{}^A(s)]^{-1}=\{m_A{}^2D_A(0)-i\Sigma_L{}^A(s)+\mathrm{Re}[i\Sigma_T{}^A(0)]\}, \quad (2.16b)$$

and analogous expressions for $\Delta'_{T,L}{}^V(s)$ and $\Delta'_P(s)$. In (2.16), we have defined

$$D_A(s)=1+\frac{1}{2\pi}\mathcal{P}\sum_j\int_{s_j}^{\infty}ds'B_j{}^A(s')\times(s'-s)^{-1}(s'-m_A{}^2)^{-1}; \quad (2.17)$$

and $B_j{}^A(s)\theta(s-s_j)$ is the discontinuity of $\Sigma_T{}^A(s)$ arising from the jth two-particle state (with threshold s_j). The wave-function renormalization constant Z_A is obtained from (2.16a) as $Z_A=D_A{}^{-1}(m_A{}^2)$.[10]

For a conserved vector current, the equations analogous to (2.10), together with the condition $\tilde{\Delta}_{(0)}{}^V(s)=0$, give $\tilde{\Delta}_L{}^V(s)=\tilde{\Delta}_T{}^V(0)$. As $\Sigma_L{}^V(s)=0$ for a conserved vector current, (2.14) then gives $\Sigma_T{}^V(0)=0$.[14] As the absorptive part of $\Sigma_T{}^V(s)$ must be positive definite, this condition requires that a dispersion representation for $\Sigma_T{}^V(s)$ must have a subtraction. We ensure that our results are consistent with the condition $\Sigma_T{}^V(0)=0$ by writing a subtracted dispersion relation for $\Sigma_T{}^V(s)$ [and requiring that $\Sigma_T{}^V(0)=0$]. For the axial-vector current, (2.10) leads to the relation $\Sigma_L{}^A(0)=\Sigma_T{}^A(0)$.

Using (2.3) and (2.4) and the above relations, we now obtain

$$i\tilde{\Delta}_{(1)}{}^A(s)=\lambda_A{}^2D_A(m_A{}^2)[(s-m_A{}^2)D_A(s)+\tfrac{1}{2}i\sum_j B_j{}^A(s)\theta(s-s_j)]^{-1}, \quad (2.18)$$

a similar expression for $i\tilde{\Delta}_{(1)}{}^V(s)$, and

$$is\tilde{\Delta}_{(0)}{}^A(s)=\lambda_A{}^2D_A(m_A{}^2)[i\Sigma_L{}^A(0)-i\Sigma_L{}^A(s)]\times[m_A{}^2+i\Sigma_T{}^A(m_A{}^2)-i\Sigma_L{}^A(s)]^{-1}\times[m_A{}^2+i\Sigma_T{}^A(m_A{}^2)-i\Sigma_T{}^A(0)]^{-1}+sF_P{}^2D_P(m_P{}^2)\times[(s-m_P{}^2)D_P(s)+\tfrac{1}{2}i\sum_j B_j{}^P(s)\theta(s-s_j)]^{-1}. \quad (2.19)$$

Here $D_V(s)$ and $D_P(s)$ are given by expressions similar to (2.17).

To proceed further, we must know the functions $\Sigma_{T,L}{}^V(s)$, $\Sigma_{T,L}{}^A(s)$, and $\Sigma_P(s)$. To obtain these, we find the contributions to the absorptive parts of these functions from $(P+P)$, $(S+S)$, $(P+S)$, $(P+V)$, and $(P+A)$ intermediate states (when these are allowed by the conservation laws).[15] For $\Sigma_P(s)$, we also include the $(V+V)$ and $(A+A)$ states. We approximate these functions by taking self-energy diagrams and including corrections to the internal lines and vertices so that the internal masses are the physical masses and the vertices are normalized approximately to the physical coupling constants on the mass shell. We assume that the vertex corrections[16] damp the absorptive parts of $\Sigma^{V,A}(s)$, etc., at large s; we take this into account by introducing a form factor.

