Inelastic Electron Scattering in the Symmetric Quark Model. II. Coulomb Potential*

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The nonrelativistic quark model with a $1/r$ binding potential is used to compute form factors for the process $e+p\to e+N^*$, where N^* is one of ten nucleon resonances. The elastic and $N^*(1236)$ predictions agree with experiment, but the predictions for higher excited states are too small. We discuss which features of the calculation should hold true in any quark model and which features depend on the binding potential used.

1. INTRODUCTION

THE nonrelativistic quark model[1] can be used to study the inelastic process[2,3]

$$e+p\to e+N^*, \quad (1)$$

where e is an electron, p is a proton, and N^* denotes a nucleon resonance. In this paper we investigate the process (1) using the symmetric quark model with a $1/r$ binding potential. The techniques used are basically the same as in Ref. 3; the difference lies in the type of binding potential assumed.

In the symmetric quark model the nucleon is pictured as a bound state of three noninteracting quarks in a potential well; a nucleon resonance is viewed as an excited state of the three-quark system. In this paper the potential well is chosen to give a good fit to the experimental elastic form factor data; the inelastic form factors and differential cross section associated with Eq. (1) are then computed.

Several features of the form factors are observed which do not depend on the particular potential chosen, and which would hold true in any quark-model calculation of this type. In particular, we note the presence of selection rules, threshold behavior, and several proportional form factors. The actual shape and magnitude of the resonance form factors, however, are found to depend strongly on the particular potential well chosen. A $1/r$ potential gives a much less rapid falloff in the form factors than does a harmonic-oscillator well, but the $1/r$ potential yields predictions that are much too small for most of the resonances, when compared with experiment.

The plan of this paper is as follows: In Sec. 2 the model is reviewed, and the $1/r$ potential is motivated. Section 3 reviews the formalism. In Sec. 4 the results are presented and discussed with respect to general and model-dependent features, and a comparison with experiment is made.

2. MODEL

In our model the nucleon is pictured as a bound state of three noninteracting point quarks in a potential well; nucleon resonances are viewed as excited states of the three-quark system. The calculation is done in a completely nonrelativistic framework. Total wave functions are required to be completely symmetric under exchange of any two quarks. Spin and isospin (as well as spatial dependence) are taken into account, and the proton and nucleon resonances are assigned[4] to representations of $SU(6)$ (see Table I [3]). With this picture of the nucleon and N^* states, we investigate the inelastic process

$$e+p\to e+N^*.$$

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[1] G. Zweig, CERN Report Nos. Th.401, 1964, and Th.412, 1964 (unpublished); M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Morpurgo, Physics **2**, 95 (1965); Y. Nambu, in *Proceedings of the Second Coral Gables Conference on Symmetry Principles at High Energy*, edited by B. Kursonoglu *et al.* (W. H. Freeman and Co., San Francisco, 1965), pp. 274–283.

[2] K. Fujimura, T. Kobayashi, T. Kobayashi, and M. Namiki, Progr. Theoret. Phys. (Kyoto) **38**, 210 (1967).

[3] N. S. Thornber, Phys. Rev. **169**, 1096 (1968).

[4] R. H. Dalitz, in *Proceedings of the Oxford International Conference on Elementary Particles, 1965* (Rutherford High-Energy Laboratory, Berkshire, England, 1966), p. 157; O. W. Greenberg, University of Maryland Report, 1967 (unpublished); O. W. Greenberg and M. Resnikoff, Phys. Rev. **163**, 1844 (1967); R. G. Moorhouse, Phys. Rev. Letters **16**, 772 (1966); the 1470 assignments were made by the author.

Other assignments for the 1470 within the framework of the nonrelativistic quark model are also possible: (1) $L=0^+$, $N=2$, **56**; (2) $L=0^+$, $N=2$, **70**; (3) $L=1^+$, $N=2$, **20** (where N is the principal quantum number). The first two assignments give nonvanishing transition matrix elements, but the third assignment (two-particle excitation) predicts zero form factors. Thus experimental observation of electroproduction (off of protons) of the $N^*(1470)$ would rule out case (3) as well as the assignments of the author. Case (3) (a two-particle excitation state) is probably to be preferred over the aurhor's assignment (one-particle excitation), since it requires the existence of far fewer nucleon states. The predictions for electroproduction of the $N^*(1470)$, however, are the same for these two assignments. Of all the assignments, the first ($L=0^+$, $N=2$, **56**) is the most economical, as emphasized by Dalitz, in Proceedings of the Topical Conference on πN Scattering, Irvine, California, 1967 (unpublished).

