PHYSICAL REVIEW VOLUME 173, NUMBER 5 25 SEPTEMBER 1968

Thermodynamic Properties of a Magnetized Fermi Gas

VITTORIO CANUTO* AND HONG-YEE CHIU†
Institute for Space Studies, Goddard Space Flight Center, National Aeronautics and Space Administration, New York, New York 10025

(Received 26 April 1968)

In a previous paper, we calculated the equation of state of a Fermi gas at arbitrary temperature in a uniform and constant magnetic field of arbitrary strength. (Such a gas is now referred to as a magnetized Fermi gas.) In the present paper, we have studied the properties of the equation of state and have obtained simplified expressions for the normal stresses and the energy and particle densities. These expressions can be used in astrophysical applications. We have found that under suitable conditions of degeneracy (the temperature approaching 0), a magnetized Fermi gas behaves exactly as a one-dimensional gas, which has been studied extensively as a theoretical problem. We have also obtained expressions for the grand partition function.

I. INTRODUCTION

IN a previous paper,[1] we calculated the equation of state of a noninteracting Fermi gas in a constant and uniform magnetic field of arbitrary strengths and at arbitrary gas temperatures. This kind of gas will be referred to hereafter as a magnetized noninteracting Fermi gas or a magnetized Fermi gas. The temperature and the chemical potential are well-defined quantities as well as being isotropic, as expected from thermodynamic considerations. We obtained the macroscopic energy-momentum tensor of a magnetized Fermi gas as a function of the chemical potential μ and the temperature T. Detailed calculations of the equation of state have been discussed in Paper I. In this paper, we shall deal mainly with the thermodynamic properties of a magnetized Fermi gas.

One of the most fundamental properties of a magnetized Fermi gas is exhibited in the anisotropy of the normal stresses. (The normal stresses become the pressure in the case of an isotropic medium.) In our case, the normal stresses (the diagonal spatial element of the energy-momentum tensor) are different in the directions parallel and perpendicular to the magnetic field. This anisotropy is directly associated with the quantization of energy levels by the presence of a magnetic field. Classically, the electron orbits are helixes or circles with axes parallel to the field. The motions in the plane perpendicular to the field are circles of constant angular velocity, and can be decomposed into the motions of two correlated simple harmonic oscillators. Let the direction of the field be taken in the z direction. When these simple harmonic oscillators are quantized, the energy levels are[2]

$$E(p_z,n',r)=\pm mc^2\left[1+\left(\frac{p_z}{mc}\right)^2+2\frac{H}{H_c}(n'+r-1)\right]^{1/2}, \quad (1)$$

where p_z is the momentum in the z direction, H is the magnetic field strength, m is the electron mass, c is the velocity of light, and $H_c=m^2c^3/e\hbar=4.414\times10^{13}$ G. The $+$ and $-$ signs refer to electrons and positrons, respectively, $r=1$, 2 and $n'=0, 1, 2, \cdots$. r and n' are two quantum numbers characterizing the spin and orbits of the electron. The energy levels are thus strongly quantized when p_z/mc is small compared to $2H/H_c$. In the limit of large n', the correspondence between n' and the x and y linear momentum is

$$\left(\frac{p_x}{mc}\right)^2+\left(\frac{p_y}{mc}\right)^2=2\frac{H}{H_c}(n'+r-1). \quad (2)$$

We have shown that in this limit (large n') the gas becomes an ordinary Fermi gas. In other words, the parameter that characterizes a classical noninteracting Fermi gas is

$$\xi=\frac{\langle p_z^2\rangle}{(mc)^2}\Big/2\frac{H}{H_c}.$$

The magnetic properties of a gas are quantized if $\xi\leq1$, and classical if $\xi\gg1$. This condition gives the following criteria for a magnetized Fermi gas (see Paper I):

$$\begin{aligned}
&kT/mc^2\lesssim 2H/H_c, && [\text{(nonrelativistic, nondegenerate)}\ T\ll5.9\times10^9\ {}^\circ\text{K}]\\
&(kT/mc^2)^2\lesssim 2H/H_c, && [\text{(relativistic, nondegenerate)}\ T\gg5.9\times10^9\ {}^\circ\text{K}]\\
&\tfrac{2}{3}(\epsilon_F/mc^2)\approx(\rho/10^7)^{2/3}\lesssim 2H/H_c, && \text{(nonrelativistic, degenerate)}\\
&\tfrac{1}{3}(\epsilon_F/mc^2)^2\approx(\rho/10^7)^{2/3}\lesssim 2H/H_c, && \text{(relativistic, degenerate)}
\end{aligned} \quad (3)$$

where ϵ_F is the Fermi energy [$\epsilon_F=mc^2(\mu-1)$ in the limit $\phi\to0$]. The temperature encountered during and before gravitational collapse is in the range $T_9=1\to100$ [$T_9\equiv T/10^9$ °K]. Hence the field strength of interest is of the order of 10^{13} G and beyond. Such a strong field may be present during gravitational collapse or in gravitationally collapsed objects (e.g.,

* On leave of absence from Department of Physics, C.I.E.A., Ap. Post. 14740, Mexico 14, D.F.

† Also at Department of Physics and Department of Earth and Space Sciences, State University of New York, Stony Brook, N. Y.

[1] V. Canuto and H. Y. Chiu, preceding paper, Phys. Rev. 173, 1210 (1968), hereafter referred to as Paper I.

[2] M. H. Johnson and B. A. Lippmann, Phys. Rev. 76, 828 (1949); H. Robl, Acta Phys. Austriaca 6, 105 (1952).

the collapsed star in the center of Crab Nebula). Figure 1 shows the regions of a magnetized gas.

The nonrelativistic properties of a magnetized Fermi gas have been extensively discussed in conjunction with solid-state physics. The most important properties of a nonrelativistic magnetized Fermi gas are associated with the Landau diamagnetism and the Pauli paramagnetism. In addition to these nonrelativistic properties, in the relativistic case we are also interested in the thermodynamic behavior (the relation between temperature and energy density, etc.). In the following, we shall study various thermodynamic properties of a magnetized Fermi gas.