By analogy with the dipole (electromagnetic) form factors of the nucleon, we take as a model for each meson vertex a form factor $(1-s/\bar{M}^2)^{-1}$ for small and moderately large s, where $\bar{M}$ is the mean mass of the 1^-, 1^+, or 0^- meson nonet (or octet).[17] (For the 1^-, 1^+, and 0^- propagators, we take $\bar{M}^2\approx36\mu_\pi{}^2$, $73\mu_\pi{}^2$, and $8\mu_\pi{}^2$, respectively.) We assume that at very large s, the form factors are damped more rapidly, so that the dipole form factor is effectively cut off at s of the order of $200m_\pi{}^2$. In our model, we retain only those contributions which are not sensitive to the value of this effective cutoff.[18]

For the 1^-, 1^+, and 0^- meson propagators, we find the discontinuity functions $B_j(s)$ to be given by the follow-

[13] Representations like (2.15) and (2.16) may be obtained using the analytic properties and unitarity for the inverse propagator. Results derived from this will be discussed in a separate paper.

[14] Alternatively, this follows from (2.14a) on noting that with a conserved vector current one must have $\Delta'_{(1)}{}^V(0)=i\int ds\, s^{-1}\bar{\rho}_{(1)}{}^V(s)=im_0{}^{-2}$. We are grateful to Professor T. Akiba for pointing this out.

[15] As Weinberg's sum rules for $SU_2\times SU_2$ (and for $SU_3\times SU_3$) express a residual chiral symmetry, we take this into account in our model by choosing the intermediate states appropriately; e.g., corresponding to $(P+P)$ states, we also include $(S+S)$ states, etc.

[16] We assume that the vertex corrections which can be included in the proper self-energy part give rise to a form factor of the type assumed, normalized approximately to the physical coupling constant when all the particles are on the mass shell.

[17] Assuming a dipole form factor for the nucleon, together with vector-meson dominance, corresponds to assuming for the vector-meson vertex a form factor $\Gamma(s)\approx(1-s/\bar{M}^2)^{-1}$. Our choice and interpretation of the form factors is very similar to that in Ref. 5.

[18] In evaluating the contribution of $(A+P)$ intermediate states to the 1^- propagator, or of $(V+P)$ states to the 1^+ propagator, writing the AVP vertex in terms of transverse and longitudinal couplings [as defined, for instance by F. Gilman and H. Harari, Phys. Rev. **165**, 1803 (1968)] gives contributions which make (2.17) sensitive to the effective cutoff. Instead, when the AVP vertex is written in terms of S-wave and D-wave couplings, the S-wave coupling is found to give a very small contribution, while the D-wave contribution is sensitive to the cutoff. We therefore omit the contributions of these intermediate states (as well as intermediate states involving particles with higher spin) in our model.

ing [where $B_j^V(s)=\bar{B}_j^V(s)|\Gamma(s)|^2$, etc., and $\Gamma(s)$ is the form factor arising from the vertex corrections]:

(i) 1^- propagator: $\bar{B}^V(PP)=\frac{4}{3}q^3s^{-1/2}$;
$$\bar{B}^V(VP)=\frac{2}{3}q^3s^{1/2}, \quad (2.20a)$$

(ii) 1^+ propagator: $\bar{B}^A(PS)=\frac{4}{3}q^3s^{-1/2}$;
$$\bar{B}^A(AP)=\frac{2}{3}q^3s^{1/2}, \quad (2.20b)$$

(iii) 0^- propagator: $\bar{B}^P(PP)=qs^{-1/2}$;
$$\bar{B}^P(VP)=4q^3s^{1/2}m_V^{-2};\quad \bar{B}^P(VV)=2q^3s^{1/2}. \quad (2.20c)$$

Here, $B^V(PP)$ is the contribution to the discontinuity in the 1^- propagator arising from a $(P+P)$ intermediate state, etc., and q is the c.m. momentum in the two-particle intermediate state with a total energy $s^{1/2}$.

In using the first equation in (2.2), we have neglected the coupling of possible scalar mesons to the nonconserved (strangeness-changing) components of the vector current $\mathcal{V}^\mu$. [To take these into account, we would add a scalar-meson term $F_S\phi_S$ to the first equation in (2.2).]