It is also possible that the $N^*(1470)$ should be assigned to an excited state in the three-triplet model (in this model there are three triplets of quarks instead of only one triplet as in the ordinary quark model). In such a case the $N^*(1470)$ could belong to a **10*** representation of $SU(3)$, for example. (Such a representation is not possible in the ordinary quark model.) The predictions for the form factors would depend in general on the particular assignments chosen. If $L=2$, then one would again predict zero matrix elements for the Coulomb and convection current operators (since S must equal $\frac{3}{2}$), whereas if $L=0$, one would *a priori* not expect these matrix elements to vanish.

TABLE I. Quantum numbers of states used in symmetric quark model. (L is the total quark orbital angular momentum, S is the total quark spin, and I is the total isospin.)

State (MeV)	J^P	L	S	I	$SU(6)$ representation
940	$\frac{1}{2}^+$	0	$\frac{1}{2}$	$\frac{1}{2}$	56
1470	$\frac{1}{2}^+$	2	$\frac{3}{2}$	$\frac{1}{2}$	70
1525	$\frac{3}{2}^-$	1	$\frac{1}{2}$	$\frac{1}{2}$	70
1570	$\frac{1}{2}^-$	1	$\frac{1}{2}$	$\frac{1}{2}$	70
1670	$\frac{5}{2}^-$	1	$\frac{3}{2}$	$\frac{1}{2}$	70
1688	$\frac{5}{2}^+$	2	$\frac{1}{2}$	$\frac{1}{2}$	56
1700	$\frac{1}{2}^-$	1	$\frac{3}{2}$	$\frac{1}{2}$	70
2190	$\frac{7}{2}^-$	3	$\frac{1}{2}$	$\frac{1}{2}$	70
1236	$\frac{3}{2}^+$	0	$\frac{3}{2}$	$\frac{3}{2}$	56
1670	$\frac{1}{2}^-$	1	$\frac{1}{2}$	$\frac{3}{2}$	70
1920	$\frac{7}{2}^+$	2	$\frac{3}{2}$	$\frac{3}{2}$	56

TABLE III. Form factors in the symmetric quark model: $2\pi|\langle J_f\|\hat{T}_J{}^{\text{mag}}\|J_i\rangle|^2$ when $V\propto 1/r$. ($|f_+|^2+|f_-|^2=2\pi|\langle J_f\|\hat{T}_J{}^{\text{mag}}\|J_i\rangle|^2+2\pi|\langle J_f\|\hat{T}_{J'}{}^{\text{el}}\|J_i\rangle|^2$.) The b's are defined by $b_l=[(l+2)/(2l+2)]b_0$, $b_0{}^2=0.71$ (BeV/c)^2.

State	I	$2\pi\|\langle J_f\|\hat{T}_J{}^{\text{mag}}\|J_i\rangle\|^2$
940	$\frac{1}{2}$	$(2\mu_p{}^2b_0{}^2)(q^*/b_0)^2(1+q^{**2}/b_0{}^2)^{-4}$
1470	$\frac{1}{2}$	0
1525	$\frac{1}{2}$	$(2\mu_p{}^2b_0{}^2)(5)(4/9)^4(q^*/b_1)^4(1+q^{*2}/b_1{}^2)^{-6}$
1570	$\frac{1}{2}$	0
1670	$\frac{1}{2}$	0
1688	$\frac{1}{2}$	$(2\mu_p{}^2b_0{}^2)(7/240)(q^*/b_2)^6(1+q^{*2}/b_2{}^2)^{-8}$
1700	$\frac{1}{2}$	0
2190	$\frac{1}{2}$	$(2\mu_p{}^2b_0{}^2)(1/56)(4/5)^{10}(q^*/b_3)^8(1+q^{*2}/b_3{}^2)^{-10}$
1236	$\frac{3}{2}$	$(2\mu_p{}^2b_0{}^2)(8/9)(q^*/b_0)^2(1+q^{*2}/b_0{}^2)^{-4}$
1670	$\frac{3}{2}$	0
1920	$\frac{3}{2}$	$(2\mu_p{}^2b_0{}^2)(2/315)(q^*/b_2)^6(1+q^{*2}/b_2{}^2)^{-8}$

This reaction is assumed to proceed via one-photon exchange, and we let the exchanged photon interact separately with each quark. (We thus investigate only the one-particle excitation piece of the N^* wave functions). Just as in elastic scattering $(e+p\rightarrow e+p)$, there are two form factors to be determined; in inelasic scattering (when only the final electron is detected) the differential cross section contains two unknown functions of momentum transfer[5]:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}}=\frac{\alpha^2\cos^2\frac{1}{2}\theta}{4\epsilon^2\sin^4\frac{1}{2}\theta\,[1+(2\epsilon/m)\sin^2\frac{1}{2}\theta]}\left[\frac{q^4}{q^{*4}}|f_c|^2+\left(\frac{q^2}{2q^{*2}}+\frac{M^2}{m^2}\tan^2\tfrac{1}{2}\theta\right)(|f_+|^2+|f_-|^2)\right].\quad(2)$$

Here θ is the electron scattering angle, ϵ is the incident electron energy, m and M are the nucleon and N^* masses, q^2 is the invariant four-momentum transfer, and q^{*2} is the square of the three-momentum transfer in the N^* rest frame. The functions $|f_c|^2$ and $(|f_+|^2+|f_-|^2)$ are the form factors that we wish to determine.