II. EQUATIONS OF STATE OF A MAGNETIZED FERMI GAS

The equations of state of a magnetized Fermi gas, as given by Eqs. (88)–(91) of Paper I, can be rewritten in a more convenient form if we first perform the summation over the index r, and then shift the summation index from n to $n+1$, and the integration on x (from $-\infty$ to $+\infty$) to a new one (from 0 to ∞). This last step gives only a factor of 2. The normalization volume can be taken equal to 1. The result is

$$P_{xx}=P_{yy}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}\sum_{n=1}^{\infty}n\int_0^\infty F(x,n)\frac{dx}{E(x,n,H)}, \quad (4)$$

$$P_{zz}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\frac{1}{2}\int_0^\infty F(x,0)\frac{x^2dx}{E(x,0,H)}+\sum_{n=1}^{\infty}\int_0^\infty F(x,n)\frac{x^2dx}{E(x,n,H)}\right], \quad (5)$$

$$U=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\frac{1}{2}\int_0^\infty F(x,0)E(x,0,H)dx+\sum_{n=1}^{\infty}\int_0^\infty F(x,n)E(x,n,H)dx\right], \quad (6)$$

$$\mathfrak{N}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}\left[\frac{1}{2}\int_0^\infty F(x,0)dx+\sum_{n=1}^{\infty}\int_0^\infty F(x,n)dx\right], \quad (7)$$

where

$$F(x,n)=\left(1+\exp\frac{E(x,n,H)-\mu}{\phi}\right)^{-1} \quad (8)$$

and

$$E(x,n,H)=[1+x^2+2(H/H_c)n]^{1/2},\quad n=0,1,2,\cdots$$
$$x=p_z/mc,\quad \phi=kT/mc^2=T/5.903\times10^9\ {}^\circ\mathrm{K}, \quad (9)$$
$$\lambda_c=h/mc=3.86\times10^{-11}\ \mathrm{cm}.$$

Here U is the energy density and $\mathfrak{N}$ is the particle energy density, and μ is the chemical potential plus the rest energy (in units of mc^2).[3] The macroscopic energy-momentum tensor $T_{\mu\nu}$ is related to P_{xx}, P_{yy}, and P_{zz} as follows:

$$T_{\mu\nu}=\begin{bmatrix}P_{xx}&0&0&0\\0&P_{yy}&0&0\\0&0&P_{zz}&0\\0&0&0&T_{44}\end{bmatrix}. \quad (10)$$

Equation (10) is valid only in the frame of reference comoving with the gas. The energy-momentum tensor components are related to each other and to the temperature T and density $\mathfrak{N}$ through Eqs. (5)–(8). The equations of state are therefore expressed as infinite series which generally cannot be approximated by an integration over n. When we approximate the sum over n by an integral over dn, we obtain the expression of the equations of state of classical nonmagnetized Fermi gas, as we have shown in Paper I.

Introducing the following transformation in the integrands:

$$v=\frac{x}{a_n},\quad a_n=\left(1+2\frac{H}{H_c}n\right)^{1/2},$$
$$\left(1+x^2+2\frac{H}{H_c}n\right)^{1/2}=a_n(1+v^2)^{1/2}, \quad (11)$$

the integrals in Eqs. (5)–(8) can be expressed in terms of the following functions:

$$C_1(\phi,\mu)\equiv\int_0^\infty\frac{dv}{(1+v^2)^{1/2}\{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]\}}, \quad (12)$$

$$C_2(\phi,\mu)\equiv\int_0^\infty\frac{v^2dv}{(1+v^2)^{1/2}\{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]\}}=C_3(\phi,\mu)-C_1(\phi,\mu), \quad (13)$$

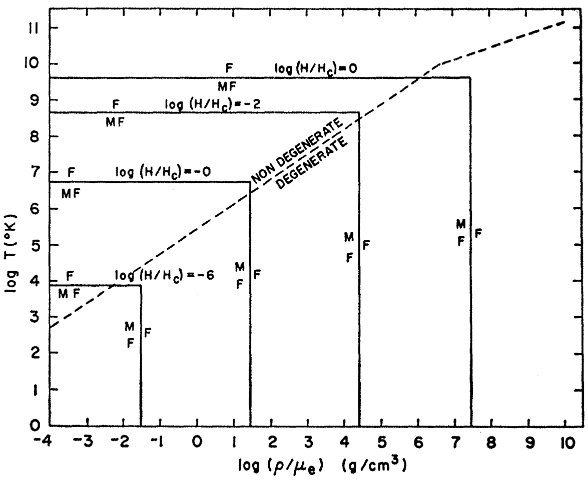

FIG. 1. Approximate regions of a magnetized Fermi gas for several field strengths, as indicated by the solid lines, marked by "MF." $H_c=4.414\times10^{13}$ G. The logarithms are to the base 10.

[3] H. Y. Chiu, *Stellar Physics* (Blaisdell Publishing Co., Waltham, Mass., 1968), Vol. I, Chap. III.

TABLE I. Values of $C_k(\phi,\mu)$, listed in the order $[C_1(\phi,\mu),\ C_2(\phi,\mu),\ C_3(\phi,\mu),\ C_4(\phi,\mu)]$, for a number of values of ϕ and μ. $\phi=kT/mc^2=T/(5.903\times10^9\ ^\circ\text{K})$, and μ is the (chemical potential+rest mass)$/mc^2$.