Substituting the expressions for the absorptive parts given by (2.20) into (2.16)–(2.19), we obtain the quantities occurring in the sum rules (2.11) in terms of coupling constants, masses, and dispersion integrals like those in (2.17). In the following sections we discuss the relations that are thus obtained.

III. EVALUATION OF THE $SU_2\times SU_2$ SUM RULES

When the expressions for $\tilde{\Delta}_{(j)}{}^V(s)$ and $\tilde{\Delta}_{(j)}{}^A(s)$ obtained in the previous section are substituted into Weinberg's sum rules (2.11), they give relations between the coupling constants and the masses. We have obtained these relations for the sum rules corresponding to asymptotic $SU_2\times SU_2$, SU_3, and $SU_3\times SU_3$.

In this section we discuss the results following from the $SU_2\times SU_2$ sum rules.

In evaluating these, we find that the first term on the right-hand side of (2.19)is negligibly small,[19] while in the second term (proportional to the 0^- meson propagator), $D_P(m_P^2)$ is found to differ negligibly from unity, so that the limit of (2.19), as $s\to\infty$, is just F_P^2.

Equations (2.11) now lead to the following sum rules:

$$(\lambda_\rho^2/m_\rho^2)D_V(m_\rho^2)D_V^{-1}(0) - (\lambda_{A_1}^2/m_{A_1}^2)D_A(m_{A_1}^2)D_A^{-1}(0)=F_\pi^2, \quad (3.1a)$$

$$\lambda_\rho^2D_V(m_\rho^2)-\lambda_{A_1}^2D_A(m_{A_1}^2)=0, \quad (3.1b)$$

where

$$D_V(0)\approx1+\frac{1}{4\pi}(0.02g_{\rho\pi\pi}^2+0.006g_{\rho K\bar{K}}^2 +0.28h_{\rho\omega\pi}^2+0.05h_{\rho\phi\pi}^2); \quad (3.2a)$$

$$D_V(m_\rho^2)\approx1+\frac{1}{4\pi}(0.027g_{\rho\pi\pi}^2+0.01g_{\rho K\bar{K}}^2 +0.49h_{\rho\omega\pi}^2+0.07h_{\rho\phi\pi}^2); \quad (3.2b)$$

$$D_A(0)\approx1+\frac{1}{4\pi}(ag_{A_1\sigma\pi}^2+0.1h_{A_1D\pi}^2 +0.03h_{A_1E\pi}^2); \quad (3.2c)$$

$$D_A(m_{A_1}^2)\approx1+\frac{1}{4\pi}(bg_{A_1\sigma\pi}^2+0.17h_{A_1D\pi}^2 +0.05h_{A_1E\pi}^2). \quad (3.2d)$$

Here E denotes a possible axial-vector state with $I=0$, $G=(-)$, at about 1420 MeV. The terms involving $g_{\rho\delta\delta}^2$ in $D_V(0)$ and $D_V(m_\rho^2)$ have been omitted,[20] as the coefficients multiplying them are very small. The numerical coefficients in $D_V(0)$, etc., are values of complicated functions of the masses. The coefficients a and b are zero if scalar mesons are not included [i.e., in $(P+S)$ intermediate states]; with a σ meson at about 765 MeV, they are -0.04 and -0.005, while for a σ meson at about 420 MeV, they are 0.1 and 0.03. We have omitted the contributions of $(\rho+\pi)$ and $(A_1+\pi)$ intermediate states to the A_1 and ρ propagators, respectively, as they are sensitive to the effective cutoff.[18]

The sum rules (3.1) give relations between the constants λ_ρ, λ_{A_1}, the masses of the ρ, A_1, and the various particles occurring in the intermediate states, and the coupling constants occurring in $D_V(0)$, $D_A(0)$, etc. These are the basic relations following from Weinberg's sum rules when unitarity corrections to pole dominance are taken into account in our approximation; they express the consequences of an asymptotic chiral symmetry and unitarity in this approximation.