The theoretical predictions for the form factors naturally depend on the model chosen. For a symmetric quark model, one must first decide which potential well

TABLE II. Form factors $|f_c|^2$ in the symmetric quark model; $V\propto 1/r$. The b's are defined by $b_l=[(l+2)/(2l+2)]b_0$, $b_0{}^2=0.71$ (BeV/c)^2.

State (MeV)	I	$\|f_c\|^2$
940	$\frac{1}{2}$	$(1+q^{*2}/b_0{}^2)^{-4}$
1470	$\frac{1}{2}$	0
1525	$\frac{1}{2}$	$16(4/9)^5(q^*/b_1)^2(1+q^{*2}/b_1{}^2)^{-6}$
1570	$\frac{1}{2}$	$8(4/9)^5(q^*/b_1)^2(1+q^{*2}/b_1{}^2)^{-6}$
1670	$\frac{1}{2}$	0
1688	$\frac{1}{2}$	$(27/640)(q^*/b_2)^4(1+q^{*2}/b_2{}^2)^{-8}$
1700	$\frac{1}{2}$	0
2190	$\frac{1}{2}$	$(1/21)(4/5)^{13}(q^*/b_3)^6(1+q^{*2}/b_3{}^2)^{-10}$
1236	$\frac{3}{2}$	0
1670	$\frac{3}{2}$	$8(4/9)^5(q^*/b_1)^2(1+q^{*2}/b_1{}^2)^{-6}$
1920	$\frac{3}{2}$	0

TABLE IV. Form factors in the symmetric quark model: $2\pi|\langle J_f\|\hat{T}_{J'}{}^{\text{el}}\|J_i\rangle|^2$ when $V\propto 1/r$. ($|f_+|^2+|f_-|^2=2\pi|\langle J_f\|\hat{T}_J{}^{\text{mag}}\|J_i\rangle|^2+2\pi|\langle J_f\|\hat{T}_{J'}{}^{\text{el}}\|J_i\rangle|^2$.) The b's are defined by $b_l=[(l+2)/(2l+2)]b_0$, $b_0{}^2=0.71$ (BeV/c)^2.

State (MeV)	I	$2\pi\|\langle J_f\|T_{J'}{}^{\text{el}}\|J_i\rangle\|^2$
940	$\frac{1}{2}$	0
1470	$\frac{1}{2}$	0
1525	$\frac{1}{2}$	$\left(\frac{b_0{}^2}{2M_q{}^2}\right)\left(\frac{4}{9}\right)^5\left(1-\frac{3g_qq^{*2}/b_1{}^2}{2(1+q^{*2}/b_1{}^2)}\right)^2\left(1+\frac{q^{*2}}{b_1{}^2}\right)^{-4}$
1570	$\frac{1}{2}$	$\left(\frac{b_0{}^2}{2M_q{}^2}\right)\left(\frac{2}{9}\right)\left(\frac{4}{9}\right)^4\left(1+\frac{3g_qq^{*2}/b_1{}^2}{1+q^{*2}/b_1{}^2}\right)^2\left(1+\frac{q^{*2}}{b_1{}^2}\right)^{-4}$
1670	$\frac{1}{2}$	0
1688	$\frac{1}{2}$	$\left(\frac{b_0{}^2}{2M_q{}^2}\right)\left(\frac{9}{2560}\right)\left(\frac{q^*}{b_2}\right)^2\left(1-\frac{4g_qq^{*2}/b_2{}^2}{3(1+q^{*2}/b_2{}^2)}\right)^2\times\left(1+\frac{q^{*2}}{b_2{}^2}\right)^{-6}$
1700	$\frac{1}{2}$	0
2190	$\frac{1}{2}$	$\left(\frac{b_0{}^2}{2M_q{}^2}\right)\left(\frac{4}{875}\right)\left(\frac{16}{25}\right)^5\left(\frac{q^*}{b_3}\right)^4\left(1-\frac{5g_qq^{*2}/b_3{}^2}{4(1+q^{*2}/b_3{}^2)}\right)^2\times\left(1+\frac{q^{*2}}{b_3{}^2}\right)^{-8}$
1236	$\frac{3}{2}$	0
1670	$\frac{3}{2}$	$\left(\frac{b_0{}^2}{2M_q{}^2}\right)\left(\frac{2}{9}\right)\left(\frac{4}{9}\right)^4\left(1-\frac{g_qq^{*2}/b_1{}^2}{1+q^{*2}/b_1{}^2}\right)^2\left(1+\frac{q^{*2}}{b_1{}^2}\right)^{-4}$
1920	$\frac{3}{2}$	0

[5] J. D. Bjorken and J. D. Walecka, Ann. Phys. (N.Y.) 38, 35 (1966).