μ \ ϕ	0.1	0.2	0.5	0.8	1.0	2.0	5.0	8.0	10.0
1.0	0.23617	0.32946	0.50279	0.61738	0.67824	0.89346	1.23319	1.42745	1.52348
	0.03157	0.09265	0.40242	0.88018	1.28702	4.34605	23.53205	57.59191	88.53946
	0.26774	0.42210	0.90520	1.49756	1.96525	5.23951	24.76525	59.01936	90.06294
	0.25074	0.36927	0.64714	0.89004	1.04446	1.78164	3.90504	6.00065	7.39328
1.5	0.94123	0.89063	0.84340	0.87085	0.89778	1.03252	1.30710	1.48009	1.56813
	0.38003	0.44944	0.84332	1.42454	1.90207	5.31564	25.55397	60.65912	92.30214
	1.32126	1.34007	1.68672	2.29538	2.79985	6.34816	26.86107	62.13922	93.87026
	1.10315	1.07314	1.13567	1.30050	1.42638	2.10203	4.18472	6.26952	7.65846
2.0	1.31029	1.28645	1.18230	1.13651	1.12924	1.17749	1.38242	1.53336	1.61319
	1.09265	1.15112	1.54949	2.19135	2.72269	6.45322	27.71887	63.86273	96.19897
	2.40294	2.43756	2.73179	3.32786	3.85194	7.63071	29.10129	65.39610	97.81216
	1.72862	1.71548	1.69364	1.77487	1.86393	2.45321	4.47694	6.54622	7.92991
3.0	1.76054	1.75353	1.69453	1.61250	1.56765	1.47335	1.53627	1.64147	1.70441
	3.37872	3.43123	3.80365	4.47762	5.06590	9.28964	32.50237	70.69472	104.40760
	5.13926	5.18477	5.49818	6.09012	6.63355	10.76299	34.03864	72.33619	106.11202
	2.82768	2.82521	2.80509	2.80677	2.83736	3.23601	5.09811	7.12289	8.49148
4.0	2.06230	2.05880	2.02978	1.97074	1.92502	1.75942	1.69256	1.75117	1.79681
	6.73124	6.78225	7.14127	7.80872	8.41276	12.95316	37.93076	78.11869	113.19015
	8.79354	8.84105	9.17105	9.77946	10.33777	14.71258	39.62332	79.86986	114.98696
	3.87270	3.87180	3.86351	3.85378	3.89178	4.10259	5.76626	7.73008	9.07769
5.0	2.29173	2.28960	2.27302	2.23587	2.20100	2.02100	1.84914	1.86182	1.89003
	11.11802	11.16840	11.52180	12.18046	12.78546	17.51365	44.04953	86.16480	122.57105
	13.40975	13.45800	13.79482	14.41633	14.98646	19.53465	45.89867	88.02661	124.46108
	4.89884	4.89841	4.89469	4.88737	4.88562	5.02584	6.47832	8.36700	9.68811
6.0	2.47741	2.47597	2.46518	2.44134	2.41685	2.25176	2.00400	1.97281	1.98373
	16.52598	16.57603	16.92673	17.57959	18.18209	23.01654	50.90085	94.86227	132.57426
	19.00339	19.05200	19.39190	20.02093	20.59893	25.26830	52.90485	96.83508	134.55800
	5.91600	5.91576	5.91381	5.90937	5.90659	5.98442	7.23072	9.03264	10.32222
7.0	2.63357	2.63252	2.62488	2.60865	2.59154	2.45156	2.15536	2.08353	2.07758
	22.94837	22.99824	23.34743	23.99682	24.59674	29.48934	58.52311	104.23926	143.22318
	25.58194	25.63076	25.97231	26.60547	27.18828	31.94090	60.67847	106.32279	145.30075
	6.92815	6.92800	6.92685	6.92419	6.92190	6.96375	8.01953	9.72583	10.97939
8.0	2.76840	2.76760	2.76187	2.75014	2.73787	2.62355	2.30177	2.19341	2.17124
	30.28127	30.43101	30.77927	31.42655	32.02451	36.94787	66.95069	114.32269	154.54049
	22.14966	33.19861	33.54114	34.17668	34.76238	39.57142	69.25247	116.51609	156.71173
	7.93772	7.93712	7.93638	7.93471	7.93307	7.95471	8.84070	10.44528	11.65891
10.0	2.99306	2.99255	2.98897	2.98194	2.97489	2.90128	2.57563	2.40858	2.35674
	48.26929	48.31889	48.66610	49.31111	49.90674	54.85283	86.33941	136.70959	179.26736
	51.26235	51.31144	51.65507	52.29305	52.88163	57.75410	88.91504	139.11817	181.62410
	9.94986	9.94981	9.94944	9.94868	9.94788	9.95240	10.56443	11.95720	13.08176

$$C_3(\phi,\mu)\equiv\int_0^\infty\frac{(1+v^2)^{1/2}dv}{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]},\tag{14}$$

$$C_4(\phi,\mu)\equiv\int_0^\infty\frac{dv}{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]}.\tag{15}$$

We find that the use of the C_k functions gives

$$\int_0^\infty\frac{dx}{E(x,n,H)\{1+\exp[(E(x,n,H)-\mu)/\phi]\}}=C_1\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right),\tag{16}$$

$$\int_0^\infty\frac{x^2dx}{E(x,n,H)\{1+\exp[(E(x,n,H)-\mu)/\phi]\}}=a_n^2C_2\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right),\tag{17}$$

$$\int_0^\infty\frac{E(x,n,H)dx}{1+\exp[(E(x,n,H)-\mu)/\phi]}=a_n^2C_3\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right),\tag{18}$$

$$\int_0^\infty\frac{dx}{1+\exp[(E(x,n,H)-\mu)/\phi]}=a_nC_4\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right).\tag{19}$$

The equations of state can now be written as

$$P_{xx}=P_{yy}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}\sum_{n=1}^{\infty}nC_1\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right),\tag{20}$$

$$P_{zz}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_2(\phi,\mu)+\sum_{n=1}^{\infty}a_n^2C_2\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right)\right],\tag{21}$$

$$U=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_3(\phi,\mu)+\sum_{n=1}^{\infty}a_n^2C_3\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right)\right],\tag{22}$$

$$\mathfrak{N}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}\left[\tfrac{1}{2}C_4(\phi,\mu)+\sum_{n=1}^{\infty}a_nC_4\left(\frac{\phi}{a_n},\frac{\mu}{a_n}\right)\right].\tag{23}$$

The C_k functions are expectation values of quantities relevant to a one-dimensional gas. $C_1(\phi,\mu)$ is the expectation value of $(E')^{-1}$, where $E'=(1+v^2)^{1/2}$ is the energy of a one-dimensional particle; $C_2(\phi,\mu)$ is the expectation value of vdv/dE, and is the pressure exerted by a one-dimensional particle; and C_3 and C_4 are the expectation values of energy E' and particle number, respectively. We can therefore say that a magnetized Fermi gas has properties similar to a combination of one-dimensional gases. Later we shall show that in one limiting case—degenerate, high field, and low density—a magnetized gas does behave as a one-dimensional gas.

The equations of state thus can be written as a sum over the same function with different arguments. Table I lists $C_k(\phi,\mu)$ for a number of values of μ and ϕ. (Note that in the above sum the ratio of the two arguments of the C_k functions is a constant.)