To examine these sum rules further, we require estimates for the coupling constants occurring in (3.2). We use the following rough estimates[21]:

$$\begin{aligned}(1/4\pi)g_{\rho\pi\pi}^2&\approx2.5\approx4[(1/4\pi)g_{\rho K\bar{K}}^2],\\ (1/4\pi)h_{\rho\omega\pi}^2&\approx0.45\approx(1/4\pi)h_{A_1D\pi}^2,\quad h_{\rho\phi\pi}^2\approx0.\end{aligned} \quad (3.3)$$

We ignore the terms involving the $A_1E\pi$ couplings, as little is known about the E meson.

[19] In evaluating the first term in (2.19), as $s\to\infty$, we note that in our model $\Sigma_L{}^A(s)\to0$ as $s\to\infty$. We write the factor $[m_A{}^2+i\Sigma_T{}^A(m_A{}^2)]$ as $[m_A{}^2+i\Sigma_T{}^A(m_A{}^2)-i\Sigma_T{}^A(0)+i\Sigma_L{}^A(0)]$, making use of the relation $\Sigma_T{}^A(0)=\Sigma_L{}^A(0)$. One can then express this factor in terms of $\Sigma_L{}^A(0)$ and a subtracted dispersion integral for $\Sigma_T{}^A(s)$, both of which are insensitive to the value of the effective cutoff.

[20] δ denotes a $I=1$, $J^P=0^+$ nonstrange meson; we have tentatively taken its mass as 960 MeV. The data on masses and decay widths used in this paper have been taken from A. H. Rosenfeld *et al.*, University of California Lawrence Radiation Laboratory Report No. UCRL-8030, revised, September, 1967 (unpublished).

[21] This value of $g_{\rho\pi\pi}^2$ is obtained by assuming $\Gamma(\rho\to\pi\pi)\approx128$ MeV, $m_\rho\approx777$ MeV, while the estimate of $g_{\rho K\bar{K}}^2$ is obtained assuming approximate SU_3 for the VPP couplings. (The Ademollo-Gatto nonrenormalization theorem would suggest that this would give a fair estimate for the vector-meson couplings.) The value of $h_{\rho\omega\pi}^2$ is obtained from the decay rate $\Gamma(\omega\to\pi+\gamma)\approx1.18$ MeV, assuming ρ dominance of the isovector electromagnetic current [M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962)]. The decay rate $\Gamma(\omega\to3\pi)\approx11$ MeV, with the Gell-Mann–Sharp–Wagner model gives $(1/4\pi)h_{\rho\omega\pi}^2\approx0.52$. The estimate $h_{A_1D\pi}^2\approx h_{\rho\omega\pi}^2$ was suggested by the relations obtained by F. Gilman and H. Harari [Phys. Rev. **165**, 1803 (1968)] from superconvergence relations with saturation. The uncertainty in $g_{\rho\pi\pi}^2$, $h_{\rho\omega\pi}^2$, etc., caused by the uncertainty in the mass and width of the ρ does not affect the results significantly.

For the σ meson, we have examined the results obtained with two alternatives: (i) $m_\sigma \approx m_\rho$, $g_{A_1\sigma\pi}^2 \approx m_{A_1}^2/F_\pi^2$, as suggested by the sum rules from superconvergence obtained by Gilman and Harari,[18] and (ii) $m_\sigma \approx 3m_\pi$, $g_{A_1\sigma\pi}^2 \approx g_{\rho\pi\pi}^2$.

With these estimates, Eqs. (3.1) give the following relations:

$$1.1\lambda_\rho^2 m_\rho^{-2} - \alpha\lambda_{A_1}^2 m_{A_1}^{-2} = F_\pi^2\,, \tag{3.4a}$$

$$1.3\lambda_\rho^2 - \beta\lambda_{A_1}^2 = 0\,, \tag{3.4b}$$

where (α,β) have the values (1.03,1.08), (1.24,1.08), and (1.08,1.16) with the assumptions of no σ meson, a σ meson at 765 MeV, and one at 420 MeV, respectively. For comparison, we note that in the pole approximation, all the numerical coefficients in (3.4) would be unity. The deviations of these from the pole-dominance values are thus within about 30%; they are, on the whole, larger for the second sum rule than for the first.