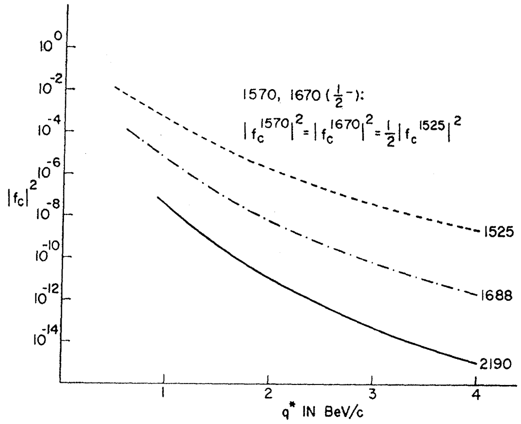

FIG. 1. $|f_c|^2$ using a $1/r$ potential; $\hat{p}$ does not include $1/M_q^2$ corrections.

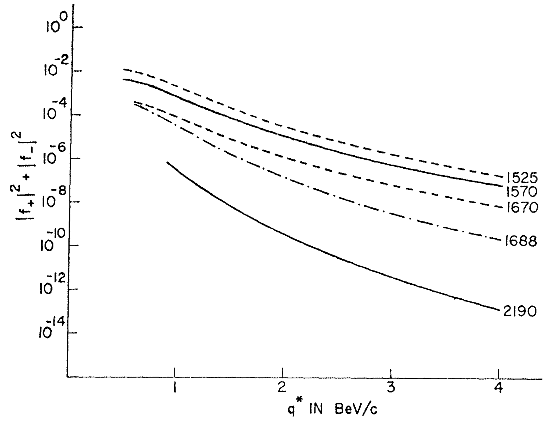

FIG. 3. $(|f_+|^2+|f_-|^2)$ using a $1/r$ potential; $M_q=\infty$. The 1236 and 1920 have the same $(|f_+|^2+|f_-|^2)$ as in Fig. 2.

to use for the quarks. If the quarks are placed in a harmonic-oscillator well,[2,3] then one naturally obtains form factors proportional to $e^{-b^2q^{*2}/8}$, since the Fourier transform of the Gaussian harmonic-oscillator wave functions yields just another Gaussian. This type of form factor, however, falls off too rapidly with increasing momentum transfer, and a less rapid falloff would be desirable.

Another model which suggests itself is that of three quarks inside a rigid-walled spherical box,[6] subject to the following conditions: (a) The c.m. of the quarks is located at the center of the sphere, (b) each quark remains inside of the sphere, and (c) otherwise the quarks are completely free. Conditions (a) and (c) are easy to satisfy, but condition (b) becomes very involved algebraically.

One is next led to ask the question[7] of which input potential in the original model will yield a theoretical dipole fit to the elastic proton form factors (i.e., which potential yields agreement with the elastic data).

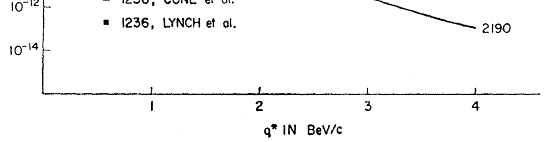

FIG. 2. $(|f_+|^2+|f_-|^2)$ using a $1/r$ potential; $M_q\cong\frac{1}{3}m_{\text{proton}}$.

[6] L. I. Schiff, Phys. Rev. **160**, 1257 (1967).

[7] The author is indebted to Dr. Y. S. Tsai and Professor L. I. Schiff for asking this question.

Since the Fourier transform of e^{-br} is proportional to

$$(1+q^2/b^2)^{-2},$$

we shall take a $1/r$ potential (this has an exponential ground-state wave function e^{-br}). The mass spectrum predicted by a $1/r$ potential does not agree with experiment (no n^{-2} behavior is observed), but since the elastic form factors for such a potential are correctly predicted, it seems interesting to study the behavior of the inelastic form factors with this potential. With a Coulomb potential, the shape of the form factors should now be greatly improved over that of the harmonic-oscillator case.

There are three parameters in a quark-model calculation of this kind: the damping parameter b in the wave function (e^{-br} here; $e^{-b'^2r^2}$ for a harmonic oscillator), the quark magnetic moment (g factor divided by quark mass), and the quark mass. The first two parameters will be determined (as in Ref. 3) by a fit to the low-q^2 behavior of the elastic form factors, and the third parameter, M_q, will first be set equal to roughly $\frac{1}{3}m_p$, and then to ∞.