III. PROPERTIES OF THE C_k FUNCTIONS

A. $\mu<1$

In the case $\mu<1$, we can expand the denominator in the definition of C_k [Eqs. (12)–(15)] into a geometrical series and we can express $C_k(\phi,\mu)$ as a sum over the McDonald functions $K_n(z)$, which are defined by[4]

$$K_n(z)=\int_0^\infty \cosh n\theta \exp(-z\cosh\theta)\,d\theta. \tag{24}$$

We find

$$\begin{aligned}C_1(\phi,\mu)&=\int_0^\infty \frac{1}{(1+v^2)^{1/2}}\frac{dv}{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]}\\&=\int_0^\infty \frac{1}{(1+v^2)^{1/2}}\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\times\exp\left(-\frac{m(1+v^2)^{1/2}}{\phi}\right)dv\\&=\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\int_0^\infty \frac{1}{(1+v^2)^{1/2}}\times\exp\left(-\frac{m(1+v^2)^{1/2}}{\phi}\right)dv.\end{aligned} \tag{25}$$

The transformation

$$v=\sinh\theta \tag{26}$$

reduces the integral in the last part of Eq. (25) to the McDonald function $K_0(m/\phi)$:

$$\int_0^\infty \exp\left(-\frac{m(1+v^2)^{1/2}}{\phi}\right)\frac{dv}{(1+v^2)^{1/2}}=\int_0^\infty \exp\left(-\frac{m}{\phi}\cosh\theta\right)d\theta=K_0\left(\frac{m}{\phi}\right). \tag{27}$$

[4] S. Chandrasekhar, *An Introduction to the Study of Stellar Structure* (Dover Publications, Inc., New York, 1957), Chap. X.

Therefore,

$$C_1(\phi,\mu)=\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)K_0\left(\frac{m}{\phi}\right). \tag{28}$$

This series converges for all values if $\mu<1$.

Similarly, using the transformation (26), we have

$$\begin{aligned}C_3(\phi,\mu)&=\int_0^\infty \frac{(1+v^2)^{1/2}dv}{1+\exp[((1+v^2)^{1/2}-\mu)/\phi]}\\&=\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\int_0^\infty (1+v^2)^{1/2}\exp\left(-\frac{m(1+v^2)^{1/2}}{\phi}\right)dv\\&=\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\int_0^\infty \exp\left(-\frac{m\cosh\theta}{\phi}\right)\cosh^2\theta\,d\theta.\end{aligned} \tag{29}$$

Using the relation $\cosh^2\theta=\frac{1}{2}(\cosh 2\theta+1)$, and the definition of $K_n(z)$ [Eq. (24)], $C_3(\phi,\mu)$ becomes

$$C_3(\phi,\mu)=\tfrac{1}{2}\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\left[K_2\left(\frac{m}{\phi}\right)+K_0\left(\frac{m}{\phi}\right)\right]. \tag{30}$$

We also have

$$\begin{aligned}C_2(\phi,\mu)&=C_3(\phi,\mu)-C_1(\phi,\mu)\\&=\tfrac{1}{2}\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)\left[K_2\left(\frac{m}{\phi}\right)-K_0\left(\frac{m}{\phi}\right)\right],\end{aligned} \tag{31}$$

$$C_4(\phi,\mu)=\sum_{m=1}^\infty \exp\left(\frac{m\mu}{\phi}\right)K_1\left(\frac{m}{\phi}\right). \tag{32}$$

The following recurring formulas for $K_n(z)$ may be useful[4]:

$$K_{n-1}(z)+K_{n+1}(z)=-2K_n'(z), \tag{33}$$

$$K_{n+1}(z)-K_{n-1}(z)=(2n/z)K_n(z). \tag{34}$$

The behavior of $K_n(z)$ at large and small values of z are

$$z\to\infty:\quad K_n(z)=\left(\frac{\pi}{2z}\right)^{1/2}e^{-z}\left[1+\frac{4n^2-1}{8z}+\frac{(4n^2-1)(4n^2-3^2)}{2(8z)^2}+\cdots\right], \tag{35}$$

$$z\to 0:\quad K_n(z)=\tfrac{1}{2}(n-1)!/(\tfrac{1}{2}z)^n,\quad K_0(z)\to\ln z. \tag{36}$$

For $\phi\ll 1$ we have, therefore,

$$C_1(\phi,\mu)=(\tfrac{1}{2}\pi)^{1/2}\sum_{m=1}^\infty\left(\frac{\phi}{m}\right)^{1/2}\exp\left(-\frac{(1-\mu)m}{\phi}\right), \tag{37}$$

$$C_2(\phi,\mu)=(\tfrac{1}{2}\pi)^{1/2}\sum_{m=1}^\infty\left(\frac{\phi}{m}\right)^{3/2}\exp\left(-\frac{(1-\mu)m}{\phi}\right), \tag{38}$$

TABLE II. $C_K(\mu)$.

μ	$C_1(\mu)$	$C_2(\mu)$	$C_3(\mu)$	$C_4(\mu)$
1	0	0	0	0
1.25	0.69315	0.12218	0.81532	0.75000
1.5	0.96242	0.35731	1.31974	1.11803
1.75	1.15881	0.67722	1.83603	1.43614
2.00	1.31696	1.07357	2.39053	1.73205
2.5	1.56680	2.08071	3.64509	2.29129
3.0	1.76275	3.36127	5.12401	2.82843
3.5	1.92485	4.90726	6.83210	3.35410
4.0	2.06344	6.71425	8.77769	3.87298
5.0	2.29243	11.10123	13.39366	4.89898
6.0	2.47789	16.50930	18.98718	5.91608
7.0	2.63392	22.93175	25.56567	6.92820
8.0	2.76866	30.36469	33.13335	7.93725
10.0	2.99322	48.25276	51.24598	9.94987

$$C_3(\phi,\mu)=(\tfrac{1}{2}\pi)^{1/2}\sum_{m=1}^{\infty}\left(\frac{\phi}{m}\right)^{1/2}\exp\left(-\frac{(1-\mu)m}{\phi}\right)=C_1(\phi,\mu), \quad (39)$$

$$C_4(\phi,\mu)=C_1(\phi,\mu). \quad (40)$$

If $(1-\mu)/\phi\gg 1$, we only need to take the first term, and thereby have

$$C_1(\phi,\mu)\to(\tfrac{1}{2}\pi)^{1/2}\phi^{1/2}\exp\left(-\frac{1-\mu}{\phi}\right), \quad (41)$$

$$C_2(\phi,\mu)\to(\tfrac{1}{2}\pi)^{1/2}\phi^{1/2}\exp\left(-\frac{1-\mu}{\phi}\right), \quad (42)$$

$$C_3(\phi,\mu)=C_4(\phi,\mu)=C_1(\phi,\mu). \quad (43)$$

There is no convenient reduction in the general case $\mu>1$.