Weinberg's sum rules for $SU_2 \times SU_2$ were originally[1] compared with experiment by combining them with the pole-dominance approximation and using the KSFR relation[22] $\mathcal{K} \equiv (2m_\rho^2 F_\pi^2/\lambda_\rho^2) = 1$ to obtain (m_{A_1}/m_ρ). As the KSFR relation appears to require more assumptions than was thought originally,[23] it seems to be more appropriate to take the ratio m_{A_1}/m_ρ from experiment, eliminate λ_{A_1} from (3.4), and obtain the value of $\mathcal{K}$. If F_π is assumed to be known, this gives the value of λ_ρ, which would lead to predictions for the rates for the leptonic decays $\rho \to l^+ + l^-$. With the values $m_{A_1} \approx 1058$ MeV, $m_\rho \approx 777$ MeV used here and with the three assumptions about the σ meson [occurring in the $(P+S)$ intermediate state], namely, (i) no σ meson, (ii) a σ meson at 765 MeV with $g_{A_1\sigma\pi}^2 \approx m_{A_1}^2/F_\pi^2$, and (iii) a σ meson at 420 MeV with $g_{A_1\sigma\pi}^2 \approx g_{\rho\pi\pi}^2$, Eqs. (3.4) give $\mathcal{K} \approx 0.87$, 0.59, and 0.9, respectively. For comparison we note that the KSFR relation gives $\mathcal{K} = 1$, while Weinberg's sum rules with pole dominance, and $m_{A_1} \approx 1058$ MeV, $m_\rho \approx 777$ MeV, give $\mathcal{K} \approx 0.92$. Since the values of m_ρ and m_{A_1} [used in obtaining the numerical coefficients in (3.4), as well as in solving these equations to obtain $\mathcal{K}$] are subject to some uncertainty, the numerical results for $\mathcal{K}$ are correspondingly uncertain.

However, these results indicate that when corrections to pole dominance are included, the results following from the $SU_2 \times SU_2$ sum rules are fairly sensitive to the assumptions made about the existence and the nature of the scalar mesons (included in the $P+S$ intermediate states). With the values of m_{A_1} and m_ρ assumed here, the results obtained with different assumptions about the σ meson differ by up to 20% among one another and by up to 30% from the pole-dominance value.

If the value of F_π is assumed to be known, one may use the value of $\mathcal{K}$ to obtain predictions for the leptonic decay rates $\Gamma(\rho \to l^+ l^-)$. These predictions differ from the pole-dominance results by up to 60–70% [because $\mathcal{K}$ occurs as $\mathcal{K}^2$ in these decay rates, e.g., $\Gamma(\rho \to e^+e^-) \approx (8\pi\alpha^2/3)F_\pi^2/(m_\rho\mathcal{K}^2)$]. Thus, unitarity corrections of 20–30% to Weinberg's sum rules can give rise to significant differences in the predictions for experimentally observable quantities. As the predictions for the ratios $\Gamma(\rho \to e^+e^-)/\Gamma(\rho \to \text{all})$ have the added uncertainty arising from the uncertainty in the ρ width, we cannot at present obtain reliable numerical estimates for these ratios. When the masses and widths of the meson resonances are better known, one may examine quantitatively whether inclusion of the unitarity corrections improves the agreement of Weinberg's $SU_2 \times SU_2$ sum rules with experiment.

IV. SU_3 SUM RULES

Equations (2.11) now lead to the following relations[24]:

$$(\lambda_\rho^2/m_\rho^2) D_V^\rho(m_\rho^2)/D_V^\rho(0) = (\lambda_{K^*}^2/m_{K^*}^2) D_V^{K^*}(m_{K^*}^2)/D_V^{K^*}(0)\,, \tag{4.1a}$$

$$\lambda_\rho^2 D_V^\rho(m_\rho^2) = \lambda_{K^*}^2 D_V^{K^*}(m_{K^*}^2)\,, \tag{4.1b}$$

where $D_V^\rho(0)$ and $D_V^\rho(m_\rho^2)$ are given by (3.2), and

$$D_V^{K^*}(0) \approx 1 + (1/4\pi)[-0.026 g_{K^*K\pi}^2 + 0.14 h_{K^*K^*\pi}^2 + 0.053 h_{K^*\omega K}^2 + 0.055 h_{K^*\rho K}^2 + 0.012 h_{K^*\phi K}^2]\,, \tag{4.2a}$$

$$D_V^{K^*}(m_{K^*}^2) \approx 1 + (1/4\pi)[0.014 g_{K^*K\pi}^2 + 0.24 h_{K^*K^*\pi}^2 + 0.076 h_{K^*\omega K}^2 + 0.08 h_{K^*\rho K}^2 + 0.016 h_{K^*\phi K}^2]\,. \tag{4.2b}$$