3. FORMALISM

The formalism used has been described previously,[3,8] and only a summary need be given here. With the differential cross section for $e+p\to e+N^*$ given by Eq. (2), we calculate $|f_c|^2$ and $|f_+|^2+|f_-|^2$ according to[8]

$$|f_c|^2=2\pi\sum_{J=0}^{\infty}|\langle J_f\|\hat{M}_J{}^{\text{Coul}}(q^*)\|\tfrac{1}{2}+\rangle|^2$$

and (3)

$$|f_+|^2+|f_-|^2=2\pi\sum_{J=1}^{\infty}[|\langle J_f\|\hat{T}_J{}^{\text{el}}(q^*)\|\tfrac{1}{2}+\rangle|^2+|\langle J_f\|\hat{T}_J{}^{\text{mag}}(q^*)\|\tfrac{1}{2}+\rangle|^2],$$

[8] T. deForest and J. D. Walecka, Advan. Phys. **15**, 1 (1966); J. D. Walecka, in *International School of Physics "Enrico Fermi," Italian Physical Society, Course* 38, edited by T. E. O. Ericson, (Academic Press Inc., New York, 1967), p. 17.

where

$$\hat{M}_{JM}{}^{\text{Coul}}(q^*)=\int d^3x\, j_J(q^*x)Y_{JM}(\Omega_x)\hat{\rho}(\mathbf{x}),$$

$$\hat{T}_{JM}{}^{\text{el}}(q^*)=\frac{1}{q^*}\int d^3x\,[\nabla\times j_J(q^*x)\mathbf{Y}_{JJ1}{}^M(\Omega_x)]\cdot\hat{\mathbf{J}}(\mathbf{x}),\tag{4}$$

$$\hat{T}_{JM}{}^{\text{mag}}(q^*)=\int d^3x\, j_J(q^*x)\mathbf{Y}_{JJ1}{}^M(\Omega_x)\cdot\hat{\mathbf{J}}(\mathbf{x}).$$

Here $\mathbf{Y}_{JJ1}{}^M(\Omega_x)$ is a vector spherical harmonic, and $\hat{\rho}$ and $\hat{\mathbf{J}}=\mathbf{j}(x)+\nabla\times\boldsymbol{\mu}(\mathbf{x})$ are the nonrelativistic charge and current densities, respectively ($\mathbf{j}$ is the convection current, and $\boldsymbol{\mu}$ is the magnetic moment operator). $\hat{\rho}$ and $\hat{\mathbf{J}}$ obey current conservation.

We let the quarks obey parastatistics (as mentioned by Dalitz in Ref. 4). Using the quantum-number assignments shown in Table I, totally symmetric p and N^* wave functions are constructed (the method is just that of Ref. 3). If Fermi statistics were assumed, and the $SU(6)$ assignments retained, the spatial wave functions would in general be more complicated. No attempt is made at separating out c.m. motion. The wave functions are then inserted into (3), and the form factors are computed. [For a harmonic-oscillator potential the effect of separating out the c.m. motion would be generally to decrease the over-all normalization of the inelastic form factors (since the amplitude of the single-particle excitation mode is decreased) and to increase the value of the oscillator parameter b^2. The first of these two effects would also be expected for a $1/r$ potential, but precise statements are difficult to make because of the algebraic complexity that arises.]

4. RESULTS AND DISCUSSION

Using the above model, we have calculated $|f_c|^2$ and $(|f_+|^2+|f_-|^2)$ for the process

$$e+p\rightarrow e+N^*,$$

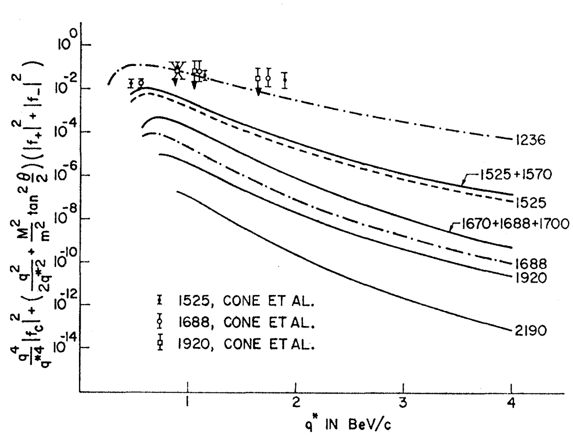

FIG. 4. Transition probabilities using a $1/r$ potential; $\theta_{\text{lab}}=31°$, $M_q=\frac{1}{3}m_{\text{proton}}$. $\hat{\rho}$ does not include any $1/M_q{}^2$ corrections.