B. The Limit $(\mu-1)\gg\phi$

This case occurs when the gas becomes degenerate. (The consequence of degeneracy is slightly different from the usual one, as will be discussed later.) In this case, the Fermi distribution function can be replaced by a step function

$$\left[1+\exp\frac{(1+v^2)^{1/2}-\mu}{\phi}\right]^{-1}=1, \quad v<(\mu^2-1)^{1/2}$$
$$=0, \quad v>(\mu^2-1)^{1/2} \quad (44)$$

as is usually done for an ordinary Fermi gas. This is equivalent to replacing the upper limit of integration in Eqs. (12)–(15) by $(\mu^2-1)^{1/2}$. Then $C_k(\phi,\mu)$ becomes a function of μ only. Let

$$C_k(\mu)=\lim_{\phi\to 0}C_k(\phi,\mu); \quad (45)$$

we find that

$$C_1(\mu)=\int_0^{(\mu^2-1)^{1/2}}\frac{dv}{(1+v^2)^{1/2}}=\ln[\mu+(\mu^2-1)^{1/2}], \quad (46)$$

$$C_3(\mu)=\int_0^{(\mu^2-1)^{1/2}}(1+v^2)^{1/2}dv=\tfrac{1}{2}\ln[\mu+(\mu^2-1)^{1/2}]+\tfrac{1}{2}\mu(\mu^2-1)^{1/2}, \quad (47)$$

$$C_4(\mu)=\int_0^{(\mu^2-1)^{1/2}}dv=(\mu^2-1)^{1/2}, \quad (48)$$

$$C_2(\mu)=C_3(\mu)-C_1(\mu)=\tfrac{1}{2}\mu(\mu^2-1)^{1/2}-\tfrac{1}{2}\ln[\mu+(\mu^2-1)^{1/2}]; \quad (49)$$

when $\mu\gg 1$, we find

$$C_1(\mu)=\ln 2\mu, \quad (50)$$

$$C_3(\mu)=C_2(\mu)=\tfrac{1}{2}\mu^2, \quad (51)$$

$$C_4(\mu)=\mu; \quad (52)$$

when $\mu-1\ll 1$, we find $(\mu=1+\xi)$

$$C_1(\mu)=(2\xi+\xi^2)^{1/2}[1-\tfrac{1}{3}\xi+(2/15)\xi^2], \quad (53)$$

$$C_2(\mu)\to\xi(2\xi+\xi^2)^{1/2}[\tfrac{2}{3}-(1/15)\xi], \quad (54)$$

$$C_3(\mu)\to(2\xi+\xi^2)^{1/2}[1+\tfrac{1}{3}\xi+(1/15)\xi^2], \quad (55)$$

$$C_4(\mu)\to(2\xi+\xi^2)^{1/2}. \quad (56)$$

Table II lists $C_k(\mu)$ as a function of μ.

IV. THERMODYNAMIC PROPERTIES OF A NONDEGENERATE MAGNETIC GAS

As we have shown in Paper I, if the sum in the equations of state (20)–(23) is contributed mainly by terms with large values of n, magnetized Fermi gas approaches a classical Fermi gas. Therefore, we only need to consider cases with small values of n. As an example, let us consider the case of the lowest term in n, namely, $n=1$. In this limit we have

$$P_{xx}=P_{yy}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}C_1\left(\frac{\phi}{a_1},\frac{\mu}{a_1}\right), \quad (57)$$

$$P_{zz}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_2(\phi,\mu)+a_1^2C_2\left(\frac{\phi}{a_1},\frac{\mu}{a_1}\right)\right], \quad (58)$$

$$U=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_3(\phi,\mu)+a_1^2C_3\left(\frac{\phi}{a_1},\frac{\mu}{a_1}\right)\right], \quad (59)$$

$$\mathfrak{N}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}\left[\tfrac{1}{2}C_4(\phi,\mu)+a_1C_4\left(\frac{\phi}{a_1},\frac{\mu}{a_1}\right)\right]. \quad (60)$$

The C_k functions are decreasing functions of ϕ. If we consider only terms small in n, we must have

$$1\ll a_1\ll a_2\ll\cdots\ll a_n. \quad (61)$$

Therefore, we can also drop $C_k(\phi/a_1, \mu/a_1)$ with respect to $C_k(\phi,\mu)$ in Eqs. (58)–(60). In the nondegenerate approximation, the factor 1 in the denominator in the Fermi distribution function may be neglected. This is equivalent to taking the first term in the series for $C_k(\phi,\mu)$ [Eqs. (28) and (30)–(32)]. We have

$$P_{xx}=P_{yy}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}\exp\left(\frac{\mu}{\phi}\right)K_0\left(\frac{a_1}{\phi}\right), \tag{62}$$

$$P_{zz}=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\exp\left(\frac{\mu}{\phi}\right)\phi K_1\left(\frac{1}{\phi}\right), \tag{63}$$

$$U=\frac{1}{4\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\exp\left(\frac{\mu}{\phi}\right)\left[K_2\left(\frac{1}{\phi}\right)+K_0\left(\frac{1}{\phi}\right)\right], \tag{64}$$

$$\mathfrak{N}=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}\exp\left(\frac{\mu}{\phi}\right)K_1\left(\frac{1}{\phi}\right). \tag{65}$$

We find, from Eqs. (63) and (65), that

$$P_{zz}=\mathfrak{N}mc^2\phi=\mathfrak{N}kT. \tag{66}$$

This means that Boyle's law is valid along the z axis, i.e., the gas behaves as a normal gas along the z axis.