Equations (4.1) thus give sum rules relating λ_ρ, λ_{K^*}, m_ρ, m_{K^*}, and the various coupling constants in (3.2a), (3.2b), (4.2a), and (4.2b). To compare these sum rules with experiment, we first substitute available estimates of the various coupling constants.

The observed decay width $\Gamma(K^* \to K + \pi) \approx 49.6$ MeV gives $(1/4\pi) g_{K^*K\pi}^2 \approx 2.5$. Using the estimate (3.3) of $h_{\rho\omega\pi}^2$ and SU_3 relations for the VVP couplings, we estimate the remaining coupling strengths, as obtained with the three models of mixing suggested in Ref. 7. We obtain

$$\frac{1}{4\pi} h_{K^*K^*\pi}^2 \approx \frac{1}{4\pi} h_{K^*\rho K}^2 \approx \begin{pmatrix} 0.3 \\ 0.28 \\ 0.4 \end{pmatrix}, \qquad \frac{1}{4\pi} h_{K^*\omega K}^2 \approx \begin{pmatrix} 0.22 \\ 0.15 \\ 0.075 \end{pmatrix}, \quad \frac{1}{4\pi} h_{K^*\phi K}^2 \approx \begin{pmatrix} 0.27 \\ 0.2 \\ 0.24 \end{pmatrix}. \tag{4.3}$$

Here the three values correspond to the current-mixing

[22] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Fayyazuddin and Riazuddin, Phys. Rev. **147**, 1071 (1966).

[23] For example, see D. A. Geffen, Phys. Rev. Letters **19**, 770 770 (1967); S. G. Brown and G. B. West, *ibid.* **19**, 812 (1967).

[24] For the strangeness-changing vector current, we again find that the first term in $is\tilde{\Delta}_{(0)}{}^V(s)$, analogous to the first term in (2.19), is negligibly small. In writing (4.1), we have neglected the possible contribution of a κ meson, so that there is no term corresponding to the second term in (2.19). The effect of including a κ meson will be discussed below.

model, the first mass-mixing model (with $\theta_Y=\theta_N\approx 32°$), and the second mass-mixing model (with $\theta_Y=\theta_N\approx 39°$) of Kroll, Lee, and Zumino.[7] Using (3.3) and (4.3), we obtain from (4.1) the following:

$$1.1(2\lambda_\rho^2/m_\rho^2)=\alpha_1\lambda_{K^*}^2/m_{K^*}^2, \quad (4.4a)$$

$$1.3(2\lambda_\rho^2)=\beta_1\lambda_{K^*}^2. \quad (4.4b)$$

Here, (α_1,β_1) take the values (1.15,1.16), (1.15,1.15), and (1.15,1.17) for the three models of mixing, and are thus insensitive to the assumptions made about $\omega=\phi$ mixing in obtaining the VVP coupling constants. The deviation of the coefficients in the K^* terms in (4.4) from the pole-dominance value of unity is now seen to be within about 20%. Assuming $m_{K^*}\approx 893$ MeV, $m_\rho \approx 777$ MeV, the first sum rule in the form (4.4a) gives

$$\lambda_{K^*}^2/2\lambda_\rho^2\approx 1.26. \quad (4.5)$$

With $m_\rho\approx 760$ MeV, this ratio is obtained to be 1.32.

The deviation from the pole-dominance value $(\lambda_{K^*}^2/2\lambda_\rho^2\approx m_{K^*}^2/m_\rho^2)$ is then within about 10% for the first SU_3 sum rule.