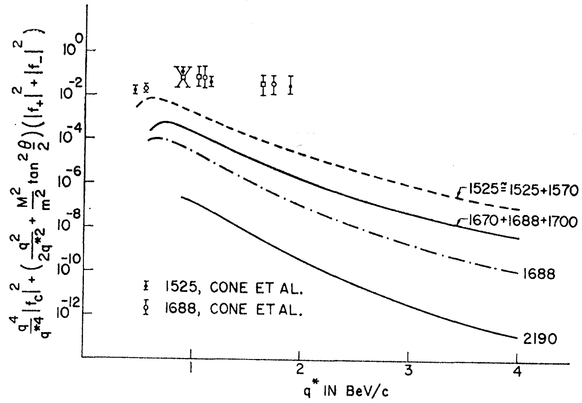

FIG. 5. Same as Fig. 4, but with $M_q=\infty$. The 1236 and 1920 states have the same transition probabilities as in Fig. 4.

where N^* is one of the 10 resonances listed in Table I. The results are listed in Tables II–VI and are plotted in Figs. 1–6. These results are expected to be less accurate for large q^*, but for the sake of easy visualization we have plotted them up to $q^*=4$ BeV/c.

There are several features of our results which are independent of the particular potential well chosen, and which would hold true in any quark-model calculation of this type. One such feature is the presence of selection rules which prohibit certain transitions (some of these rules have already been pointed out by Moorhouse[4]). An example of such a rule is the vanishing of the Coulomb form factor f_c for the 1236 resonance. The Coulomb form factor arises from charge excitation only and does not involve any quark-spin operators; thus any resonance with quark spin unequal to the proton quark spin (such as the 1236 state) will have a vanishing Coulomb form factor. In a similar fashion one can look at the other vanishing form factors (Tables II–IV); by means of arguments involving only parity, spin, and isospin (independently of the form of the radial wave function) one can deduce the zero results.[3] The conclusion is that given the quantum-number assignments of Table I, all of the zero form factors of Tables II–IV follow, independently of the particular potential assumed.

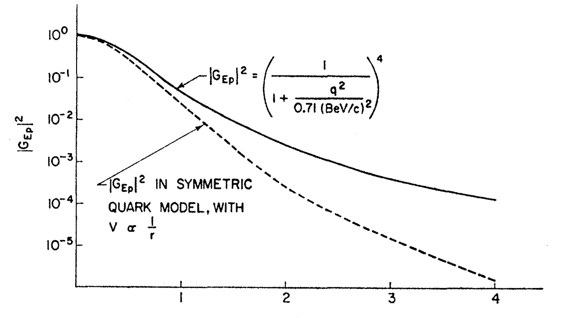

FIG. 6. $|f_c|^2/(1+q^2/4m^2)=|G_{Ep}{}^{\text{theory}}|^2$ for elastic scattering in the symmetric quark model, using a $1/r$ potential. Also plotted is the dipole expression for $|G_{Ep}|^2$.

TABLE V. Differential cross sections in inelastic electron scattering ($d\sigma/d\Omega$ is measured in units of cm^2/sr; four-momenta are in BeV/c; $V \propto 1/r$).

State (MeV)	ϵ	q^2	q^*	$d\sigma/d\Omega$[a]	$d\sigma/d\Omega$[b]	$d\sigma/d\Omega$[c]	$d\sigma/d\Omega$[d]	$d\sigma/d\Omega$[e]
1236	2.358	0.99	1.01	0.71×10^{-32}		0.71×10^{-32}		0.83×10^{-32}
	2.988	1.55	1.30	0.18×10^{-32}		0.18×10^{-32}		0.25×10^{-32}
	4.874	3.62	2.25	0.004×10^{-32}		0.004×10^{-32}		0.012×10^{-32}
1525		0	0.475	0.34×10^{-32}	0.78×10^{-32}	0.36×10^{-32}	0.48×10^{-32}	2×10^{-32}
	2.358	0.79	0.91	0.028×10^{-32}	0.050×10^{-32}	0.032×10^{-32}	0.044×10^{-32}	0.88×10^{-32}
	2.988	1.30	1.14	0.006×10^{-32}	0.010×10^{-32}	0.0064×10^{-32}	0.0087×10^{-32}	0.29×10^{-32}
	4.874	3.29	1.91	0.60×10^{-36}	1.03×10^{-36}	0.70×10^{-36}	0.94×10^{-36}	0.0099×10^{-32}
1688		0	0.58	0.0049×10^{-32}	0.0145×10^{-32}	0.0057×10^{-32}	0.013×10^{-32}	1.5×10^{-32}
	2.358	0.65	0.90	4.8×10^{-36}	0.0038×10^{-32}	5.6×10^{-36}	0.0046×10^{-32}	0.80×10^{-32}
	2.988	1.14	1.10	0.88×10^{-36}	0.0008×10^{-32}	1.0×10^{-36}	0.0010×10^{-32}	0.26×10^{-32}
	4.874	3.05	1.78	0.61×10^{-38}	5.5×10^{-38}	0.73×10^{-38}	1.9×10^{-36}	0.011×10^{-32}
1920	2.358	0.43	0.90	7.4×10^{-36}		7.4×10^{-36}		$\leq 0.08 \times 10^{-32}$
	2.988	0.87	1.06	0.19×10^{-36}		0.19×10^{-36}		$\leq 0.03 \times 10^{-32}$
	4.874	2.70	1.64	0.003×10^{-36}		0.003×10^{-36}		$\leq 0.016 \times 10^{-32}$