The anisotropy factor τ may be defined as

$$\tau=\frac{P_{xx}}{P_{zz}}=\frac{2H}{\phi H_c}\frac{K_0(a_1/\phi)}{K_1(1/\phi)}. \tag{67}$$

From Eqs. (35) and (36) we find that, when $\phi\gg1$ and $\phi\ll a_1$,

$$\tau=\frac{2H}{H_c}\left(\frac{\pi}{2}\frac{\phi}{a_1}\right)^{1/2}\exp\left(\frac{-a_1}{\phi}\right)=\left(\frac{H}{H_c}\right)\left(\frac{2\pi\phi}{a_1}\right)^{1/2}\exp\left(\frac{-a_1}{\phi}\right). \tag{68}$$

[The case $\phi\gg a_1$ corresponds to a classical Fermi gas. See Eq. (3).] For the case $\phi\ll1$ (nonrelativistic case), we have

$$\tau=\frac{H}{H_c}\frac{2}{\phi\sqrt{a_1}}\exp\left[\frac{-(a_1-1)}{\phi}\right]. \tag{69}$$

The relation between P_{zz} and U is worked out as follows: For the relativistic case $\phi\gg1$, we find, from Eqs. (35) and (36),

$$\frac{P_{zz}}{U}=\frac{2\phi K_1(1/\phi)}{K_2(1/\phi)+K_0(1/\phi)}\to\frac{2\phi^2}{2\phi^2+\ln\phi}\to1-\frac{\ln\phi}{2\phi^2}, \tag{70}$$

but $P_{zz}/U<1$ if $\phi<\infty$. The velocity of sound in the z direction is given by[5]

$$v_z=c\left(\frac{dP_{zz}}{dU}\right)^{1/2}=c\left(\frac{2}{2+(1/2\phi^2)\ln\phi}\right)^{1/2}, \tag{71}$$

which approaches the velocity of light as $\phi\to\infty$. In a nonmagnetized Fermi gas the limiting velocity of sound is $c/\sqrt{3}$ only.

For the nonrelativistic case $\phi\ll1$, and from Eqs. (35) and (36) we find that ($\phi\ll1$)

$$\frac{P_{zz}}{U}=\frac{2\phi K_1(1/\phi)}{K_2(1/\phi)+K_0(1/\phi)}\to\phi\equiv\frac{kT}{mc^2}. \tag{72}$$

However, U also includes the rest energy mc^2, so that in the limit $kT/mc^2\gg1$ Eq. (72) is identical with Eq. (66). We are really interested in the ratio of $P_{zz}/(U-mc^2)$, which is the thermodynamic energy. We find

$$\begin{aligned}U-\mathfrak{N}mc^2&=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}K_2\left(\frac{1}{\phi}\right)+\tfrac{1}{2}K_0\left(\frac{1}{\phi}\right)-K_1\left(\frac{1}{\phi}\right)\right]\exp\left(\frac{\mu}{\phi}\right)\\&=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\tfrac{1}{2}\phi\left(\frac{\pi}{2/\phi}\right)^{1/2}\exp\left(\frac{\mu-1}{\phi}\right),\end{aligned} \tag{73}$$

which gives

$$P_{zz}/(U-\mathfrak{N}mc^2)\to2. \tag{74}$$

Equation (74) again differs from the classical result that $P/(U-\mathfrak{N}mc^2)\to\frac{2}{3}$ by a factor of 3. The velocity of sound is accordingly greater by a factor of $\sqrt{3}$.

The velocity of sound in the direction perpendicular to the field is

$$v_\perp=c\left(\frac{d(\tau P_{zz})}{dU}\right)^{1/2}\sim c\tau^{1/2}\left(\frac{dP_{zz}}{dU}\right)^{1/2}\ll v_z. \tag{75}$$

The ratio $U/\mathfrak{N}$ or $(U-\mathfrak{N}mc^2)/\mathfrak{N}$ in the nonrelativistic case is

$$\frac{U}{\mathfrak{N}}=\tfrac{1}{2}mc^2\frac{K_2(1/\phi)+K_0(1/\phi)}{K_1(1/\phi)}\to mc^2\phi=kT, \tag{76}$$

and using Eq. (73), we find

$$(U-\mathfrak{N}mc^2)/\mathfrak{N}=\tfrac{1}{2}mc^2\phi=\tfrac{1}{2}kT, \tag{77}$$

which are to be compared with the corresponding value of a nonmagnetized gas of $3kT$ ($\phi\to\infty$) and $\frac{3}{2}kT$

[5] The sound velocity is not the Alfvén velocity. The Alfvén velocity is the velocity of transverse electromagnetic waves. To compute the Alfvén velocity one needs to know the dielectric constant.

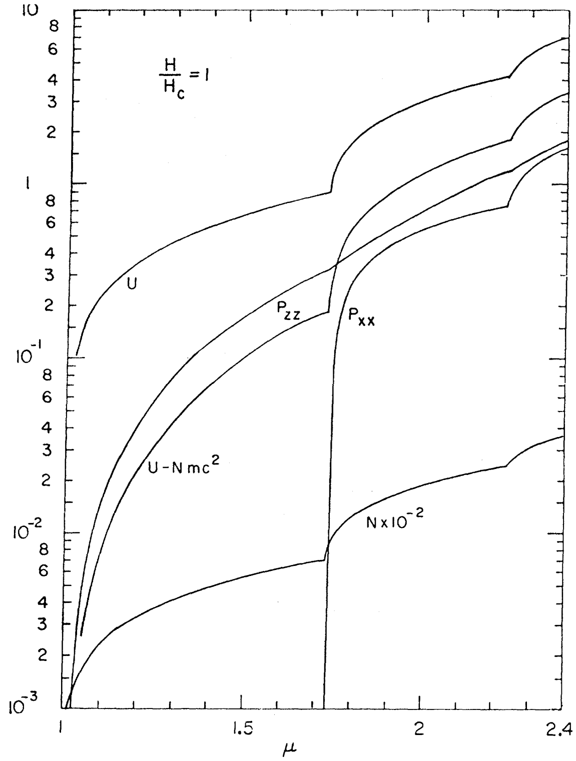

FIG. 2. The dependence of U, P_{xx}, P_{zz}, $U-Nmc^2$, and N as functions of μ in the degenerate limit of the case $H/H_c=1$.

($\phi\rightarrow 0$), respectively.[6] We can thus conclude that only one out of the three degrees of freedom is excited in a strongly magnetized gas.

When terms other than $n=0$ are considered in P_{zz}, U, $\mathfrak{N}$, and P_{xx}, respectively, the anisotropy decreases and eventually $U/\mathfrak{N}\rightarrow 3kT$, or $(U-\mathfrak{N}mc^2)/\mathfrak{N}\rightarrow \frac{3}{2}kT$ in the limit of $n\rightarrow\infty$. The speed of sound also decreases. The gas eventually becomes a three-dimensional gas.

In reality, when ϕ is not zero, states other than the lowest one are always excited to some extent. Hence considerations made here are only of limited interest. The general case can be studied by resorting to more complicated formulas (20)–(23) and applying numerical methods.