On the other hand, we may combine (4.4a) and (4.4b) to eliminate $\lambda_\rho^2/\lambda_{K^*}^2$ to obtain $m_{K^*}^2/m_\rho^2\approx 1.16$ to 1.18. Pole dominance would give $m_{K^*}^2/m_\rho^2=1$. Comparing with the experimental value for this ratio, which ranges from 1.32 to 1.38, we see that although the unitarity corrections in Weinberg's second sum rule make the agreement with experiment better, they are not sufficient.

One possible explanation would be that the strangeness-changing vector current is coupled to a κ meson, and that neglecting this is not a good approximation. Equation (4.4a) should then be modified by the addition of a term F_κ^2 to the right-hand side (assuming, by analogy with the 0^- meson propagator, that the pole contribution dominates the 0^+ propagator). Equation (4.4b) and the sum rule replacing (4.4a), together with the experimental value of $m_{K^*}^2/m_\rho^2$, may now be used for obtaining an estimate for F_κ^2:

$$F_\kappa^2\approx 0.008\lambda_\rho^2. \quad (4.6)$$

This would give a value for F_κ^2 of the order of 0.25.[25]

An alternative possibility is that Weinberg's second sum rule for SU_3 itself needs to be modified. That such a modification was necessary in order to take into account symmetry breaking,[26] at least with the pole-dominance approximation, was suggested by Das, Mathur, and Okubo and by Sakurai (see Ref. 6).

A phenomenological modification of the second sum rule, suggested by Das, Mathur, and Okubo (see Ref. 6), gives

$$\int ds'[\rho_3(s')+3\rho_8(s')-4\rho_4(s')]=0. \quad (4.7)$$

To obtain the spectral functions for the ω and ϕ propagators, one must consider a two-channel problem, including the (off-diagonal) mixed (ω-ϕ) propagator. We have estimated these spectral functions in a crude model in which we have neglected the mixed ω-ϕ propagator and have evaluated the ϕ and ω propagators in a chain approximation. The coefficients in the sum rules that then follow from (4.7) show a large deviation from the pole-dominance values; however, these sum rules [together with (4.1a)] do not lead to physical solutions for $\lambda_\omega^2/\lambda_\rho^2$, $\lambda_\phi^2/\lambda_\rho^2$, etc., indicating that one must solve the complete two-channel ϕ-ω problem in order to obtain a satisfactory approximation. We hope to consider this elsewhere.

Finally, we remark that the inclusion of a scalar κ meson in the first sum rule and a modification of the second sum rule may both be necessary to take into account in an adequate manner the effects of symmetry breaking.

V. $SU_3\times SU_3$ SUM RULES

For the strangeness-changing vector and axial-vector currents, the equations analogous to (2.11) lead to the following sum rules:

$$\frac{\lambda_{K^*}^2}{m_{K^*}^2}\frac{D_V^{K^*}(m_{K^*}^2)}{D_V^{K^*}(0)}-\frac{\lambda_{K_A}^2}{m_{K_A}^2}\frac{D_A^{K_A}(m_{K_A}^2)}{D_A^{K_A}(0)}=F_K^2-F_\kappa^2 \quad (5.1)$$

if the scalar meson κ is included in the sum rules. In (5.1), $D_V^{K^*}(0)$ and $D_V^{K^*}(m_{K^*}^2)$ are given by (4.2), and

$$D_A^{K_A}(0)\approx 1+(1/4\pi)(-0.027g_{K_A\pi\kappa}^2+\bar{\alpha}g_{K_A\sigma K}^2 +0.12h_{K_AK_A\pi}^2+0.11h_{K_AA_1K}^2); \quad (5.2a)$$

$$D_A^{K_A}(m_{K_A}^2)\approx 1+(1/4\pi)(0.033g_{K_A\pi\kappa}^2+\bar{\beta}g_{K_A\sigma K}^2 +0.3h_{K_AK_A\pi}^2+0.25h_{K_AA_1K}^2). \quad (5.2b)$$

Here κ denotes a possible $I=\frac{1}{2}$, $S=1$ scalar meson, the mass of which has been taken to be about 725 MeV, and $(\bar{\alpha},\bar{\beta})$ take the values (0,0), (0.043, -0.01), and (-0.03, 0.05) for the three assumptions about the σ meson (see Sec. III).