[a] Theory; $M_q = \frac{1}{3}m_p$; 1570, 1670, and 1700 states are omitted.
[b] Theory; $M_q = \frac{1}{3}m_p$; 1525+1570 and 1670+1688+1700 contributions.
[c] Theory; $M_q = \infty$; 1570, 1670, and 1700 states are omitted.
[d] Theory; $M_q = \infty$; 1525+1570 and 1670+1688+1700 contributions.
[e] Experimental values (Ref. 9).

A second general feature is the q^* dependence for small q^*. This threshold behavior has been derived on general grounds by Bjorken and Walecka[5] (it is true in *any* model):

Normal-parity transitions ($\frac{1}{2}^+ \to \frac{3}{2}^-, \frac{5}{2}^+, \cdots$)

$$f_c \sim (q^*)^{J-\frac{1}{2}}; \quad f_\pm \sim (q^*)^{J-\frac{3}{2}}.$$

Abnormal-parity transitions ($\frac{1}{2}^+ \to \frac{1}{2}^-, \frac{3}{2}^+, \cdots$)

$$f_c \sim (q^*)^{J+\frac{1}{2}}; \quad f_\pm \sim (q^*)^{J-\frac{1}{2}}.$$

A third feature to note is that whenever two states have the same orbital angular-momentum value L, then the Coulomb or magnetic multipole matrix elements of the two resonances (if nonvanishing) are proportional to each other. [For example, the $N^*(1236)$ and the proton both have the same $L(L=0)$; hence the magnetic multipole matrix element for these two states is the same up to a constant factor.] This general rule arises because the q^{*2} dependence of the form factors comes from the **r** dependence of the wave function; thus resonances with the same **r** dependence will have form factors that are proportional. The rule does not depend on the potential well chosen for the quarks.

TABLE VI. Photoproduction amplitudes; $1/r$ potential.

State (MeV)	I	Ratio	Predictions of symmetric quark model ($V \propto 1/r$) $M_q = \frac{1}{3}M_p$	$M_q = \infty$	Walker (Ref. 11)
1525	$\frac{1}{2}$	M_2^-/E_2^-	3.2	-2.7	0.53 ± 0.2
1570	$\frac{1}{2}$	M_1^-/E_1^-	0	0	
1670	$\frac{1}{2}$	E_2^+/M_2^+	0/0	0/0	-0.5 ± 0.5
1688	$\frac{1}{2}$	M_3^-/E_3^-	5.9	-2.6	0.5 ± 0.3
1700	$\frac{1}{2}$	M_1^-/E_1^-	0/0	0/0	
2190	$\frac{1}{2}$	M_4^-/E_4^-	25.2	-1.9	
1236	$\frac{3}{2}$	E_1^+/M_1^+	0	0	-0.04 ± 0.08
1670	$\frac{3}{2}$	M_1^-/E_1^-	0	0	
1920	$\frac{3}{2}$	E_3^+/M_3^+	0	0	
Ratios of $\|f_+\|^2 + \|f_-\|^2$					
$\frac{1}{2}^-(1570)/\frac{3}{2}^-(1525)$			1.1	4.3	0.15 ± 0.2
$\frac{5}{2}^-(1670)/\frac{5}{2}^+(1688)$			0	0	0.24 ± 0.3

The actual variable appearing in the form factors in this type of calculation is not unique. One could, for example, choose q^2 instead of q^{*2}; the two have the same limit for small momentum transfers in elastic scattering. q^2 is the invariant four-momentum transfer, and q^{*2} is the three-momentum transfer as seen in the N^* rest frame[5]:

$$q^{*2} = q^2 + (1/4M^2)(q^2 - M^2 + m^2)^2.$$

The choice of q^{*2} seems perhaps the most natural to us; q^2 would give better agreement with the elastic data (Fig. 6) for a $1/r$ potential, but on the other hand, it would give vanishing form factors at $q^2 = 0$ for resonances such as the $N^*(1688)$, in contradiction with experiment.[9]