V. DEGENERATE MAGNETIZED FERMI GAS. $(\mu-1)/\phi\gg 1$

In this limit the equations of state become

$$P_{xx}=P_{yy}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}\sum_{n=1}^{s} nC_1\left(\frac{\mu}{a_n}\right), \tag{78}$$

$$P_{zz}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_2(\mu)+\sum_{n=1}^{s} a_n^2C_2\left(\frac{\mu}{a_n}\right)\right], \tag{79}$$

$$U=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}\left[\tfrac{1}{2}C_3(\mu)+\sum_{n=1}^{s} a_n^2C_3\left(\frac{\mu}{a_n}\right)\right], \tag{80}$$

$$\mathfrak{N}=\frac{1}{\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}\left[\tfrac{1}{2}C_4(\mu)+\sum_{n=1}^{s} a_nC_4\left(\frac{\mu}{a_n}\right)\right]. \tag{81}$$

It can very easily be shown that if $\mu/a_n\leq 1$,

$$\lim_{\phi\rightarrow 0} C_k(\phi,\mu)\rightarrow 0,\quad k=1, 2, 3, 4.$$

Thus the sum over n does not extend to infinity but terminates at s such that

$$a_s\leq\mu<a_{s+1}, \tag{82}$$

where s is an integer.

$\mathfrak{N}$, U, P_{xx}, and P_{zz} have now discontinuous derivatives with respect to μ. The curves of $\mathfrak{N}$, U, P_{xx}, and P_{zz} versus μ and $\mathfrak{N}$ will contain kinks, as is shown in Figs. 2 and 3. However, these kinks disappear at large values of s and also when one regards U as a function of $\mathfrak{N}$, etc. These kinks signify the fact that the energy states are one by one excited as μ increases.

These kinks, maxima, and minima in the thermodynamic variables can be understood in terms of the

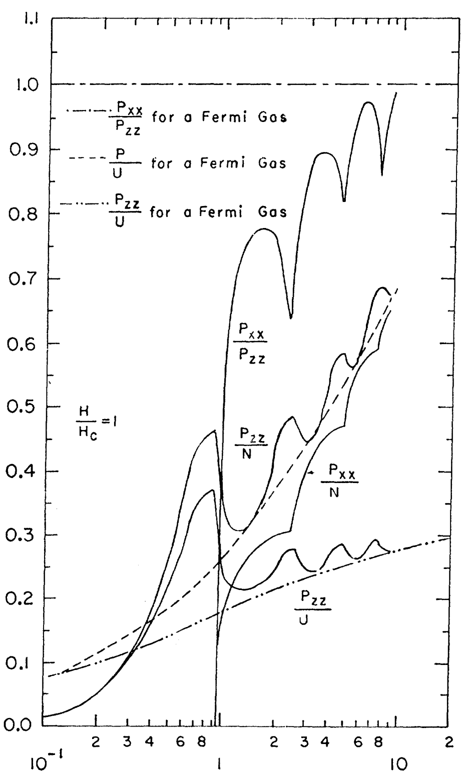

FIG. 3. Functional dependence of P_{xx}/P_{zz}, P_{zz}/N, P_{xx}/N, and P_{zz}/N or N for the degenerate case with $H/H_c=1$. The corresponding functions for a Fermi gas are also shown for comparison.

[6] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962), 2nd ed., pp. 87–99.

behavior of the density of states $\eta(E)$. $\eta(E)$ is given by

$$\eta(E)=\frac{dN(E)}{dE}=\frac{d\mathfrak{N}(\mu)}{d\mu}=\mathfrak{N}'(\mu),$$

where $N(E)$ is the number of states with energy below energy E. $N(E)$ coincides with $\mathfrak{N}(\mu)$ if we set $E=\mu$. Figure 4 shows $\eta(E)$ as a function of E. $\eta(E)$ shows sharp peaks as each magnetic state is excited.[7]

As before, we now consider the case $s=0$. This case represents a physical example of a one-dimensional gas which has been discussed extensively in the literature.[8]

$$P_{xx}=P_{yy}=0, \tag{83}$$

$$P_{zz}=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}C_2(\mu), \tag{84}$$

$$U=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{mc^2}{\lambda_c^3}C_3(\mu), \tag{85}$$

$$\mathfrak{N}=\frac{1}{2\pi^2}\left(\frac{H}{H_c}\right)\frac{1}{\lambda_c^3}C_4(\mu). \tag{86}$$

In this case there is no lateral stress. A gas with no lateral stress is unstable against collapse. However, a small amount of pressure will always be present due to the finiteness of the temperature. Since

$$\begin{aligned} &C_1(\phi,\mu)\to K_0(1/\phi)\exp(\mu/\phi), \\ &K_0(1/\phi)\to(\tfrac{1}{2}\pi)^{1/2}\phi^{1/2}\exp(-\phi)\quad(\phi\to 0), \end{aligned} \tag{87}$$

we find the residual lateral pressure to be

$$P_{xx}=P_{yy}\sim\frac{1}{(2\pi^3)^{1/2}}\left(\frac{H}{H_c}\right)^2\frac{mc^2}{\lambda_c^3}\left(\frac{\phi}{(1+2H/H_c)^{1/2}}\right)^{1/2}\times\exp\left(-\frac{(1+2H/H_c)^{1/2}-\mu}{\phi}\right). \tag{88}$$

This residual pressure can, under suitable conditions, prevent collapse in the direction perpendicular to the field.

Since $C_2(\mu)$, $C_3(\mu)$, $C_4(\mu)$ are proportional to the pressure, energy, and densities of a one-dimensional gas, we conclude that a degenerate electron gas in an intense magnetic field can behave almost exactly as a *one-dimensional gas*. The critical density corresponding to the one-dimensional behavior is such that

$$\mu\leqslant(1+2H/H_c)^{1/2}. \tag{89}$$

At $H/H_c=1$, Eq. (89) gives a critical density of the order of 10^6 g/cm³ for a composition of helium.

[7] D. C. Mattis, *The Theory of Magnetism* (Harper and Row, New York, 1965).

[8] E. H. Lieb and D. C. Mattis, *Mathematical Physics of One Dimension* (Academic Press Inc., New York, 1966).