When estimates become available for the coupling constants involved in these sum rules, we may again examine how the inclusion of unitarity corrections affects the results obtained with pole dominance.

VI. CONCLUSIONS

In this paper we have examined the nature of unitarity corrections to the sum rules obtained from an asymptotic symmetry (in particular, an asymptotic chiral symmetry) for two-point functions and the pole-

[25] For comparison, we may note the estimate $F_\kappa\approx 0.73F_\pi$ of Glashow, Schnitzer, and Weinberg (see Ref. 2), using $SU_3\times SU_3$ sum rules; this gives a value of F_κ^2 of the order of 0.5. If independent estimates of F_κ indicate that its value is quite different from (4.6), this would suggest that the inclusion of the κ and a modification of the second sum rule are both necessary.

[26] We remark here that the inclusion of a scalar meson κ coupled to the divergence of the strangeness-changing vector current is itself one way of incorporating a part of SU_3-symmetry breaking.

dominance approximation. We have here evaluated a particular class of corrections, namely, that given by assuming the field-current identities (2.2) and evaluating the 1^-, 1^+, and 0^- meson propagators in a chain approximation, assuming that the vertex functions involved are damped rapidly at large p^2. The main uncertainty in our results arises from the uncertainty in the nature of the form factors assumed at the vertices and that in the input values of the masses and couplings of the meson resonances. However, we believe that our results indicate correctly the qualitative nature of the corrections considered here.

Our main results are the following. Weinberg's sum rules (and their analogs for SU_3 and $SU_3 \times SU_3$) when evaluated in our model give sum rules relating the various coupling constants and masses. These are the basic relations following from our model. To examine the validity of these sum rules, we have rewritten them as relations involving λ_ρ^2, m_ρ^2, $\lambda_{A_1}^2$, and $m_{A_1}^2$ for the $SU_2 \times SU_2$ sum rules, and similarly for the other sum rules, using available estimates for the coupling constants. (As these coupling constants enter only in the correction terms to the pole-dominance results, the uncertainty in the estimates of these couplings causes a comparatively small uncertainty in the results.) The deviations of the coefficients in these relations from their pole-dominance values are found to be relatively small—within 20 or 30%—but not negligible. When these equations are solved and used for predicting experimental decay rates or mass ratios, it is found that the deviations from the pole-dominance predictions can be significant.

On the whole, the corrections to pole dominance are found to be larger for the second sum rule than for the first. For the $SU_2 \times SU_2$ sum rules, the results are fairly sensitive to the assumptions made about the scalar meson retained in the intermediate states. For the SU_3 sum rules, using both the sum rules (without modification) gives a value for $m_{K^*}^2/m_\rho^2$ that agrees better with the experimental value than the pole-dominance prediction; however, the agreement is not sufficiently improved by the inclusion of the unitarity corrections. This suggests that one should modify one or both of the SU_3 sum rules. If one assumes that including the κ meson in the first SU_3 sum rule is adequate, then one obtains an estimate of F_κ.

To find the effect of the unitarity corrections on the second sum rule as modified to take into account symmetry breaking, it is necessary to obtain the ϕ and ω propagators in a two-channel model in which mixing effects are treated in some detail; we hope to discuss this elsewhere.

The detailed sum rules obtained from our model, relating the coupling constants and masses, may be tested when reliable estimates of the various coupling constants become available.

Finally, we stress that we have here evaluated a particular class of corrections to pole dominance, namely, those which arise from corrections to the propagators of the 1^-, 1^+, and 0^- mesons, when these are evaluated in a chain approximation. The chain diagrams by themselves are not expected to be the ones that dominate the propagators at large p^2, and other corrections may be important. We have in our model assumed that the inclusion of vertex corrections will damp the propagators strongly at large p^2, so that corrections that may otherwise be important at high energies are damped out. The question of the nature of these vertex corrections at large p^2 and of other possible corrections to the propagator is being investigated.

We have here examined in some detail unitarity corrections to the sum rules obtained with two-point functions. A related question is the incorporation of unitarity corrections to the generalized pole approximation or tree approximation for n-point functions that has been found useful in connection with work using chiral Lagrangians. Such questions are being studied, and we hope to discuss them elsewhere.

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