Having thus explored the model-independent features of our quark-model calculation, we now turn to the model-dependent aspects. For a harmonic-oscillator potential well, one obtains[3] form factors proportional to $e^{-q^{*2}b^2/3}$. This type of behavior, which is to be expected for the Gaussian harmonic-oscillator wave functions, shows too rapid a falloff with increasing momentum transfer. The predictions for this potential tend to agree with the data for small q^2, but are too small for $q^2 > 1$ $(\text{BeV}/c)^2$.[3] On the other hand, a much less rapid falloff is obtained in the present case, using a $1/r$ potential well. Instead of an exponential decrease, one simply obtains a power falloff in q^{*2} for large q^{*2}. Thus

[9] A. A. Cone, K. W. Chen, J. R. Dunning, Jr., G. Hartwig, Norman F. Ramsey, J. K. Walker, and Richard Wilson, Phys. Rev. **156**, 1490 (1967); **163**, 1854(E) (1967).

the shape of the form factors, when using a $1/r$ potential, is much improved over the harmonic-oscillator case.

The absolute normalization in the $1/r$ case, however, is much too small for most of the resonances. (See Table V and Figs. 1–5.) (The only resonance having form factors that agree with experiment[9,10] is the 1236 resonance.) The small normalization factors come about, roughly speaking, because of the energy-level dependence of the exponential e^{-br} in a Coulomb potential: The constant b is inversely porportional to n, where n is the label of the energy level of the excited quark. Thus for large n (high-lying resonances) the exponential does not damp as strongly, and to normalize the wave function one must divide by a larger number. This effect was not present in the harmonic-oscillator case, and agreement for small q^2 was obtained. Thus we conclude that the magnitude of the form factors, as well as their shape, depends on the potential chosen.

[10] H. L. Lynch, J. V. Allaby, and D. M. Ritson, Phys. Rev. **164**, 1635 (1967).

We also note that whereas for a harmonic-oscillator well the form factors are all proportional to the elastic form factors,[3] this is no longer true for a Coulomb potential. Finally, we present in Table VI the quark-model predictions for various photoproduction amplitudes.[11] The predicted magnitudes are in general too large, but the signs (when $M_q = \frac{1}{3} m_{\text{proton}}$) agree with experiment. The agreement is better (when $M_q = \frac{1}{3} m_{\text{proton}}$) for a $1/r$ potential than for the harmonic-oscillator well.

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[11] P. L. Pritchett and J. D. Walecka, Phys. Rev. **168**, 1638 (1968).

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Theory of Currents, σ Model, and the Spherical Top in the Internal Space*

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A Lagrangian field theory is constructed which gives a canonical realization of the recently proposed theory of currents. It is very similar to Gell-Mann and Levy's σ model, but with some crucial differences. It is the second-quantized theory of the spherical top in the internal space, thus implying some connection the strong-coupling theory.

1. INTRODUCTION

RECENTLY a simple nontrivial model field theory in which only currents appear as the coordinates was proposed.[1] The vector and axial-vector currents were taken to satisfy the algebra of fields implied by the massive Yang-Mills theory.[2] Then the energy-momentum tensor was given in terms of these currents:

$$\theta_{\mu\nu} = (1/2C)[\{V_\mu{}^i V_\nu{}^i + V_\nu{}^i V_\mu{}^i - g_{\mu\nu}(V_\rho{}^i V_\rho{}^i)\} + (V \to A)]. \quad (1)$$

This form of $\theta_{\mu\nu}$ determines the theory completely and it was shown that the theory does not contain any internal inconsistencies. In this theory we do not have canonical variables explicitly. The reason for this was studied by Bardakci, Frishman, and Halpern.[3] It turned out that this theory is a peculiar limit of the Yang-Mills theory. Nevertheless, we might still be able to find some canonical realization of the theory.

We indeed found a Lagrangian field theory which is equivalent to the original theory of currents, at least when the internal symmetry is SU_2 or $SU_2 \times SU_2$. A very important feature of this Lagrangian theory is that, although we have canonical variables in it, we cannot attach particles directly to them because of their transformation property in the internal space. Actually, the theory is quite similar to the "σ model" of Gell-Mann and Lévy[4] except for the difference in the isospin rotation. Thus our theory is very much like the currently popular phenomenological Lagrangian theory,[5] at least in appearance. We can easily extend

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[1] H. Sugawara, Phys. Rev. **170**, 1659 (1968). The first explicit suggestion of this kind of theory was made by M. Gell-Mann in *Proceedings of the Thirteenth International Conference on High-Energy Physics, 1966, Berkeley* (University of California Press, Berkeley, 1967), p. 3.

[2] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[3] K. Bardakci, Y. Frishman, and M. B. Halpern, Phys. Rev. **170**, 1353 (1968).

[4] M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

[5] See, e.g., P. Chang and F. Gürsey, Phys. Rev. **164**, 1752 (1967).