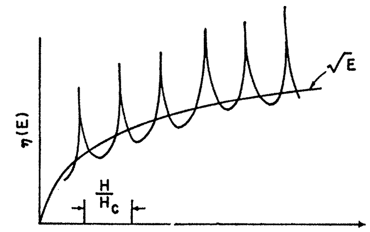

FIG. 4. The density of states $\eta(E)$ as a function of E for the nonrelativistic case (from Reference 7).

At the high-density limit $(\mu-1)\gg1$, we then have

$$P_{zz}=\frac{1}{4\pi^2}\frac{H}{H_c}\frac{mc^2}{\lambda_c^3}\mu^2=U\propto\mu^2, \tag{90}$$

$$\mathfrak{N}=\frac{1}{2\pi^2}\frac{H}{H_c}\frac{1}{\lambda_c^3}\mu\propto\mu, \tag{91}$$

$$P_{zz}\propto\mathfrak{N}^2. \tag{92}$$

Other properties of this gas resemble a noninteracting one-dimensional Fermi gas. All relevant quantities can be calculated from the general equation of state. For example, the specific heat of a degenerate magnetized Fermi gas can be calculated, using the following formula[4]:

$$\int_0^\infty\frac{[d\varphi(u)/du]du}{e^{(u-u_0)}+1}=\varphi(u_0)+2[c_2\varphi''(u_0)+c_4\varphi^{(\mathrm{IV})}(u_0)+\cdots], \tag{93}$$

where

$$C_\nu=\sum_{n=1}\frac{(-)^{n+1}}{n^\nu}. \tag{94}$$

Equation (93) is accurate to the order e^{-u_0}.

VI. PAIR-CREATION EQUILIBRIUM

One might think that since classically the spin energy of an electron in a magnetic field is of the order of $-\mu_BH$, where $\mu_B=eh/2mc$, when $H\geq 2H_c$ then the spin energy will be greater than $2mc^2$; it might be possible to create an electron pair with suitable spin directions at the expense of the field energy. However, because of the form of energy states [Eq. (1)], the separation between positive and negative energy states is always at least $2mc^2$. This means that pair creation will take place, not at the expense of the field energy, but at the expense of other forms of energy (e.g., thermodynamic energy). This conclusion is valid only for the case of a uniform and constant magnetic field. The question of whether pair creation will take place in a nonuniform or time-dependent magnetic field can only be answered by a study of the quantum origin of magnetic fields.

The chemical reaction of pair creation is

$$\gamma \rightleftarrows e^- + e^+ , \tag{95}$$

where γ stands for one or more photons. Equation (95) leads to the equation

$$\mu_\gamma = \mu_- + \mu_+ + 2mc^2 , \tag{96}$$

where μ_γ, μ_-, and μ_+ refer to the chemical potential of the photon, electron, and positrons, respectively. (Note that $\mu = \mu_- + mc^2$ is the chemical potential for the electron used in previous discussions in this paper.) Since $\mu_\gamma = 0$, we find that

$$\mu_- + mc^2 = \mu = -(\mu_+ + mc^2) . \tag{97}$$

The charge-conservation law requires that

$$\mathfrak{N}_- - \mathfrak{N}_+ = \mathfrak{N} , \tag{98}$$

where $\mathfrak{N}_-$ ($\mathfrak{N}_+$) refers to the electron (positron) number densities and $\mathfrak{N}$ is the number density of excessive electrons. Equations (97) and (98) are identical to those equations for pair creation in the absence of a magnetic field. Equations (97) and (98) together with Eqs. (20)–(23) can be solved to give $\mathfrak{N}_-$ and $\mathfrak{N}_+$ as functions of $\mathfrak{N}$ and T.

When the anomalous magnetic moment is taken into account, the lowest-energy eigenvalue of Dirac equation for an electron in a magnetic field vanishes for values of H equal to $4\pi\alpha^{-1}H_c$, and therefore pair creation phenomena can take place. The problem is fully discussed in Ref. 9.

VII. GRAND PARTITION FUNCTION

The grand partition function $\mathfrak{Z}$ is given by

$$\ln \mathfrak{Z} = \ln \mathrm{Tr} \exp(-\tilde{\beta}\hat{\mathcal{H}}) , \tag{99}$$

where Tr means trace, $\tilde{\beta} = (kT)^{-1}$, and $\hat{\mathcal{H}}$ is the Hamiltonian [defined in Eq. (80) of Paper I]

$$\hat{\mathcal{H}} = \sum_{n,r} E_{n,r} a_r^\dagger(n) a_r(n) , \tag{100}$$

where

$$E_{n,r} = \pm mc^2 \left[1 + \left(\frac{p_z}{mc}\right)^2 + 2\frac{H}{H_c}(n+r-1) \right]^{1/2} \tag{101}$$

and $a_r^\dagger(n)$ and $a_r(n)$ are the creation operators for the state n. Equation (99) is easily reduced to (see Paper I for techniques used)

$$\begin{aligned} \ln \mathfrak{Z} &= \ln \prod_{n,r,p_z} [1+\lambda \exp(-\tilde{\beta}E_{n,r})]^{\omega_{n,r}} \\ &= \sum_{n,p_z} \omega_{n,1} \ln[1+\lambda \exp(-\tilde{\beta}E_{n,1})] \\ &\quad + \sum_{n,p_z} \omega_{n,2} \ln[1+\lambda \exp(-\tilde{\beta}E_{n,2})] , \end{aligned} \tag{102}$$

where $\omega_{n,r}$ is the degeneracy factor discussed in Paper I:

$$\omega_{n,r} = \Omega^{2/3} eH/2\pi\hbar c \tag{103}$$

and

$$\lambda = \exp(\tilde{\beta}\tilde{\mu}) . \tag{104}$$

VIII. SUMMARY

We have calculated the thermodynamic properties of a magnetized Fermi gas (a Fermi gas of arbitrary temperature in a magnetic field of arbitrary strength) and have obtained convenient expressions for the energy and normal stresses. We have found that in the limit of low quantum numbers a magnetized gas behaves as a one-dimensional gas. In the limit of zero temperature, the gas behaves exactly as a one-dimensional gas for certain density ranges.

We have also considered the pair-creation phenomena and calculated the grand partition function for a magnetized gas.

ACKNOWLEDGMENTS

One of us (V.C.) acknowledges the support of an NAS-NRC resident research associateship.

[9] H. Y. Chiu, V. Canuto, and L. Fassio-Canuto, Phys. Rev. (to be published).